# Modern Geometry,

# From *Local* to *Global*,
# From *Smooth* to *Rough*,
# From *Static* to *Dynamic*

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Jean-Pierre BOURGUIGNON

(CNRS & IHÉS)

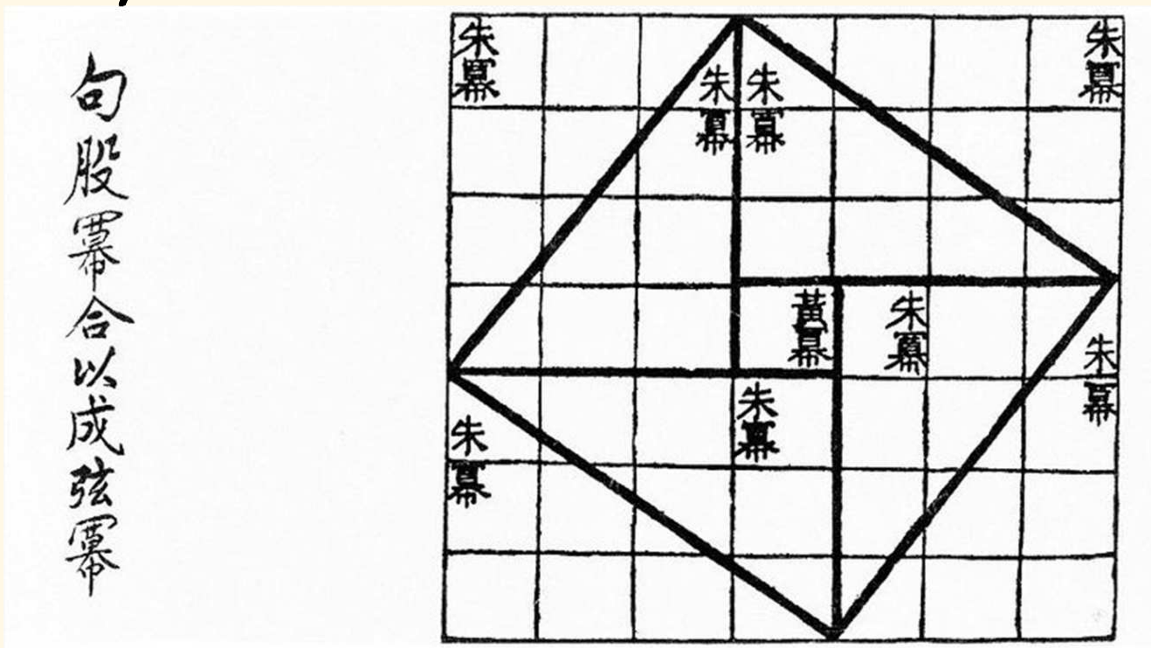
## Subtitle

*Over the course of History, Geometry had undergone major transformations by broadening its scope and its methods.*

# 0. From Old Times on

# Geometry means «measuring the Earth»

- This question mobilized many civilisations (Egypt, Mesopotamia, China, ...).
- in Ancient China, geometric knowledge was developped as, shown by this « proof » of Pythagoras theorem.



We briefly concentrate on the Greek one :

- *for its long lasting contribution,*
- *for the variety of visions expressed,*
- *for the model of intellectual creation it provided.*

# Euclide's Elements:

- *probably the non religious book with the longest influence in the History of Mankind,*
- *provides a model for Geometry and a method to establish it firmly through the introduction of the axiomatic method,*
- *develops a number of important results in Geometry on basic objects : lines, planes, circles, conic sections,...*

# A revolution is due to René DESCARTES

René DESCARTES

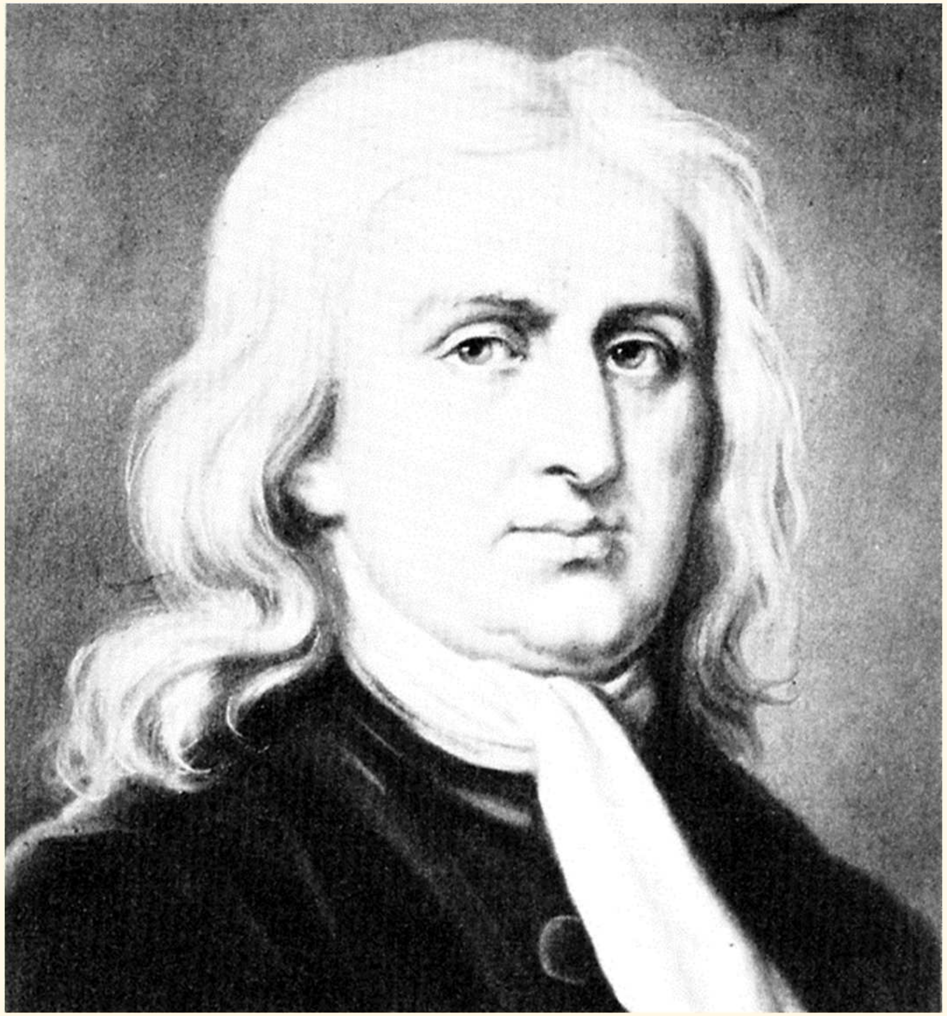
A revolution is due to René DESCARTES who

- *through Analytic Geometry, mixed numbers and geometric figures with the result of:*

    - ♉ *broadening considerably figures that can be considered;*

    - ♉ *laying the foundations for a systematic analytic handling of geometric problems.*

# A new dimension is due to Isaac NEWTON



Isaac NEWTON

A new dimension is due to Isaac NEWTON who

- *wrote a most influential book, the Philosophia Naturalis Principia Mathematica in 1687*

# PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

AUCTORE

ISAACO NEWTONO, EQ. AURATO.

*Perpetuis Commentariis illustrata, communi studio*

PP. THOMÆ LE SEUR & FRANCISCI JACQUIER

*Ex Gallicanâ Minimorum Familiâ,*

*Matheseos Professorum.*

TOMUS PRIMUS.



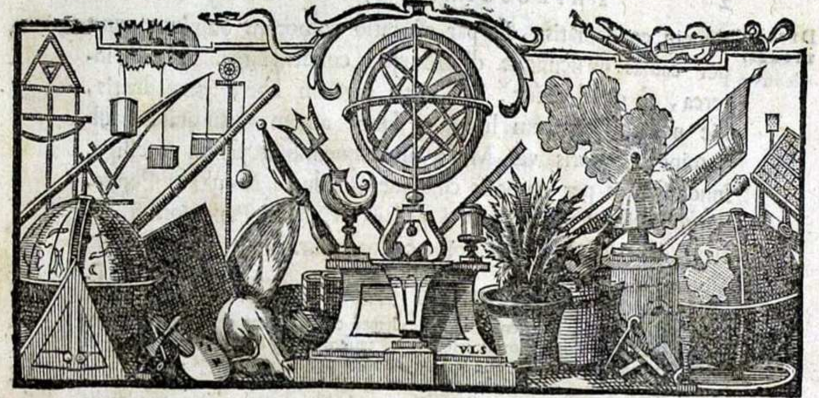
GENEVÆ,

Typis BARRILLOT & FILII Bibliop. & Typogr.

MDCCXXXIX.

A new dimension is due to Isaac NEWTON who

- *wrote a most influential book, the Philosophia Naturalis Principia Mathematica in 1687 that mimicks Euclid's Elements*



# PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

## *DEFINITIONES.*

### DEFINITIO I. (a)

*Quantitas Materiæ eſt menſura ejuſdem orta ex illius Denſitate & Magnitudine conjunctim.*

AER, denſitate duplicata, in ſpatio etiam duplicato fit quadruplus; in triplicato ſextuplus. Idem intellige de Nive & Pulveribus per compreſſionem vel liquefactionem

Tom. I. A

*Licet primæ definitiones* NEWTONIANÆ *vix aliquam poſtulare videantur explicationem; in ipſo tamen operis noſtri limine, nonnulla levioris momenti præmittenda judicamus, quæ ad majora viam ſternunt. Prima quæ in poſterùm ſæpiùs recurrent Mechanices principia interſerere non abs re erit, tùm ut Lectorum labori parcamus, tùm ut magis continua ſervetur noſtrarum demonſtrationum ſeries.*

(a) 1. Materia eſt ſubſtantia trinâ dimenſione prædita, ſolida ſeu impenetrabilis, mobilis, diviſibilis. Spatium purum eſt illa immenſa, penetrabilis, ſui ubique

A new dimension is due to Isaac NEWTON who

- *wrote a most influential book, the Philosophia Naturalis Principia Mathematica in 1687 that mimicks Euclid's Elements*

- *and in which he does three things in the same course of development:*

  - ♉ *He formulates the differential calculus;*
  - ♉ *He states the fundamental law of mechanics;*
  - ♉ *He states the law of gravitation.*

A new dimension is due to Isaac NEWTON who

- *wrote another most influential book, the Philosophia Naturalis Principia Mathematica in 1687 that mimicks Euclid's Elements*

- *and this results in the possibility of discussing smooth objects, broadening even further the variety of models available, opening the way to Differential Geometry.*

We owe to three main figures the definition of non-Euclidean geometries:

- *the first is Carl-Friedrich GAUSS,*

Carl Friedrich GAUSS

We owe to three main figures the definition of non-Euclidean geometries:

- *the first one is Carl-Friedrich GAUSS, who in the fundamental essay*

# DISQUISITIONES GENERALES

CIRCA

# SUPERFICIES CURVAS

AUCTORE

## CAROLO FRIDERICO GAUSS

SOCIETATI REGIAE OBLATAE D. 8. OCTOB. 1827

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COMMENTATIONES SOCIETATIS REGIAE SCIENTIARUM
GOTTINGENSIS RECENTIORES. VOL. VI. GOTTINGAE MDCCCXXVIII

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GOTTINGAE

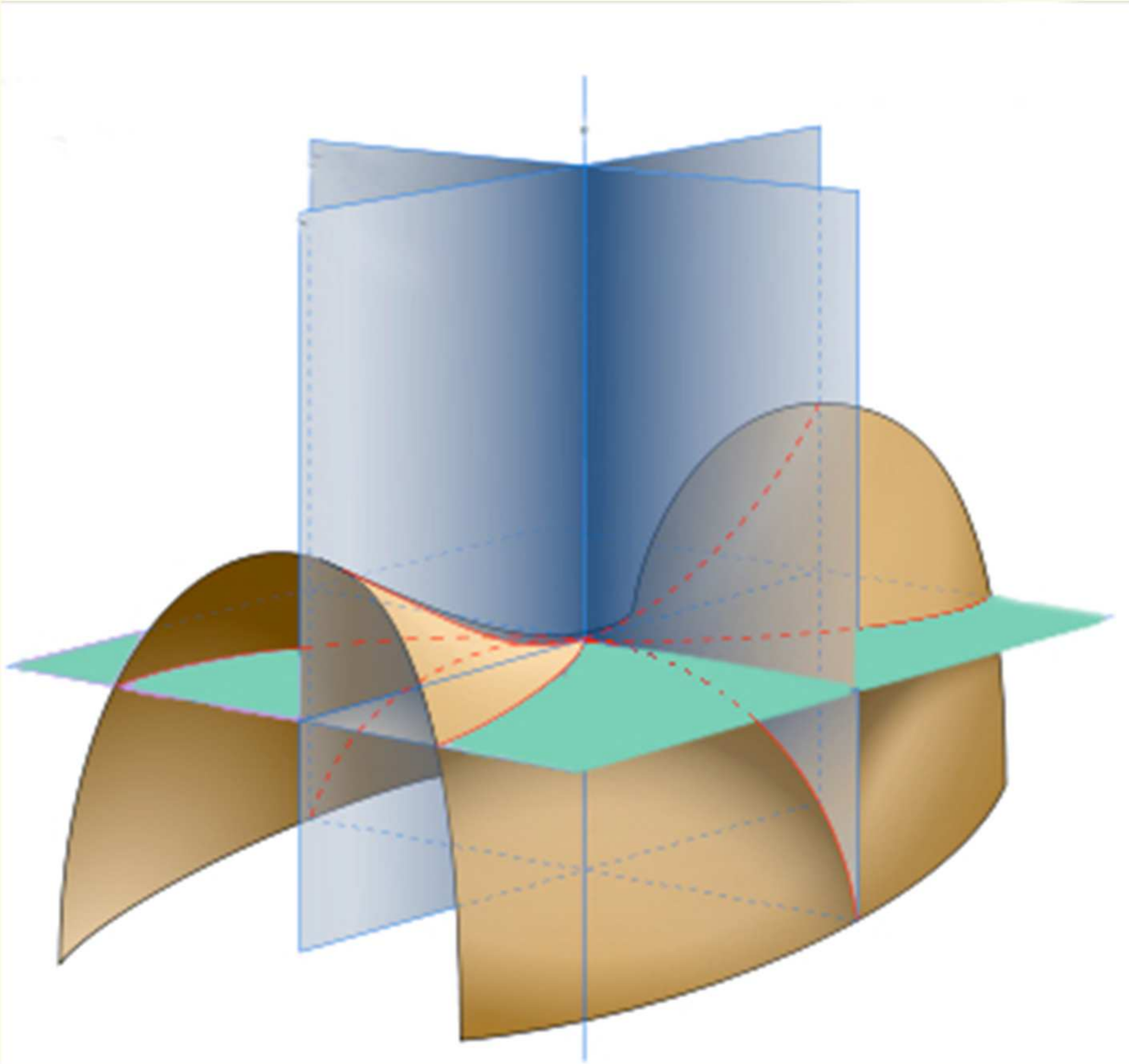
TYPIS DIETERICHIANIS

MDCCCXXVIII

We owe to three main figures the definition of non-Euclidean geometries:

- *the first one is Carl-Friedrich GAUSS, who in the fundamental essay « Disquisitiones Generales circa superficies curvas » introduced the concept of intrinsic curvature*

# connected to principal curvatures as follows



We owe to three main figures the definition of non-Euclidean geometries:

- *the first one is Carl-Friedrich GAUSS, who in the fundamental essay « Disquisitiones Generales circa superficies curvas » introduced the concept of intrinsic curvature that he could derive analytically in his Theorema Egregium*

Quodsi iam has expressiones diversas in formula pro mensura curvaturae in fine art. praec. eruta substituimus, pervenimus ad formulam sequentem, e solis quantitatibus $E$, $F$, $G$ atque earum quotientibus differentialibus primi et secundi ordinis concinnatam:

$$4(EG-F^2)^2 k = E\left(\frac{\partial E}{\partial q}\cdot\frac{\partial G}{\partial q} - 2\frac{\partial F}{\partial p}\cdot\frac{\partial G}{\partial q} + \left(\frac{\partial G}{\partial p}\right)^2\right)$$
$$+F\left(\frac{\partial E}{\partial p}\cdot\frac{\partial G}{\partial q} - \frac{\partial E}{\partial q}\cdot\frac{\partial G}{\partial p} - 2\frac{\partial E}{\partial q}\cdot\frac{\partial F}{\partial q} + 4\frac{\partial E}{\partial p}\cdot\frac{\partial F}{\partial q} - 2\frac{\partial F}{\partial p}\cdot\frac{\partial G}{\partial p}\right)$$
$$+G\left(\frac{\partial E}{\partial p}\cdot\frac{\partial G}{\partial p} - 2\frac{\partial E}{\partial p}\cdot\frac{\partial F}{\partial q} + \left(\frac{\partial E}{\partial q}\right)^2\right) - 2(EG-F^2)\left(\frac{\partial^2 E}{\partial q^2} - 2\frac{\partial^2 F}{\partial p\,.\,\partial q} + \frac{\partial^2 G}{\partial p^2}\right)$$

12.

Quum indefinite habeatur

$$\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2 = E\mathrm{d}p^2+2F\mathrm{d}p\,.\,\mathrm{d}q+G\mathrm{d}q^2$$

patet, $\sqrt{(E\mathrm{d}p^2+2F\mathrm{d}p\,.\,\mathrm{d}q+G\mathrm{d}q^2)}$ esse expressionem generalem elementi linearis in superficie curva.

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- *but Janosz BOLYAI*

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- *the first one is Carl-Friedrich GAUSS,*
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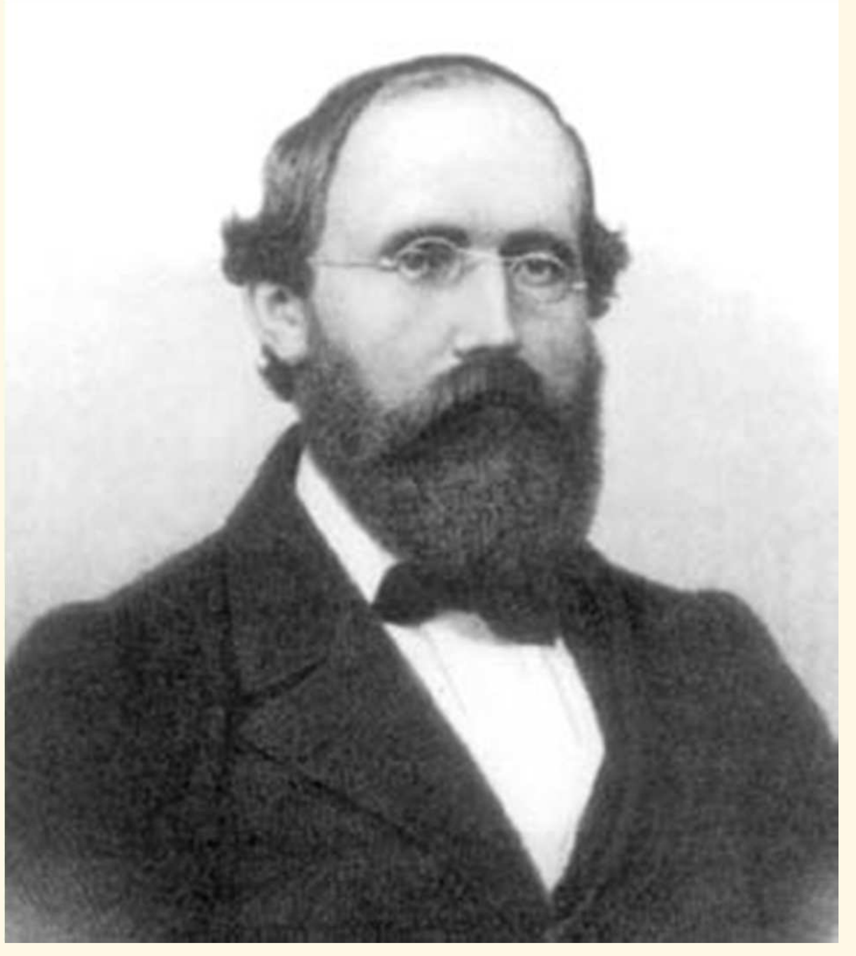
- *but Janosz BOLYAI should not be forgotten.*

The next generalisation of Geometry came from the genius of Bernhard RIEMANN



Bernhard RIEMANN

The next generalisation of Geometry came from the genius of Bernhard RIEMANN in

# Ueber die Hypothesen, welche der Geometrie zu Grunde liegen.

(Aus dem dreizehnten Bande der Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen.)*)

## Plan der Untersuchung.

Bekanntlich setzt die Geometrie sowohl den Begriff des Raumes, als die ersten Grundbegriffe für die Constructionen im Raume als etwas Gegebenes voraus. Sie giebt von ihnen nur Nominaldefinitionen, während die wesentlichen Bestimmungen in Form von Axiomen auftreten. Das Verhältniss dieser Voraussetzungen bleibt dabei im Dunkeln; man sieht weder ein, ob und in wie weit ihre Verbindung nothwendig, noch a priori, ob sie möglich ist.

Diese Dunkelheit wurde auch von Euklid bis auf Legendre, um den berühmtesten neueren Bearbeiter der Geometrie zu nennen, weder von den Mathematikern, noch von den Philosophen, welche sich damit beschäftigten, gehoben. Es hatte dies seinen Grund wohl darin, dass der allgemeine Begriff mehrfach ausgedehnter Grössen, unter welchem die Raumgrössen enthalten sind, ganz unbearbeitet blieb. Ich habe mir daher zunächst die Aufgabe gestellt, den Begriff einer mehrfach ausgedehnten Grösse aus allgemeinen Grössenbegriffen zu construiren. Es wird daraus hervorgehen, dass eine mehrfach ausgedehnte Grösse verschiedener Massverhältnisse fähig ist und der Raum also nur einen besonderen Fall einer dreifach ausgedehnten Grösse bildet. Hiervon aber ist eine nothwendige Folge, dass die Sätze der

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*) Diese Abhandlung ist am 10. Juni 1854 von dem Verfasser bei dem zum Zweck seiner Habilitation veranstalteten Colloquium mit der philosophischen Facultät zu Göttingen vorgelesen worden. Hieraus erklärt sich die Form der Darstellung, in welcher die analytischen Untersuchungen nur angedeutet werden konnten; einige Ausführungen derselben findet man in der Beantwortung der Pariser Preisaufgabe nebst den Anmerkungen zu derselben.

The next generalisation of Geometry came from the genius of Bernhard RIEMANN in « *Über die Hypothesen, welche der Geometrie zu Grunde liegen* » published after his death in 1868:

- it is founded on the variability of the line element g, *that is a scalar product on tangent vectors at each point*

$$g = g_{ij}(x^k)\, dx^i\, dx^j ,$$

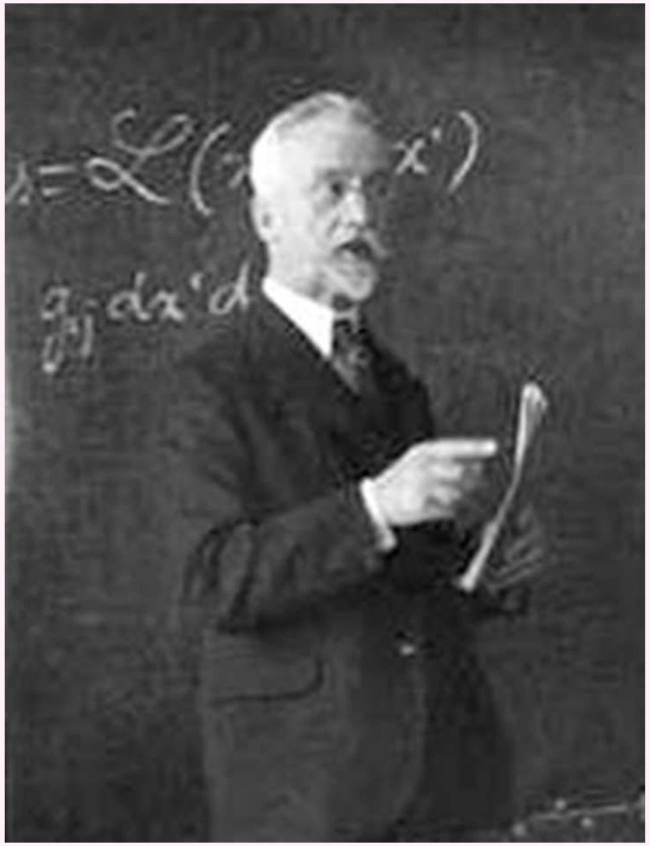
- the intrinsic curvature of GAUSS is vastly generalized by a 4-tensor $R_{ijk}{}^{l}$ whose vanishing characterizes Euclidean metrics.

# 1. From Local to Global

The need to view spaces in a global way was already considered by RIEMANN when he introduced the concept of Riemann surfaces.

It took quite some time to formalize it to a satisfactory level of generality into that of a manifold.

A great geometer such as Élie CARTAN



Élie CARTAN

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It took quite some time to formalize it to a satisfactory level of generality into that of a manifold.

A great geometer such as Élie CARTAN started an article in the 1920s by saying: « *The concept of a manifold is a subtle concept. Let M be a manifold...* ».

It was finally formalized by Hassler WHITNEY in the 1930s.

It was then recognized that spaces having locally a product structure between a piece of a manifold and a model space were particularly interesting. This is the bundle approach, due in particular to Charles EHRESMANN

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Identifying invariants that would allow to detect whether a bundle is globally a product or not was important.

This connected with some earlier work by Henri POINCARÉ

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Identifying invariants that would allow to detect whether a bundle is globally a product or not was important.

This connected with some earlier work by Henri POINCARÉ, who created a new branch of Mathematics, that he called Analysis Situs and is now called Topology.

JOURNAL

DE

L'ÉCOLE POLYTECHNIQUE.

# ANALYSIS SITUS;

Par M. H. POINCARÉ.

## INTRODUCTION.

La Géométrie à $n$ dimensions a un objet réel; personne n'en doute aujourd'hui. Les êtres de l'hyperespace sont susceptibles de définitions précises comme ceux de l'espace ordinaire, et si nous ne pouvons nous les représenter, nous pouvons les concevoir et les étudier. Si donc, par exemple, la Mécanique à plus de trois dimensions doit être condamnée comme dépourvue de tout objet, il n'en est pas de même de l'Hypergéométrie.

La Géométrie, en effet, n'a pas pour unique raison d'être la description immédiate des corps qui tombent sous nos sens : elle est avant tout l'étude analytique d'un groupe; rien n'empêche, par conséquent, d'aborder d'autres groupes analogues et plus généraux.

Mais pourquoi, dira-t-on, ne pas conserver le langage analytique et le remplacer par un langage géométrique, qui perd tous ses avantages dès que les sens ne peuvent plus intervenir. C'est que ce langage nouveau est plus concis; c'est ensuite que l'analogie avec la Géométrie ordinaire peut créer des associations d'idées fécondes et suggérer des généralisations utiles.

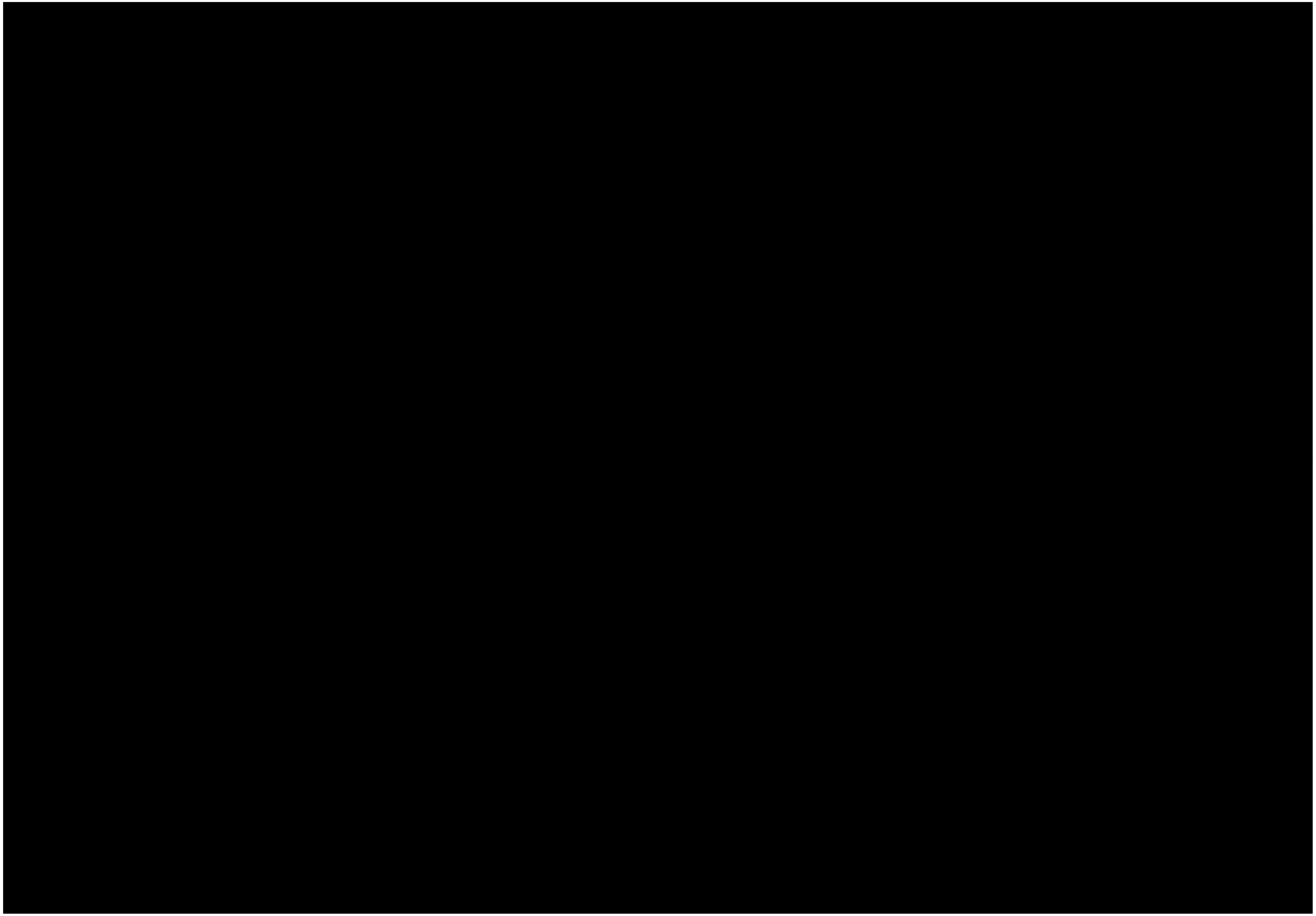
Some find the origin of Topology in the work of LEIBNIZ and/or EULER in connection with the famous problem of the seven bridges of Königsberg

A formula linking together Geometry and Topology appeared already in the 19th century: the Gauss-Bonnet formula.

It says that on a compact surface M endowed with a line element $g$

$$\int_M K_g \, vol_g = 2\pi \, \mathrm{Euler}(M)$$

where $K_g$ denotes Gauss' intrinsic curvature and Euler(M) a topological invariant connected to the number of holes of M .



This formula can be vastly generalized and Shiing Shen CHERN provided several ways for this, notably through the Chern Classes.

One example is valid for a compact manifold M of dimension 2n endowed with a line element $g$

$$\int_M P_n(R_g)\,\mathrm{vol}_g = a_n\,\mathrm{Euler}(M)$$

where $P(R_g)$ denotes a polynomial in Riemann's curvature tensor and $a_n$ a universal constant.

Remarks on the differential geometry of fiber bundles.

1. Given a real or complex $C^\infty$ vector bundle over a manifold $X$. Let $\Omega$ be the curvature matrix obtained from a connection. Then the coefficients in

$$\det(I + \lambda\Omega) = 1 + c_1(\Omega)\lambda + \cdots + c_k(\Omega)\lambda^k + \cdots$$

define cohomology classes in $X$, independent of the choice of the connection.

2. If the group of the bundle can be reduced to $S(p,q)$, the subgroup of $GL(n;\mathbb{R})$, consisting of all transformations of determinant $+1$ which leave invariant a quadratic form of signature $(p,q)$, $p+q=n$, then the pfaffian

$$\sqrt{\det(F\Omega)},$$

where $F$ = matrix of scalar products of the vectors of a frame (so that $F\Omega$ is skew-symmetric), defines, up to a factor, the Euler class of the bundle. In particular, this gives the Gauss-Bonnet formula for a pseudo-riemannian manifold.

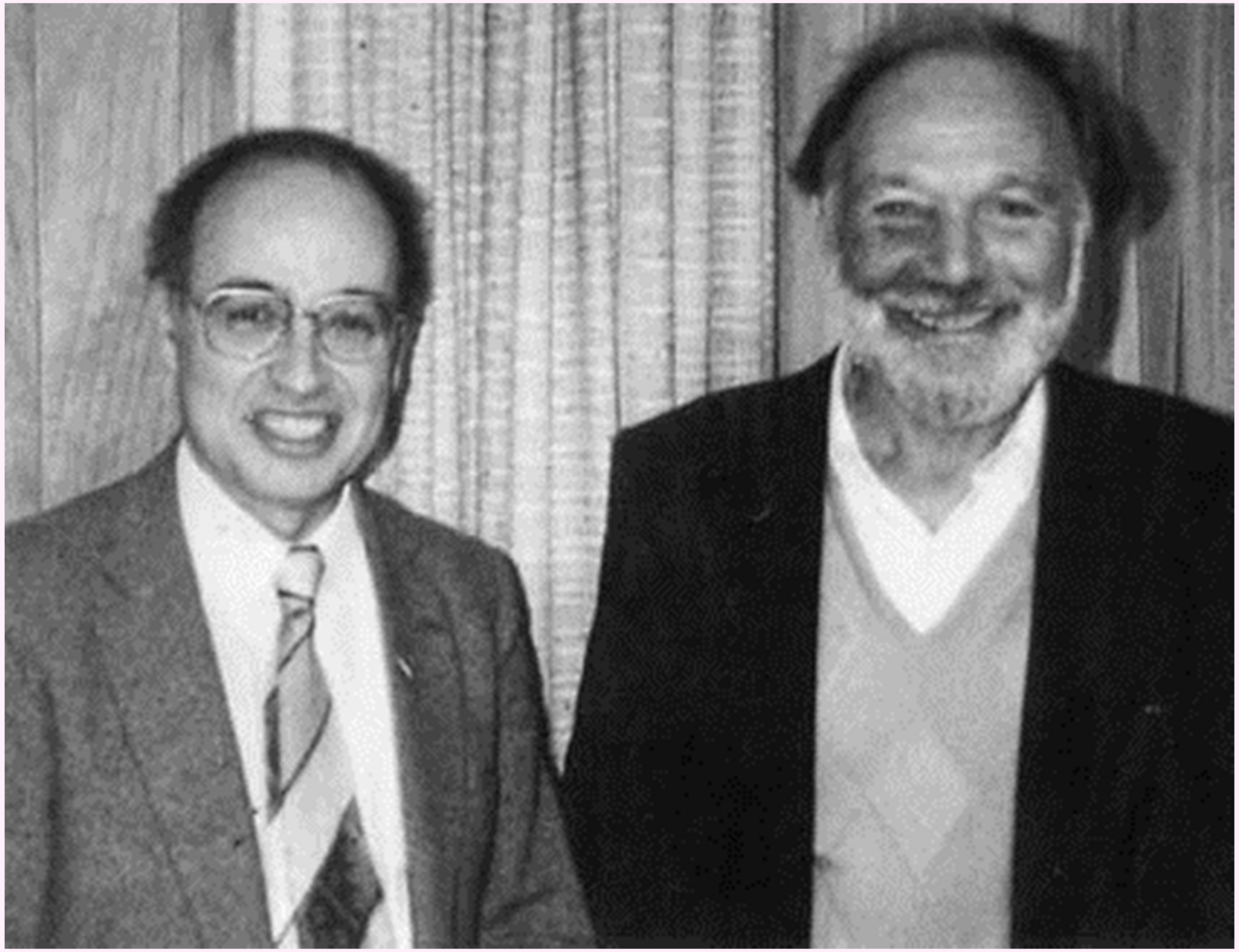
3. $X$ = complex manifold, bundle is holomorphic. Then the theory can be refined relative to the $d'd''$-operator. This has important application to the question of equi-distribution of the zeros of holomorphic sections.

S. S. Chern

Berkeley, California, USA

The theme « *Links between Curvature and Topology* » has been most active in Geometry.

The Gauss-Bonnet formula has been vastly generalized through the Index Theorem due to Michael ATIYAH and Isadore SINGER



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The theme « *Links between Curvature and Topology* » has been most active in Geometry.

The Gauss-Bonnet formula has been vastly generalized through the Index Theorem due to Michael ATIYAH and Isadore SINGER.

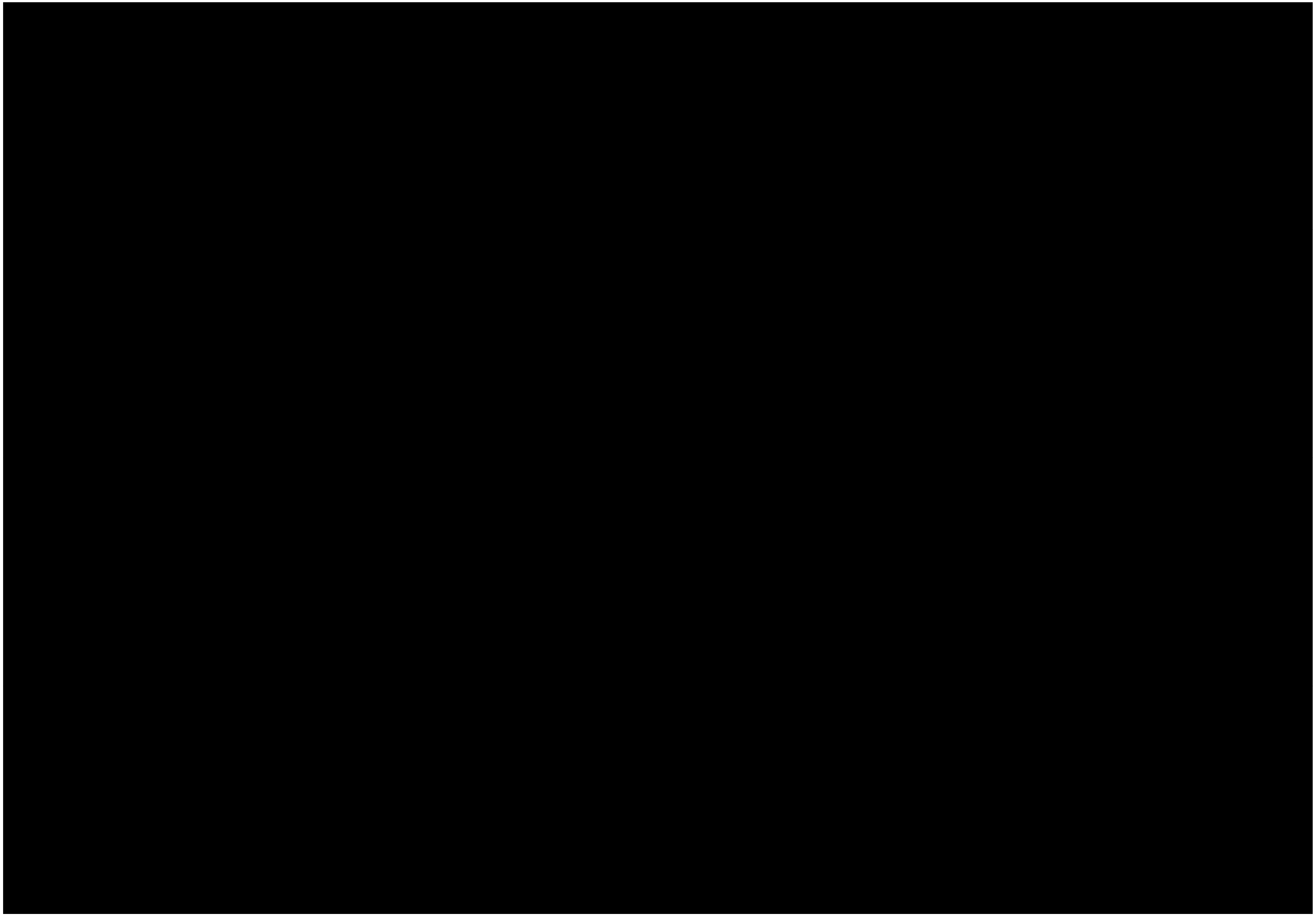
It says that for any elliptic operator between sections of bundles over a compact manifold, its analytic index, that can be computed from its leading term involving its geometry, and its topological index, involving the Chern classes of the bundle.

# 2. From Smooth to Rough

Several other mathematicians have developed this idea to look at spaces that are less smooth than manifolds:

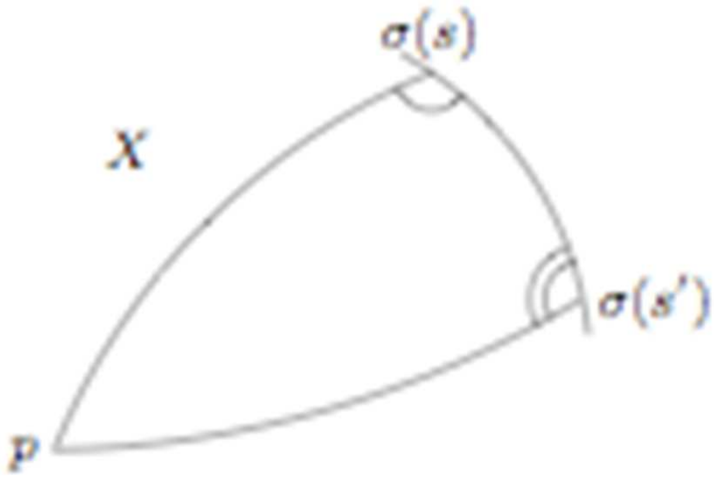
- Alexander ALEXANDROV

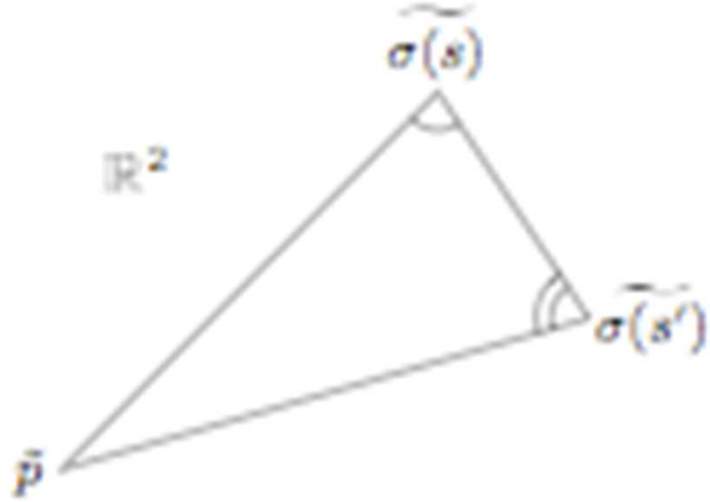
Alexander ALEXANDROV



Several other mathematicians have developed this idea to look at spaces that are less smooth than manifolds:

- Alexander ALEXANDROV focused his attention on less regular spaces. He introduced in particular a notion of spaces with lower and upper bounds for their curvature provided one can define shortest paths by comparing to model spaces with constant curvature;


$\sigma(s)$
$X$
$\sigma(s')$
$p$


$\widetilde{\sigma(s)}$
$\mathbb{R}^2$
$\widetilde{\sigma(s')}$
$\tilde{p}$

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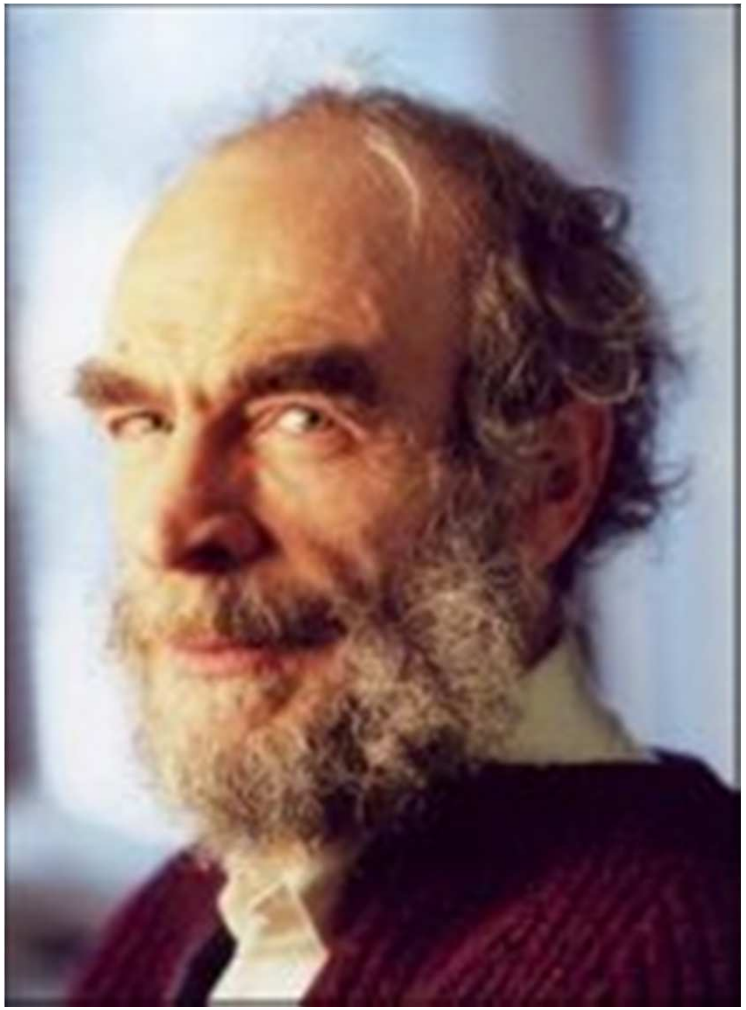
- Mikhail GROMOV



Mikhail GROMOV

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- Alexander ALEXANDROV focused his attention on less regular spaces. He introduced in particular a notion of spaces with lower and upper bounds for their curvature provided one can define shortest paths in the space by comparing to model spaces with constant curvature;
- Mikhail GROMOV went much further and considered general families of metric spaces.

Mikhail GROMOV introduced the very efficient tool of a distance on the space of metric spaces, the Gromov-Hausdorff distance allowing to study the convergence of geometric properties on a family of spaces:

- He proves that: «spaces having an upper bound on their diameter and a lower bound on their curvature form a precompact set in the Gromov-Hausdorff topology»;
- He also found completely unexpected applications to Group Theory, relating their algebraic structure to properties of a metric space he attaches to them.

The Gromov-Hausdorff distance has been used in shape analysis in computer science:

# One Small Step for Gromov, One Giant Leap for Shape Analysis

A window into the 2009 Abel Laureate's contribution in computer vision and computer graphics

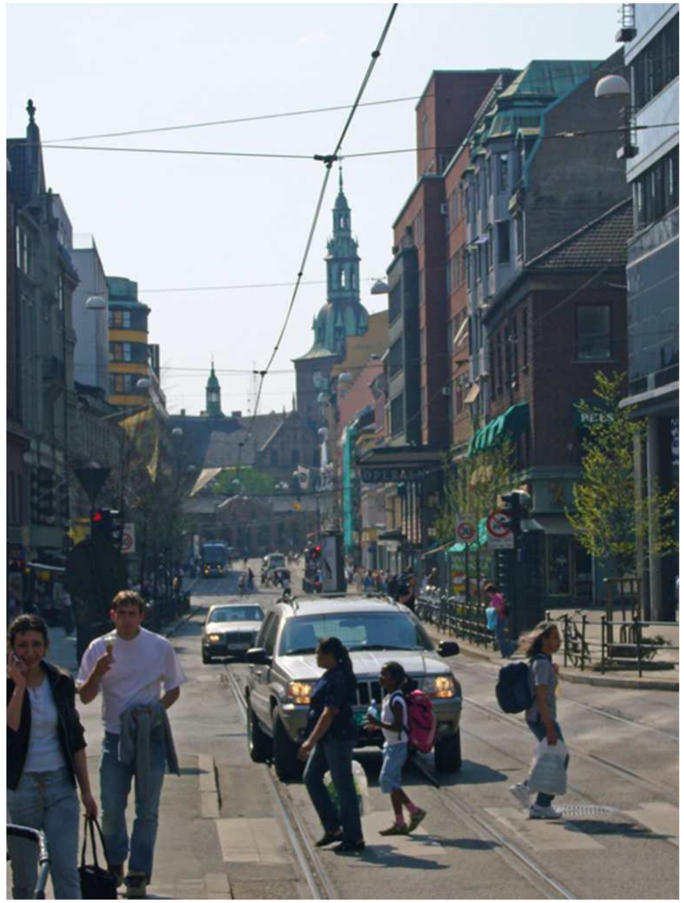
Guillermo SAPIRO

University of Minnesota

JPB-Cech Lecture

Prague, October 31, 2014

The Gromov-Hausdorff distance has been used in shape analysis in computer science: What is the goal?



# 3. From Static to Dynamic

In 1905, Albert EINSTEIN introduced his Theory of Special Relativity, that forced to unify space and time.

The basic mathematical concept behind it is the use of generalized metrics, the Lorentzian metrics, such as the Minkowski metric

$$g = -c^2\, dt^2 + dx^2 + dy^2 + dz^2.$$

Later, Albert EINSTEIN went one step further in his General Theory of Relativity, that revolutionised the theory of gravitation.

For that, he was greatly helped in his mathematical quest by his colleague at the ETH Zurich  Marcel GROSSMANN

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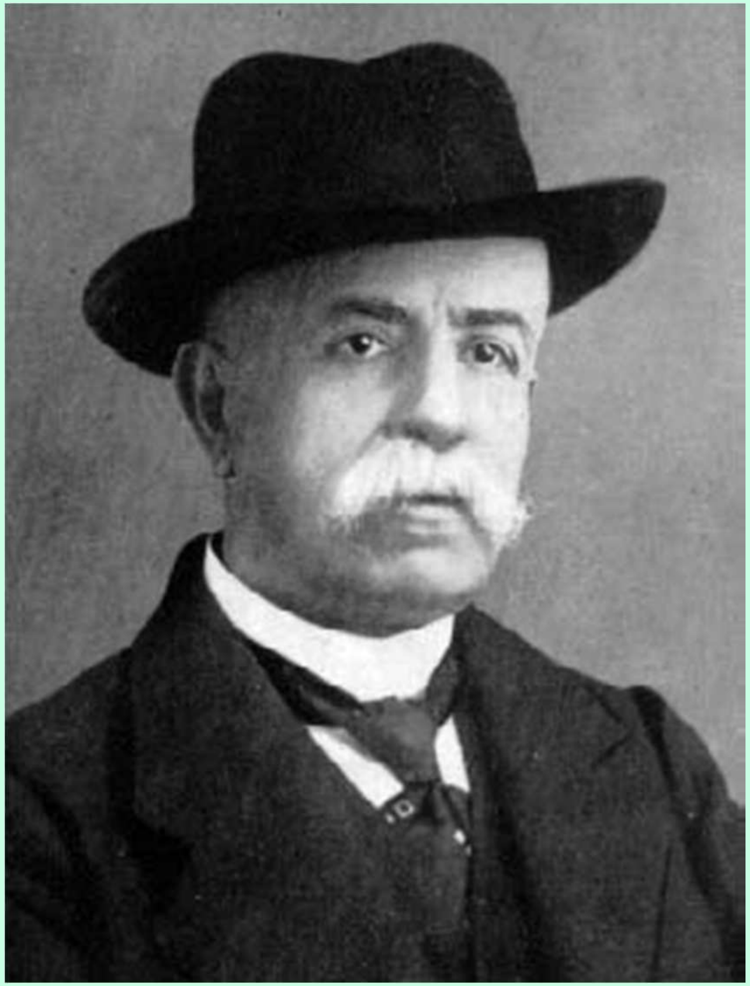
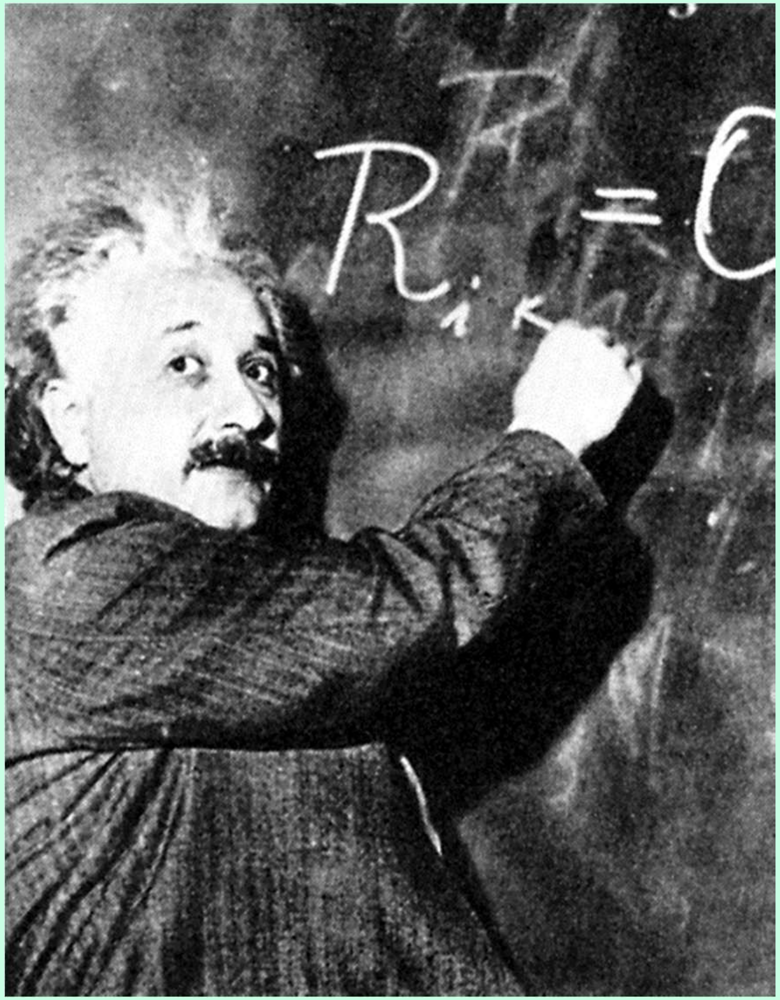


Grigorio RICCI-CURBASTRO

As you know, Albert EINSTEIN went later one step further in his General Theory of Relativity, that revolutionized the theory of gravitation.

For that, he was greatly helped in his mathematical quest by his colleague at the ETH Zurich  Marcel GROSSMANN, who introduced him to a variant of the Riemann curvature due to Grigorio RICCI-CURBASTRO, the Ricci curvature, introduced in 1904 for strictly mathematical purposes.



Albert EINSTEIN

In this theory, the metric is determined by the position of the matter and other physical fields.

Because of the signature of the metric (-+++) the field equations can be viewed as a dynamic equation on the family of Riemannian metrics induced on space hypersurfaces, that are 3-dimensional manifolds.

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Because of the signature of the metric (-+++) the field equations can be viewed as a dynamic equation on the family of Riemannian metrics induced on space hypersurfaces, that are 3-dimensional manifolds.

The leading term of this dynamic is the Ricci curvature of the space hypersurfaces. This lead to the idea of «*deforming the metric in the direction of its Ricci curvature*».

The first one to use this idea in Mathematics has been Thierry AUBIN in 1970 who was looking for an infinitesimal result.

In 1979, in a conference in Berlin at the invitation of Udo SIMON, I asked the following question:

3.24 QUESTION.– *Does the local flow theorem hold for the vector fields* $\mathrm{Ric}_g - k\,\mathrm{Scal}_g\, g$ *on the space of metrics? What is the global behaviour of the integral curves if they exist?*

In 1982, the existence of the flow of - Ricci was proved by Richard HAMILTON

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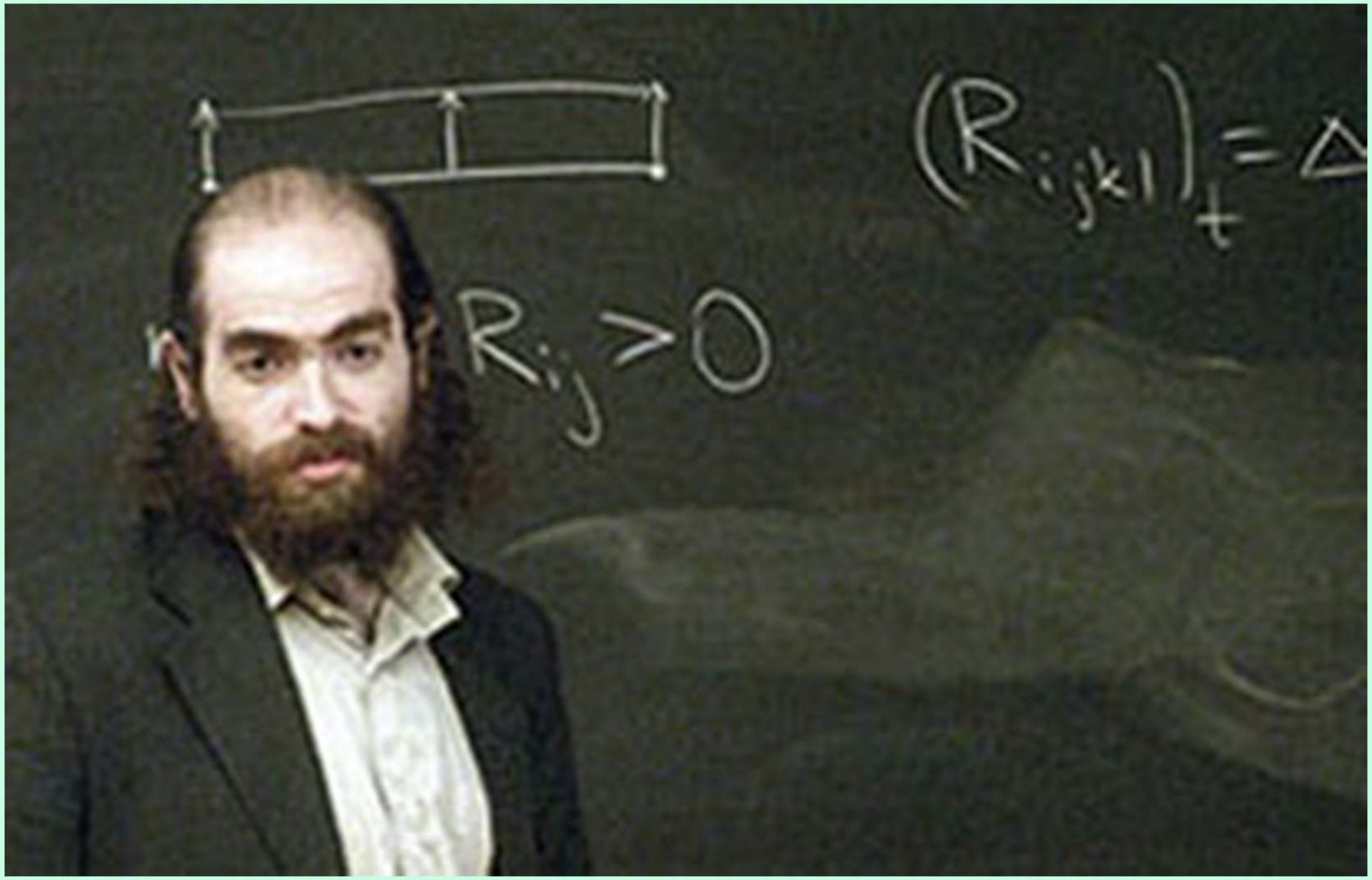
In a series of papers around the turn of the century, Grigory PERELMAN



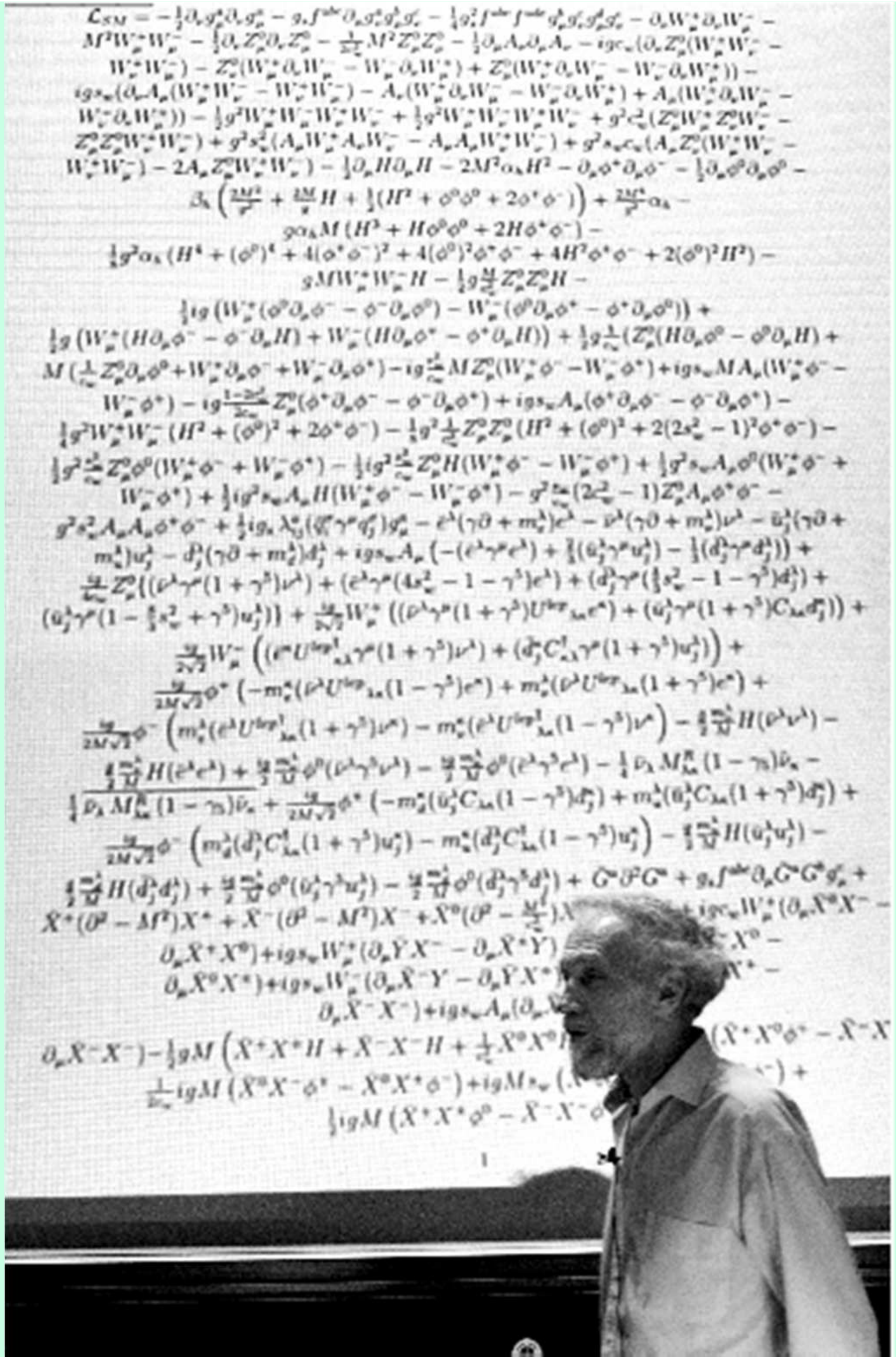

Grigory PERELMAN

In a series of papers around the turn of the century, Grigory PERELMAN shows that, without any curvature assumptions, the Ricci flow can develop singularities that can be controled.

He shows that, on a 3-dimensional simply connected manifold, the metric converges to a metric with constant curvature, hence providing a proof of the Poincaré conjecture.

This is a wonderful success for Modern Geometry and its evolution from local to global, from smooth to rough, from static to dynamic.

Another geometric development that was triggered by Physics is the development of Non-Commutative Geometry due to Alain CONNES



Alain CONNES

Another geometric development also triggered by Physics, namely Quantum Mechanics, is due to Alain CONNES:

- *he starts from the algebra of observables, that he does not suppose to be commutative as is the algebra of functions on a usual space;*
- *he notices that non-commutativity creates an internal dynamic to this generalized Geometry;*
- *here again, continuous and discrete are reunified allowing new models for Physics and Number Theory.*

# Concluding Remarks

The broadening of Geometry will certainly go on and benefit from many influences:

- *the role of Physics has been mentioned several times;*
- *other disciplines such as Computer Science and Biology will certainly play an increasing role in the future;*
- *the trends stressed in the lecture will certainly be also present.*

This was also an opportunity to go through a gallery of portraits of mathematicians who made outstanding contributions:

- *they all pushed the boundary of the discipline;*
- *they often showed exceptional technical ability in solving difficult problems;*
- *one should stress that some of the most important developments came from pure curiosity and search for beauty.*

*I thank you for your attention.*

Jean-Pierre BOURGUIGNON
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This should not make us forget about the human dimension. From this point of view, Shiing Shen CHERN is undisputably a model, because of his special attention to young students and colleagues.

Here is a special picture of him from a lecture at the Mathematisches Institut Oberwolfach.


IHÉS
Institut des Hautes Études Scientifiques
Centenaire de Shiing Shen CHERN
陳省身
Célébration en France
17 novembre 2011
Centre de conférences Marilyn et James Simons
Bures-sur-Yvette
11h00 Friedrich HIRZEBRUCH (Max-Planck-Institut für Mathematik, Bonn, Allemagne)
Chern Classes
12h00 Claire VOISIN (CNRS-Institut de Mathématiques de Jussieu, Paris)
Coniveau of Cohomology and Algebraic Cycles
14h30 ZHANG Weiping (Chern Institute of Mathematics, Nankai Univ., Tianjin, R.P. Chine)
Dirac Operators and Vanishing Theorems
16h00 George CSICSERY
Taking the Long View.
The Life of Chern Shiing-Shen *
Première projection du film en Europe
17h00 Présentation du Fonds Chern à l'IHÉS
en présence de May CHU, Présidente de la Fondation Chern
17h20 Lancement officiel du Fonds Huawei à l'IHÉS
Information et inscription :
www.ihes.fr
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