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How Does Public IPR Protection Affect its Private Counterpart? Copyright and the Firms' Own IPR Protection in a Software Duopoly[1]

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Abstract

We study how the strength of public intellectual property rights (IPR) protection against software piracy (copyright protection) affects private IPR protection (that software developers may themselves undertake to protect their IPR). There are two software developers that offer a product variety of differing (exogenously given) quality and compete in prices for heterogeneous users, who make a choice whether to buy a legal version, use an illegal copy (if they can), or not use a product at all. Using an illegal version violates IPR and is thus punishable when disclosed. If a developer considers the level of piracy as high, he can introduce a form of physical protection for his software or digital product. The main aim of our analysis is to study how the level and the change of public IPR protection affect the pricing and IPR protection strategies of software developers. In particular, we are interested in establishing when the two forms of IPR protection (public and private) are complements to each other, when are they substitutes and when a change in public IPR has no impact on private IPR protection.

Keywords: Vertically differentiated duopoly, Software Piracy, Bertrand competition, Copyright protection, Private and public intellectual property rights protection

JEL Classification: D43, L11, L21, O25, O34

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Abstrakt

Zkoumáme, jak úroveň veřejné ochrany práv duševního vlastnictví (IPR) proti softwarovému pirátství, známa též jako ochrana autorských práv, ovlivňuje úroveň firemní ochrany těchto práv. Firemní ochranou IPR máme na mysli ty kroky, které mohou vývojáři softwaru sami podniknout, aby ochránili svá práva. Uvažujeme model se dvěma softwarovými společnostmi nabízejícími rozdílné produkty, jenž se liší exogenně danou kvalitou. Tyto společnosti mezi sebou soutěží cenou o heterogenní uživatele. Uživatelé se rozhodují, zda zakoupí legální verzi, zda použijí nelegální kopii, je-li to možné, anebo zda si produkt vůbec nepořídí. Pokud uživatel použije nelegální verzi, porušuje tím IPR, což s sebou přináší riziko odhalení a následného trestu. Kromě toho, softwarová společnost navíc může sama zavést vlastní formu ochrany proti kopírování pro svůj software nebo digitální produkt. Hlavním cílem naší analýzy je studovat, jak úroveň veřejné ochrany IPR a její změna ovlivňuje cenovou strategii firem a jejich strategii ohledně ochrany softwaru proti kopírování. Zajímáme se zejména o schopnost určit, kdy jsou uvažované dvě formy ochrany IPR (veřejná a soukromá) komplementární, a kdy se vzájemně substituují a dále kdy změna ve veřejné ochraně IPR nemá žádný vliv na úroveň firemní ochrany IPR.

1 Introduction

A typical characteristic of software and other information products (such as movies, music and e-books) is that it is rather hard to exclude others, especially non-payers, from using these products. The reason for that lies in the low costs and low technical requirements to acquire these products. So, it is no wonder that these "information" products (also known as digital content products) are the easiest target for illegal imitations nowadays. Imitations of these products are often fully identical to the original and the direct costs of copying might be negligible. The Business Software Alliance reported that the share of pirated software as a percentage of total software installed in 2008 mounted to 41%, resulting in a global loss in excess of $50 billion. Top of the list are countries like Georgia, Pakistan, Indonesia, and China where 80% and more of installed software is illegal. Even in the US, roughly 20% of software is installed illegally. The corresponding figure for Western Europe is around one third.

The fast spread of broadband internet along with the expansion of DVD burners, has tremendously increased the opportunity for illegal copying and is also eliminating mass illegal producers from the market. Thus these days, illegal copies are typically made by the end users themselves who make them only for themselves,[1] and so this feature dramatically changes the essentials of the fight against piracy and against intellectual property rights (IPR) violation. Contrary to the situation in, for instance, pharmaceuticals, luxury goods, or electronics markets (where end users are often perceived as victims of counterfeiting and fraud), in "information" markets, they are the ones that actually carry out IPR violation. Thus, the fight against IPR violation in digital content markets these days is targeted mainly against end users (both retail and corporate).

Our focus is on such a digital content market (like the software market) where only end users violate IPR. More specifically, we analyze strategic interactions among software developers who compete in prices but may also undertake private IPR product protection against

[1] In this paper, we do not consider commercial piracy where, say, bootleggers sell pirated DVDs/CDs or software. These kinds of piracy experienced a boom more than a decade ago and are now declining rapidly, especially in developed countries. For a survey of models with commercial piracy, see Belleflamme and Peitz (2012).

end users' piracy. On the other hand, public IPR protection (in the form of copyrights) also exists. So the core of our analysis is to study how public (copyright) protection affects pricing and the private IPR protection strategies of software developers. For that purpose, we developed a dynamic two-stage duopoly game. In the last stage of the game, two developers compete in prices for users with different price sensitivity on the same market. That is, we rely on a quality competition model (see, for instance, Shaked and Sutton, 1982, Sutton, 1991 and Tirole, 1988). In the last (second) stage of the game, each developer has an option to choose a level of its private IPR protection. Like most of the literature, we assume that the government's punishment (copyright protection) is broad-based in a sense that it raises the piracy costs for all consumers[2].

As for the developers' private protection, we assume that it comes in the form of costly physical product protection. That is, a developer, for instance, protects his software by means of special CDs (or encryption against cracking) like in games, where copies created on a standard DVD burner do not work anymore. Such kind of protection is always imperfect since there is always a fraction of skillful consumers who are capable of overcoming this protection and enjoying the copied software to its full value, much like the legal users. The developers, however, could incur larger effort and costs to reduce this fraction of skillful consumers but cannot fully eliminate it. These kinds of private protection measures are known in the literature as "technical protection measures" (see Scotchmer, 2004, for a survey on this topic) and are also closely related to the so called DRM (Digital Rights Management) system [3].

To capture the regulator's role in a simple manner, we assume that imposing a penalty on the IPR violators is the only instrument for reducing or eliminating the illegal use of the product that is under copyright protection. So the height of the (expected) penalty serves

[2] However, there is also an alternative approach in which public protection mostly targets institutional and corporate users rather than individual users, see Harbaugh and Khemka, 2010 and the relevant literature cited there on such an approach.

[3] DRM is an umbrella term for various technologies that limit the usage of digital content in an unintended way by the developer. Most major content providers such as Microsoft, Sony, Amazon, or Apple used to exploit DRM. Nowadays most content providers experiment with DRM-free alternatives, mainly in music (see more on DRM in Belleflamme and Peitz, 2010 and also Scotchmer, 2004).

as the measure of the strength of copyright protection.

As for the related literature and relation of our paper to it, it is important to stress that our approach is somewhat different from the current literature on software piracy. According to Belleflamme and Peitz's comprehensive surveys (2012 and 2014), the vast majority of papers that analyze the economic issues of digital piracy make the simplifying assumption that software is supplied by a single developer. The reason for this is that consumers perceive software products as highly differentiated so a change of one product's price hardly affects the demand of the other products (see Belleflamme and Peitz, 2014). While this may roughly be true in some cases, we claim that a more realistic analysis of the software market should rely on competition among software developers. More specifically, the concept of vertical product differentiation looks appropriate here because typically there is a software product that is perceived to have superior quality than the product of its competitor and so it is priced much more than its closest substitute. Thus, if both softwares are offered at the same price, most (or even all) consumers would choose the product that would be considered of higher quality. For instance, in a market for vector graphic editing software, there are two relevant products: Adobe Illustrator and CorelDRAW. The first one (Adobe) has a higher consumer rating and its price is 2.5 times higher than the Corel software indicating that products might be perceived as vertically differentiated.

To put our approach further into perspective, we use the Belleflamme and Peitz (2012) classification, according to which our paper belongs, to i) end-user piracy models that ii) includes the competitive effects, meaning that there are two producers of substitutable and piratable digital products that directly compete with each other. As Belleflamme and Peitz (2012) noted, there are only a few articles dealing with digital piracy while explicitly tackling direct competition among firms. Moreover, these papers mostly rely on the notion of horizontal product differentiation. The article that is somewhat related to our analysis is our companion paper, Žigić et al. (2013), that deals with the interaction of private and public IPR protection. The form of private IPR protection, however, is radically different there and it comes in the form of a simple and costless service restriction like denying various services related to the efficient use of software, restricting access to users' manuals, etc.)

Moreover, the focus of Žigić et al. (2013) is on normative analysis, that is, on optimal public IPR protection and its interaction with private protection rather than on developers' IPR protection and pricing strategies like in this paper. Other related paper are the works of Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010). Unlike these papers, we focus on direct strategic interaction between the developers where the two firms compete in prices in a vertically differentiated market, whereas the strategic interactions in Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010) are indirect ones stemming from different copying technologies. Secondly, in addition to the different focus (direct versus indirect competition), the other key difference between our set-up and that of Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010) is that in their settings the original products have the same quality, while in our set-up, the original products are vertically differentiated and thus have distinct qualities to begin with. Thirdly, since we focus on the software market, we do not allow for a different copying technology as it is typically the case with multiple, initially independent digital products. Thus, the cost of consuming illegal copies is constant in our setting, while it may be decreasing with the number of different originals copied in the settings of Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010).

Perhaps the very first article on this subject that introduced the competitive effect is one by Shy and Thisse (1999), who analyze piracy in the Hotelling-type duopoly competition where users have exogenous preferences for a particular developer[4]. They show that a developer's decision to introduce protection against illegal copying depends mainly on the network effects (NEs), and that under strong NEs, each developer decides not to implement protection in order to make his software more attractive and to raise the users base. Jain (2008) builds upon the model of Shy and Thisse (1999) and assumes that firms can choose a level of IPR protection so that only a proportion of consumers with low product valuations (who are, by assumption, the only consumers interested in copying) can copy its product. In the absence of NE, Jain shows that, in such a set-up, piracy can change the structure of the market and, thereby, reduce price competition between firms. The reason is that copying by low, more price-sensitive types enables firms to credibly charge higher prices to

[4]There is, however, a mistake in the article; see Peitz, (2004) for the correction of the mistake.

the segment of consumers that do not copy. Furthermore, this positive effect of piracy on firms' profits can sometimes outweigh the negative impact due to lost sales. So, even in the absence of network effects, firms may prefer weak copyright protection in equilibrium. Finally, there is a recent paper by Minnitti and Vergari (2010), who also rely on the Hotelling differentiated-product duopoly framework. They, however, deal with a rather specific form of piracy similar to a private file sharing community and study how its presence affects the pricing behavior and profitability of producers of digital products.

Finally, there are by now numerous scholarly articles that deal with the issue of digital piracy in a monopoly set-up or a dominant firm set-up (constrained by competitive fringe like Harbaugh and Khemka, 2010). As for the paper that exploits a monopoly set-up, see, for instance, Yoon, 2002, Banerjee, 2003; King and Lampe, 2003; Kúnin, 2004; Bae and Choi, 2006, Banerjee, et al., 2008. Takeyama, 2009, Ahn, and Shin, 2010. Thus, for instance, King and Lampe (2003) show that a monopoly allows illegal users in cases where a network effect is present, while Takeyama (2009) shows that under asymmetric information about product quality, the copyright has to be imperfect in order to avoid adverse selection. Kúnin (2004) provides an explanation as to why a software manufacturer may tolerate widespread copyright infringement in developing countries and often even offer local versions of their software. He showed that if NEs are present and there is an expected improvement in copyright, then software manufacturers enter the market even if they incur losses in the beginning when copyright enforcement is weak. For a deeper and systematic review of the literature on the piracy of digital products, the interested reader is advised to look at the excellent and comprehensive surveys in Peitz and Waelbroeck (2006) and Belleflamme and Peitz (2012) and (2014).

The structure of the article is the following: In the second section, we put forward our set-up and we conclude this section with a brief analysis of the monopoly market structure. The third section contains a key analysis of duopoly competition in prices and the impact of copyright strength on the IPR and pricing strategies of software developers. Finally, we make some concluding remarks in the fourth and final section.

2 The Model

2.1 Industry set-up

Consider two developers A and B that compete in prices on a particular market and offer product varieties of different quality. Developer A releases a product of quality q_A, while the quality of developer B is q_B and we assume, without loss of generality, in the rest of the article that developer A offers higher quality ($q_A > q_B$). Product qualities q_A, q_B, in the whole article are assumed to be exogenous and cannot be changed by the developers[5]. The unit variable costs are assumed to be constant and normalized to zero. One may think about developer A as an already established and known software producer that already operates on other markets. This fact is, in turn, reflected in the preferences of the consumers, who strictly prefer software A over software B if offered at the same price. Similarly, developer B can be thought of as a local developer offering lower quality. In other words, we assume that both developers already existed before meeting and competing on the market under consideration. Consequently, both developers are assumed to have already incurred set-up fixed costs and fixed costs associated with software development (R&D costs). These fixed costs are, from our perspective, general and not related to the developer's presence on the particular market under consideration, and therefore, we leave them out of the profit function. We, however, may allow for the fixed costs of entry to the particular market under consideration, so we denote as F_A and F_B these entry or set-up costs respectively (sinking these costs can be considered to take place at the first stage of the game). We will, however, omit these fixed costs from the profit functions for the purpose of transparency and assume that the developers' profits are positive net of these costs.

To summarize, we simply assume that:

1. Initially, both developers A and B already exist with established quality levels of their respective varieties.

[5] In the more elaborated versions of this kind of models, there is also a choice of quality preceding the pricing decision. In this case, it is standard to assume that the bulk of the costs of generating quality falls on fixed costs so that quality or R&D costs are in fact endogenously determined (see, for instance, Shaked and Sutton, 1982 and 1983; Kúnin and Žigić, 2006). For each case that we analyze, it should be clear how to relax the model and allow the developers to choose and compete in qualities too.

2. The focus is on a particular software market, which is not interrelated with the other markets on which developers may operate ("segmented market hypothesis").

Perhaps it might be convenient to think that developers compete (or may compete) on some third market (that is, a market that is not their home market). An important implication of these two assumptions is that in our set-up one or even both developers may not be active on the market under consideration. The reason for this is that due to the absence of the developers' own IPR protection and the possible lack of IPR protection by the side of the regulator, it may not be profitable for the developer(s) to operate on the market under consideration. We, however, assume that even if a developer does not enter the market, the users are still able to obtain an illegal version via copying. This, in turn, makes entry deterrence not viable. We use a sub-game perfect equilibrium as a solution concept throughout this paper in all multi-period games under consideration.

2.2 Private protection against copying–physical protection

As we already mentioned, we aim to study here the economic impact of so called "physical protection". By physical protection we understand that installing an illegal version of the software is more difficult either because of low availability of the illegal version or because of a high requirement of user skill to install (or use) the illegal version. An example of such protection is a DVD with games where a version coming from standard copying with a DVD burner cannot be installed on a PC any longer[6]. Another example is requiring users to authenticate their copy on the developer's web pages during installation, which could be technically complicated to avoid (e.g., only by installing a "crack" to a particular directory and a set of steps to complete the installation). All such tools create obstacles in installing an illegal version, and thus limit its availability to common users. After installation, however, a user often may not distinguish an illegal version from the legal one. As already mentioned in

[6]The illegal copy does not work since the original DVD is intentionally produced with certain kinds of mistakes, and during copying, these mistakes are always corrected by the "burning" software. At the same moment, during the installation process, those mistakes are mandatory for the successful completion of the installation.

the introduction, some forms of DRM can also serve as examples of such protection. Thus, a user's perception of software quality is often intact.

2.3 The regulator's role

We introduce a very simple regulator whose role is limited to monitoring software usage and to the penalization of those users, who use products illegally and are disclosed. The probability of being caught using an illegal version is the same for all users, and the level of the penalty is fixed. The penalty and the probability of being caught is known and independent on used product and product prices, thus all users and both developers could calculate the expected penalty for using an illegal version, that we denote as X. Moreover, while we implicitly assume that the regulator choice of the optimal IPR is governed by an underlying objective function like the maximization of social welfare, we do not explicitly study the optimal choice of expected penalty since we focus on the forms of the developers' pricing and IPR protection strategies and their economic implications[7]. Thus, the whole regulator's framework is very simple in our model and translates into one parameter: expected penalty X for illegal users that also captures the strength of copyright protection (see Varian's, 2005 survey on the economics of copyrights).

2.4 Developers' problem

We assume that both developers have access to technology that allows product protection against copying and illegal use [8]. The developers' decisions are dependent only on the profitability of such a step. The protection against copying is imperfect, which means that a fraction of the users still have access to the illegal version[9]. This fraction of users is uniformly distributed over the whole interval $\langle 0, \bar{\theta} \rangle$. We say that a developer implements protection at

[7]For instance, if the government maximizes social welfare, we would need to know which of the developers is the domestic one and which is not in order to write down the objective functions. While these considerations are interesting per se, they are not the focus of the essay. For the analysis of the optimal IPR from the side of the regulator in a similar set-up, see for instance Žigić et al. (2013).

[8]Neither legal nor licence restrictions are assumed for the developer in the case of implementing protection against copying.

[9]By eliminating public availability we mean neither access to an illegal version nor access to an illegal version accompanied by the limited user's skill to install/use the illegal version.

level c, if for each $\theta \in \langle 0, \bar{\theta} \rangle$ the fraction of users with the ability to use the illegal version is $(1 - c)$, and the remaining fraction of users (c) could only use the legal version. Protection c is from interval $\langle 0, 1 \rangle$, and if c tends to 1 we say that protection becomes perfect, while c tending to 0 represents full public availability of an illegal version[10]. We further assume that both developers could implement this kind of protection, and that they could differ from each other in the protection level c. Formally, there is a two-stage game in which one or both developers choose the level of private protection in the first stage, and then they compete in prices in the second stage.

Implementing physical protection is costly, and these costs rise more than proportionally as c increases tending to infinity as c approaches 1. Thus, the costs of implementing protection c, labelled as $C = h(c)$, possess the following properties:

1. $h(0) = 0$, $\lim_{c \to 1} h(c) = +\infty$;

2. $\frac{\partial}{\partial c} h(0) = 0$, $\frac{\partial}{\partial c} h(c) > 0$;

3. $\frac{\partial^2 h(c)}{\partial c^2} > 0$;

4. $\Pi_i^* = \pi_i^*(c_i) - h(c_i)$ is a concave function reaching its maximum at $c_i^* \in (0, 1)$. (We use the symbol Π for net profit, when protection costs are accounted for, while π stands for the price-competition stage profit.), and

5. $\left| \frac{\partial^2 \pi_i^*}{\partial c_i \partial c_i} \right| > \left| \frac{\partial^2 \pi_i^*}{\partial c_i \partial c_j} \right|$. This standard assumption guarantees the uniqueness of the equilibrium values of c_A^* and c_B^* as well as "stability" (see Vives, 2000).

2.5 The consumer problem

We assume that only some users have access to both a legal and an illegal version, while some users have access only to a legal version. The users with access to both versions prefer

[10] The availability of an illegal version and the ability to break it differs significantly among users and is more dependent on technical skill than on the sensitivity to price θ. Uniform distribution is an analytical simplification not harming the nature of the paper

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the legal version only if the utility from it is higher and their proportion is $1-c$. The utility function of user θ is the following:

$$U_P(\theta) = \begin{cases} \theta q_i - p_i & \text{... if he buys the legal version of the software.} \\ \theta q_i - X & \text{... if he uses the software illegally.} \\ 0 & \text{... if he does not use the software at all.} \end{cases} \quad (1)$$

We also assume that if the price of the legal version of a product exactly equals the expected punishment for using the illegal one, $p_i = X$, then the consumers strictly prefer the legal version—in other words, second-order stochastic dominance applies.

Users without access to the illegal version could compare only the expected utility from purchasing the legal version and not using it at all. Their proportion is c, and the utility function of user θ is:

$$U(\theta) = \begin{cases} \theta q_i - p_i & \text{... if he buys the legal version of the software.} \\ 0 & \text{... if he does not use the software at all.} \end{cases} \quad (2)$$

2.6 The market environment

As we already noted, both developers could implement physical protection for their product, and so three basic combinations of product protection could occur on the market :

1. None of the developers implement protection. This situation arises when X does not bind in the maximization problems of either A or B so that in the equilibrium, we have $p_B^* \leq p_A^* \leq X$.

2. Developer A implements protection while developer B does not. This situation occurs when pure Bertrand equilibrium is not possible because X would be binding for developer A since $p_B^* \leq X \leq p_A^*$.

3. Both developers implement protections.[11] Finally for low X, both developers would have to introduce protection since pure Bertrand equilibrium would result in $X \leq p_B^* \leq p_A^*$.

[11] Note that the case in which only developer B implements protection never occurs. If B has to implement protection due to the low expected penalty X, then developer A must also implement physical protection because his product would be the primary target of illegal usage.

Before analyzing the above cases in more detail, we start with the monopoly case that helps us to illustrate the flavor of the model.

2.7 Monopoly

A monopoly case helps us to illustrate the flavor of the model. Consider now developer A who introduces a level of protection at c for his product q_A and sets the price p_M. In analyzing monopolist behavior, we could focus only on the case when the expected penalty is such that $X < p_M$, since the case where $X > p_M$ no user has the incentive to use an illegal version. Users' demand for the legal product of monopoly developer A is $D_A = c\left(\bar{\theta} - \frac{p_M}{q_A}\right)$ and it leads to the following market coverage:

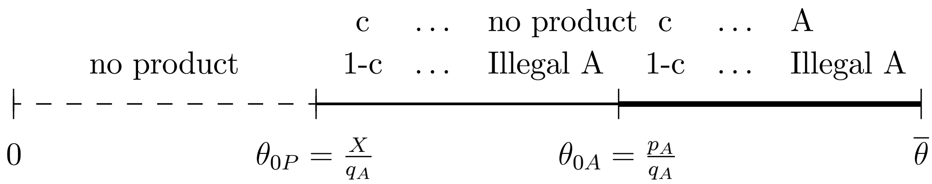

Figure 1: Monopoly market with product protection c

Monopoly equilibrium could be easily derived to yield:

$$p^*_M = \frac{1}{2}\bar{\theta}q_A, \;\; \pi^*_M = c\frac{1}{4}\bar{\theta}^2 q_A. \tag{3}$$

Note that under the assumptions regarding $h(c)$, $\Pi^*_M = \pi^*_M - h(c)$ has a unique maximum, $c^*_M \in (0,1)$. A monopoly developer A always has an option to decrease the price to X instead of implementing protection c. By comparing developer A's profit in the case of lowering the price to X with his profit after implementing protection, we find out that developer A prefers physical protection as long as the expected penalty, X, is below a certain critical level. More specifically, even with protection costs $h(c) = 0$, it is more profitable to lower the price to X instead of implementing protection if $X > \bar{\theta}q_A\frac{1-\sqrt{1-c^*_M}}{2}$.

3 Optimal pricing and private IPR protection in a Duopoly

Our core analysis focuses on the optimal pricing and private IPR protection in a duopoly as a function of the strength of copyright protection captured by the size of X. We omit the case when the expected penalty X is high enough ($p_B^o \leq p_A^o \leq X$), and developers have no incentives to introduce physical protection against copying[12]. Thus, we first focus on the case where only developer A has the incentive to introduce protection $p_B^* \leq X \leq p_A^*$ and then, finally, on the case where both developers have such incentives, that is, $X \leq p_B^* \leq p_A^*$. Note that in our set-up, prices are as typically strategic complements (see Tirole, 1989, and Bulow et al., 1985), that is, $\frac{\partial^2 \pi_i}{\partial p_B \partial p_A} > 0$. The first case ($p_B^* \leq X \leq p_A^*$) seems to be relevant for middle and, perhaps, some high per capita income countries, while the situation associated with zero or very low effective strength of copyright protection is typical in developing countries (see Fig. 1 in Varian 2005).

3.1 Only developer A implements protection c

We start with solving the model backward. Thus in the last (second) stage we analyze optimal pricing as a function of the strength of copyright protection. In the case, where $p_B^* \leq X \leq p_A^*$, only developer A has the incentive to implement physical protection since the product of developer B would only be used legally. As we already mentioned in our model set-up, the illegal version of product A is available only to the fraction $1-c$ of the user base. Product A is used illegally only by users with $\frac{X}{q_A} \leq \theta$, while users with $\theta \leq \frac{X}{q_A}$ prefer not to use the product at all. The demand for product B consists of users with low sensitivity θ to purchasing product A, who, at the same time, have no access to an illegal version of A, but their θ is high enough to buy product B. These users have $\theta \in (\frac{p_B}{q_B}, \frac{p_A - p_B}{q_A - q_B})$, and their fraction is c. As for the users with access to an illegal version of product A, there are two sub-cases that could occur in equilibrium depending on the size of the expected penalty:

1. The first sub-case occurs when there are some users who have illegal access to product A but still want to buy product B, or more formally, the measure of these users is strictly positive with $\theta \in \left(\frac{p_B}{q_B}, \frac{X - p_B}{q_A - q_B}\right)$, and so, $\frac{X - p_B}{q_A - q_B} > \frac{p_B}{q_B}$. These users would like to purchase product B if X is "large enough" (in the sense that $X > p_B \frac{q_A}{q_B}$). Looking at it from the developers' point of view, developer B competes for the consumers that have illegal

[12] The prices in the pure Bertrand equilibrium are given as follows: $p_A^o = 2\bar{\theta} q_A \frac{(q_A - q_B)}{4q_A - q_B}$, $p_B^o = \bar{\theta} q_B \frac{(q_A - q_B)}{4q_A - q_B}$.

access to software (so called "non-controlled" consumers) by aggressively charging a low price so that $p_B^* < \frac{q_B}{q_A}X$.The market coverage is given in Figure 2 .

2. The second sub-case occurs when illegal users always prefer an illegal version of A to the legal version of B, that is, when $\theta q_A - X > \theta q_B - p_B$ for all θ since illegal usage is then more profitable even for the consumer with the lowest valuation. So, X has to be "low" enough, that is, $\frac{X-p_B}{q_A-q_B} \leq \frac{p_B}{q_B}$ (or equivalently $X \leq p_B \frac{q_A}{q_B}$) given that $p_B^* \leq X$ still holds. From the perspective of the developers, developer $B's$ price is "too high" to attract the non-controlled consumers and in this situation his profit fully depends on the protection of developer A. The market coverage of this case is presented in Figure 3 .

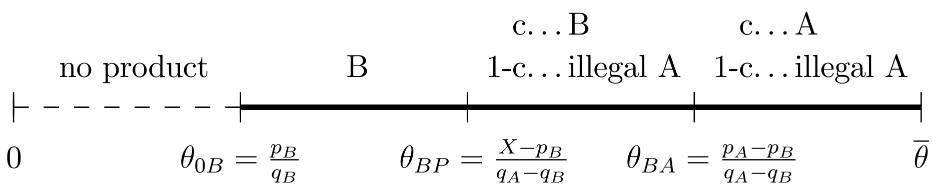

Figure 2: BC, when developer A introduces protection c (Case 1).

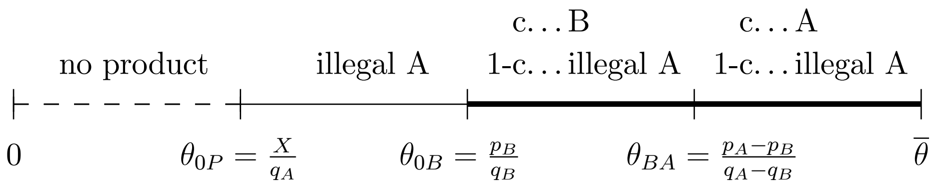

Figure 3: BC, when developer A introduces protection c (Case 2).

As for sub-case 1, we obtain demand for legal versions of both products by putting all fractions of users together:

$$D_A = c\left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right), \tag{4}$$

$$D_B = c\left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) + (1-c)\left(\frac{X - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) =$$

$$= \frac{cp_A + (1-c)X - p_B}{q_A - q_B} - \frac{p_B}{q_B}.$$

In sub-case 2, only the users without access to an illegal version of A buy product B so the demand functions are now:

$$D_A = c\left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right),$$
$$D_B = c\left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right).$$

Note that sub-case 2 is practically identical to the pure Bertrand case yielding the same equilibrium prices, and yielding the same market coverage as well as the equilibrium profits that are only sized down by factor c. Interestingly enough, the change in the strength of copyright protection does not affect (at the margin) either developers' pricing or the IPR protection strategy of developer A. The reason is that for the particular values of the strength of copyright protection, developer B does not find it optimal to compete for the illegal ("non-controlled") users of product A but instead focuses (or free rides) on the (lower segment of) users whom developer A prevents from using the software illegally by means of physical protection. So the only target of both firms is the so called "controlled" consumers who legally buy the products and whose fraction is c in both segments of the market. Thus, the equilibrium prices are, as we saw, identical to those of the pure Bertrand set-up with X having no impact on either the prices or the equilibrium IPR strategy.

So we focus on the more interesting sub-case 1. We start with determining the range of the expected penalty values X such that this sub-case is the Nash equilibrium in prices. Namely, sub-case 1 is not an equilibrium if (i) at least one developer's profit, given the other developer's price choice, does not have a local maximum in the relevant price range. Moreover, it is also not an equilibrium if (ii) there is a local maximum in the relevant range, but at least one developer is better off deviating to a price outside the range (e.g. developer A can be better off deviating to $p_A = X$). Intuitively, for developer A to charge a high price $p_A > X$, the value of X should be small enough so that developer A prefers introducing protection to simply lowering the price to X. As for developer B charging a low price $p_B < X\frac{q_B}{q_A}$, X should be large enough so that developer B prefers charging a low price to both charging an intermediate price $X\frac{q_B}{q_A} \leq p_B \leq X$ or charging a high price $p_B > X$ and introducing protection.

For (i) not to hold, we show that a necessary condition on X is $X_{cl} < X < X_{cu}$, where $X_{cl} = \frac{\bar{\theta} c q_A (q_A - q_B)}{2(1+c) q_A - c q_B}$, and $X_{cu} = 2\bar{\theta} q_A \frac{q_A - q_B}{4q_A - q_B}$; (see Appendix 2.3.4). Note that the upper bound X_{cu}, intuitively, coincides with p_A^o that is the equilibrium price in the case of the pure Bertrand equilibrium. Then both developers' profits reach the internal local maxima in the price ranges corresponding to our sub-case 1, with the prices equal to

$$p_A^* = \frac{X(1-c) q_B + 2\bar{\theta} q_A (q_A - q_B)}{4q_A - c q_B}, \; p_B^* = q_B \frac{2X(1-c) + \bar{\theta} c (q_A - q_B)}{4q_A - c q_B}. \tag{5}$$

For (ii) not to hold, we have to verify that neither developer has an incentive to unilaterally deviate given that the other developer sets the equilibrium price, p_i^*. For developer A, it can be profitable to deviate to $p_A = X$ (given that developer B sets p_B^*) if the decrease in price from p_A^* to X is more than compensated for by an increase in the number of consumers that is no longer confined to fraction c, and for X large enough, such a deviation would yield a higher profit than choosing protection, even without protection costs (that is, $h(c) = 0$). As for developer B, if p_B^* is close enough to $X \frac{q_B}{q_A}$, then it may pay off to jump to a higher price $p_B \in (X \frac{q_B}{q_A}, X)$ given that developer A sets p_A^* as in this case, the effect of such a price increase would more than offset the loss of the consumer base. The analysis in Appendix 2.3.4 shows that for an interior equilibrium to exist, X should not be "too large" for developer A, so that $X < X_c^+ < X_{cu}$, nor should it be "too small" for developer B, so that $X > X_c^- > X_{cl}$. While values X_{cl} and X_{cu} always define a non-empty range, the condition $X_c^- < X < X_c^+$ defines a non-empty set only if $c > c^o = \frac{\sqrt{5}-1}{2} \approx 0.618034$[13]. If $X \in (X_c^-, X_c^+)$, then none of the developers have an incentive to deviate, and the prices above constitute an equilibrium. We coin this equilibrium as the piracy, no "full dependence" equilibrium since developer B does not fully depend on A's protection but also competes for the non-controlled consumers. The comparative statics analysis with respect to c is straightforward in this equilibrium: equilibrium prices $p_A^*(c)$, $p_B^*(c)$ and the profit $\pi_B^*(c)$ increase as the level of physical protection c increases, so developer A acts strategically and softens the price competition and (in jargon) displays pacifistic "fat cat" behavior (see Fudenberg and Tirole, 1984).

[13]If the quality ratio is not too high, then the lower bound on c can be improved to $c > \underline{c} \approx 0.704402$. Here "not too high" means that q_B/q_A is below the threshold value, which is itself above 0.9, so we can be almost sure that this is the case and consider it as the general situation.

Now we can briefly move to the first stage of the game in which developer A chooses the optimal private protection, c^*, by maximizing his profit function: $\Pi_A^* = \pi_A^*[p_A^*(c), p_B^*(c), c] - h(c)$.This, in turn, enables us to move on our key issue of how private and public protection interact. More specifically, we study the effect of the expected penalty X on the optimal developer $A\prime s$ protection strategy, c^*.

Recall that we are primarily interested in the interaction of the expected penalty X with the developer's protection c^* rather than in the very value of optimal private protection, c^*. That is, we wish to study how the regulator's change in the level of public protection affects the optimal private IPR protection strategies (and, consequently, equilibrium prices, profits, and market coverage). In order to address this key issue, we first have to identify relevant features (like, say, the very need for private protection, character of competition for the consumers, etc.) that could appear in the above set-up and that affect possible equilibrium candidates and structures. For instance, as already mentioned, it may be optimal for developer A not to use private protection at all but set $p_A = X$ instead, when public protection is large enough. On the other hand, for X "low enough", it may be the case that developer B does not compete for the non-controlled consumers but sets $p_B > \frac{q_B}{q_A}X$.

More generally, there are three features that affect the possible equilibrium structure in the above setting: a) the need for private protection to be exercised in equilibrium b) the status of product B for non-controlled consumers, that is, whether developer B competes for them or fully depends on the developer's A IPR protection, and c) the character of the optimal solution, that is, whether the profits attained their maxima at corner or at the interior solution. Given these features, there are five possible equilibrium outcomes that may occur in the set up under considerations (see Appendix 2.3.7 for the brief descriptions of these five possible equilibrium outcomes).

Having all this in mind, we could now start to analyze the effect of public IPR protection X on the optimal IPR strategy c^*. First, recall that X affects c^* only if it affects its marginal profitability $\frac{\partial \Pi_A^*}{\partial c}$. More technically, the effect of the change in X on the choice of c^* is non-zero only when $\frac{\partial^2 \Pi_A^*}{\partial c \partial X} \neq 0$. In other words, this cross-derivative is non-zero only if the gross equilibrium profit depends on both c and X, which only holds for the two (out of the five) possible equilibrium outcomes: 1) the piracy no "full dependence" equilibrium and, 2) so called the corner "full dependence" equilibrium, that is, the equilibrium where $p_A^* > X$, while $p_B^* = X$. In this last case, all consumers not controlled by developer A use product P (or nothing) and, like in outcome 1, both X and c enter both developers' profits (see Appendix 2.3.7) .

Proposition 1 summarizes the main findings:

Proposition 1 *When there is the piracy no "full dependence" equilibrium then private and public protection are strategic substitutes, that is,* $\frac{dc^*}{dX} < 0$. *When, on the other hand, we have the corner "full dependence" equilibrium, then private and public protection are strategic complements, that is,* $\frac{dc^*}{dX} > 0$.*In all other possible equilibrium outcomes a change of the public IPR protection (at the margin) does not affect the optimal IPR strategy of the developer A, that is,* $\frac{dc^*}{dX} = 0$.

Proof. see Appendix 2.3.5 and Appendix 2.3.7 ■

Let us focus first on the piracy, no "full dependence" equilibrium where the interval (X_c^-, X_c^+) exists and $X \in (X_c^-, X_c^+)$. As we stated above, the necessary condition for interval (X_c^-, X_c^+) to be non-empty is that $c^* > c^o$, and this, in turn, implies (or is sufficient for) $\frac{\partial^2 \pi_A^*}{\partial c \partial X} < 0$. This situation is described in jargon as "strategic substitutability" between c^* and X so that $\frac{dc^*}{dX} < 0$. For c^* being "large", it must be that the protection marginal cost function is not "too steep" so that it crosses the marginal revenue at c^* such that $c^* > c^o$. Note also that developer A by his optimal response to increase in X, harms developer B (recall that $\frac{d\pi_B^*(c)}{dc} > 0$).

The nature of the interaction between the private and public IPR protection enables us to further study the comparative statics effects of X on equilibrium prices and profits.

Lemma 1 *The effect of* X *on* $p_A^*(X)$ *and* $p_B^*(X)$ *is a priori undetermined.*

Proof. Note that $\frac{dp_i}{dX}(c(X), X) = \frac{\partial p_i}{\partial c}\frac{dc}{dX} + \frac{\partial p_i}{\partial X}$. Straightforward differentiation shows that the direct effect of X on prices is positive, that is, $\frac{\partial p_i}{\partial X} > 0$. From the analysis above, we know that $\frac{\partial p_i}{\partial c} > 0$, but $\frac{dc}{dX} < 0$. Thus, the indirect effect, $\frac{\partial p_i}{\partial c}\frac{dc}{dX} < 0$. ■

Lemma 2 *The effect of X is positive on $\Pi_A^*(X)$ but the respective effect on $\pi_B^*(X)$ is a priori unclear.*

Proof. *Note that* $\frac{d\Pi_A^*(X)}{dX}(c(X), X) = \frac{\partial \Pi_A^*}{\partial X} > 0$. *Note further that* $\frac{d\pi_B^*(X)}{dX}(c(X), X) = \frac{\partial \pi_B^*(X)}{\partial c}\frac{dc^*}{dX} + \frac{\partial \pi_B^*}{\partial X}$, *where* $\frac{\partial \pi_B^*(X)}{\partial c}\frac{dc}{dX} < 0$ *since* $\frac{dc^*}{dX} < 0$ *and* $\frac{\partial \pi_B^*}{\partial c} > 0$. *Thus, the direct and indirect effects have a conflicting impact on developer B's profit.* ■

As we can see, developer A reacts aggressively on an increase in X and cuts back in his private protection in response to increased public protection. As for developer B, if the net outcome of the above two conflicting (direct and indirect) effects is negative, the profit of developer B and equilibrium prices fall making price competition tougher. As a result, a "fat cat" strategy in this case becomes a little diluted due to the enhanced public protection while, on the other hand, consumers of both goods benefit due to the decrease in equilibrium prices[14].

Finally, the second and the last equilibrium structure where X affects the optimal choice c^* is the corner "full dependence" equilibrium. It is straightforward to show that in this case

$$\Pi_A^* = \frac{c\left(\bar{\theta}\left(q_A - q_A\right) + X\right)^2}{4\left(q_A - q_B\right)} - h(c),$$

implying that $\frac{\partial^2 \Pi_A^*}{\partial c \partial X} > 0$ and hence $\frac{dc}{dX} > 0$. In this equilibrium structure, developer B chooses $p_B^* = X$, i.e., the maximum price this developer can charge without implementing protection. This situation may occur when X is sufficiently low so that it is too costly for developer B to charge a lower price, whether in the range $p_B < X\frac{q_B}{q_A}$ or in the range $X\frac{q_B}{q_A} < p_B < X$. It is straightforward to show that in this case $p_A^* = \frac{\bar{\theta}(q_A - q_B) + X}{2}$ and $p_B^* = X$.

Lemma 3 *The effect of X on $p_A^*(X)$ and $p_B^*(X)$ is positive.*

Proof. *Note that* $\frac{dp_i}{dX}(c(X), X) = \frac{\partial p_i}{\partial c}\frac{dc}{dX} + \frac{\partial p_i}{\partial X} = \frac{\partial p_i}{\partial X} > 0$ *since* $\frac{\partial p_i}{\partial c} = 0$. ■

Lemma 4 *The effect of X is positive on both $\Pi_A^*(X)$ and on $\pi_B^*(X)$.*

Proof. *Note that* $\frac{d\Pi_A^*(X)}{dX}(c(X), X) = \frac{\partial \Pi_A^*}{\partial X} > 0$. *Note further that* $\frac{d\pi_B^*(X)}{dX}(c(X), X) = \frac{\partial \pi_B^*(X)}{\partial c}\frac{dc^*}{dX} + \frac{\partial \pi_B^*}{\partial X} > 0$, *where* $\frac{\partial \pi_B^*(X)}{\partial c}\frac{dc}{dX} > 0$ *since* $\frac{dc^*}{dX} > 0$ *and* $\frac{\partial \pi_B^*}{\partial c} > 0$ *and* $\frac{\partial \pi_B^*}{\partial X} > 0$. ■

[14] It is straightforward to show that entry deterrence by means of c is not feasible in the set-up under consideration.

Unlike in the case of the piracy no full dependence equilibrium, developer A does not react aggressively here. He strengthens his private protection in response to increased public protection and developer B benefits from both this and from the enhanced public protection. Thus, the "fat cat" strategy that developer A adopts in his choice of private protection becomes reinforced by the stricter public protection.

3.2 Both developers A and B implement protection

If the regulator sets up a very low expected penalty ($X < p_B^* < p_A^*$), then, naturally, both developers have to either implement physical protection or decrease prices to X; otherwise, they would be out of the market.

We denote protection used by developer A as c_A and protection used by developer B as c_B. Furthermore, we assume that users may have access either to an illegal version of product A, an illegal version of product B, or to both illegal versions. Moreover, we assume that access to an illegal version of product A and B are mutually independent so there are users on the market that have access to illegal versions of product A but not to illegal versions of product B and vice versa. Then there are the following fractions of users on the market:

1. $c_A c_B$ ... The fraction of users with access only to legal products;
2. $c_A(1-c_B)$ ... The fraction of users with access to an illegal version of product B;
3. $(1-c_A)c_B$ ... The fraction of users with access to an illegal version of product A;
4. $(1-c_A)(1-c_B)$ ... The fraction of users with access to illegal versions of both products.

We have now the following types of users:

1. $\theta \in (\frac{p_A-p_B}{q_A-q_B}, \bar{\theta})$ ... Users who buy product A if they do not have access to any illegal version;
2. $\theta \in (\frac{p_B}{q_B}, \frac{p_A-p_B}{q_A-q_B})$ ... Users who buy product B if they do not have access to an illegal version of A;
3. $\theta \in (\frac{X}{q_A}, \bar{\theta})$ ... Users who use an illegal version of A if they have access to it;
4. $\theta \in (\frac{p_A-X}{q_A-q_B}, \bar{\theta})$ ... Users who buy A if they have access only to an illegal version of B.

Given the above set-up, it seems that two sub-cases could arise. The first one would be such that $\bar{\theta} \leq \frac{p_A-X}{q_A-q_B}$, implying that there is no user who would buy product A if he has illegal access to product B. This, however, never occurs since in the equilibrium, developer A sets the price low enough that users with θ close to $\bar{\theta}$ always prefer to buy the legal version of A (see Appendix 2.4.1). The second situation appears when $\frac{p_A - X}{q_A - q_B} < \bar{\theta}$, implying that such users exist, and their number is higher than zero. So next, we discuss this only feasible sub-case in which both competitors introduce physical protection and $\frac{p_A - X}{q_A - q_B} < \bar{\theta}$.

In this case, there are users who prefer the legal version of the higher quality product q_A even though they have access to the illegal version of product B, but not of product A. This leads to the following market coverage:

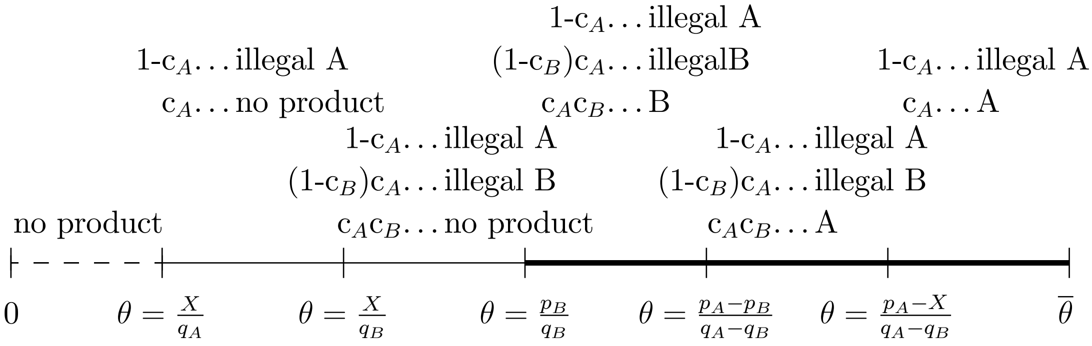

Figure 4: Both developers introduce protection, and $\frac{p_A - X}{q_A - q_B} < \bar{\theta}$

From the distribution of users on the market, we obtain the following demand for the individual products:

$$
\begin{aligned}
D_A &= c_A c_B \left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B} \right) + c_A (1 - c_B) \left(\bar{\theta} - \frac{p_A - X}{q_A - q_B} \right) \\
&= \frac{c_A \left(X(1 - c_B) + \bar{\theta}(q_A - q_B) + c_B p_B - p_A \right)}{q_A - q_B}, \\
D_B &= c_A c_B \left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B} \right).
\end{aligned} \tag{6}
$$

As in the previous section, we start with determining the range of the expected penalty values X such that this sub-case is a Nash equilibrium in prices. Recall that for the existence of a price equilibrium in the case when only developer A adopts protection, X has to be low enough from the perspective of developer A, but it has to be high enough from the viewpoint of developer B. Now in the case under consideration, there are no such opposing requirements on X, since for both developers to charge high prices (above X), they both "need" X to be low[15]. Intuitively, if X is close to zero, then both developers would implement protection and charge prices above X rather than adjust their prices to X or below. We show in Appendix 2.4.4 that a strictly positive $\underline{X} < \underline{X}_0 = \frac{\bar{\theta} q_B (q_A - q_B)}{4q_A - q_B}$ (note that $\underline{X}_0$ equals p_B^o of the pure Bertrand equilibrium) exists such that the following prices constitute an equilibrium:

$$p_A^* = 2q_A \frac{\bar{\theta}\,(q_A - q_B) + X\,(1 - c_B)}{4q_A - c_B q_B}, \quad (7)$$
$$p_B^* = \frac{\bar{\theta}\,(q_A - q_B) + X\,(1 - c_B)}{4q_A - c_B q_B} q_B.$$

As for a comparative statics analysis with respect to c_A and c_B, it is straightforward to show that equilibrium prices do not depend on c_A and increase in c_B. While the positive effect of c_B is not unexpected, the independence of the equilibrium prices on c_A might seem less intuitive. However, if both developers charge prices above X, any consumer not controlled by developer A would use an illegal version of product A, and a small change in c_A would only have a market size effect, i.e. both demands would change proportionally to the change in c_A. As there are no production costs, the change in marginal incentives will be also proportional to the change in c_A, so that the prices do not change (see the expression, 6). Alternatively, both gross profits of A and B, (that is, $p_A D_A$ and $p_B D_B$) can be re-scaled (divided) by c_A and thus both profits and, consequently, equilibrium prices become independent of c_A. Note also that both developers prefer the good protection of a competitor's product, that is $\frac{\partial \Pi_A^*}{\partial c_B} > 0$ and $\frac{\partial \Pi_B^*}{\partial c_A} > 0$. The intuition is that an increase in either c_A or c_B increases the number of legal users for both developers, as it can be seen by visual inspection that $\frac{\partial D_A}{\partial c_B} > 0$ and $\frac{\partial D_B}{\partial c_A} > 0$ and also by looking at the market coverage in Figure 4 .

Before proceeding to the central issue of our analysis—the interaction between the private and public IPR protection—we make an additional assumption that $c_B^* \leq \frac{1}{2}$. The reason

[15] Certainly, if the developers could costlessly choose X, they would set it sufficiently high as to exclude illegal use, so "need" is used in the sense of pure mathematical conditions for an equilibrium in the given range. Also note that since these mathematical conditions for both developers stipulate an upper bound, the analysis is to some extent simpler than in the case of developer A alone implementing protection as it is impossible here that the intersection of conflicting requirements on X results in an empty set.

for this might be a rather tough price competition in the vertically differentiated market. Consequently, the lower quality producer charges a substantially lower price and usually earns only a small fraction of the high-quality developer's profit in equilibrium. Thus, developer B cannot afford to expand c_B much above zero due to the increasing marginal cost of private protection (recall that $\frac{\partial^2 h}{\partial c_i^2}(c_i) > 0$). In addition, the cost function might be rather steep and that reinforces the "tough competition" argument thus yielding the "low" optimal values of c_B^*.

Proposition 2 *For* $X \in (0, \underline{X}]$. *an increase in* X *leads to an increase in the optimal protection of both developers, that is,* $\frac{dc_A^*}{dX} > 0$ *and* $\frac{dc_B^*}{dX} > 0$. *Thus, private and public IPR protections are strategic complements.*

Proof. see Appendix 2.4.6

■

The sign and the size of interaction between the public and private IPR protection, $\frac{dc_i^*}{dX}$, depends on the impact of the expected penalty, X, on the marginal profitability of both developers' private protection, or, more technically, on the signs of both $\frac{\partial^2 \pi_A^*}{\partial c_A \partial X}$ and $\frac{\partial^2 \pi_B^*}{\partial c_B \partial X}$. It turns out that $\frac{\partial^2 \pi_A^*}{\partial c_A \partial X} > 0$ for all permissible values, and $\frac{\partial^2 \pi_B}{\partial c_B \partial X} > 0$ for (at least) all values of c_B such that $c_B \leq \frac{1}{2}$ (see Appendix 2.4.6).

So, in the situation when the expected penalty is low (that is, $X \in (0, \underline{X}]$), there is strategic complementarity not only between the private and public protections but also between the two private protections that reinforce each other (recall that $\frac{\partial^2 \pi_i^*}{\partial c_A \partial c_B} > 0$). In this case, an increase in the private protection of one developer induces an increase in the optimal protection of the other developer. Thus, the situation here is rather different from piracy, no full dependence equilibrium (see section 3.1) because here an increase in X leads to an increase of both c_A and c_B causing an upward spiral in private protections until the new equilibrium is reached.

As before, the nature of the interaction between private and public IPR is the key ingredient in analyzing the comparative statics effects of X on equilibrium prices and profits.

Lemma 5 *An increase in* X *leads to a rise in both prices and profits for both developers.*

Proof. Directly from equilibrium prices (5) and from profit comparison (in a Mathematica file). ■

Note also that as both protections c_A, c_B tend to perfect protections, the equilibrium prices and profits go to profit from pure Bertrand competition.

4 Conclusion

In this article, we focus on the effect of increased copyright protection on the pricing and private IPR protection strategies of software developers. Predictably, the initial size of the expected penalty plays a decisive role in shaping the behavior of the market participants. Thus, if X is zero or small, as is typically the case in developing countries, then both developers introduce protection, and a "small" increase in X reinforces the private protection of both developers implying that the regulator's and developers' IPR protections are strategic complements. Moreover, an increase in the strength of copyright protection enables both developers to raise prices and earn larger profits. It is important to note that even for a zero or low expected punishment, it is never the case that all of the users that have access to the illegal versions would use only these illegal versions in equilibrium[16]. Thus, in an equilibrium with low X, some of the users with a high appreciation for quality who have illegal access to product B would still buy legal versions of product A. An increase in X would make product A more attractive for those users. As an optimal response, developer A would increase c_A that would in turn lead to larger profit. At the same time an increase in X would leave more room for developer B to increase his prices and profit via an increase in c_B.

For some intermediate values of X, only developer A introduces IPR protection. Unlike in the case of zero or small X, here the analysis of the pricing and strategic response to the level and change of copyright strength is more complex. We identified the two possible equilibrium structures in which public protection affects (at the margin) private IPR protection. In the first case that we focus on ("piracy no full dependence equilibrium") developer's A

[16] If this were the case, X would have no impact on the users' and consequently no impact on the developers' decisions on either c_A or c_B.

optimal reaction to the change of X is to curb his own protection implying that private and public protections are strategic substitutes. This situation occurs, roughly speaking, when X assumes the value from the mid of the relevant interval while the firm's costs of preventing piracy do not rise "very steeply" with the strength of the adopted private protection, c, so that the optimal level of this protection assumes a rather large value (exceeding a critical value of c^o) . The second equilibrium outcome is the situation when the developer B does not compete for illegal users and sets the price exactly at the level of public IPR protection (the corner full dependence equilibrium). Such a situation appears when X is low and it does not pay off for developer B to charge an even lower price. In this case the two forms of protections act as strategic complements; the change in public protection positively affects the private IPR protection of developer A.

The common feature of both equilibrium structures that occurs when only developer A adopts protection is the *fat cat strategy.* Developer A displays "friendly" behavior through strategically enlarging the controlled customer base from which developer B benefits as well (or in words of Fudenberg and Tirole, 1984, "..the large captive market makes the incumbent (developer A in our set-up) pacifistic "fat cat".. "). The remarkable difference in the two equilibria, however, is that an increase in the strength of copyright protection makes the fat cat strategy stronger for "low" values of X (the corner full dependence equilibrium) while it likely makes it weaker for X assuming intermediate values (piracy no full dependence equilibrium).

Finally, when the expected punishment is equal or exceeds a pure duopoly price of a software A, there is no need for protection by any developers, so the regulator's IPR protection is in a sense an effective full substitute for the private developers' IPR protection.

As for the possible extensions of our analysis, it might be insightful to study the regulator's strategy of setting the optimal copyright punishment. In other words, the optimal regulator's choice of IPR protection and its economic impacts would be an issue. This would, in turn, require putting "more structure" in our model and consequently specifying the regulator's objective function. Since, in our context, it was suitable to think of two foreign developers competing on a third host market, the simplest case would be that the host regulator maximizes the consumer surplus net of the costs of implementing a particular level of expected penalty. This would further mean that the regulator would prefer to induce the most competitive set-up by means of the expected penalty, given the costs of reaching a particular level of expected penalty (whereby the costs of reaching a particular level are convex, that is above proportionally increasing in it). However, in our set-up where the users have access to an illegal version of the product, the choice of an optimal expected penalty seems to be trivial; in order to maximize the consumer surplus, the regulator will simply set the expected penalty to zero (or to some minimal level if zero is not feasible due to, say, an international standard and requirements for a minimal IPR protection). Thus, the set-up in which one or both developers are the domestic ones would be surely more interesting to analyze.

APPENDIX

1 Basic Model

1.1 General notes for all appendices

Most of the calculations in this paper were performed using *Mathematica* and other similar software. The *Mathematica* file is available upon request.

In almost all model situations here, profit functions are concave (quadratic, or, in singular cases, linear) in the respective choice variables, so that an interior solution is always a (local) maximum. In the remaining situations, profit functions are explicitly assumed concave in the main text. Thus, second-order conditions always hold in equilibrium, so they are omitted everywhere below.

1.2 Indifferent users

From the user utility function it follows that indifferent users are characterized by the following quality sensitivities. The notation θ_{YZ}, where Y and Z can be one of $\{0, A, B\}$ implies that the users with $\theta < \theta_{YZ}$ strictly prefer Y to Z, and the users with $\theta > \theta_{YZ}$ strictly prefer Z to Y. Then

$$\theta_{0A} = \frac{p_A}{q_A},\ \theta_{0B} = \frac{p_B}{q_B},\ \theta_{BA} = \frac{p_A - p_B}{q_A - q_B}.$$

For the situations wherein developer B competes with either developer A's product priced at X or the illegal version thereof, also priced at X, we use the threshold $\theta_{BP} = \frac{X - p_B}{q_A - q_B}$.

1.3 Bertrand competition

1.3.1 Pure Bertrand competition

Profit functions are $\pi_A = \left(\bar{\theta} - \theta_{BA}\right) p_A$, and $\pi_B = \left(\theta_{BA} - \theta_{0B}\right) p_B$, and from F.O.C., it follows that

$$p_A^o = 2\bar{\theta} q_A \frac{(q_A - q_B)}{4q_A - q_B},\ \ p_B^o = \bar{\theta} q_B \frac{(q_A - q_B)}{4q_A - q_B},$$

so that the equilibrium profits are

$$\pi_A^o = 4\bar{\theta}^2 q_A^2 \frac{q_A - q_B}{(4q_A - q_B)^2},\ \pi_B^o = \bar{\theta}^2 q_A q_B \frac{q_A - q_B}{(4q_A - q_B)^2}.$$

1.3.2 Bertrand competition, where only developer B makes profit

The profit function of developer B is $\pi_B = (\theta_{BP} - \theta_{0B})\, p_B$, so that

$$p_B^* = \frac{q_B}{2q_A} X,\ \pi_B^* = X^2 \frac{q_B}{4q_A (q_A - q_B)}. \tag{8}$$

1.3.3 Bertrand competition with binding price p_A equal to X

Developer A is limited to setting the price $p_A^* = X$. Thus, the profit functions are $\pi_A = \left(\bar{\theta} - \theta_{BP}\right) X$, and $\pi_B = (\theta_{BP} - \theta_{0B})\, p_B$, so that p_B^*, π_B^* are the same as in (8), and

$$\pi_A^* = X \frac{2\bar{\theta} q_A (q_A - q_B) - X (2q_A - q_B)}{2q_A (q_A - q_B)}.$$

2 Developers implement physical protection

2.1 Indifferent users

As usual, the notation θ_{YZ}, where Y and Z can be one of $\{0, A, P, B, I\}$ implies that the users with $\theta < \theta_{YZ}$ strictly prefer Y to Z, and the users with $\theta > \theta_{YZ}$ strictly prefer Z to Y. Throughout this appendix, "product P" refers to the illegal version of product A, and "product I" refers to the illegal version of product B.

As in the basic model, for thresholds not involving the illegal products,

$$\theta_{0A} = \frac{p_A}{q_A},\ \theta_{0B} = \frac{p_B}{q_B},\ \theta_{BA} = \frac{p_A - p_B}{q_A - q_B}.$$

For thresholds involving product P, note that all consumers prefer P to I, and the decision between P and A is made on the basis of prices alone. The remaining thresholds are

$$\theta_{0P} = \frac{X}{q_A},\ \theta_{BP} = \frac{X - p_B}{q_A - q_B}.$$

For thresholds involving product I, note that the decision between I and B is made on the basis of prices alone. The remaining thresholds are

$$\theta_{0I} = \frac{X}{q_B},\ \theta_{IA} = \frac{p_A - X}{q_A - q_B}.$$

Also recall that the illegal products are available only to the fractions of consumers not controlled by the corresponding firms.

2.1.1 The price-quality ratio rule

The following general result can be easily shown to hold.

Lemma 6 *If there is a good of quality q_A available at price p_A and a good of quality $q_B < q_A$ available at price p_B, then a necessary condition exists for consumers to buy good B, namely the price per unit of quality is strictly lower for the lower quality good, i.e., $\frac{p_B}{q_B} < \frac{p_A}{q_A}$.*

Proof. The claim directly follows from $\theta_{BA} - \theta_{0B} > 0$. ∎

This result was implicitly used in previous chapters, and the equilibrium prices complied with it. However, in this chapter, profit functions are not unimodal, and an analysis of deviations requires the Lemma above explicitly.

Corollary 1 *No consumer with access to P prefers B to P if $p_B \geq X\frac{q_B}{q_A}$.*

Corollary 2 *No consumer with access to I prefers I to A if $p_A \leq X\frac{q_A}{q_B}$.*

2.2 Duopoly: general notes

Recall that the physical protection settings imply that every consumer is controlled by firm A with probability c_A, and independently by firm B with probability c_B. Thus, four groups of consumers exist. (In all cases, it is assumed that $\bar{\theta}$ is high enough.)

1. Consumers controlled by both firms, $c_A c_B$

   These consumers view the market as a standard duopoly, so that the following applies according to the price-quality ratio rule:

   (a) If $\frac{p_B}{q_B} < \frac{p_A}{q_A}$, then the consumers with $\theta < \theta_{0B}$ use nothing, those with $\theta_{0B} < \theta < \theta_{BA}$ buy product B, and those with $\theta_{BA} < \theta < \bar{\theta}$ buy product A.

   (b) If $\frac{p_B}{q_B} \geq \frac{p_A}{q_A}$, then the consumers with $\theta < \theta_{0A}$ use nothing, and those with $\theta_{0A} < \theta < \bar{\theta}$ buy product A.

2. Consumers controlled by firm A alone, $c_A(1-c_B)$

   If $p_B \leq X$, then product I is irrelevant, and the outcome is a standard duopoly as in group 1. If $p_B > X$, then these consumers choose between A and I so that the following applies:

   (a) If $p_A > X\frac{q_A}{q_B}$, then the consumers with $\theta < \theta_{0I}$ use nothing, those with $\theta_{0I} < \theta < \theta_{IA}$ use product I, and those with $\theta_{IA} < \theta < \bar{\theta}$ buy product A.

   (b) If $p_A \leq X\frac{q_A}{q_B}$, then the consumers with $\theta < \theta_{0A}$ use nothing, and those with $\theta_{0A} < \theta < \bar{\theta}$ buy product A.

3. Consumers controlled by firm B alone, $(1-c_A)c_B$

   If $p_A \leq X$, then product P is irrelevant, and the outcome is a standard duopoly as in group 1. If $p_A > X$, then these consumers choose between P and B so that the following applies:

   (a) If $p_B < X\frac{q_B}{q_A}$, then the consumers with $\theta < \theta_{0B}$ use nothing, those with $\theta_{0B} < \theta < \theta_{BP}$ buy product B, and those with $\theta_{BP} < \theta < \bar{\theta}$ use product P.

   (b) If $p_B \geq X\frac{q_B}{q_A}$, then the consumers with $\theta < \theta_{0P}$ use nothing, and those with $\theta_{0P} < \theta < \bar{\theta}$ use product P.

4. Consumers controlled by neither firm, $(1-c_A)(1-c_B)$

   The outcome in this group is the same as in group 3 due to the price-quality ratio rule. Namely, all consumers not controlled by firm A have access to a good of quality q_A at a price of no more than X. Then no such consumer will be interested in a product of quality q_B if offered at a price above $X\frac{q_B}{q_A} < X$, so it is irrelevant whether these consumers are controlled by firm B.

Thus, the last two groups can be united into a single group of those not controlled by A, with the total measure of $1-c_A$. Also note that if $p_A \leq X$, then the outcome is that of a standard duopoly as both illegal products are dominated by product A.

Note that in this model, the duopoly is always viable in the sense that the low-quality developer can always set a price such that the demand for B is strictly positive, e.g., $p_B = \frac{\min\{p_A, X\} q_B}{2q_A}$. Therefore, situations such that developer B is out of the market, e.g., $p_B \geq p_A$, can be neglected except in reaction functions.

From the above, it follows that every consumer depending on the firms controlling and the relative position of the prices w.r.t. X, faces one of the following three situations.

- Case I: a standard duopoly, the choice between A at p_A and B at p_B.
- Case II: the choice between P at X and B at p_B.
- Case III: the choice between A at p_A and I at X.

The correspondence between these three cases, the consumer groups, and price settings, is the following ($p_B < p_A$ assumed).

	$p_A \leq X$	$p_B \leq X < p_A$	$X < p_B$
$c_A c_B$	I	I	I
$c_A(1-c_B)$	I	I	III
$1-c_A$	I	II	II

Note that the situation faced by a consumer not controlled by developer A is solely determined by whether $p_A \leq X$ or not. When $p_B < p_A \leq X$, then no consumer uses any illegal product so that the standard duopoly applies regardless of control by either developer. When $p_A > X$, then all consumers not controlled by developer A have access to P at price X, and, therefore, such consumers will not use I regardless of control by developer B. Thus, situation II *de facto* applies to such consumers even if $p_B > X$, in which case all consumers not controlled by developer A will use P, which is consistent with situation II.

The approach to equilibrium verification is the following. First, the reaction functions are investigated, where it is assumed that the other developer's price satisfies the given constraints, and then it is checked whether it is optimal for this developer to charge a price in the relevant range. Second, equilibrium prices are computed from the corresponding first-order conditions, and constraints on parameters are finalized. This approach is necessary as the profit functions feature discontinuity and non-unimodality.

2.3 Bertrand competition where only A implements protection $c_A = c$

As stated in Chapter 4, we are primarily interested in the sub-case $p_B < X\frac{q_B}{q_A}$, $X < p_A$.

2.3.1 Reaction function of developer A

Let $p_B < X\frac{q_B}{q_A}$. Then developer A's demand function is described by the following.

1. Case (D): If $X < p_A \leq p_B + \bar{\theta}(q_A - q_B)$, then the situation that we focus on in the main text takes place,

$$D_A = c\left(\bar{\theta} - \theta_{BA}\right).$$

2. Case (d): If $p_B\frac{q_A}{q_B} < p_A \leq X$, then the outcome is that of an unconstrained duopoly,

$$D_A = \bar{\theta} - \theta_{BA}.$$

3. Case (m): If $p_A \leq p_B\frac{q_A}{q_B}$, then developer A is unconstrained,

$$D_A = \bar{\theta} - \theta_{0A}.$$

Given the range of p_B, this demand function is continuous between cases (d) and (m) but not at $p_A = X$ unless $c = 1$. The resulting profit function $\pi_A = p_A D_A$ is unimodal between (d) and (m), and is discontinuous at $p_A = X$.

An interior solution in case (D) can occur only if

$$X < X_d = \frac{\bar{\theta}(q_A - q_B)\, q_A}{2q_A - q_B}.$$

(Note, however, that X_d is always larger than the pure Bertrand duopoly price, that is $X_d > p_A^o = X_{cu}$.)

In this case, the reaction function and the corresponding profit are given by

$$r_A(p_B) = \frac{\bar{\theta}(q_A - q_B) + p_B}{2}, \quad \pi_A(p_B) = \frac{c\left(\bar{\theta}(q_A - q_B) + p_B\right)^2}{4(q_A - q_B)},$$

and an interior solution in (D) implies here that the maximum outside (D) is reached at $p_A = X$. Therefore, the profit above has to be compared with the profit in case (d), which equals

$$\pi_A^d = X\left(\bar{\theta} - \frac{X - p_B}{q_A - q_B}\right).$$

While it is possible to make a direct comparison between $\pi_A(p_B)$ and π_A^d and obtain the conditions such that there is no deviation to (m), the calculation of it would be rather cumbersome, so we postpone it to the equilibrium analysis. However, it is immediately clear that the protection duopoly profit is higher at $X = 0$ unless $c = 0$.

2.3.2 Reaction function of developer B

Let $X < p_A$. Then developer B's demand function is described by the following.

1. Case (X): If $X\frac{q_B}{q_A} \leq p_B < X$, then no user not controlled by A buys B as all such users prefer P,

$$D_B = c\left(\theta_{BA} - \theta_{0B}\right).$$

2. Case (D): If $p_B < X\frac{q_B}{q_A}$, then the situation that we focus on in the main text takes place,

$$D_B = c\left(\theta_{BA} - \theta_{0B}\right) + (1 - c)\left(\theta_{BP} - \theta_{0B}\right).$$

Strictly speaking, this analysis should include situation $p_B < p_A - \bar{\theta}(q_A - q_B)$, but in equilibrium $p_A < \bar{\theta}(q_A - q_B)$, so this can be neglected.

This demand function is continuous; however, the resulting profit function $\pi_B = p_B D_B$ is generally non-unimodal between (X) and (D).

An interior solution in case (D) occurs if $p_A < \left(1 + \frac{1}{c}\right) X$, in which case the reaction function and the corresponding profit are given by

$$r_B\left(p_A\right) = \frac{q_B}{2q_A}\left(cp_A + (1 - c)X\right), \; \pi_B\left(p_A\right) = \frac{q_B\left(cp_A + (1 - c)X\right)^2}{4q_A\left(q_A - q_B\right)}.$$

However, in (X), where the reaction function is the pure Bertrand reaction function $r_B\left(p_A\right) = \frac{q_B}{2q_A}p_A$, the condition $X\frac{q_B}{q_A} \leq p_B < X$ means that an interior maximum occurs if $2X < p_A <$

$2\frac{q_A}{q_B}X$, so that π_B is not unimodal around $p_B = X\frac{q_B}{q_A}$ if $2X < p_A < \left(1+\frac{1}{c}\right)X$. If the constraint $p_B \leq X$ is neglected, then the global maximum of π_B is attained in (D) when $p_A \leq \left(1+\frac{1}{\sqrt{c}}\right)X$. Then it can be shown that if $\left(1+\frac{1}{\sqrt{c}}\right)X \leq 2\frac{q_A}{q_B}X$, i.e., if $c \geq \left(\frac{q_B}{2q_A-q_B}\right)^2$, then the condition $p_A \leq \left(1+\frac{1}{\sqrt{c}}\right)X$ for the global maximum in (D) is both necessary and sufficient. If $c < \left(\frac{q_B}{2q_A-q_B}\right)^2$, then the global maximum occurs in (D) for $p_A \leq \bar{p}_A^D$, where $\left(1+\frac{1}{\sqrt{c}}\right)X < \bar{p}_A^D < \left(1+\frac{1}{c}\right)X$ and

$$\pi_B\left(\bar{p}_A^D\right) = \pi_B^X\left(\bar{p}_A^D\right) = cX\left(\frac{\bar{p}_A^D - X}{q_A - q_B} - \frac{X}{q_B}\right),$$

which is the profit from deviation to $p_B = X$.

2.3.3 Equilibrium calculation

Assuming that all conditions on the prices hold, the equilibrium prices and profits are the following.

$$\begin{aligned}
p_A^* &= \frac{2\bar{\theta}q_A\left(q_A - q_B\right) + X\left(1-c\right)q_B}{4q_A - cq_B}, \\
p_B^* &= q_B\frac{2X\left(1-c\right) + \bar{\theta}c\left(q_A - q_B\right)}{4q_A - cq_B}, \\
\pi_A^* &= c\frac{\left(2\bar{\theta}q_A\left(q_A - q_B\right) + q_BX(1-c)\right)^2}{\left(4q_A - q_Bc\right)^2\left(q_A - q_B\right)}, \text{ and} \\
\pi_B^* &= q_Aq_B\frac{\left(2X\left(1-c\right) + \bar{\theta}c\left(q_A - q_B\right)\right)^2}{\left(4q_A - q_Bc\right)^2\left(q_A - q_B\right)}.
\end{aligned}$$

2.3.4 Derivation of bounds on X and c

All conditions for these prices and profits to be interior local maxima are met if

$$c\frac{\bar{\theta}q_A(q_A - q_B)}{2(1+c)q_A - cq_B} = X_{cl} < X < X_{cu} = 2\frac{\bar{\theta}q_A(q_A - q_B)}{4q_A - q_B},$$

where $X < X_{cu}$ follows from $p_A^* > X$, and $X > X_{cl}$ follows from $p_B^* < X\frac{q_B}{q_A}$, with the latter equivalent to $p_A^* < X\left(1+\frac{1}{c}\right)$. (Note that $X_{cl} < X_{cu}$.) It remains to be checked whether these maxima are global, i.e. that no developer prefers switching to a price corresponding to another market structure.

Developer A can be shown *not* to switch to $p_A = X$ given $p_B = p_B^*$ if

$$X \leq X_c^+ = \frac{2\bar{\theta}q_A\left(q_A - q_B\right)\left(4q_A - c(2-c)q_B - \sqrt{1-c}\left(4q_A - cq_B\right)\right)}{16q_A^2 - 8q_Aq_B + \left(3c - 3c^2 + c^3\right)q_B^2},$$

which is smaller than X_{cu} when $c < 1$. It turns out that $X_{cl} \lesseqgtr X_c^+$ iff $c \gtreqless c^o = \frac{\sqrt{5}-1}{2} \approx 0.618034$, i.e., the (sub)case in question cannot occur if $c \leq \frac{\sqrt{5}-1}{2}$.

As for developer B, cases $c \geq \left(\frac{q_B}{2q_A - q_B}\right)^2$ and $c < \left(\frac{q_B}{2q_A - q_B}\right)^2$ are distinguished. In the former case, the condition to check is $p_A^* \leq X\left(1 + \frac{1}{\sqrt{c}}\right)$, which is equivalent to

$$X \geq X_c^- = 2\frac{\sqrt{c}\bar{\theta}q_A(q_A - q_B)}{(1+\sqrt{c})(4q_A - \sqrt{c}q_B)},$$

which is bigger than X_{cl} when $c < 1$. It can be shown that $X_c^- \lesseqgtr X_c^+$ iff $c \gtreqless \underline{c}$, where

$$\underline{c} = \frac{1}{3}\left(4 - 8\left(6\sqrt{33} - 26\right)^{-1/3} + \left(6\sqrt{33} - 26\right)^{1/3}\right) \approx 0.704402,$$

so the lower bound on c can be improved to $\underline{c}$ when $c \geq \left(\frac{q_B}{2q_A - q_B}\right)^2$. In the other case, $c < \left(\frac{q_B}{2q_A - q_B}\right)^2$, a direct comparison between π_B^* and $\pi_B^X\left(p_A^*\right)$ yields a lower bound on X located between X_{cl} and X_c^-, which translates into a lower bound on c located between $\frac{\sqrt{5}-1}{2}$ and $\underline{c}$. Note that given the lower bounds on c, case $c \geq \left(\frac{q_B}{2q_A - q_B}\right)^2$ occurs with certainty if $\frac{q_B}{q_A}$ is not too high, namely, if $\frac{q_B}{q_A} \leq\approx 0.912622$.

2.3.5 The effect of X on c

By the implicit function theorem,

$$\frac{dc}{dX} = -\frac{\frac{\partial^2\Pi_A}{\partial c\partial X}}{\frac{\partial^2\Pi_A}{\partial c\partial c}},$$

so that the sign of $\frac{dc}{dX}$ is the same as the sign of:

$$\frac{\partial^2\Pi_A^*}{\partial c\partial X} = 2q_B\frac{2\bar{\theta}q_A\left(q_A - q_B\right)\left(4q_A + cq_B - 8cq_A\right) + Xq_B\left(1-c\right)\left((4-12c)q_A + (c+c^2)q_B\right)}{\left(q_A - q_B\right)\left(4q_A - cq_B\right)^3}.$$

The sign of this expression depends on the sign of $(4q_A + cq_B - 8cq_A)$ and $((4-12c)q_A + (c+c^2)q_B)$. As $q_B < q_A$, both of these expressions can be shown to be negative for $c \geq \frac{4}{7} \approx 0.571429$. Since it is shown above that the sub-case in question can occur only if $c \geq \frac{\sqrt{5}-1}{2} > \frac{4}{7}$, both $\frac{\partial^2\Pi_A^*}{\partial c\partial X}$ and $\frac{dc}{dX}$ are negative.

2.3.6 The impact of X on prices and profits

First observe that $\frac{d\Pi_A^*}{dX}$ is clearly positive since $\frac{\partial \Pi_A^*}{\partial c} = 0$ at the point of optimum. Thus,

$$\frac{d\Pi_A^*}{dX} = \frac{\partial \Pi_A^*}{\partial c}\frac{dc}{dX} + \frac{\partial \Pi_A^*}{\partial X} = \frac{\partial \Pi_A^*}{\partial X} > 0.$$

In the case of developer B, the impact of X on developer B's profit is

$$\frac{d\Pi_B^*}{dX} = \frac{\partial \Pi_B^*}{\partial c}\frac{dc}{dX} + \frac{\partial \Pi_B^*}{\partial X}.$$

Since the indirect effect is negative and the direct one is positive, it cannot be told *a priori* which effect dominates. The same applies to both equilibrium prices.

2.3.7 The possible equilibrium structures when only the high-quality developer considers protection (that is, $c_A = c$ and $c_B = 0$)

The following equilibrium structures can occur for different values of qualities, c, and X. Note that in any equilibrium both legal goods have a positive market share (as stated above, developer B can guarantee a positive market share by setting $p_B = \frac{\min\{p_A, X\} q_B}{2q_A}$; as for developer A, $p_A = \min\{p_B, X\}/2$ does so), which also means that $p_B^* \leq \min\{p_A^*, X\}$ in any equilibrium with $c_A = c$ and $c_B = 0$. There are three properties in which the equilibrium structures differ:

- *Need for protection*: the issue here is whether there is physical protection at the equilibrium prices at all. Namely, if $p_A^* \leq X$, then no consumer uses any illegal product (recall that we assume that if a price of a legal product equals X, then the legal product is strictly preferred) so there is no need to implement protection, whereas if $p_A^* > X$, then developer A has to implement protection.

- *Status of product B for non-controlled consumers*: The question here is whether developer B competes for such consumers (this matters when $p_A^* > X$, otherwise all consumers are *de facto* controlled). While it is clear that controlled consumers choose between A and B, non-controlled consumers have access to P at price X. Then when $p_B^* < X\frac{q_B}{q_A}$, it means that developer B chooses to compete for non-controlled consumers, whereas $p_B^* > X\frac{q_B}{q_A}$ means that developer B "fully depends" on developer A's protection by ignoring non-controlled consumers completely. Note that $p_B^* = X\frac{q_B}{q_A}$ cannot occur as shown above in the analysis of the reaction function of developer B for $p_A > X$, which applies in this case.

- *Interior or corner solution*: The point here is where the developers' profits attain their maxima. Here two kinds of corner solutions in equilibria are possible: $p_A^* = X$ and $p_B^* = X$ (or neither, but not both), as the only other hypothetical threshold $p_B = X\frac{q_B}{q_A}$ cannot occur.

Thus, the following equilibrium structures are possible in the above case:

1. Unconstrained duopoly: $p_A^* < X$, which also implies an interior solution for developer A. Then protection is not needed, developer B's profit maximum is also interior, and the outcome coincides with that of the pure Bertrand duopoly.

2. Constrained duopoly: $p_A^* = X$, with a corner solution for developer A. Then protection is not needed, developer B's profit maximum is interior, and the outcome coincides with that of the constrained Bertrand duopoly with $p_A^* = X$.

3. Piracy, no "full dependence": $p_A^* > X$, $p_B^* < X\frac{q_B}{q_A}$. This is the case we focus on in our analysis.

4. Piracy, interior "full dependence": $p_A^* > X$, $X\frac{q_B}{q_A} < p_B^* \leq X$. Then all consumers not controlled by developer A use product P (or nothing), and the equilibrium prices coincide with those of the pure Bertrand duopoly. However, the protection level c now enters both developers' profits.

5. Piracy, corner "full dependence": $p_A^* > X$, $p_B^* = X$. Then all consumers not controlled by developer A use product P (or nothing), and the equilibrium prices are given by $p_B^* = X$ and $p_A^* = \frac{\bar{\theta}(q_A - q_B) + X}{2}$. Here both X and c enter both developers' profits.

Note that due to non-continuity and non-unimodality of the profit functions, there are parameter constellations such that more than one equilibrium type can occur.

As for the effect of changes in X on the choice of c, recall that the sign of $\frac{dc}{dX}$ coincides with that of $\frac{\partial^2 \Pi_A^*}{\partial c \partial X}$. This cross-derivative is non-zero only if the gross equilibrium profit depends on both c and X, which only holds for piracy equilibria with no "full dependence" or corner "full dependence" (equilibrium structures 3 and 5 listed above). If the equilibrium prices coincide with those of the pure Bertrand duopoly, then Π_A^* does not depend on X (at all), and in the constrained duopoly outcome, the gross equilibrium profit depends on X but not on c. Thus, $\frac{dc}{dX} = 0$ if the equilibrium structure is non-piracy duopoly or piracy with interior "full dependence". For piracy with no "full dependence," we have shown above that $\frac{dc}{dX} < 0$. As for piracy with corner "full dependence", it is straightforward to show that

$$\Pi_A^* = \frac{c\left(\bar{\theta}\left(q_A - q_B\right) + X\right)^2}{4\left(q_A - q_B\right)} - h\left(c\right),$$

which means $\frac{\partial^2 \Pi_A^*}{\partial c \partial X} > 0$ and hence $\frac{dc}{dX} > 0$.

2.4 Bertrand competition where both developers implement protection

As stated in Chapter 4, this case occurs if $X < p_B < p_A$.

2.4.1 The non-existence of sub-case $p_A \geq X + \bar{\theta}(q_A - q_B)$

In this sub-case, only the users controlled by both developers buy any legal products, so that the demands for the products are constant multiples of the standard duopoly demands, $D_A = c_A c_B \left(\bar{\theta} - \theta_{BA}\right)$ and $D_B = c_A c_B \left(\theta_{BA} - \theta_{0B}\right)$. Therefore, if the solution is interior, then the equilibrium prices are identical to the standard duopoly equilibrium prices. In particular,

$$p_A^* = 2\bar{\theta} q_A \frac{q_A - q_B}{4q_A - q_B} < \bar{\theta}(q_A - q_B) \leq X + \bar{\theta}(q_A - q_B),$$

which is a contradiction. Hence, the solution must be corner with $\frac{\partial \pi_A}{\partial p_A} < 0$ at $p_A = X + \bar{\theta}(q_A - q_B) + 0$. However, it can be shown that this implies $\frac{\partial \pi_A}{\partial p_A} < 0$ at $p_A = X + \bar{\theta}(q_A - q_B) - 0$ as well for $p_B \leq X + \bar{\theta}(q_A - q_B)$ (see the analysis of the profit and reaction functions below), so that $p_A \geq X + \bar{\theta}(q_A - q_B)$ is never optimal.

2.4.2 The reaction function of developer A

Let $X < p_B < \frac{q_B}{q_A}\left(X + \bar{\theta}\left(q_A - q_B\right)\right)$. (The upper limit on p_B here follows from $p_A < X + \bar{\theta}(q_A - q_B)$ and the price-quality ratio rule.) Then developer A's demand function is described by the following.

1. Case (d): If $p_A \geq X + \bar{\theta}\left(q_A - q_B\right)$, then all users of product A are completely controlled,
$$D_A = c_A c_B \left(\bar{\theta} - \theta_{BA}\right).$$

2. Case (D): If $p_B \frac{q_A}{q_B} < p_A < X + \bar{\theta}\left(q_A - q_B\right)$, then the situation that we focus on in the main text takes place,
$$D_A = c_A c_B \left(\bar{\theta} - \theta_{BA}\right) + c_A \left(1 - c_B\right)\left(\bar{\theta} - \theta_{IA}\right).$$

3. Case (I): If $X \frac{q_A}{q_B} < p_A \leq p_B \frac{q_A}{q_B}$, then no one uses B,
$$D_A = c_A c_B \left(\bar{\theta} - \theta_{0A}\right) + c_A \left(1 - c_B\right)\left(\bar{\theta} - \theta_{IA}\right).$$

4. Case (M): If $X < p_A \leq X \frac{q_A}{q_B}$, then no one uses B or I,
$$D_A = c_A \left(\bar{\theta} - \theta_{0A}\right).$$

5. Case (m): if $X \geq p_A$, then developer A is unconstrained,
$$D_A = \left(\bar{\theta} - \theta_{0A}\right).$$

Given the range of p_B, this demand function is continuous between cases (d) and (M) but not at $p_A = X$ unless $c_A = 1$. The resulting profit function $\pi_A = p_A D_A$ is strictly decreasing in p_A in (d), unimodal between (d) and (M), and is discontinuous at $p_A = X$.

Denote $X^A = X\left(1 - c_B\right) + \bar{\theta}\left(q_A - q_B\right)$. For cases (d), (D), (I), and (M), an interior solution in case (D) can occur only if

$$X < X_D = \frac{\bar{\theta}\left(q_A - q_B\right) q_B}{2q_A - q_B},\ X < p_B < p_B^D = \frac{q_B}{2q_A - c_B q_B} X^A.$$

In this case, the reaction function and the corresponding profit are given by

$$r_A(p_B) = \frac{X^A + c_B p_B}{2},\ \pi_A(p_B) = \frac{c_A\left(X^A + c_B p_B\right)^2}{4(q_A - q_B)}.$$

Now these values have to be compared with the monopoly profit in case (m). Since $X < X_D$ implies $X < \frac{\bar{\theta} q_B}{2}$ in the relevant case, the monopoly profit is maximized at the highest p_A in the range, i.e.,

$$\pi_A^m = X\left(\bar{\theta} - \frac{X}{q_A}\right).$$

While it is possible to make a direct comparison between $\pi_A(p_B)$ and π_A^m and obtain the maximal value $\bar{X}(p_B)$ such that there is no deviation to (m), the result is rather cumbersome. However, it is immediately clear that the duopoly profit is higher at $X = 0$.

2.4.3 The reaction function of developer B

Let $X\frac{q_A}{q_B} < p_A < X + \bar{\theta}(q_A - q_B)$. Then developer B's demand function is described by the following.

1. Case (D): If $X < p_B < p_A\frac{q_B}{q_A}$, then the situation that we focus on in the main text takes place,

$$D_B = c_A c_B(\theta_{BA} - \theta_{0B}).$$

2. Case (X): If $X\frac{q_B}{q_A} \leq p_B \leq X$, then no one uses I,

$$D_B = c_A(\theta_{BA} - \theta_{0B}).$$

3. Case (x): If $p_B < X\frac{q_B}{q_A}$, then there are consumers who prefer B to P (cf. the case when only A implements protection),

$$D_B = c_A(\theta_{BA} - \theta_{0B}) + (1 - c_A)(\theta_{BP} - \theta_{0B}).$$

Strictly speaking, this analysis should include situations $p_B < p_A - \bar{\theta}(q_A - q_B)$ and even $p_B < X - \bar{\theta}(q_A - q_B)$, but in equilibrium $X < p_A < \bar{\theta}(q_A - q_B)$, so these can be neglected.

This demand function is continuous between cases (X) and (x) but not at $p_B = X$ unless $c_B = 1$. The resulting profit function $\pi_B = p_B D_B$ is discontinuous at $p_B = X$ and can be non-unimodal between (X) and (x).

An interior solution in case (D) can occur only if $X < X_D$ (same as for developer A), in which case the reaction function and the corresponding profit have the same form as under a standard duopoly and are given by

$$r_B(p_A) = \frac{q_B}{q_A}\frac{p_A}{2},\ \pi_B(p_A) = c_A c_B \frac{p_A^2 q_B}{4q_A(q_A - q_B)}.$$

If the maximum in (D) is interior, then the maximum in (X) must be corner and the profit in (X) is maximized at $p_B = X$, i.e.,

$$\pi_B^X = c_A X\left(\frac{p_A q_B - X q_A}{(q_A - q_B)q_B}\right).$$

As for (x), the maximum is interior there if $p_A < X\left(1 + \frac{1}{c_A}\right)$, then $\pi_B^x = \frac{q_B(c_A(p_A - X) + X)^2}{4q_A(q_A - q_B)}$. It can be shown that if $p_A < X\left(1 + \frac{1}{c_A}\right)$ and $c_A > c_B$, then deviation to (x) from (D) is always profitable (note that deviation to (X) can be even more profitable). If $p_A \geq X\left(1 + \frac{1}{c_A}\right)$, then π_B strictly increases in p_B in (x), so that the maximal deviation profit is π_B^X above.

2.4.4 Equilibrium calculation

Assuming that all conditions on the prices hold, the equilibrium prices and profits are the following.

$$\begin{aligned}
p_A^* &= 2q_A \frac{\bar{\theta}(q_A - q_B) + X(1 - c_B)}{4q_A - c_B q_B},\\
p_B^* &= \frac{\bar{\theta}(q_A - q_B) + X(1 - c_B)}{4q_A - c_B q_B} q_B,\\
\pi_A^* &= 4c_A q_A^2 \frac{\left(\bar{\theta}(q_A - q_B) + X(1 - c_B)\right)^2}{(4q_A - q_B c_B)^2 (q_A - q_B)},\ \text{and}\\
\pi_B^* &= c_A c_B q_A q_B \frac{\left(\bar{\theta}(q_A - q_B) + X(1 - c_B)\right)^2}{(4q_A - q_B c_B)^2 (q_A - q_B)}.
\end{aligned}$$

All conditions for these prices and profits to be interior local maxima are met if

$$X < \underline{X}_0 = \frac{\bar{\theta} q_B (q_A - q_B)}{4q_A - q_B}.$$

It remains to check whether these maxima are global, i.e. that no developer prefers switching to a price corresponding to another market structure. As developer B will always switch to a price below $X\frac{q_B}{q_A}$ if $p_A < X\left(1+\frac{1}{c_A}\right)$ and $c_A > c_B$, a necessary condition for no such deviation at $p_A = p_A^*$ is

$$X < \frac{2c_A q_A (q_A - q_B)\bar{\theta}}{2(2 + c_A + c_A c_B) q_A - (1 + c_A) c_B q_B},$$

which is below $\underline{X}_0$ when $c_A < \frac{q_B}{2q_A - q_B}$.

As for deviations to $p = X$ by either developer, let $\delta_A(X) = \pi_A^*(X) - \pi_A^m(X)$ and $\delta_B(X) = \pi_B^*(X) - \pi_B^X(X)$ be the differences between the duopoly and deviation profits. The functions $\delta_i(X)$ are positive at $X = 0$ and decreasing in X for $0 < X < \underline{X}_0$. If c_A is high enough, then it is possible that developer A does not switch for all applicable X; however, developer B always switches at $X = \underline{X}_0$, i.e. $\delta_B\left(\underline{X}_0\right) < 0$. From this, it follows that $\exists \underline{X}$, $0 < \underline{X} < \underline{X}_0$, such that the prices and profits above form an equilibrium.

2.4.5 The effect of protection on prices and profits

From the expressions for the equilibrium prices and profits, it is immediately seen that c_A has no effect on prices. By algebraic derivation it can be shown that if $X < \underline{X}_0$ (and recall that the actual boundary is $\underline{X} < \underline{X}_0$), then both equilibrium prices and the net profit $\Pi_A^* = \pi_A^* - h(c_A)$ increase in c_B, and that the net profit Π_B^* increases in c_A.

2.4.6 The effect of X on c_A and c_B

Applying the implicit function theorem, we obtain:

$$\begin{aligned}
\frac{\partial \Pi_A^*}{\partial c_A}\left(c_A\left(X\right), c_B\left(X\right), X\right) \equiv 0 &\Longrightarrow \frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_A}\frac{dc_A}{dX} + \frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_B}\frac{dc_B}{dX} + \frac{\partial^2 \Pi_A^*}{\partial c_A \partial X} \equiv 0, \\
\frac{\partial \Pi_B^*}{\partial c_B}\left(c_A\left(X\right), c_B\left(X\right), X\right) \equiv 0 &\Longrightarrow \frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_A}\frac{dc_A}{dX} + \frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_B}\frac{dc_B}{dX} + \frac{\partial^2 \Pi_B^*}{\partial c_B \partial X} \equiv 0;
\end{aligned}$$

or, in matrix form:

$$\begin{pmatrix} \frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_A} & \frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_B} \\ \frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_A} & \frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_B} \end{pmatrix} \begin{pmatrix} \frac{dc_A}{dX} \\ \frac{dc_B}{dX} \end{pmatrix} = \begin{pmatrix} -\frac{\partial^2 \Pi_A^*}{\partial c_A \partial X} \\ -\frac{\partial^2 \Pi_B^*}{\partial c_B \partial X} \end{pmatrix}.$$

For simplicity, denote the first matrix as H; thus, $H = \begin{pmatrix} \frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_A} & \frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_B} \\ \frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_A} & \frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_B} \end{pmatrix}$. Applying Cramer's rule:

$$\frac{dc_A}{dX} = \frac{|H_A|}{|H|} = \frac{1}{|H|} \begin{vmatrix} -\frac{\partial^2 \Pi_A^*}{\partial c_A \partial X} & \frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_B} \\ -\frac{\partial^2 \Pi_B^*}{\partial c_B \partial X} & \frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_B} \end{vmatrix},$$

$$\frac{dc_B}{dX} = \frac{|H_B|}{|H|} = \frac{1}{|H|} \begin{vmatrix} \frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_A} & -\frac{\partial^2 \Pi_A^*}{\partial c_A \partial X} \\ \frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_A} & -\frac{\partial^2 \Pi_B^*}{\partial c_B \partial X} \end{vmatrix}.$$

Differentiating the equilibrium profits yields $\frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_A} = -h''(c_A) < 0$, $\frac{\partial^2 \Pi_A^*}{\partial c_A \partial X} > 0$, and $\frac{\partial^2 \Pi_A^*}{\partial c_A \partial c_B} > 0$ for $X < \underline{X}_0$, and by our assumptions $\frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_B} < 0$ as well. As for $\frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_A} =$

$$q_A q_B \left(\bar{\theta}(q_A - q_B) + X(1 - c_B)\right) \frac{\bar{\theta}(q_A - q_B)(4q_A + q_B c_B) - X\left(12 c_B q_A - c_B^2 q_B - 4q_A - q_B c_B\right)}{\left(4q_A - q_B c_B\right)^3 (q_A - q_B)},$$

which looks ambiguous, note that $\frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_A} = \frac{\partial^2 \pi_B^*}{\partial c_B \partial c_A}$, and $\frac{\partial^2 \pi_B^*}{\partial c_B \partial c_A} = \frac{1}{c_A} \frac{\partial \pi_B^*}{\partial c_B}$; then, F.O.C. $\frac{\partial \Pi_B^*}{\partial c_B} = 0$ implies $\frac{\partial \pi_B^*}{\partial c_B} = h'(c_B)$, so that $\frac{\partial^2 \Pi_B^*}{\partial c_B \partial c_A} > 0$. Finally, for $\frac{\partial^2 \Pi_B^*}{\partial c_B \partial X}$,

$$= -2 q_B q_A c_A \frac{\bar{\theta}(q_A - q_B)(4q_A(2c_B - 1) - q_B c_B) + (1 - c_B) X \left(4q_A(3c_B - 1) - q_B c_B(1 + c_B)\right)}{\left(4q_A - q_B c_B\right)^3 (q_A - q_B)},$$

it can be shown that for $X < \underline{X}_0$ and $c_B \leq 1/2$, $\frac{\partial^2 \Pi_B^*}{\partial c_B \partial X} > 0$. While the condition $c_B \leq 1/2$ cannot be loosened, this is a typical situation that we expect to occur in equilibrium, in which clearly $c_B^* < c_A^*$. Thus, we postulate $c_B^* < 1/2$ so that $\frac{\partial^2 \Pi_B^*}{\partial c_B \partial X}(c_B^*) > 0$.

Now consider the matrix H and recall that $|H| = \frac{\partial^2 \pi_A^*}{\partial c_A \partial c_A} \frac{\partial^2 \pi_B^*}{\partial c_B \partial c_B} - \frac{\partial^2 \pi_A^*}{\partial c_A \partial c_B} \frac{\partial^2 \pi_B^*}{\partial c_B \partial c_A}$. The first term is always positive since $\frac{\partial^2 \pi_A^*}{\partial c_A \partial c_A} < 0$ and $\frac{\partial^2 \pi_B^*}{\partial c_B \partial c_B} < 0$. The second term is also always positive since $\frac{\partial^2 \pi_B^*}{\partial c_B \partial c_A} > 0$ and $\frac{\partial^2 \pi_A^*}{\partial c_A \partial c_B} > 0$. Thus, we make a standard stability assumption here that $\left|\frac{\partial^2 \pi_i^*}{\partial c_i \partial c_i}\right| > \left|\frac{\partial^2 \pi_i^*}{\partial c_i \partial c_j}\right|$, which ensures that $|H| > 0$. Given the above, the determinants $|H_A|$ and $|H_B|$ are positive, so that $\frac{dc_A}{dX} > 0$ and $\frac{dc_B}{dX} > 0$.

2.4.7 The effect of X on equilibrium prices and profits

As for the prices,

$$\frac{dp_A^*}{dX}\left(c_A(X), c_B(X), X\right) = \frac{\partial p_A^*}{\partial c_A} \frac{dc_A}{dX} + \frac{\partial p_A^*}{\partial c_B} \frac{dc_B}{dX} + \frac{\partial p_A^*}{\partial X},$$

$$\frac{dp_B^*}{dX}\left(c_A(X), c_B(X), X\right) = \frac{\partial p_B^*}{\partial c_A} \frac{dc_A}{dX} + \frac{\partial p_B^*}{\partial c_B} \frac{dc_B}{dX} + \frac{\partial p_B^*}{\partial X};$$

since $\frac{\partial p_A^*}{\partial c_A} = \frac{\partial p_B^*}{\partial c_A} = 0$, and the remaining terms are strictly positive (as is shown above or can be shown by direct differentiation), $\frac{dp_A^*}{dX} > 0$ and $\frac{dp_B^*}{dX} > 0$.

As for the profits,

$$\frac{d\Pi_A^*}{dX} = \frac{\partial \Pi_A^*}{\partial c_A}\frac{dc_A}{dX} + \frac{\partial \Pi_A^*}{\partial c_B}\frac{dc_B}{dX} + \frac{\partial \Pi_A^*}{\partial X},$$
$$\frac{d\Pi_B^*}{dX} = \frac{\partial \Pi_B^*}{\partial c_A}\frac{dc_A}{dX} + \frac{\partial \Pi_B^*}{\partial c_B}\frac{dc_B}{dX} + \frac{\partial \Pi_B^*}{\partial X};$$

by virtue of the envelope theorem, $\frac{\partial \Pi_A^*}{\partial c_A} = 0$ and $\frac{\partial \Pi_B^*}{\partial c_B} = 0$, and the remaining terms are again strictly positive, so that $\frac{d\Pi_A^*}{dX} > 0$ and $\frac{d\Pi_B^*}{dX} > 0$.

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