THE ABEL PRIZE 2023

The Norwegian Academy of Science and Letters awards
the Abel Prize for 2023 to

Luis A. Caffarelli,

University of Texas at Austin, USA,

“for his seminal contributions to regularity theory for nonlinear partial differential equations including free-boundary problems and the Monge–Ampère equation.”

Partial differential equations arise naturally as laws of nature, whether to describe the flow of water or the growth of populations. These equations have been a constant source of intense study since the days of Newton and Leibniz. Yet, despite substantial efforts by mathematicians over centuries, fundamental questions concerning stability or even uniqueness, and the occurrence and type of singularities of some key equations, remain unresolved.

Over a period of more than 40 years, Luis Caffarelli has made ground-breaking contributions to ruling out or characterizing singularities. This goes under the name of regularity theory and captures key qualitative features of the solutions beyond the original functional analytic set-up. It is conceptually important for modelling – is for instance the assumption of macroscopically varying fields self-consistent? – and informs discretization strategies and is thus crucial for efficient and reliable numerical simulation. Caffarelli’s theorems have radically changed our understanding of classes of nonlinear partial differential equations with wide applications. The results go to the core of the matter, the techniques show at the same time virtuosity and simplicity, and cover many different areas of mathematics and its applications.

A large part of Caffarelli’s work concerns so-called free-boundary problems. Consider for instance the problem of ice melting into water. Here the free boundary is the interface between water and ice; it is part of the unknown that is to be determined. Another example is provided by water seeping through a porous medium – again the interface between the saturated and unsaturated part of the medium is to be understood.

A particular class of free-boundary problems are denoted as obstacle problems. An example is given by a balloon pressing against a wall or an elastic body resting on a surface. Caffarelli has given penetrating solutions to these problems with applications to solid-liquid interfaces, jet and cavitational flows, and gas and liquid flows in a porous media, as well as financial mathematics.

Caffarelli's regularity results rely on zooming in on the free boundary, and classifying the resulting blow-ups, where non-generic blow-ups correspond to singularities of the free boundary.

The incompressible Navier–Stokes equations model fluid flow, such as water. The regularity of solutions of these equations in three dimensions is one of the open Clay Millennium Problems. In 1983, based on Scheffer's previous work, Caffarelli, with Kohn and Nirenberg, showed that sets of singularities of suitable weak solutions cannot contain a curve, that is, they have to be very "small".

Caffarelli's regularity theorems from the 1990s represented a major breakthrough in our understanding of the Monge–Ampère equation, a highly nonlinear, quintessential partial differential equation, that for instance is used to construct surfaces of prescribed Gaussian curvature. Important existence results were established by Alexandrov, and earlier central properties had been shown by Caffarelli in collaboration with Nirenberg and Spruck, with further key contributions by Evans and Krylov. Caffarelli however closed the gap in our understanding of singularities by proving that the explicitly known examples of singular solutions are the only ones.

Caffarelli has – together with collaborators – applied these results to the Monge–Kantorovich optimal mass transportation problem, based on previous work by Brenier. Caffarelli and Vasseur gave deep regularity results for the quasi-geostrophic equation in part by applying the exceptionally influential paper by Caffarelli and Silvestre on the fractional Laplacian.

Furthermore, Caffarelli has made seminal contributions to the theory of homogenization, where one seeks to characterize the effective or macroscopic behaviour of media that have a microstructure, for instance because they are formed by a composite material. A typical problem regards a porous medium – like a hydrocarbon reservoir – where one has a solid rock with pores, posing a complex and – to a large degree – unknown structure through which fluids flow.

Caffarelli is an exceptionally prolific mathematician with over 130 collaborators and more than 30 PhD students over a period of 50 years. Combining brilliant geometric insight with ingenious analytical tools and methods, he has had and continues to have an enormous impact on the field.