Fairness Under Risk: Insights from Dictator Games

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Fairness Under Risk: Insights from Dictator Games

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Abstract

Recent theories of fairness (e.g., Bolton & Ockenfels, 2000; Fehr & Schmidt, 1999) have typically used the assumption of ex ante known pie size. Here I explore theoretically the ramifications of pie size being unknown ex ante. Using a simple allocation problem known as dictator game, I find that attitude to fairness is systematically and intuitively related to risk and risk attitude. Results from informal experiments support the model proposed here.

Abstrakt: Současné teorie spravedlnosti (např. Bolton, Ockenfels, 2000, a Fehr, Schmidt, 1999) typicky využívají předpokladu předem známé velikosti celkového koláče. V tomto článku teoreticky zkoumám důsledky případu, kdy velikost koláče předem není známa. Na jednoduché přerozdělovací úloze známé jako diktátorská hra ukazuji, že postoj ke spravedlnosti je systematicky a intuitivně svázán s rizikem a postojem k němu. Navržený model podporují i výsledky prvních neformálních experimentů.

Keywords: inequity aversion, dictator game, risk, expected utility, constant relative risk-aversion

JEL classification: D63, D64, D81

1 Introduction

Important recent papers (Bolton & Ockenfels, 2000; Fehr & Schmidt, 1999) have tried to explain the results of pie distribution experiments which suggest that many subjects do not behave in the purely selfish manner postulated by standard economic theory[1]. Both models incorporate other-regarding behavior in the form of inequity aversion as their key explanatory component. They are also constructed under the assumption of ex ante known pie sizes. The world, however, is not always fully known ex ante.

Take the situation of a couple who want to be together for the rest of their lives. While deeply in love, she is rational enough to know that there is a - ever so slightly - chance that things will not work out as planned. She is the better prospect commercially (being as it is, a hot-shot lawyer, fresh out of a top-notch law school) while he is a sensitive guy who writes wonderful poems but is unlikely to eke out more than a meager living from his profession. Hence, she wants a prenuptial agreement. He has no choice but to accept in its entirety whatever it is that she wants.

Clearly, this is a one-shot dictator game. It is also a dictator game under uncertainty or risk (dependent on whether we assume the range of possible outcomes to be known or not) because the dictator does not know what the size of the pie will be if, contrary to today's blissful expectations of living happily ever after, push would come to shove. What will the dictator do in

[1]For example, the game-theoretic prediction in dictator and ultimatum games suggests zero giving using standard selfish preferences. Experimental studies, however, provide clear evidence on positive giving for both games; the transfer to the recipient amounts to about 20% of the pie size for the dictator game and more than twice that for the ultimatum game.

such a situation? In this paper, based on results from informal experiments[2], I assume that she has a preference for relative rather than for "absolute" giving, and I investigate how the variance of possible pie sizes, i.e. the risk associated with the distribution, will affect her offers. I also explore how this decision is related to her risk attitude.

2 ERC[3] analysis of the game

Both the Fehr-Schmidt model and the Bolton-Ockenfels ERC model study interactions of n people. In both models, people care about their own payoffs. The difference lies in the modelling of inequity aversion. In Fehr & Schmidt (1999), it is expressed as some linear function of the difference of one's own payoff and the various payoffs of other actors, while in the ERC model it is expressed as some function of the relative payoff, i.e. the ratio of one's own payoff to the sum of all payoffs[4].

For two-person games the distribution of payoffs is fully, and conveniently, determined by either the absolute or the relative payoff of a single agent. Consequently, with any sum of total payoffs, the Fehr-Schmidt utility can be viewed as a special case of the ERC motivation function (the difference in absolute payoffs is equal to the difference in relative payoffs times a constant

[2]And also based on the key behavioral assumption of most models of reciprocity and fairness.

[3]ERC = Equity, Reciprocity, and Competition

[4]For example, if the payoffs are 6, 3 and 1 for the other players and 5 for oneself, then inequity aversion according to Fehr & Schmidt is measured as 6-5=1 on one side and (5-3)+(5-1)=6 on the other side; for the final inequity aversion both of these values (weighted by possibly different fairness sensitivity parameters) are used. For the ERC model, the difference of the relative payoff to the equal division matters, i. e. $\frac{5}{15} - \frac{1}{4}$, where the second number normalizes the relative payoff with respect to the equal standing.

representing the size of the pie to be distributed[5]). Hence, the following ERC analysis of the game can be easily translated into the corresponding Fehr-Schmidt analysis.

Let the motivation function be additively separable:

$$v(y, \sigma) = u(y) - kf(\sigma)$$

where y is the absolute payoff of the player we are interested in (the dictator) and σ is her relative payoff (i.e. the ratio of her absolute payoff to the sum of all payoffs). To fulfill the assumptions of ERC theory, let u be a continuous increasing concave function (i.e. the marginal utility of one's own payoff is decreasing), f be a continuous strictly convex function attaining its minimum at $\sigma = 0.5$ (the disutility which a player experiences from her relative position in the game is minimized when her payoff equals that of the other player), and $k > 0$ be a constant (the coefficient k quantifies how much she cares about her relative payoff). As $k \to 0$, she cares less and less about her relative standing and becomes, in the limit, a selfish actor with utility function u postulated by standard economic theory.

Let C be a random variable which determines the size of the pie to be distributed. Let p be the proportion of the pie that the dictator is willing to

[5]Take for example payoffs 5 and 3. For the Fehr-Schmidt model, inequity aversion is evaluated as 5-3=2, and the difference of relative payoffs from ERC is also $\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$, 8 is the total size of the pie in the game.

transfer to the recipient. Then, the dictator's maximization problem is[6]:

$$\max_p Ev((1-p)C, 1-p) = \max_p [Eu((1-p)C) - kf(1-p)].$$

Note that for the utility maximizing decision holds $p \in [0, 0.5]$ since the allocation $p = 0.5$ is always preferred to any allocation $p' > 0.5$. Note also that the level of inequity aversion is the same for all realizations of the random variable C since the dictator's decision determines the relative payoffs no matter what the actual size of the pie will be.

Since under our assumptions above (concavity of both components of the motivation function, and hence of the motivation function itself), the second-order condition is automatically fulfilled, the optimal dictator giving p follows from the first-order condition:

$$\frac{d(Eu((1-p)C))}{dp} + kf'(1-p) = 0.$$

To be able to compute comparative static results, and for the sake of computational convenience, I assume that u is a function of the constant relative risk aversion variety, namely $u(x) = sgn(r)x^r$ with $r \leq 1$, $r \neq 0$[7]. (Recall that the coefficient of relative risk aversion is equal to $r - 1$ for such functions). I can then rewrite the first-order condition as

$$(1-p)^{1-r} f'(1-p) = \frac{r}{k} sgn(r) E(C^r). \tag{1}$$

[6] I assume for now that agents are expected utility maximizers. I am aware that this assumption is a topic of ongoing disputes on which I remain agnostic. My interest here is, within the framework of previous studies, to analyze the ramifications of decision making under risk. I am in the process of extending my analysis to prospect theory.

[7] See for example experimental results by Holt & Laury, 2002.

Note that if the right hand side does not belong to the interval $(0, f'(1))$ then border dictator giving occurs - either none or half of the pie will be transferred.

For any distribution (here, of pie sizes), the value of $E(C^r)$ represents a risk associated with a given distribution. For example, in the case of $EC_1 = EC_2$ it is easy to see that $VarC_1 < VarC_2 \iff E(C_1^2) < E(C_2^2)$ since $VarC_i = E(C_i^2) - (EC_i)^2$. Similarly, for symmetric distributions it is always true (and in fact it is typically true for arbitrary distributions) that if $EC_1 = EC_2$ and $VarC_1 < VarC_2$ then also $E(C_1^r) < E(C_2^r)$ for all $r < 0$ and $E(C_1^r) > E(C_2^r)$ for all $r \in (0, 1)$. This is due to the convexity/concavity of function x^r. Thus, the right-hand side of equality (1) is increasing or decreasing with the increasing "risk" of a given distribution depending on the relative risk-aversion of the economic agent, i.e., whether $r < 0$ or $r \in (0, 1)$, respectively. Note that for $r = 1$ (i.e., risk-neutral players) the decision p depends only on the expected size of the pie.

An analogous analysis for the constant relative risk-aversion function corresponding to $r = 0$, i.e. for $u(x) = \log x$, yields the following first-order condition:

$$kf'(1-p) = \frac{1}{1-p}.$$

Note that the dictator's decision under this functional specification does not depend on the size of the pie distributed in the dictator game.

It is possible to refine the above analysis further if we also assume f to be of the constant relative risk-aversion variety (although this term is not about risk aversion but reflects rather inequity aversion), i.e.

$$f(\sigma) = (\sigma - 0.5)^{\gamma}$$

where $\gamma > 1$ to assure its strict convexity. Consequently, $f'(1-p) = \frac{\gamma(1-2p)^{\gamma-1}}{2^{\gamma-1}}$ and dictator giving, using (1), now satisfies the equality

$$(1-p)^{1-r}(1-2p)^{\gamma-1} = \frac{r}{l} sgn(r) E(C^r) \qquad (2)$$

where $l = \frac{\gamma k}{2^{\gamma-1}}$ is a constant.

Under the given parameter assumptions ($r < 1$, $\gamma > 1$), it is easy to show that the left-hand side of (2) is a decreasing function of p. This means that dictator giving is lower when the right-hand side is higher and vice-versa. Together with my analysis of the effects of risk attitude on the right-hand side of (1) above, I prove the following proposition:

Proposition 1 *Within the ERC framework, people characterized by a coefficient of relative risk aversion below -1 will decrease their dictator giving for any given pie size as risk increases, and people with a coefficient of relative risk aversion above -1 will increase their dictator giving in such a situation. Decisions of people with relative risk aversion equal to -1, as well as decisions of risk-neutral agents, will be unaffected by risk when pie size is unknown ex ante.*

What is the intuition behind this theoretical result? As the coefficient of relative risk aversion decreases from 1, people are willing to be more altruistic up to a certain level; they substitute risk aversion for fairness. However, after that level, risk aversion prevails and people start to treat risk and fairness attributes as complements, decreasing the giving with higher risk. When the coefficient of relative risk aversion is equal to -1, the behavior crosses the neutral point as it was in the starting coefficient of 1.

Remark 1: A similar comparative statics analysis can be done with respect to changes in the size of the pie if it is not uncertain. In that case, all expected value operators will disappear and, as a result, the ERC model suggests higher (lower) dictator giving for larger sizes of the pie for coefficients of relative risk aversion below (above) -1, and no influence of the pie size in the case of logarithmic utility. Similarly, risk-neutral people would decrease their offers with increasing pie.

Remark 2: A similar analysis can also be done for a generalized model of the Fehr-Schmidt type where the argument in function f is now the difference of the payoffs of the players, i.e. $(1-2p)C$. The situation is then somewhat more complicated since $E(C^{\gamma})$ also enters into the denominator of the right-hand side in (1) and hence plays a role in the behavior of dictator giving in such a model. But then, since the term $E(C^{\gamma})$ in the denominator decreases the right-hand side in equations (1) and (2) when the risk increases, the only difference in the new specification of the model is that the critical value of parameter r (i.e. the value when the dependence of behavior on risk switching is similar to that in the proposition above) is lower than 0 (or, equivalently, the critical coefficient of relative risk aversion is lower than -1); it decreases even more with increasing parameter γ and such a change may also differ for different types of probability distributions.

Remark 3: The analysis is applicable also to risk-loving agents, i.e. those with convex selfish utility u. However, the second-order condition can then be invalid and in such a case, the dictator giving $p \in [0, \frac{1}{2}]$ will not be the interior point. Typically, in such a situation the model predicts zero dictator giving in riskier conditions for these kinds of agents.

In fact, such a result conforms to the intuition that very high payoffs are really attractive for risk-loving agents and, thus, these agents do not like to share such payoffs with others, at least compared to lower payoffs which are more likely to happen.

3 Discussion

I chose to analyze the dictator game because giving behavior in this game depends only on a single person's preferences. I thus could study preferences in their purest form. The results of the informal experiments I conducted in Prague and Jena demonstrate, quite intuitively, that risk aversion matters and hence ought to be incorporated into models explaining other-regarding behavior. The pilot experiments suggest that people prefer relative over absolute offers under risky conditions. This fact conforms to the intuition that, if there is a choice, risk-averse agents prefer to share risk over bearing it themselves. The pilot experiments also suggest that on average, again quite in line with intuition, decision makers want to keep a certain risk premium, and that subjects decrease their giving (both absolute and relative) with increasing risk, suggesting that for the average subject the coefficient of relative risk aversion is less than -1[8]. Of course, people are heterogeneous so the actual giving behavior is different for some people. That said, for almost all subjects both risk and fairness attitudes factor into their giving behavior. My model above formalizes this result.

The present research can be expanded in various ways. First, the expected utility approach can be replaced by an approach based on prospect theory (Kahneman & Tversky, 1979). Such an extension allows for modeling divergent perceptions of gains and losses and results in different predictions for different types of people, which also conforms to different behavioral patterns observed in various experiments.

Second, an extension of my analysis to more complicated games such as the ultimatum game seems desirable even though the experimental results for such a setting are going to be noisier due to beliefs playing a role in the decision making. Also desirable seems experimental work that tries to assess empirically the correlation between risk and fairness attitudes.

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[8]For example, an econometric analysis of auction data suggests this coefficient to be around 0.5. However, the difference here can be affected by different settings of the game or other assumptions.

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