INTERGENERATIONAL BARGAINING IN TECHNOLOGY ADOPTION

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Abstract

I study the choice of technology adoption in an environment where human capital is transmitted from the old to the young generation, but the young generation can opt out for a new technology. The adoption and matching decisions are made in a sequential intergenerational bargaining. Since technology adoption benefits future generations who do not participate in the bargaining, there is an inherent bias toward preserving the current technology. The main result is that economic integration (i.e., the sharing of frontier technology among countries) promotes growth while political integration (i.e., the merging of countries into a single bargaining) promotes stagnation.

Abstrakt

V této práci studuji přijímání technologií v prostředí, kde je lidský kapitál předáván ze starší generace na mladší, přičemž mladší generace se může vyvázat a přijmout novou technologii. Přijímání nových technologií a její vybírání si je realizováno v sekvenčním mezigeneračním vyjednávání. Jelikož přijímání technologií obohacuje budoucí generace, které se neúčastní vyjednávání, existuje zde bias směrem k zachovávání stávajících technologií. Hlavní výsledek práce je ten, že ekonomická integrace (sdílení špičkových technologií mezi zeměmi) prospívá růstu, zatímco politická integrace (začlenění zemí pod jednoho vyjednavače) vede spíše ke stagnaci.

JEL classification: O40; E10

Keywords: technology adoption; growth; stagnation; bargaining; generation; human capital; economic integration; political fragmentation

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1. Introduction

I study the choice of technology adoption in an environment where human capital is transmitted from the old to the young generation, but the young generation can opt out for a new technology. The adoption of new technology raises the productivity of the young and future generations while it depreciates the value of existing human capital. The choice of technology adoption is made in each period in an intergenerational bargaining. Since technology adoption benefits future generations who do not participate in the bargaining, there is an inherent bias toward preserving the current technology. The current generations have the incentive to preserve the current technology if the average human capital of the old generation is close enough to the frontier technology.

I examine the variations of the environment along two dimensions. First, I consider economic integration, that is, the sharing of the frontier technology among countries initially separated from each other. A condition for perpetual growth in economic integration is the diversity of human capital distribution among countries: In each period, the world economy grows as long as the country furthest from the frontier technology has the incentive to adopt the frontier technology. When the diversity is limited, the world economy may still grow due to a coordination failure among countries: Every country adopts the frontier technology since, from the perspective of an individual country, stagnation is advantageous only when other countries stagnate as well. Second, I consider political integration, that is, the merging of countries into a political union with a single intergenerational bargaining. The political union corrects the coordination failure and aligns the incentives of individual countries for stagnation by means of side payments that are implicit in bargaining. Thus, political integration always leads to a stagnation.

The modern economic growth of the world may reflect the growth imperative that each country faces under economic integration and political fragmentation. The economic integration defined as the sharing of the frontier technology was probably at a continental scale a couple of hundred years ago. The sustained growth of European countries at that time may have to do with political fragmentation. A politically integrated China, on the other hand, stagnated then and in preceding centuries. This view can be found in Rosenberg and Birdzell (1986)[1] and Mokyr (1990)[2] among others. Jones (1981)[3] hints at the uncontrolled diffusion as a mechanism for a sustained advancement of technology in the politically fragmented Europe. More recently, economic integration is at the global scale and stagnation is not a sustainable option for any countries. Waves of growth-oriented reforms have swept through the world over the decades starting with East Asian economies and including the former-communist countries.

The modeling exercise builds on the vintage human capital model of Chari and Hopenhayn (1991) and the political economy model of Krusell and Rios-Rull (1996). In the latter, a segment of population with human capital in existing technologies may outvote the others

[1] "One difference between East and West, heavily emphasized in the writings of economic historians, was that the West was politically fragmented between more or less autonomous units that competed for survival, wealth, and power. Long before the states system emerged, Europe was a pluralistic and diverse society in which many political units jealously guarded their independence from each other" (p. 206, Mokyr 1990).

[2] "Unlike China and the ancient empires, the Europe of the late medieval city-states and the early monarchies came to the age of discovery without a central authority strong enough to check the determination of its merchants to gain access to profitable trading opportunities, even though some satrap or other had forbidden such access or claimed it as a private preserve. The central authorities which eventually emerged did not take the form of a single monolithic empire, but of a group of nation-states which continued, among themselves, the early city-state competition for trade" (p. 60, Rosenberg and Birdzell 1986).

[3] "All in all, the competitiveness and 'genetic variety' of the states system helped to generalize best practices without, in the event, the penalties that morally may have been right. This was done by voluntary and involuntary movements of capital and labour. Thus the culture, science, technology and commercial practice of the Italian city-states, Antwerp, Amsterdam, and London were passed from each to the next, and were diffused across agrarian and backward economies that showed little sign of attaining the same level on their own. Galileo's trial silenced Italian scientists, but the 'scientific revolution' continued in Protestant lands. Books might be smashed by mobs, entrepreneurs banished and investors expropriated by governments, but Europe as a whole did not experience technological regression. The multi-cell system possessed a built-in ability to replace its local losses, a vigorous recombination, regrowth or substitution effect. The system had its own signature and was more than the sum of its parts" (p. 123-4, Jones 1981).

and block the adoption of a new technology, thereby creating rents for themselves. Subsequent related works explored alternative political mechanisms. In Bellettini and Ottaviano (2005), the young and the old bid for and against the upgrading of the aggregate technology to a one-period lasting regulator who maximizes the collection of bids. Similarly, in Bridgman, Livshits, and MacGee (2007), a myopic government decides on the adoption of new technologies across industries weighing the aggregate output and the bribes from industry lobbies.

In this paper, the society-wide decision process is modeled as a bargaining between the old and the young generations. The bargaining delivers efficient adoption and matching behavior *for the current generations* and assigns a payoff to each generation. The aggregate path of the economy is independent of individual activities and payoffs as long as they aggregate to the bargaining outcome. I consider efficient bargaining as a means of abstracting from the variations in the details of economic and political institutions that add up to deliver efficient adoption and matching behavior for the current generations. This abstraction allows me to focus on the effects of the sequential structure of intergenerational bargaining and the international integration and fragmentation on the growth path. Conversely, the incentives for international integration and fragmentation are due to the growth dynamics under the bargaining structure. In comparison, the models of integration and fragmentation such as Alesina and Spolaore (1997) and Bolton and Roland (1997) have focused on the incentives due to the provision of public goods and re-distribution in a static setting. These models are in part motivated by events such as the formation of the European Union and the break-up of the Soviet Union in an economically integrated world. This paper has little to say about such a regional integration and fragmentation since an economically integrated world needs a worldwide political union, a possibility in the future, to hold back the advancement of technology.

2. The Model Economy: A Single Country

There are two overlapping generations in each period. There are many people in each generation whose number is normalized to one. Human capital embodies technology with a higher level of h associated with a superior technology. Technologies are ordered such that the ratio of the human capital levels associated with any two adjacent technologies is fixed at $\lambda > 1$. Let $\{h_s\}_{s\geq 0}$ denote the ordered set of human capital levels, where h_0 is associated with the frontier technology available for adoption, and $h_s/h_{s+1} = \lambda$ for all s. Let n_s denote the fraction of old people with human capital h_s. I have $\sum_{s\geq 1} n_s = 1$. An old person can work alone or work with a young person; a young person is endowed with no human capital and can work with an old person or adopt the frontier technology alone. A team of an old person with human capital h_s and a young person produces h_s units of output. Within a team, the young person inherits the human capital of the old person in the next period. An old person with human capital h_s alone produces ϕh_s units of output. A young person alone produces no output but obtains human capital h_0 in the next period. Assume that $\phi < 1$, which implies that team production has an advantage over individual production, holding technology.

The frontier technology advances to the next level in the next period if at least one young person adopts the current frontier technology. Let τ denote the technology adoption decision at the aggregate level: $\tau = 1$ if at least one young person adopts the current frontier technology; $\tau = 0$ if not. Let $\tilde{n}_s$, $s \geq 0$, denote the fraction of young people who will have human capital h_s when old. Let $n \equiv (n_1, n_2, n_3, \ldots)$ and $\tilde{n} \equiv (\tilde{n}_0, \tilde{n}_1, \tilde{n}_2, \ldots)$. The feasibility requires: $\tilde{n}_s \leq n_s$ for all $s \geq 1$, and $\tilde{n} = (0, n)$ if $\tau = 0$.[4] The aggregate

[4] It is feasible to have $\tilde{n} = (0, n)$ and $\tau = 1$. Strictly speaking, this implies that the frontier technology can advance even if no young person adopts the current frontier technology. Since there are an infinite number of young people, the country as a whole can trivially advance the frontier technology by having one young person adopt the current frontier technology. In other words, the intergenerational bargaining outcome and the other equilibrium properties, as will be discussed, are not substantively altered by modeling the technology adoption by one young person. I abstract from this modeling detail for a technical convenience.

output is:

$$Y(h_0, n, \tilde{n}) = \sum_{s=1}^{\infty} \big(h_s \tilde{n}_s + \phi h_s (n_s - \tilde{n}_s)\big). \tag{1}$$

Let $V_o(h_0, n)$ denote the aggregate utility of the old generation, and $V_y(h_0, n)$ the aggregate utility of the young generation. Let $W(h_0, n, \tilde{n}, \tau)$ denote the aggregate utility across the young and the old generations conditional on τ and $\tilde{n}$:

$$W(h_0, n, \tilde{n}, \tau) = Y(h_0, n, \tilde{n}) + \beta(\tau \cdot V_o(\lambda h_0, \tilde{n}) + (1 - \tau) \cdot V_o(h_0, n)), \tag{2}$$

where β is the discount rate. Note that the aggregate utility includes the discounted utility of the young generation's utility when it becomes old.

The adoption and matching behavior and the utilities of generations are determined in a sequential, intergenerational bargaining. Let $\bar{V}_o(h_0, n)$ denote the reservation utility of the old generation; and $\bar{V}_y(h_0)$ the reservation utility of the young generation: $\bar{V}_o(h_0, n) = Y(h_0, n, \bar{n}) = \sum_{s=1}^{\infty} \phi h_s n_s$ and $\bar{V}_y(h_0) = \beta V_o(\lambda h_0, \bar{n})$, where $\bar{n} \equiv (1, 0, 0, \ldots)$. The reservation utilities are obtained when there are no matches across the generations: Each old person works alone and each young person adopts the frontier technology. Let $g(h_0, n)$ denote the adoption and matching behavior, that is, a function from $\{(h_0, n)\}$ to $\{\tilde{n}\}$. This function and the utilities of generations solve the Nash bargaining problem: For all h_0 and n,

$$(g(h_0, n), q(h_0, n), V_o(h_0, n), V_y(h_0, n)) = \underset{\tilde{n}, \tau, v_o, v_y}{\operatorname{argmax}} \big\{(v_o - \bar{V}_o(h_0, n))^{\mu} (v_y - \bar{V}_y(h_0))^{1-\mu}\big\}, \tag{3}$$

where the maximization is subject to $v_o + v_y \leq W(h_0, n, \tilde{n}, \tau)$. Note that bargaining is across the generations; the individual utilities do not need to be specified. One interpretation is collective bargaining between the generations. Given the ex-ante homogeneity among young people, I could assume that the young generation's aggregate lifetime utility $V_y(h_0, n)$ is divided equally among the young people, which may involve an unequal division of output in each of the two periods of the lifetime. An equilibrium is the value functions, W, V_o, and V_y, and the policy functions, $g(h_0, n)$ and $q(h_0, n)$, which solve (2) and (3).

I make two restrictions on the equilibrium. In order to ensure the existence of equilibrium, I restrict the parameter values as follows.

Assumption 1: $1 - \beta\mu - \beta\lambda\mu > 0$.

Further, I restrict the equilibrium to have the property that the re-scaling of human capital distribution changes the equilibrium path only proportionately.

Assumption 2: For any h_0, n, and $\theta > 0$, $g(\theta h_0, n) = g(h_0, n)$, $q(\theta h_0, n) = q(h_0, n)$, and $V_o(\theta h_0, n)/V_o(h_0, n) = V_y(\theta h_0, n)/V_y(h_0, n) = \theta$.

In other words, h_0 can be normalized to one. Given the transferability of utility across the generations, (3) is equivalent to maximizing the aggregate utility and dividing the utility by μ and $1 - \mu$ shares across the generations. I can rewrite the value functions as:

$$\hat{W}(n) = (1 - \hat{q}(n)) \cdot \hat{W}_1(n) + \hat{q}(n) \cdot \max_{\tilde{n}}\{\hat{W}_2(n, \tilde{n})\}, \tag{4}$$

where

$$\hat{W}_1(n) = \hat{Y}(n, (0, n)) + \beta\hat{V}_o(n); \tag{5}$$

$$\hat{W}_2(n, \tilde{n}) = \hat{Y}(n, \tilde{n}) + \beta\lambda\hat{V}_o(\tilde{n}); \tag{6}$$

$$\hat{V}_o(n) = \mu \cdot (\hat{W}(n) - \beta\lambda\hat{V}_o(\bar{n})) + (1 - \mu) \cdot \hat{Y}(n, \bar{n}); \tag{7}$$

$\hat{Y}(n, \tilde{n}) \equiv Y(1, n, \tilde{n})$; $\hat{q}(n) \equiv q(1, n) = 1$ if there is a technology adoption; and $\hat{q}(n) = 0$ if not. Let $\hat{g}(n) \equiv (\hat{g}_0(n), \hat{g}_1(n), \ldots) \equiv g(1, n)$ denote the policy function on the human capital distribution.

I will characterize the equilibrium mainly in terms of the productivity parameter ϕ and the distribution of human capital n since the value functions are linear in ϕ and n, and analytical results are more easily obtained in terms of these parameters. Intuitively, a higher value of ϕ implies a lower output loss from production by an unmatched old person and would make the young people more likely to adopt the frontier technology. A human capital distribution that is more skewed toward the frontier technology would make the young people more likely to match with the old people instead of adopting the frontier technology. The effects of the innovation size parameter λ, the discount rate β, and the old generation's bargaining power μ on the equilibrium are more difficult to characterize explicitly. Intuitively, a higher value of λ or β would make the young more likely to adopt the frontier technology. The effect of the old generation's bargaining power μ is less obvious and perhaps surprising. Generally, a higher value of μ makes the young more likely to adopt the frontier technology since the young can expropriate a greater share of the return from adopting the frontier technology when they become old.

Throughout the characterization of the equilibrium below, I make use of the stagnation premium, i.e., the aggregate utility gain from holding the frontier technology:

$$SP(n) \equiv \hat{W}_1(n) - \max_{\tilde{n}}\{\hat{W}_2(n, \tilde{n})\}. \tag{8}$$

The intuition for a possibly positive premium is as follows. Today's technology adoption allows tomorrow's young generation to adopt an even better technology, strengthening their bargaining position and possibly reducing the production surplus that accrues to tomorrow's old generation or, equivalently, to today's young generation. Current generations can avoid such a loss by not adopting the frontier technology. In other words, advancing the frontier technology carries a positive externality to future generations, which the current generations do not take into consideration in their decision making.

For a detailed derivation of the equilibrium, see Steps 1 to 14 in the Appendices. In Step 7, I show that there is a unique equilibrium. The parameter space can be divided into five zones, A to E. Figure 1 at the end of the paper illustrates the zones when $\beta = .5$ and $\lambda = 1.5$. The upper bound of μ, which is 0.8 for the illustration, is given by the condition that $1 - \beta\mu - \beta\lambda\mu > 0$ in Assumption 1.

Zone A. Unconditional Growth Equilibrium (UGE):

$$\hat{q}(n) = 1 \text{ and } \hat{g}(n) = (1, 0, 0, \ldots) \text{ for all } n \tag{9}$$

if $\phi \geq \bar{\phi}$, where $\bar{\phi} \equiv 1/(1 - \beta + \beta\lambda)$.

In this equilibrium, there are no matches across the generations: Each old person works alone and each young person adopts the frontier technology regardless of n. The return from adopting the frontier technology dominates any other considerations in this range of parameters. Given the initial distribution of human capital n, I can derive the equilibrium path by repeatedly updating $\tilde{n}$ and n using $\hat{q}(n)$ and $\hat{g}(n)$. I have

Result 1: Under Unconditional Growth Equilibrium (UGE), the country grows perpetually starting from any n.

Zone B. Conditional Growth Equilibrium 1 (CGE 1):

$$\begin{aligned} &\hat{q}(n) = 0 \text{ and } \hat{g}(n) = (0 \qquad , n_1, n_2, \ldots) \text{ for } n \in \Upsilon_{stag1}, \\ &\hat{q}(n) = 1 \text{ and } \hat{g}(n) = (1 \qquad , 0 \ , 0 \ , \ldots) \text{ for } n \in \Upsilon_{stag2}, \text{ and} \\ &\hat{q}(n) = 1 \text{ and } \hat{g}(n) = (1 - n_1, n_1, 0 \ , \ldots) \text{ for } n \in \Upsilon_{stag3} \cup \Upsilon_{grow} \end{aligned} \tag{10}$$

if $\hat{\phi} \leq \phi \leq \bar{\phi}$, where $\hat{\phi} \equiv (1 - \beta\lambda\mu)/(1 - \beta + \beta\lambda - \beta\lambda\mu)$; $\Upsilon_{stag1} \equiv \{n : SP(n) \geq 0\} \subset \{n : n_1 \geq \check{n}\}$; $\Upsilon_{stag2} \equiv \{n : n_1 \leq \hat{n}\}$; $\Upsilon_{stag3} \equiv \{n : n_1 \geq 1 - \hat{n}\} - \Upsilon_{stag1} - \Upsilon_{stag2}$; $\Upsilon_{grow} \equiv \{n : \hat{n} < n_1 < 1 - \hat{n}\}$; $\hat{n} \equiv \beta^2\lambda\mu^2/((1 - \beta\lambda\mu)(1 - \beta\mu + \beta\lambda\mu))$; $\check{n} \equiv ((1 - \beta\mu + \beta\lambda\mu - \beta^2\lambda^2\mu^2 + \phi\beta^2\lambda^2\mu^2(1 - \beta + \beta\lambda))/(1 - \beta\mu + \beta\lambda\mu) - \phi(1 - \beta + \beta\lambda^2))/(1 - \beta\lambda\mu - \phi(1 - \beta + \beta\lambda^2 - \beta\lambda\mu(1 - \beta + \beta\lambda)))$; $1/2 \leq 1 - \hat{n} \leq \check{n} \leq 1$; $\check{n}$ is increasing in ϕ; $\check{n} = 1$ when $\phi = \bar{\phi}$; and $\check{n} = 1 - \hat{n}$ when $\phi = \hat{\phi}$.

In this equilibrium, the adoption and matching decisions depend on the distribution of human capital. The country stagnates perpetually if $n \in \Upsilon_{stag1}$; it stagnates perpetually after one period of growth if $n \in \Upsilon_{stag2}$; it stagnates perpetually after two periods of growth if $n \in \Upsilon_{stag3}$; and it grows perpetually if $n \in \Upsilon_{grow}$. I have

Result 2: Under Conditional Growth Equilibrium 1 (CGE 1), the country grows perpetually if $n_1 \in [\hat{n}, 1-\hat{n}]$; the country stagnates after, at most, two periods of growth if $n_1 \in [0,\hat{n}] \cup [1-\hat{n}, 1]$.

Zone C. Conditional Growth Equilibrium 2 (CGE 2):

$$\begin{aligned} &\hat{q}(n) = 0 \text{ and } \hat{g}(n) = (0 \qquad , n_1, n_2, \ldots) \text{ for } n \in \Omega_{stag1} \text{ and} \\ &\hat{q}(n) = 1 \text{ and } \hat{g}(n) = (1-n_1, n_1, 0 \;\;, \ldots) \text{ for } n \in \Omega_{stag2} \cup \Omega_{stag3} \cup \Omega_{grow} \end{aligned} \tag{11}$$

if $\tilde{\phi}_w \leq \phi \leq \hat{\phi}$, where $\tilde{\phi}_w \equiv (1-\beta\mu-\beta\lambda^2\mu(1-\beta\lambda\mu))/((1-\beta)(1-\beta\mu)+\beta\lambda^2(1-\mu)(1-\beta\lambda\mu))$; $\Omega_{grow} \equiv \{n : 1-\check{n} < n_1 < \check{n}\}$; $\Omega_{stag1} \equiv \{n : SP(n) \geq 0\} \subset \{n : n_1 \geq \check{n}\}$; $\Omega_{stag2} \equiv \{n : n_1 \leq 1-\check{n}\} - \Upsilon_{stag1} \supset \{n : n_1 \leq \min\{\check{n}, 1-\check{n}\}\}$; $\Omega_{stag3} \equiv \{n : n_1 \geq \max\{\check{n}, 1-\check{n}\}\} - \Upsilon_{stag1}$; $\check{n}$ is increasing in ϕ; $\check{n} = 1-\hat{n}$ if $\phi = \hat{\phi}$; $\check{n} = (1-\phi+\beta\phi-\beta\lambda^2(\phi+\mu(1-\phi))(1+\beta^2\lambda\mu^2 - \beta^2\lambda^2\mu^2)/(1-\beta\mu+\beta\lambda\mu))/((1-\beta\lambda\mu)(1-\phi+\beta\phi-\beta\lambda^2(\phi+\mu(1-\phi))))$ when $\check{\phi}_a \leq \phi \leq \hat{\phi}$; $\check{n} = 1/2$ when $\phi = \check{\phi}_a \equiv (1-\beta\mu-\beta\lambda\mu(1-\beta\lambda\mu)(\lambda-1))/((1-\beta)(1-\beta\mu+\beta\lambda\mu)+\beta\lambda^2(1-\mu)(1+\beta\mu-\beta\lambda\mu))$; $\check{n} = (1-\beta\mu-\phi(1-\beta)(1-\beta\mu)+\beta\lambda\mu(1-\phi+\beta\phi)-\beta\lambda^2(\phi+\mu(1-\phi)))/((1-\beta\mu+\beta\lambda\mu)(1-\phi+\beta\phi-\beta\lambda^2(\phi+\mu(1-\phi))))$ when $\tilde{\phi}_w \leq \phi \leq \check{\phi}_a$; and $\check{n} > 0$ when $\phi = \tilde{\phi}_w$.

This equilibrium is similar to Conditional Growth and Stagnation Equilibrium 1. A major difference is that the no-match-all-adopt case does not occur regardless of n. The value of productivity parameter ϕ for a single-old-person-production is low enough, equivalently, the productivity loss, $1-\phi$, from a single-old-person-production production is high enough for a young person to match with an old person with the best technology in use instead of adopting the frontier technology. The country stagnates perpetually if $n \in \Omega_{stag1}$; it stagnates perpetually after one period of growth if $n \in \Omega_{stag2}$; it stagnates perpetually after two periods of growth if $n \in \Omega_{stag3}$; and it grows perpetually if $n \in \Omega_{grow}$. I have

Result 3: Under Conditional Growth Equilibrium 2 (CGE 2), the country grows perpetually if $n_1 \in [1-\check{n}, \check{n}]$; the country stagnates after, at most, two periods of growth if $n_1 \in [0, 1-\check{n}] \cup [\check{n}, 1]$.

Zone D. Unconditional Stagnation Equilibrium 1 (USE 1):

$$\begin{aligned} &\hat{q}(n) = 0 \text{ and } \hat{g}(n) = (0 \qquad\qquad , n_1, n_2, \ldots) \text{ for } n \in \Theta_{stag1} \\ &\hat{q}(n) = 1 \text{ and } \hat{g}(n) = (1-n_1 \qquad , n_1, 0\ \ , \ldots) \text{ for } n \in \Theta_{stag2} \text{ and} \\ &\hat{q}(n) = 1 \text{ and } \hat{g}(n) = (1-n_1-n_2, n_1, n_2, \ldots) \text{ for } n \in \Theta_{stag3} \end{aligned} \tag{12}$$

if $\tilde{\phi}_a \leq \phi \leq \tilde{\phi}_w$, where $\tilde{\phi}_a \equiv (1-\beta\mu+\beta\lambda\mu-\beta\lambda^2\mu)/((1-\beta)(1-\beta\mu+\beta\lambda\mu)+\beta\lambda^2(1-\mu))$; $\Theta_{stag1} \equiv \{n : SP(n) \geq 0\} \subset \{n : n_1 \geq \check{n}\}$; $\{n : n_1 \leq \check{n}\} \subset \Theta_{stag2} \equiv \{n : n_2 \leq \zeta(n_1)\} - \Theta_{stag1} \subset \{n : n_1 \leq 1-\check{n}\}$; $\{n : n_1 \geq 1-\check{n}\} - \Theta_{stag1} \subset \Theta_{stag3} \equiv \{n : n_2 \geq \zeta(n_1)\} - \Theta_{stag1} \subset \{n : n_1 \geq \check{n}\}$; $\check{n} = (1-\beta\mu-\phi(1-\beta)(1-\beta\mu)+\beta\lambda\mu(1-\phi+\beta\phi)-\beta\lambda^2(\phi+\mu(1-\phi)))/((1-\beta\mu+\beta\lambda\mu)(1-\phi+\beta\phi-\beta\lambda^2(\phi+\mu(1-\phi))))$; $\check{n}$ is increasing in ϕ; $\check{n} < 1/2$ when $\phi = \check{\phi}_a$; $\check{n} = 0$ when $\phi = \tilde{\phi}_a$; ζ is linear and decreasing in n_1; $\zeta(1-\check{n}) = 0$; $\zeta(\check{n}) = 1-\check{n}$; and $\check{n} < \check{n} \equiv (1-\beta\mu)/(1-\beta\mu+\beta\lambda\mu) < 1-\check{n}$.

This equilibrium is similar to Conditional Growth and Stagnation Equilibrium 2 in the segment of $\tilde{\phi}_w \leq \phi \leq \check{\phi}_a$: The country stagnates eventually starting from any n. A major difference is that an old person with the second best technology in use may be matched with a young person. The value of productivity parameter ϕ for a single-old-person-production is low enough, equivalently the productivity loss, $1-\phi$, from a single-old-person-production production is high enough, for a young person to match with an old person with the second best technology in use instead of adopting the frontier technology in some cases. The country stagnates perpetually if $n \in \Theta_{stag1}$; it stagnates perpetually after one period of growth if $n \in \Theta_{stag2}$; and it stagnates perpetually after two periods of growth if $n \in \Theta_{stag3}$. I have

Result 4: Under Unconditional Stagnation Equilibrium 1 (USE 1), the country stagnates after, at most, two periods of growth starting from any n.

Zone E. Unconditional Stagnation Equilibrium 2 (USE 2):

$$\hat{q}(n) = 0 \text{ and } \hat{g}(n) = (0, n_1, n_2, \ldots) \text{ for all } n \in \Psi_{stag} \tag{13}$$

if $\phi \leq \tilde{\phi}_a$, where $\Psi_{stag} \equiv \{n : SP(n) \geq 0\} \supset \{n : \sum_{s \leq u} n_s = 1\}$; $u = \max\{s | \phi \leq \varphi(s)\} \geq 2$; and $\varphi(s) \equiv (1 - \beta\mu + \beta\lambda\mu - \beta\lambda^s\mu)/((1-\beta)(1 - \beta\mu + \beta\lambda\mu) + \beta\lambda^s(1-\mu))$.

In this equilibrium, everyone is matched and no young person adopts the frontier technology if $\sum_{s \leq u} n_s = 1$. In other words, the country stagnates perpetually if the initial distribution is concentrated on u number of the best technologies in use. Further, u increases as ϕ declines: Stagnation becomes more attractive as the productivity loss, $1 - \phi$, from a single-old-person-production production rises. In particular, the country stagnates if $n_1 + n_2 = 1$ since $\phi \leq \tilde{\phi}_a = \varphi(2)$. I do not have a characterization of the equilibrium path starting from $n \neq \Psi_{stag}$. Thus, I cannot rule out the possibility of perpetual growth starting from some initial distributions.

Result 5: Under Unconditional Stagnation Equilibrium 2 (USE 2), the country stagnates perpetually if $n_1 + n_2 = 1$.

3. The Model Economy: Multiple Countries

There are I number of countries, indexed by i, each of which faces the same adoption and matching environment as in Section 2, except that the frontier technology is shared across countries. Let n, $\tilde{n}$, and τ denote the vectors of human capital distributions and technology adoption decisions across countries: $\tau \equiv (\tau^1, \tau^2, ..., \tau^I)$; $n \equiv (n^1, n^2, ..., n^I)$;

$n^i \equiv (n^i_1, n^i_2, n^i_3, ...)$; $\tilde{n} \equiv (\tilde{n}^1, \tilde{n}^2, ..., \tilde{n}^I)$; and $\tilde{n}^i \equiv (\tilde{n}^i_0, \tilde{n}^i_1, \tilde{n}^i_2, ...)$. The feasibility requires: $\tilde{n}^i_s \leq n^i_s$ for all $s \geq 1$; and $\tilde{n}^i = (0, n)$ if $\tau^i = 0$.[5] The aggregate output of country i is:

$$Y(h_0, n^i, \tilde{n}^i) = \sum_{s=1}^{\infty} \big(h_s \tilde{n}^i_s + \phi h_s (n^i_s - \tilde{n}^i_s)\big). \tag{1$'$}$$

Let $V^i_o(h_0, n)$, $V^i_y(h_0, n)$, and $W^i(h_0, n^i, \tilde{n})$ denote the aggregate utilities. The aggregate utility of country i is:

$$W^i(h_0, n, \tilde{n}, \tau) = Y(h_0, n^i, \tilde{n}^i) + \beta(\max\{\tau^i\} \cdot V^i_o(\lambda h_0, \tilde{n}) + (1 - \max\{\tau^i\}) \cdot V^i_o(h_0, n)\}). \tag{2$'$}$$

Let $g^i(h_0, n)$ and $q^i(h_0, n)$ denote the adoption and matching behavior, that is, a function from $\{(h_0, n)\}$ to $\tilde{n}^i$ and to τ^i, respectively. Let $G^{-i}(h_0, n, \tilde{n}^i) \equiv (\ldots, g^{i-1}(h_0, n), \tilde{n}^i, g^{i+1}(h_0, n), \ldots)$ and $Q^{-i}(h_0, n, \tau^i) \equiv (\ldots, q^{i-1}(h_0, n), \tau^i, q^{i+1}(h_0, n), \ldots)$. The reservation utilities are: $\bar{V}_o(h_0, n^i) = Y(h_0, n^i, \bar{n}) = \sum_{s=1}^{\infty} \phi h_s n^i_s$ and $\bar{V}^i_y(h_0, n) = \beta V^i_o(\lambda h_0, G^{-i}(h_0, n, \bar{n}))$. These functions solve the Nash bargaining problem: for all h_0 and n,

$$(g^i(h_0,n), q^i(h_0,n), V^i_o(h_0,n), V^i_y(h_0,n)) = \underset{\tilde{n}^i, \tau^i, v_o, v_y}{\operatorname{argmax}} \{(v_o - \bar{V}_o(h_0,n^i))^{\mu} (v_y - \bar{V}^i_y(h_0,n))^{1-\mu}\}, \tag{3$'$}$$

where the maximization is subject to $v_o + v_y \leq W^i(h_0, n^i, G^{-i}(h_0, n, \tilde{n}^i), Q^{-i}(h_0, n, \tau^i))$. An equilibrium is the value functions, $\{W^i\}$, $\{V^i_o\}$, and $\{V^i_y\}$, and the policy functions, $\{g^i(h_0, n)\}$ and $\{q^i(h_0, n)\}$, that solve (2)$'$ and (3)$'$.

I maintain the same restrictions on the equilibrium as in Section 2, that is, Assumption 1 as in Section 2 and Assumption 2 rewritten as

Assumption 2$'$: For any h_0, n, and $\theta > 0$, $g^i(\theta h_0, n) = g^i(h_0, n)$, $q^i(\theta h_0, n) = q^i(h_0, n)$, and $V^i_o(\theta h_0, n)/V^i_o(h_0, n) = V^i_y(\theta h_0, n)/V^i_y(h_0, n) = \theta$.

Further, I restrict the equilibrium to be symmetric across countries.

Assumption 3: $W^i = W^j$, $V^i_o = V^j_o$, $g^i = g^j$, and $q^i = q^j$ for all i and j.

[5] It is feasible to have $\tilde{n}^i = (0, n^i)$ and $\tau^i = 1$. See footnote 4.

Now the value functions can be rewritten as:

$$\check{W}^i(n) = (1 - \max_j\{\check{q}^j(n)\}) \cdot \check{W}_1^i(n) + \max_j\{\check{q}^j(n)\} \cdot \max_{\tilde{n}^i}\{\check{W}_2^i(n, \tilde{n}^i)\}, \tag{4$'$}$$

where

$$\check{W}_1^i(n) = \check{Y}(n^i, (0, n^i)) + \beta \check{V}_o^i(n); \tag{5$'$}$$

$$\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda \check{V}_o^i(\check{G}^{-i}(n, \tilde{n}^i)); \tag{6$'$}$$

$$\check{V}_o^i(n) = \mu \cdot (\check{W}^i(n) - \beta\lambda \check{V}_o^i(\check{G}^{-i}(n, \bar{n}))) + (1 - \mu) \cdot \check{Y}(n^i, \bar{n}); \tag{7$'$}$$

$\check{G}^{-i}(n, \tilde{n}^i) \equiv G^{-i}(1, n, \tilde{n}^i)$; $\check{Y}(n, \tilde{n}) \equiv Y(1, n, \tilde{n})$; $\check{q}^i(n) \equiv q^i(1, n) = 1$ if there is a technology adoption in country i; and $\check{q}^i(n) = 0$ if not. Let $\check{g}^i(n) \equiv (\check{g}_0^i(n), \check{g}_1^i(n), \ldots) \equiv g^i(1, n)$ denote the policy function on the human capital distribution. The stagnation premium is:

$$SP^i(n) = \begin{cases} 0 & \text{if } \max_{j \neq i}\{\check{q}^j(n)\} = 1; \\ \Lambda^i(n) & \text{if } \max_{j \neq i}\{\check{q}^j(n)\} = 0, \end{cases} \tag{8$'$}$$

where $\Lambda^i(n) \equiv \check{W}_1^i(n) - \max_{\tilde{n}^i}\{\check{Y}(n^i, \tilde{n}^i) + \beta\lambda \check{V}_o^i(((0, n), \ldots, \tilde{n}^i, \ldots, (0, n)))\}$. The function Λ^i is the premium conditional on all the other countries not adopting the frontier technology. Note that the stagnation premium is zero if at least one country adopts the frontier technology: It takes only one country to advance the frontier technology. Thus, there can be a negative externality of technology adoption by one country on the current generations in the other countries. On the other hand, a technology adoption carries a positive externality to the future generations in the world for the same reason as in Section 2.

As in Section 2, I will characterize the equilibrium mainly in terms of the productivity parameter ϕ and the distribution of human capital n. For a detailed derivation of the equilibrium, see Steps 15 to 22 in the Appendices. Unlike the equilibrium of a country in autarky, the equilibrium of the multiple country world is not unique. This is perhaps not surprising given that each country makes the adoption and matching decisions taking as given the adoption and matching behavior of the other countries as well as that of the future generations in the country. Multiple equilibria can result, in particular, in terms of the coordination failure among countries as will be discussed below. My strategy is, on the one hand, to look for an equilibrium that is heuristically similar to that in autarky and, on the other hand, to consider the equilibrium with a complete coordination failure.

Zone A. Unconditional Growth World Equilibrium 1 (UGWE 1):

$$\check{q}^i(n) = 1 \text{ and } \check{g}^i(n) = (1, 0, 0, \ldots) \text{ for all } n \tag{14}$$

if $\phi \geq \bar{\phi}$.

In this equilibrium, there are no matches across the generations: Each old person works alone and each young person adopts the frontier technology regardless of n. The return from adopting the frontier technology dominates any other considerations in this range of parameters, as in the Unconditional Growth Equilibrium in Section 2. Given the initial distribution of human capital n, I can derive the equilibrium path by repeatedly updating $\tilde{n}$ and n using $\check{q}(n)$ and $\check{g}(n)$. I have

Result 6: Under Unconditional Growth World Equilibrium 1 (UGWE 1), the world economy grows perpetually starting from any n.

Zones B and C. Unconditional Growth World Equilibrium 2 (UGWE 2):

$$\check{q}^i(n) = 1 \text{ and } \check{g}^i(n) = (1 - n_1^i, n_1^i, 0, ...) \text{ for all } n \tag{15}$$

if $\tilde{\phi}_w \leq \phi \leq \bar{\phi}$.

In this equilibrium, the old people with the best technology in use are matched with the young while the remaining old people are not matched and the remaining young people adopt the new technology regardless of n. Consequently, the world economy grows perpetually. This equilibrium is different from any equilibria in Section 2. From the perspective of a single country, the frontier technology advances exogenously regardless of its own adoption decision. Therefore, there is no stagnation premium and every country adopts the frontier technology, which in turn validates the advancement of the frontier technology. This equilibrium hinges on a coordination failure: For some n, the current generations in all countries may prefer the world economy to stagnate but they are unable to coordinate their decisions: $\{n : SP^j(n) \geq 0 \text{ for all } j\} = \emptyset$ while $\{n : \Lambda^j(n) \geq 0 \text{ for all } j\} \neq \emptyset$.[6]

Result 7: Under Unconditional Growth World Equilibrium 2 (UGWE 2), the world economy grows perpetually starting from any n.

Zone B. Conditional Growth World Equilibrium 1 (CGWE 1):

$$\begin{aligned}
&\check{q}^i(n) = 0 \text{ and } \check{g}^i(n) = (0, n_1^i, n_2^i, \ldots) \text{ for } n \in \tilde{\Upsilon}_{stag1},\\
&\check{q}^i(n) = 1 \text{ and } \check{g}^i(n) = (1, 0, 0, \ldots) \text{ for } n \in \tilde{\Upsilon}_{stag2},\\
&\check{q}^i(n) = 1 \text{ and } \check{g}^i(n) = (1 - n_1^i, n_1^i, 0, \ldots) \text{ for } n \in \tilde{\Upsilon}_{stag3} \cup \tilde{\Upsilon}^i_{grow1}, \text{ and}\\
&\check{q}^i(n) = 1 \text{ and } \check{g}^i(n) = (\hat{m}, 1 - \hat{m}, 0, \ldots) \text{ for } n \in \tilde{\Upsilon}^i_{grow2}
\end{aligned} \tag{16}$$

if $\hat{\phi} \leq \phi \leq \bar{\phi}$, where $\tilde{\Upsilon}^i_{grow1} \equiv \{n : n_1^j > \hat{m} \text{ for some } j,\ n_1^j < 1 - \hat{\rho}\hat{m} \text{ for some } j\} - \tilde{\Upsilon}^i_{grow2}$; $\tilde{\Upsilon}^i_{grow2} \equiv \{n : n_1^j > \hat{m} \text{ for some } j,\ 1 - \hat{m} < n_1^i < 1 - \hat{\rho}\hat{m}, \text{ and } n_1^j > n_1^i \text{ for all } j \neq i\}$; $\tilde{\Upsilon}_{stag1} \equiv \{n : SP^j(n) \geq 0 \text{ for all } j\} = \{n : n_1^j = 1 \text{ for all } j\} \subset \{n : \Lambda^j(n) \geq 0 \text{ for all } j\}$;

[6] The extreme case of the coordination failure is when $n^i = \bar{n}$ for all i. I have $\check{g}^i(n) = (0, \bar{n})$ for all i while $\check{q}^i(n) = 1$ for all i. By choosing not to adopt the frontier technology, the current generations in any country are not worse off regardless of technology adoption decisions of the other countries, and would be better off if all the other countries chose not to adopt the frontier technology either. Recall that I assumed away any costs of technology adoption at the aggregate level: It takes one person out of an infinite number of young people to advance the frontier technology so as to allow the choice of $\tilde{n}^i = (0, n^i)$ and $\tau^i = 1$ for any i (see footnotes 4 and 5). If there is a non-negligible cost of technology adoption at the aggregate level, the perpetual growth path may not be sustainable for some n. On the other hand, there are costs of stagnation from which I have abstracted, too. For example, suppose that not all of the young people are able to adopt the frontier technology. Then, the world economy would never be at or near the state of $n^i = \bar{n}$ for all i, and the current generations in any country would be always better off by having some young people adopt the frontier technology as long as the other countries adopt the frontier technology, too. All these additional modeling details would not add substance to the logic of stagnation in the equilibrium as modeled here.

$\tilde{\Upsilon}_{stag2} \equiv \{n : n_1^j \leq \hat{m}$ for all $j\}$; and $\tilde{\Upsilon}_{stag3} \equiv \{n : n_1^j \geq 1 - \hat{\rho}\hat{m}$ for all $j\} - \tilde{\Upsilon}_{stag1} - \tilde{\Upsilon}_{stag2}$; $\hat{m} \equiv \beta^2\lambda\mu^2/(1 - \beta\mu - \beta^2\lambda^2\mu^2) > \hat{n}$; and $\hat{\rho} \equiv 1/(1 + \beta\lambda\mu)$.

In this equilibrium, the adoption and matching decisions depend on the distribution of human capital in ways analogous to Conditional Growth Equilibrium 1 in Section 2.[7] The world economy stagnates perpetually if $n \in \tilde{\Upsilon}_{stag1}$; it stagnates perpetually after one period of growth if $n \in \tilde{\Upsilon}_{stag2}$;[8] it stagnates perpetually after two periods of growth if $n \in \tilde{\Upsilon}_{stag3}$; and it grows perpetually if $n \in \tilde{\Upsilon}^i_{grow1} \cup \tilde{\Upsilon}^i_{grow2}$. Note that the world economy stagnates perpetually only if all of the old people embody the best technology in use ($n_1^j = 1$) in all countries. If some old people do not embody the best technology in use in some countries, the current generations in all countries may still prefer the world economy to stagnate. In other words, there are some coordination failures: $\{n : SP^j(n) \geq 0$ for all $j\} = \{n : n_1^j = 1$ for all $j\} \subset \{n : \Lambda^j(n) \geq 0$ for all $j\}$. The extreme threshold for stagnation is a technical convenience in constructing an equilibrium analogous to Conditional Growth Equilibrium 1: The threshold can be lowered at a considerable cost of complication due to the discontinuity of value functions. However, the coordination failures are not sufficient to generate perpetual growth except possibly when $n \in \tilde{\Upsilon}^i_{grow2}$ for some i: $n \in \tilde{\Upsilon}_{stag1} \cup \tilde{\Upsilon}_{stag2} \cup \tilde{\Upsilon}_{stag3}$ if $n \in \{n : \Lambda^j(n) \geq 0$ for all $j\} - \tilde{\Upsilon}^i_{grow2}$ for any i. Thus, the growth bias of the coordination failures is limited. I have

[7] The analogy is not complete in two aspects. First, note that some old people with the best technology in use may not be matched in one country along a perpetual growth path ($\tilde{\Upsilon}^i_{grow2}$). See footnote 8 for a discussion. Second, note that it requires all of the old people in all countries to embody the best technology in use for the world economy to stagnate ($\tilde{\Upsilon}_{stag1}$), while a country in autarky can stagnate even when some old people do not embody the best technology in use (Υ_{stag1}). See the discussion below.

[8] Country i with $n \in \tilde{\Upsilon}^i_{grow2}$ chooses to have a sufficient fraction, $\hat{m}$, of young people to adopt the frontier technology in order to be on a perpetual growth path, leaving some old people with the best technology in use unmatched. The peculiar adoption and matching decisions of the country are due to a discontinuity of value functions. Note that the set $\tilde{\Upsilon}^i_{grow2}$ selects one country, i, among all countries, j's, with $1 - \hat{m} < n_1^j < 1 - \hat{\rho}\hat{m}$. It is of no significance which country is chosen for the growth path of the world economy. For simplicity, I have chosen a country with the smallest fraction of the old people with the best technology in use. Strictly speaking, this leaves selection indeterminate when two countries share the smallest fraction of the old people with the best technology in use. I could add the randomization of the chosen country to address this technical issue. I have not done so for the sake of saving on notation.

Result 8: Under Conditional Growth World Equilibrium 1 (CGWE 1), the world economy grows perpetually if $n_1^j > \hat{m}$ for some j and $n_1^j < 1 - \hat{\rho}\hat{m}$ for some j; the world economy stagnates perpetually after, at most, two periods of growth if $n_1^j \leq \hat{m}$ for all j, or if $n_1^j \geq 1 - \hat{\rho}\hat{m}$ for all j.

Zone C. Conditional Growth World Equilibrium 2 (CGWE 2):

$$\begin{aligned} &\check{q}^i(n) = 0 \text{ and } \check{g}^i(n) = (0, n_1^i, n_2^i, \ldots) \text{ for } n \in \tilde{\Omega}_{stag1} \\ &\check{q}^i(n) = 1 \text{ and } \check{g}^i(n) = (1 - n_1^i, n_1^i, 0, \ldots) \text{ for } n \in \tilde{\Omega}_{stag2} \cup \tilde{\Omega}^i_{stag4} \cup \tilde{\Omega}_{stag5} \cup \tilde{\Omega}_{grow} \\ &\check{q}^i(n) = 1 \text{ and } \check{g}^i(n) = (\check{m}, 1 - \check{m}, 0, \ldots) \text{ for } n \in \tilde{\Omega}^i_{stag3} \end{aligned} \quad (17)$$

if $\tilde{\phi}_w < \phi \leq \hat{\phi}$, where $\tilde{\Omega}_{grow} \equiv \{n : n_1^j > 1 - \check{m}$ for some j and $n_1^j < \check{m}$ for some $j\}$; $\tilde{\Omega}_{stag1} \equiv \{n : SP^j(n) \geq 0$ for all $j\} = \{n : \Lambda^j(n) \geq 0$ for all $j\} \subset \{n : n_1^j \geq \check{m}\}$; $\tilde{\Omega}_{stag2} \equiv \{n : n_1^j \leq 1 - \check{m}$ for all $j\}$; $\tilde{\Omega}^i_{stag3} \equiv \{n : 1 - \check{m} \leq n_1^i \leq 1 - \check{\rho}\check{m}$ and $\check{m} \leq n_1^j \leq 1 - \check{m}$ for all $j \neq i\} - \tilde{\Omega}_{stag1}$; $\tilde{\Omega}^i_{stag4} \equiv \{n : 1 - \check{m} \leq n_1^k \leq 1 - \check{\rho}\check{m}$ for some $k \neq i$ and $\check{m} \leq n_1^j \leq 1 - \check{m}$ for all $j \neq k\} - \tilde{\Omega}_{stag1}$; $\tilde{\Omega}_{stag5} \equiv \{n : n_1^j \geq \check{m}$ for all $j\} - \tilde{\Omega}_{stag1} \cup \tilde{\Omega}_{stag2} \cup \tilde{\Omega}_{stag3} \cup \tilde{\Omega}_{stag4}$; $\check{m} \equiv (1 + \beta\lambda\mu)(1 - \beta\mu - \beta\lambda^2\mu(1 - \beta\lambda\mu) - \phi((1 - \beta)(1 - \beta\mu) + \beta\lambda^2(1 - \mu)(1 - \beta\lambda\mu)))/(1 - \beta\lambda^2\mu - \phi(1 - \beta + \beta\lambda^2(1 - \mu)))/(1 - \beta\mu - \beta^2\lambda^2\mu^2) < \check{n}$; $\check{m}$ is increasing in ϕ; $\check{m} = 1 - \hat{m}$ when $\phi = \hat{\phi}$; $\check{m} \to 0$ as $\phi \to \tilde{\phi}_w$; $\check{\rho} \equiv (1 - \beta\mu)(1 - \phi(1 - \beta + \beta\lambda))/(1 + \beta\lambda\mu)/(1 - \beta\mu - \beta\lambda\mu + \beta^2\lambda^2\mu^2 - \phi((1 - \beta)(1 - \beta\mu) + \beta\lambda(1 - \mu)(1 - \beta\lambda\mu)))$.

This equilibrium is analogous to Conditional Growth Equilibrium 2 in Section 2.[9] The world economy stagnates perpetually if $n \in \tilde{\Omega}_{stag1}$; it stagnates perpetually after one period of growth if $n \in \tilde{\Omega}_{stag2} \cup \tilde{\Omega}^i_{stag3} \cup \tilde{\Omega}^i_{stag4}$;[10] it stagnates perpetually after two periods of growth if $n \in \tilde{\Omega}_{stag5}$; and it grows perpetually if $n \in \tilde{\Omega}_{grow}$.

[9] The analogy is not complete in that some old people with the best technology in use may not be matched in one country along the path leading to a stagnation ($\tilde{\Omega}^i_{stag3}$). See footnote 10 below for a discussion.

[10] Country i with $n \in \tilde{\Omega}^i_{stag3}$ chooses to have a sufficient fraction, $\check{m}$, of young people to adopt the frontier technology in order to induce a stagnation in the world economy in the following period, leaving some old people with the best technology in use unmatched. The peculiar adoption and matching decisions of the country are due to a discontinuity of value functions as in the case of $n \in \tilde{\Upsilon}^i_{grow2}$ (see footnote 8). Note that $\tilde{\Omega}^i_{stag3}$ is non-empty only if $\check{m} \leq 1/2$.

Result 9: Under Conditional Growth World Equilibrium 2 (CGWE 2), the world economy grows perpetually if $n_1^j > 1 - \check{m}$ for some j and $n_1^j < \check{m}$ for some j; the world economy stagnates perpetually after, at most, two periods of growth if $n_1^j \leq 1 - \check{m}$ for all j, or if $n_1^j \geq \check{m}$ for all j.

For Zones D and E ($\phi \leq \tilde{\phi}_w$), it is more complicated to characterize the equilibrium. In particular, the symmetric equilibrium does not exist depending on the parameter values.

Reflecting on Results 8 and 9, note that the diversity of countries in terms of human capital distribution promotes growth in the world. The intuition is that with diversity come differential gains from adopting the frontier technology across countries and the frontier technology advances as long as it is advantageous to, at least, one country. Also, comparing Results 7 and 8 or comparing Results 7 and 9, note the multiple equilibria[11] when the diversity is limited: Stagnation is advantageous to a country as long as other countries stagnate, but growth is advantageous if others grow. This is a consequence of the coordination failure: The current generations of a country can enjoy the stagnation premium by not adopting the frontier technology only if the other countries do not adopt the frontier technology either.

4. The Comparison of World Economies

I compare three versions of the world economy. The first world economy is a collection of countries, each in autarky as described in Section 3. The second world economy is a collection of countries that share the frontier technology but are separated in the intergenerational bargaining as described in Section 4. The third world economy is a political union with a single frontier technology and a single bargaining, equivalent to a single autarky in Section 3. I consider economic integration as the transition from the first to the

[11] I mean economically significant variations in equilibrium, aside from minor variations due to technical details.

second world economy and political integration as the transition from the second to the third world economy. I assume that the transition is an unexpected or a small-probability event so that the evaluation of the transition can be done in terms of the value functions in Sections 2 and 3 in approximation. Comparing the adoption and matching behavior in Section 2 (Results 1 to 3) and that in Section 3 (Results 6 to 9), I have the following variations of transition in Zones A to C:

Table: Variations of Transition

	Perpetual Growth in economic integration and political fragmentation	Eventual Stagnation in economic integration and political fragmentation
Perpetual Growth in autarky or in political union	UGE to UGWE 1 CGE 1 to UGWE 2 CGE 2 to UGWE 2 CGE 1 to CGWE 1 CGE 2 to CGWE 2	— — — CGE 1 to CGWE 1 CGE 2 to CGWE 2
Eventual Stagnation in autarky or in political union	— CGE 1 to UGWE 2 CGE 2 to UGWE 2 CGE 1 to CGWE 1 CGE 2 to CGWE 2	— — — CGE 1 to CGWE 1 CGE 2 to CGWE 2

4.1 Economic Integration

First, consider the transition of the world economy from a collection of countries, $\{1, 2, ..., I\}$, each in autarky as described in Section 2 to those sharing the frontier technology as described in Section 3.

Zone A

In this zone ($\phi \geq \bar{\phi}$), a country grows unconditionally both in autarky and in the economically integrated world. Technically, there is no equilibrium in which a country stagnates in some states even in the economically integrated world (see Step 17 in Appendices).

Zone B

In this zone ($\hat{\phi} \leq \phi \leq \bar{\phi}$), a country may grow perpetually or stagnate eventually depending on the distribution of human capital in autarky. In the economically integrated world, there are variations of equilibrium in terms of the coordination failure: The country grows perpetually regardless of the distribution of human capital across countries if there is a complete coordination failure; it may grow perpetually or stagnate eventually depending on the distribution of human capital across countries if the coordination failure is limited. I highlight the stagnation-to-growth and the growth-to-stagnation transitions:

Result 10: Under the transition from Conditional Growth Equilibrium 1 (CGE 1) to Unconditional Growth World Equilibrium 2 (UGWE 2), country i, for any i, stagnates eventually in autarky but grows perpetually in the economically integrated world if $n_1^i \in [0, \hat{n}] \cup [1 - \hat{n}, 1]$.

Result 11: Under the transition from Conditional Growth Equilibrium 1 (CGE 1) to Conditional Growth World Equilibrium 1 (CGWE 1), (a) country i, for any i, stagnates eventually in autarky but grows perpetually in the economically integrated world if $n_1^i \in [0, \hat{n}] \cup [1 - \hat{n}, 1]$, $n_1^j \in [\hat{m}, 1]$ for some j, and $n_1^j \in [0, 1 - \hat{\rho}\hat{m}]$ for some j; (b) country i, for any i, grows perpetually in autarky but stagnates eventually in the economically integrated world if $n_1^i \in [\hat{n}, 1 - \hat{n}]$ and '$n_1^j \in [0, \hat{m}]$ for all j or $n_1^j \in [1 - \hat{\rho}\hat{m}, 1]$ for all j'.

Zone C

In this zone ($\tilde{\phi}_w \leq \phi \leq \hat{\phi}$), the characterization of the transition is similar to Zone B in terms of whether there will be a perpetual growth or an eventual stagnation although it may differ in terms of the equilibrium dynamics over a few periods. Again, I highlight the stagnation-to-growth and the growth-to-stagnation transitions.

Result 12: Under the transition from Conditional Growth Equilibrium 2 (CGE 2) to Unconditional Growth World Equilibrium 2 (UGWE 2), country i, for any i, stagnates eventually in autarky but grows perpetually in the economically integrated world if $n_1^i \in [0, 1 - \check{n}] \cup [\check{n}, 1]$.

Result 13: Under the transition from Conditional Growth Equilibrium 2 (CGE 2) to Conditional Growth World Equilibrium 2 (CGWE 2), (a) country i, for any i, stagnates eventually in autarky but grows perpetually in the economically integrated world if $n_1^i \in [0, 1 - \check{n}] \cup [\check{n}, 1]$, $n_1^j \in [1 - \check{m}, 1]$ for some j, and $n_1^j \in [0, \check{m}]$ for some j; (b) country i, for any i, grows perpetually in autarky but stagnates eventually in the economically integrated world if $n_1^i \in [1 - \check{n}, \check{n}]$ and '$n_1^j \in [0, 1 - \check{m}]$ for all j or $n_1^j \in [\check{m}, 1]$ for all j'.

Zone D

In this zone ($\tilde{\phi}_a \leq \phi \leq \tilde{\phi}_w$), every country stagnates eventually in autarky (USE 1) while, if an equilibrium exists, there may be a perpetual growth in the economically integrated world depending on parameter values and the distribution of human capital across countries. A consequence is that economic integration will *not* turn a perpetual growth to an eventual stagnation.

Zone E

In this zone ($\phi \leq \tilde{\phi}_a$), country i, for any i, stagnates in autarky (USE 2) if $\sum_{s \leq u} n_s^i = 1$, where $u \geq 2$ and u is increasing as ϕ is decreasing, while, if an equilibrium exists, there may be perpetual growth in the economically integrated world depending on parameter values and the distribution of human capital across countries. A consequence is that economic integration will *not* turn perpetual growth to an eventual stagnation if $\sum_{s \leq u} n_s^j = 1$ for all j.

Reflecting on case (a) in Results 11 and 13, note that the diversity of the human capital distribution across countries can turn a stagnating country into a growing one upon economic integration. Reflecting on case (b) in Results 11 and 13, also note that it is possible for economic integration to turn a growing country into a stagnating one when the diversity across countries is limited.[12]

Now consider the effect of economic integration on the aggregate utility of the country in transition. Assume that $\tilde{\phi}_w \leq \phi \leq \bar{\phi}$ and Unconditional Growth World Equilibrium 2 holds after economic integration. In order to motivate the choice of Unconditional Growth World Equilibrium 2 as the equilibrium, note the fundamental asymmetry between the advancement and the stagnation of the frontier technology when there are multiple countries: It takes only one country to advance the technology while it takes all of the countries to hold the technology. Loosely speaking outside the current model, random

[12] The growth-to-stagnation transition requires that the fraction of the old people with the best technology in use is neither too high nor too low ($n_1^i \in [\hat{n}, 1-\hat{n}]$ or $n_1^i \in [1-\check{n}, \check{n}]$) in the country while it is either high enough ($n_1^j \in [1-\hat{\rho}\hat{m}, 1]$ for all j or $n_1^j \in [\check{m}, 1]$ for all j) or low enough ($n_1^j \in [0, \hat{m}]$ for all j or $n_1^j \in [0, 1-\check{m}]$ for all j) in all countries. Such a distribution of human capital exists since $\hat{n} < \hat{m}$ and $\check{n} > \check{m}$. Further, it requires that $\phi \geq \check{\phi}_a$ so that $\check{n} \geq 1/2$. In order to gain intuition, consider a country in autarky, call it country A and a country in economic integration, call it country B. Assume that the two countries, A and B, and the rest of the countries in economic integration all have the same human capital distribution. The equality of human capital distribution between country B and the rest of countries in economic integration is to eliminate any diversity. Suppose that both countries stagnate in equilibrium ($n_1^A = n_1^B = 1$ if $\phi \geq \hat{\phi}$; $n_1^A = n_1^B \geq \check{n}$ if $\phi \leq \hat{\phi}$). If the young generation were to exercise the outside option of an all-out-technology-adoption, it would lead to stagnation in country A in the next period, but it would lead to growth in country B in the next period by triggering the growth in the other countries. In other words, country A would enjoy the stagnation premium but country B would not. Thus, the reservation utility of the young generation is higher and the old generation's utility is lower in country A than in country B. Since the same situation is repeated every period in stagnation, the aggregate utility, i.e., the current output plus the discounted utility of the young generation's utility when they become old, is lower in country A than in country B. The best alternative to stagnation is to grow for two periods and then stagnate in both country A and country B, assuming that $\phi \geq \check{\phi}_a$. There would be no stagnation premium in both countries in the next period. In fact, the aggregate utility of this alternative is the same between the countries. Therefore, the advantage of stagnation is smaller in country A than in country B. The smaller advantage of stagnation makes the eventual stagnation less likely for a country in autarky than a country in economic integration, holding the other determinants of stagnation.

factors can disrupt stagnation more easily than growth in a multi-country world. Further, I focus on the distribution of human capital that is spread over the two best technologies in use: $n_1^i + n_2^i = 1$ for the country i in transition.

Let $\tilde{W}(n^i) \equiv \check{W}^i(n) = \check{W}^i(..., n^{i-1}, n^i, n^{i+1}, ...)$ under Unconditional Growth World Equilibrium 2. Note that $\tilde{W}(n^i)$ is independent of $\{n^j\}_{j \neq i}$. I can show that:

$$\begin{aligned}
&\hat{W}(n^i) \geq \tilde{W}(n^i) \quad \text{for all } n^i \text{ with } n_1^i + n_2^i = 1 \text{ and } n_1^i \in [\check{x}, 1]; \\
&\hat{W}(n^i) \leq \tilde{W}(n^i) \quad \text{for all } n^i \text{ with } n_1^i + n_2^i = 1 \text{ and } n_1^i \in [\hat{x}, \check{x}]; \\
&\hat{W}(n^i) \geq \tilde{W}(n^i) \quad \text{for all } n^i \text{ with } n_1^i + n_2^i = 1 \text{ and } n_1^i \in [0, \hat{x}],
\end{aligned} \tag{18}$$

where $\check{x} = ((1-\beta\mu+\beta\lambda\mu)(1+\beta\lambda\mu-\beta\lambda^2\mu)-\beta^3\lambda^2\mu^3-\phi(\beta\lambda(1+\beta\lambda\mu)(\mu(1-\beta)+\lambda(1-\mu))+(1-\beta\mu)((1-\beta)(1+\beta\lambda\mu(1+\beta\lambda\mu))+\beta^3\lambda^3\mu^2)))/(1-\beta\mu+\beta\lambda\mu)/(1-\beta\lambda^2\mu-\phi(1-\beta+\beta\lambda^2-\beta\lambda^2\mu))$; $\check{x}$ is increasing in ϕ if $\tilde{\phi}_w \leq \phi \leq \bar{\phi}$; $\check{x} = 1$ when $\phi = \bar{\phi}$; and $\check{x} = \hat{x}$ when $\phi = \check{\phi}_3 \equiv (1-\beta\mu+2\beta\lambda\mu-\beta\lambda^2\mu-\beta^2\lambda\mu^2)/((1-\beta)(1-\beta\mu+\beta\lambda\mu)+\beta\lambda^2(1-\mu)+\beta\lambda\mu(1-\beta\mu)(1-\beta+\beta\lambda))$; $\hat{x} = \beta^2\lambda\mu^2/(1-\beta\mu+\beta\lambda\mu)$ if $\hat{\phi} \leq \phi \leq \bar{\phi}$; $\hat{x} = \beta\lambda\mu(1-\phi(1-\beta+\beta\lambda))(1-\beta\mu)/(1-\beta\mu+\beta\lambda\mu)/(\phi(1-\beta+\beta\lambda^2(1-\mu))-(1-\beta\lambda^2\mu))$ if $\tilde{\phi}_w \leq \phi \leq \hat{\phi}$; $\hat{x}$ is decreasing in ϕ if $\tilde{\phi}_w \leq \phi \leq \hat{\phi}$; and $\hat{x} = \check{x}$ when $\phi = \check{\phi}_3$. Figure 2 shown at the end of the paper illustrates $\hat{W}(n^i)$ and $\tilde{W}(n^i)$ when $n_1^i + n_2^i = 1$, $\beta = .5$, $\lambda = 1.5$, $\mu = .5$, and $\phi = .65$. I have

Result 14: Under the transition from Conditional Growth Equilibrium 1 or 2 (CGE 1 or CGE 2) to Conditional Growth World Equilibrium 2 (CGWE 2), economic integration raises the aggregate utility of country i, for any i, if $n_1^i \in (\hat{x}, \check{x})$ and $n_1^i + n_2^i = 1$; economic integration lowers the aggregate utility of country i, for any i, if $n_1^i \in [0, \hat{x}) \cup (\check{x}, 1]$ and $n_1^i + n_2^i = 1$.

I can show that $\hat{x} \leq \hat{n}$ and $\check{n} \leq \check{x} \leq 1$ if $\hat{\phi} \leq \phi \leq \bar{\phi}$; $\hat{x} \leq 1 - \check{n}$ and $\check{n} \leq \check{x} \leq 1 - \hat{x}$ if $\check{\phi}_3 \leq \phi \leq \hat{\phi}$; and $\check{n} = \check{x} = \hat{x} = 1/(2+\beta\lambda\mu) < 1/2$ if $\phi = \check{\phi}_3$. Consulting Results 2 and 3, I have

Result 15: Under the transition from Conditional Growth Equilibrium 1 or 2 (CGE 1 or CGE 2) to Conditional Growth World Equilibrium 2 (CGWE 2), country i, for any i, would have stagnated perpetually after at most one period of growth in autarky if economic integration lowers the aggregate utility of the country and if $n_1^i + n_2^i = 1$.

The intuition is that economic integration eliminates the option to hold the frontier technology and to enjoy the stagnation premium. Loosely speaking, the current generations of the country would prefer to isolate itself from the world permanently. However, this is not feasible. Since the rest of the world will continue to grow, the advantage of adopting the frontier technology would eventually outweigh the loss of the stagnation premium.

4.2 Political Integration

Now consider the transition of an economically integrated but politically fragmented world as described in Section 3 to a single political union as described in Section 2. The distribution of human capital in the union is:

$$n^u = \sum_i \eta^i n^i, \tag{18}$$

where η^i is country i's share of the world population: $\sum_i \eta^i = 1$. The characterization of transition is equivalent to Zones A to E in Section 4.1 except that the change is in the opposite direction, and country i is replaced by political union u. I highlight the growth-to-stagnation transitions starting from Unconditional Growth World Equilibrium 2 (UGWE 2).

Result 16 (Reverse of Result 10): Under the transition from Unconditional Growth World Equilibrium 2 (UGWE 2) to Conditional Growth Equilibrium 1 (CGE 1), the world economy grows perpetually in political fragmentation but stagnates eventually in political union if $n_1^u \in [0, \hat{n}] \cup [1 - \hat{n}, 1]$.

Result 17 (Reverse of Result 12): Under the transition from Unconditional Growth World Equilibrium 2 (UGWE 2) to Conditional Growth Equilibrium 2 (CGE 2), the world economy grows perpetually in political fragmentation but stagnates eventually in political union if $n_1^u \in [0, 1 - \check{n}] \cup [\check{n}, 1]$.

Now consider a once-and-for-all formation of political union. As in Section 4.1, assume that $\tilde{\phi}_w \leq \phi \leq \bar{\phi}$ and Unconditional Growth World Equilibrium 2 holds in political fragmentation, and focus on the distribution of human capital that is spread over the two best technologies in use: $n_1^u + n_2^u = 1$. I can show that $\tilde{W}(n^i)$ is linear in n^i so that:

$$\sum\nolimits_i \eta^i \tilde{W}(n^i) = \tilde{W}\left(\sum\nolimits_i \eta^i n^i\right) = \tilde{W}\left(n^u\right) \tag{19}$$

for all $\{\eta^i\}$ and $\{n^i\}$. Thus, the utility comparison of a politically fragmented world and a political union is analogous to the utility comparison of a country in autarky and a country in economic integration in Section 4.1.

Result 18 (Reverse of Result 14): Under the transition from Unconditional Growth World Equilibrium 2 (UGWE 2) to Conditional Growth Equilibrium 1 or 2 (CGE 1 or CGE 2), political integration raises the worldwide aggregate utility if $n_1^u \in [0, \hat{x}) \cup (\check{x}, 1]$ and $n_1^u + n_2^u = 1$; political integration lowers the worldwide aggregate utility if $n_1^u \in (\hat{x}, \check{x})$ and $n_1^u + n_2^u = 1$.

Result 19 (Reverse of Result 15): Under the transition from Unconditional Growth World Equilibrium 2 (UGWE 2) to Conditional Growth Equilibrium 1 or 2 (CGE 1 or CGE 2), the political union, once formed, stagnates perpetually after at most one period of growth if political integration raises the worldwide aggregate utility and $n_1^u + n_2^u = 1$.

Now suppose that the opportunity to form or break up a political union is a small probability event in every period so that the utility comparison can be done as if the formation or the break-up is permanent in approximation. Assume that the decision of forming or breaking up a political union is made through cross-country bargaining and that the cross-country bargaining is efficient: The bargaining outcome maximizes the worldwide aggregate utility of the current generations. Further, maintain that $\tilde{\phi}_w \leq \phi \leq \bar{\phi}$; Unconditional Growth World Equilibrium 2 holds in political fragmentation; and $n_1^u + n_2^u = 1$.

According to Result 18, the political union would form if and only if $n_1^u \in [0, \hat{x}) \cup (\check{x}, 1]$. Let $\xi^t(n^i) \equiv (\xi_1^i(n^i), \xi_2^i(n^i), \xi_3^i(n^i), \ldots)$ denote the evolution of human capital distribution in country i, for any i, starting from n^i in a politically fragmented world: $\xi^0(n^i) \equiv n^i$ and $\xi^{t+1}(n^i) = \check{g}^i(\xi^t(n^i))$, where $\check{g}^i$ is given by Equation 15. I have

$$\hat{W}\left(\sum_i \eta^i \xi^t(n^i)\right) \leq \sum_i \eta^i \tilde{W}\left(\xi^t(n^i)\right) \tag{20}$$

for all t if $n^u = \sum_i \eta^i n^i \in [\max\{\hat{x}, 1 - \check{x}\}, \min\{\check{x}, 1 - \hat{x}\}]$. Thus, under this condition there will never be an incentive to form a political union. On the other hand, Equation 20 will be violated at least once every two periods if $n_1^u \in [0, \max\{\hat{x}, 1 - \check{x}\}) \cup (\min\{\check{x}, 1 - \hat{x}\}, 1]$. Under this condition, the political union will form eventually.

Now let $\tilde{\xi}^t(n^u) \equiv (\tilde{\xi}_1^t(n^u), \tilde{\xi}_2^t(n^u), \tilde{\xi}_3^t(n^u), \ldots)$ denote the evolution of human capital distribution in the political union starting from $n^u \in [0, \hat{x}) \cup (\check{x}, 1]$: $\tilde{\xi}^0(n^u) = n^u$; $\tilde{\xi}^{t+1}(n^u) = \tilde{\xi}^t(n^u)$ if $\hat{q}(n^u) = 0$; and $\tilde{\xi}^{t+1}(n^u) = \hat{g}(\tilde{\xi}^t(n^u))$ if $\hat{q}(n^u) = 1$. Consulting Results 2 and 3 and using the properties that $\hat{x} \leq \hat{n}$ and $\check{n} \leq \check{x} \leq 1$ if $\hat{\phi} \leq \phi \leq \bar{\phi}$; $\hat{x} \leq 1 - \check{n}$ and $\check{n} \leq \check{x} \leq 1 - \hat{x}$ if $\check{\phi}_3 \leq \phi \leq \hat{\phi}$; and $\check{n} = \check{x} = \hat{x} = 1/(2 + \beta\lambda\mu) < 1/2$ if $\phi = \check{\phi}_3$, I have $\tilde{\xi}^t(n^u) = \tilde{\xi}^1(n^u)$ and $\tilde{\xi}_1^t(n^u) > \check{x} > \check{n}$ for all $t \geq 1$. This repeats the reasoning for Results 16 and 19. Further, it implies that $\hat{W}(\tilde{\xi}^t(n^u)) \geq \tilde{W}(\tilde{\xi}^t(n^u))$ for all t. Combined with (19), I have

$$\hat{W}(\tilde{\xi}^t(n^u)) \geq \sum_i \tilde{\eta}_t^i \tilde{W}(m_t^i) \tag{21}$$

for all t and for any $\{\tilde{\eta}_t^i\}$ and $\{m_t^i\}$ that satisfy $\tilde{\xi}^t(n^u) = \sum_i \tilde{\eta}_t^i m_t^i$. Thus, there is no incentive to break up a political union once it is formed. In summary, I have

Result 20: Assume that the opportunity to form or break up the political union is a small probability event and that the formation and the break-up are the transitions between Unconditional Growth World Equilibrium 2 (UGWE 2) and Conditional Growth Equilibrium 1 or 2 (CGE 1 or CGE 2) in approximation. The political union will form eventually if $n_1^u \in [0, \max\{\hat{x}, 1-\check{x}\}) \cup (\min\{\check{x}, 1-\hat{x}\}, 1]$ and $n_1^u + n_2^u = 1$. Once formed, the political union will stagnate perpetually after, at most, one period of growth, and it will never break-up. The world economy will always remain politically fragmented and grow perpetually if $n^u \in [\max\{\hat{x}, 1-\check{x}\}, \min\{\check{x}, 1-\hat{x}\}]$ and $n_1^u + n_2^u = 1$.

That the formation of the political union leads to a perpetual stagnation underlines its nature. Recall that the growth in the politically fragmented world is driven by the diversity of countries in terms of the incentives to adopt the frontier technology and the coordination failure among countries that desire to hold the advancement of technology. The political union addresses both factors: It corrects the coordination failure and adjusts the incentives of individual countries by means of side payments that are implicit in the cross-country bargaining. The outcome is a worldwide perpetual stagnation through the political union if it is advantageous to the current generations in the world. The caveat is that the perpetual stagnation is never efficient in terms of maximizing the discounted utilities of the current and future generations in the world: the utility gain by the current generations is at the greater expense of future generations.[13]

[13] I do not attach a great significance to the efficiency of growth when taking into consideration the utilities of the future generations. This is due to the absence of any costs in advancing the frontier technology at the aggregate level although there are costs of adopting the frontier technology at the individual level. The perpetual stagnation can be improved on by advancing the frontier technology continuously and discarding the old technologies eventually. This is a simple modeling choice that allows a focus on the effects of intergenerational and cross-country bargaining on growth and stagnation; it is simplistic in view of assessing the welfare effects of growth on the current and future generations as a whole.

5. Conclusion

In summary, the economically integrated but politically fragmented world is the most conducive to long-run growth, while an economy in autarky or a political union is more likely to stagnate. Consequently, economic integration (i.e., the sharing of frontier technology among countries) promotes growth while political integration (i.e., the merging of countries into a single bargaining) promotes stagnation. Since a political union is an economy in autarky, political integration is essentially a reverse of economic integration. The implicit asymmetry is that economic integration is exogenous to a country while political integration is a collective choice by all countries.

The model is a formalization of what may be termed 'the political fragmentation hypothesis' in understanding European growth in contrast to Chinese stagnation prior to its adoption of European technology. I have modeled economic integration as the unstoppable diffusion of technology: The unstoppable diffusion of technology defines the boundary of an economically integrated area. I have modeled political integration as efficient bargaining among the current generations in an economically integrated area. As mentioned earlier, economic integration as such was probably at the continental scale prior to the European growth validating the China-Europe comparison in terms of political integration and fragmentation.[14] Since then, economic integration has expanded to a global scale so that nothing short of a global political union can withhold the advancement of technology.

[14] In considering China as an economically integrated area, I have ignored the periphery countries like Korea, Japan, and Vietnam. In terms of the model, it is arguable that China had such a dominant influence on the periphery countries in the centuries preceding European growth that the diversity was limited, and the coordination was easier to achieve in East Asia than in Europe. No single European country held such a dominant position.

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APPENDICES

Derivation of the Equilibrium in Section 2

[1] From (4) to (7), I have

$$\hat{W}(n) = (1 - \hat{q}(n)) \cdot \hat{W}_1(n) + \hat{q}(n) \cdot \max_{\tilde{n}}\{\hat{W}_2(n, \tilde{n})\}, \tag{A1}$$

where

$$\hat{W}_1(n) = \hat{Y}(n, (0, n)) + \beta\mu(\hat{W}(n) - \beta\lambda\hat{V}_o(\bar{n})) + \beta(1 - \mu) \cdot \hat{Y}(n, \bar{n}) \tag{A2}$$

and

$$\hat{W}_2(n, \tilde{n}) = \hat{Y}(n, \tilde{n}) + \beta\lambda\mu(\hat{W}(\tilde{n}) - \beta\lambda\hat{V}_o(\bar{n})) + \beta\lambda(1 - \mu) \cdot \hat{Y}(\tilde{n}, \bar{n}). \tag{A3}$$

Policy function $\hat{q}(n)$ indicates the technology adoption: $\hat{q}(n) = 1$ if there is a technology adoption; $\hat{q}(n) = 0$ if not. Let $\hat{g}(n) \equiv (\hat{g}_0(n), \hat{g}_1(n), \ldots)$ denote the policy function on the human capital distribution. Note that $\hat{g}(n) = (0, n)$ if $\hat{q}(n) = 0$. If $\hat{W}_1(n) \geq \max_{\tilde{n}}\{\hat{W}_2(n, \tilde{n})\}$, $\hat{W}(n) = \hat{W}_1(n) = (\hat{Y}(n, (0, n)) - \beta^2\lambda\mu\hat{V}_o(\bar{n}) + \beta(1-\mu) \cdot \hat{Y}(n, \bar{n}))/(1-\beta\mu)$ so that (A2) can be re-written without a loss of generality:

$$\hat{W}_1(n) = (\hat{Y}(n, (0, n)) - \beta^2\lambda\mu\hat{V}_o(\bar{n}) + \beta(1 - \mu) \cdot \hat{Y}(n, \bar{n}))/(1 - \beta\mu). \tag{A2$'$}$$

[2] I have $\beta\lambda\mu < 1$ from Assumption 1.

[3] Suppose that there exists a $\hat{W}$ that solves (A1) to (A3), and that $\hat{W}(n) < \hat{W}(m)$ for some n and m with $\sum_{s \leq u} n_s \geq \sum_{s \leq u} m_s$ for all $u \geq 1$. I have $\hat{q}(m) = 1$ since $\hat{W}(n) - \hat{W}(m) \geq \hat{W}_1(n) - \hat{W}_1(m) \geq 0$ if $\hat{q}(m) = 0$. There is a z with $\sum_{s \leq u} z_s \geq \sum_{s \leq u} g_s(m)$ for all $u \geq 1$ and $\hat{Y}(n, z) \geq \hat{Y}(m, \hat{g}(m))$. Let $\tilde{z}(n, m)$ be an arbitrary function that satisfies this property. I have $\hat{W}(n) - \hat{W}(m) \geq \hat{Y}(n, \tilde{z}(n, m)) - \hat{Y}(m, \hat{g}(m)) + \beta\lambda\mu(\hat{W}(\tilde{z}(n, m)) -$

$\hat{W}(g(m))) + \beta\lambda(1-\mu)(\hat{Y}(\tilde{z}(n,m),\bar{n}) - \hat{Y}(g(m),\bar{n}))$ so that $\hat{W}(\tilde{z}(n,m)) < \hat{W}(g(m))$. Repeating the above reasoning, $\hat{q}(\hat{g}(m)) = 1$, $\hat{q}(\hat{g}(\hat{g}(m))) = 1$, and so on. I have

$$
\begin{aligned}
\hat{W}(n) &\geq \hat{Y}(n,\tilde{z}(n,m)) + \beta\lambda(1-\mu)\cdot\hat{Y}(\tilde{z}(n,m),\bar{n}) + \beta\lambda\mu\hat{W}(\tilde{z}(n,m)) \\
&\geq \hat{Y}(n,\tilde{z}(n,m)) + \beta\lambda(1-\mu)\cdot\hat{Y}(\tilde{z}(n,m),\bar{n}) \\
&\quad + \beta\lambda\mu(\hat{Y}(\tilde{z}(n,m),\tilde{z}(\tilde{z}(n,m),g(m))) + \beta\lambda(1-\mu)\cdot\hat{Y}(\tilde{z}(\tilde{z}(n,m),g(m)),\bar{n})) \\
&\quad + (\beta\lambda\mu)^2(\hat{Y}(\tilde{z}(\tilde{z}(n,m),g(m)),\tilde{z}(\tilde{z}(\tilde{z}(n,m),\hat{g}(m)),\hat{g}(\hat{g}(m)))) \\
&\qquad\qquad + \beta\lambda(1-\mu)\cdot\hat{Y}(\tilde{z}(\tilde{z}(\tilde{z}(n,m),\hat{g}(m)),\hat{g}(\hat{g}(m))),\bar{n})) \\
&\quad + \dots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A4)} \\
&\geq \hat{Y}(m,g(m)) + \beta\lambda(1-\mu)\cdot\hat{Y}(\hat{g}(m),\bar{n}) \\
&\quad + \beta\lambda\mu(\hat{Y}(\hat{g}(m),\hat{g}(\hat{g}(m))) + \beta\lambda(1-\mu)\cdot\hat{Y}(\hat{g}(\hat{g}(m)),\bar{n})) \\
&\quad + (\beta\lambda\mu)^2(\hat{Y}(\hat{g}(\hat{g}(m)),\hat{g}(\hat{g}(\hat{g}(m)))) + \beta\lambda(1-\mu)\cdot\hat{Y}(\hat{g}(\hat{g}(\hat{g}(m))),\bar{n})) \\
&\quad + \dots \\
&= \hat{W}(m).
\end{aligned}
$$

This is a contradiction. Therefore, $\hat{W}(n) \geq \hat{W}(m)$ for any n and m with $\sum_{s\leq u} n_s \geq \sum_{s\leq u} m_s$ for all $u \geq 1$.

[4] Suppose that there exists a $\hat{W}$ that solves (A1) to (A3), and that $\hat{q}(n) = 1$ and $\hat{g}_s(n) > 0$ for some n and s. I have

$$
\begin{aligned}
\hat{W}(n) \geq\ & \hat{Y}(n,(\hat{g}_0(n)+\hat{g}_s(n),\hat{g}_1(n),\dots,\hat{g}_{s-1}(n),0,\hat{g}_{s+1}(n),\dots)) \\
& + \beta\lambda(1-\mu)\cdot\hat{Y}((\hat{g}_0(n)+\hat{g}_s(n),\hat{g}_1(n),\dots,\hat{g}_{s-1}(n),0,\hat{g}_{s+1}(n),\dots),\bar{n}) \\
& + \beta\lambda\mu\hat{W}(\hat{g}_0(n)+\hat{g}_s(n),\hat{g}_1(n),\dots,\hat{g}_{s-1}(n),0,\hat{g}_{s+1}(n),\dots) \qquad \text{(A5)} \\
=\ & \hat{W}(n) - \hat{g}_s(n)\cdot\phi(1+\beta\lambda(1-\mu))/\lambda^{s-1} \\
& + \beta\lambda\mu(\hat{W}(\hat{g}_0(n)+\hat{g}_s(n),\hat{g}_1(n),\dots,\hat{g}_{s-1}(n),0,\hat{g}_{s+1}(n),\dots) - \hat{W}(\hat{g}(n))).
\end{aligned}
$$

If $q(g(n)) = 0$,

$$
\begin{aligned}
&\hat{W}(\hat{g}_0(n) + \hat{g}_s(n), \hat{g}_1(n), \ldots, \hat{g}_{s-1}(n), 0, \hat{g}_{s+1}(n), \ldots) - \hat{W}(\hat{g}(n)) \\
\geq\ & g_s(n) \cdot (1 + \beta\phi(1-\mu))(1 - 1/\lambda^s)/(1 - \beta\mu).
\end{aligned} \tag{A6}
$$

If $q(g(n)) = 1$,

$$
\begin{aligned}
&\hat{W}(\hat{g}_0(n) + \hat{g}_s(n), \hat{g}_1(n), \ldots, \hat{g}_{s-1}(n), 0, \hat{g}_{s+1}(n), \ldots) - \hat{W}(\hat{g}(n)) \\
\geq\ & (\hat{g}_{s+1}(\hat{g}(n)) + \phi(\hat{g}_s(n) - \hat{g}_{s+1}(\hat{g}(n))) \cdot (1 - 1/\lambda^s) \\
& + \hat{W}(\hat{g}_0(\hat{g}(n)), \hat{g}_1(\hat{g}(n)) + \hat{g}_{s+1}(\hat{g}(n)), \hat{g}_2(\hat{g}(n)), \ldots \hat{g}_s(\hat{g}(n)), 0, \hat{g}_{s+2}(\hat{g}(n)), \ldots) \\
& - \hat{W}(g(g(n)) \\
\geq\ & \hat{g}_s(n) \cdot \phi(1 - 1/\lambda^s)
\end{aligned} \tag{A7}
$$

since $\hat{W}(n)$ is increasing in n (consult Step 3). From (A5)-(A7), I have $\beta\lambda^s\mu \leq 1 + \beta\mu + \beta\lambda(1-\mu)$. This is a contradiction if s is large enough. Therefore, there exists an S so that for all n and $s > S$, $\hat{g}_s(n) = 0$ if $n_s = 0$.

[5] Set $n \equiv (n_1, \ldots, n_S)$, where S satisfies the above condition.

[6] Let T denote the operator that maps a $\hat{W}$ to a new $\hat{W}$ according to (A1) to (A3) given $V_o(\bar{n})$:

$$
\begin{aligned}
T(\hat{w}|\bar{v}_o)(n) &= \max\{\bar{T}(n|\bar{v}_o), \max_{\tilde{n}}\{\tilde{T}(\hat{w})(n, \tilde{n}|\bar{v}_o)\} \\
&= \max\{\bar{T}(n|\bar{v}_o), \max_{\tilde{n}}\{\hat{Y}(n, \tilde{n}) + \beta\lambda\mu(\hat{w}(\tilde{n}) - \beta\lambda\bar{v}_o) + \beta\lambda(1-\mu) \cdot \hat{Y}(\tilde{n}, \bar{n})\}\},
\end{aligned} \tag{A8}
$$

where $\bar{T}(n|\bar{v}_o) = (\hat{Y}(n, (0, n)) - \beta^2\lambda\mu\bar{v}_o + \beta(1-\mu) \cdot \hat{Y}(n, \bar{n}))/(1 - \beta\mu)$. Note that $\bar{T}$ is continuous in n since $\hat{Y}$ is continuous. Further, if $\hat{w}$ is continuous in n, $\max_{\tilde{n}}\{\tilde{T}(\hat{w})(n, \tilde{n}|\bar{v}_o)\}$ is continuous in n since the choice set of $\tilde{n}$, $\Gamma(n) = \{\tilde{n} : \sum_s \tilde{n}_s = 1$ and $\tilde{n}_s \leq n_s$ for all $s \geq 1\}$, is compact-valued and continuous, according to the Theorem of the Maximum (consult Step 5). I have, for any $\hat{w}$, $\hat{\omega}$, and n, $T(\hat{w}|\bar{v}_o)(n) - T(\hat{\omega}|\bar{v}_o)(n) \leq 0$ if $\bar{T}(n|\bar{v}_o) \geq \max_{\tilde{n}}\{\tilde{T}(\hat{w})(n, \tilde{n}|\bar{v}_o)\}$; and $T(\hat{w}|\bar{v}_o)(n) - T(\hat{\omega}|\bar{v}_o)(n) \leq \beta\lambda\mu(\hat{w}(\arg\max_{\tilde{n}}\{\tilde{T}(\hat{w})(n, \tilde{n}|\bar{v}_o)\}) - \hat{\omega}(\arg\max_{\tilde{n}}\{\tilde{T}(\hat{w})(n, \tilde{n}|\bar{v}_o)\}))$ if $\bar{T}(n|\bar{v}_o) \leq \max_{\tilde{n}}\{\tilde{T}(\hat{w})(n, \tilde{n}|\bar{v}_o)\}$. Conversely, $T(\hat{\omega}|\bar{v}_o)(n) -$

$T(\hat{w}|\bar{v}_o)(n|\bar{v}_o) \leq 0$ if $\bar{T}(n|\bar{v}_o) \geq \max_{\tilde{n}}\{\tilde{T}(\hat{\omega})(n,\tilde{n}|\bar{v}_o)\}$; and $T(\hat{\omega}|\bar{v}_o)(n) - T(\hat{w})(n|\bar{v}_o) \leq \beta\lambda\mu(\hat{\omega}(\arg\max_{\tilde{n}}\{\tilde{T}(\hat{\omega})(n,\tilde{n}|\bar{v}_o\}) - \hat{w}(\arg\max_{\tilde{n}}\{\tilde{T}(\hat{\omega})(n,\tilde{n}|\bar{v}_o))$ if $\bar{T}(n|\bar{v}_o) \leq \max_{\tilde{n}}\{\tilde{T}(\hat{\omega})(n,\tilde{n}|\bar{v}_o)\}$. Then, $\sup_n |T(\hat{w}|\bar{v}_o)(n) - T(\hat{\omega}|\bar{v}_o)(n)| \leq \beta\lambda\mu \sup_n |\hat{w}(n) - \hat{\omega}(n)|$. Mapping T is a contraction, and there is a unique $\hat{W}$ that solves $T(\hat{w}|\bar{v}_o) = \hat{w}$, according to the Contraction Mapping Theorem. Further, $\hat{W} = \lim_{t\to\infty} T^t(\hat{w}|\bar{v}_o)$ for any $\hat{w}$, where $T^{t+1}(\hat{w}|\bar{v}_o) \equiv T(T^t(\hat{w}|\bar{v}_o)|\bar{v}_o)$.

[7] For any $\hat{w}$, $\bar{v}_o$, and $\bar{\nu}_o$ with $\bar{v}_o < \bar{\nu}_o$, $0 \leq T(\hat{w}|\bar{v}_o)(\bar{n}) - T(\hat{w}|\bar{\nu}_o)(\bar{n}) \leq (\bar{\nu}_o - \bar{v}_o) \cdot \max\{\beta^2\lambda\mu/(1-\beta\mu), \beta^2\lambda^2\mu\}$ and

$$\begin{aligned}
0 &\leq T^{t+1}(\hat{w}|\bar{v}_0)(\bar{n}) - T^{t+1}(\hat{w}|\bar{\nu}_o)(\bar{n}) \\
&\leq \max\{(\bar{\nu}_o - \bar{v}_o) \cdot \beta^2\lambda\mu/(1-\beta\mu), \\
&\qquad (\bar{\nu}_o - \bar{v}_o) \cdot \beta^2\lambda^2\mu + \beta\lambda\mu(T^t(\hat{w}|\bar{v}_0)(\bar{n}) - T^t(\hat{w}|\bar{\nu}_o)(\bar{n}))\} \qquad \text{(A9)} \\
&\leq (\bar{\nu}_o - \bar{v}_o) \cdot \max\{\beta^2\lambda\mu/(1-\beta\mu), \beta^2\lambda^2\mu\} + \beta\lambda\mu(T^t(\hat{w}|\bar{v}_0)(\bar{n}) - T^t(\hat{w}|\bar{\nu}_o)(\bar{n})) \\
&\leq (\bar{\nu}_o - \bar{v}_o) \cdot \max\{\beta^2\lambda\mu/(1-\beta\mu), \beta^2\lambda^2\mu\} \cdot (1 - \beta^t\lambda^t\mu^t)/(1-\beta\lambda\mu).
\end{aligned}$$

Then, $0 \leq T^\infty(\hat{w}_2|\bar{v}_0)(\bar{n}) - T^\infty(\hat{w}_2|\bar{\nu}_o)(\bar{n}) \leq \max\{\beta^2\lambda\mu/(1-\beta\mu), \beta^2\lambda^2\mu\}/(1-\beta\lambda\mu)$. This implies that $\hat{W}(\bar{n})$ is continuous and non-increasing in $\bar{v}_o$. On the other hand, $\hat{W}(\bar{n})$ is continuous and increasing in $\bar{v}_o$ without an upper or a lower bound in (6). Therefore, there is a unique set of $\hat{W}$ and $\hat{V}_o$ that solve (A1) to (A3) and (6).

[8] For any n, $s \geq 1$, $u \geq 1$, and $\delta \in [0, n_s]$ with $n_s \geq \delta$, let: $\Delta(n,s,u,\delta) \equiv \max\{\Delta^1(n,s,u,\delta), \Delta^2(n,s,u,\delta)\}$; $\Delta^1(n,s,u,\delta) \equiv \hat{W}_1(n) - \hat{W}_1(n_1,\ldots,n_s-\delta,\ldots,n_u+\delta,\ldots))$; and $\Delta^2(n,s,u,\delta) \equiv \hat{W}_2(n,\hat{g}(n)) - \hat{W}_2((n_0,\ldots,n_s-\delta,\ldots,n_u+\delta,\ldots), g(n_1,\ldots,n_s-\delta,\ldots,n_u+\delta,\ldots))$. I have

$$\Delta^1(n,s,u,\delta) = \delta(1/\lambda^s - 1/\lambda^u) \cdot (1+\beta\phi(1-\mu))/(1-\beta\mu) \qquad \text{(A10)}$$

and

$$
\begin{aligned}
&\Delta^2(n,s,u,\delta)\\
&\le \hat{W}_2(n,\hat{g}(n)) - \hat{W}_2((n_1,...,n_s-\delta,...,n_u+\delta,...),\\
&\qquad\qquad\qquad (\hat{g}_0(n),...,\hat{g}_s(n)-\min[\hat{g}_s(n),\delta],...,\hat{g}_u(n)+\min[\hat{g}_s(n),\delta],...))\\
&\le \delta(1/\lambda^s - 1/\lambda^u)\cdot(1+\beta\phi(1-\mu))\\
&\quad + \beta\lambda\mu(\hat{W}(\hat{g}(n)) - \hat{W}((\hat{g}_0(n),...,\hat{g}_s(n) - \min[\hat{g}_s(n),\delta],...,\hat{g}_u(n) + \min[\hat{g}_s(n),\delta],...))\\
&\le \delta(1/\lambda^s - 1/\lambda^u)\cdot(1+\beta\phi(1-\mu))\\
&\quad + \beta\lambda\mu\cdot\Delta(\hat{g}(n),s+1,u+1,\min[\hat{g}_s(n),\delta])\\
&\le \delta(1/\lambda^s - 1/\lambda^u)\cdot(1+\beta\phi(1-\mu))\\
&\quad + \beta\lambda\mu\max\{\delta(1/\lambda^{s+1} - 1/\lambda^{u+1})\cdot(1+\beta(1-\mu))/(1-\beta\mu),\\
&\qquad\qquad \delta(1/\lambda^{s+1} - 1/\lambda^{u+1})\cdot(1+\beta(1-\mu))\\
&\qquad\qquad + \beta\lambda\mu\Delta(\hat{g}(\hat{g}(n)),s+2,u+2,\min[\hat{g}_{s+1}(\hat{g}(n)),\delta])\\
&\le \dots\\
&\le \delta(1/\lambda^s - 1/\lambda^u)\cdot(1+\beta\phi(1-\mu))/(1-\beta\mu)\\
&= \Delta^1(n,s,u,\delta).
\end{aligned}
\tag{A11}
$$

This implies that for any n^1 and n^2 with $\sum_{s\le u} n_s^1 \ge \sum_{s\le u} n_s^2$ for all $u \ge 1$, $\hat{W}_1(n^1) \ge \max_{\tilde{n}}\{\hat{W}_2(n^1,\tilde{n})\}$ if $\hat{W}_1(n^2) \ge \max_{\tilde{n}}\{\hat{W}_2(n^2,\tilde{n})\}$, and $\hat{W}_1(n^2) \le \max_{\tilde{n}}\{\hat{W}_2(n^2,\tilde{n})\}$ if $\hat{W}_1(n^1) \le \max_{\tilde{n}}\{\hat{W}_2(n^1,\tilde{n})\}$.

[9] Consider the updating of $\hat{w}$ by operator T defined above. Let $\bar{n}_s$ denote the human capital distribution concentrated on h_s: $n_s = 1$ and $n_u = 0$ for all $u \ne s$. Note that $\bar{n}_1 = \bar{n}$. Set $\bar{v}_o$ to be the equilibrium utility $\hat{V}_o(\bar{n})$, and the initial $\hat{w}$ to be the equilibrium utility function of stagnation $\hat{W}_1$. I have $\bar{T}(n|\bar{v}_o) = \bar{T}(n|\hat{V}_o(\bar{n})) = \hat{W}_1(n)$. Since $\hat{W}_1$ is linear in

n, the adoption and matching behavior is identical in the population if $n = \bar{n}_s$ for some s: $T(\hat{W}_1)(\bar{n}_s) = \max\{\hat{W}_1(\bar{n}_s), \tilde{T}(\hat{W}_1)(\bar{n}_s, \bar{n}_{s+1})), \tilde{T}(\hat{W}_1)(\bar{n}_s, \bar{n})\}$. Further,

$$
\begin{aligned}
&\max_{\tilde{n}}\{\tilde{T}(\hat{W}_1)(n, \tilde{n})\} \\
&= \max_{\tilde{n}}\Big\{\hat{Y}(n, \tilde{n}) + \beta\lambda\mu(\hat{W}_1(\tilde{n}) - \beta\lambda\hat{V}_o(\bar{n})) + \beta\lambda(1-\mu)\cdot\hat{Y}(\tilde{n}, \bar{n})\Big\} \\
&= \max_{\{\sigma_s\}}\Big\{\sum_s n_s\Big(\sigma_s(\hat{Y}(\bar{n}_s, \bar{n}_{s+1}) + \beta\lambda(1-\mu)\cdot\hat{Y}(\bar{n}_{s+1}, \bar{n})) \\
&\qquad\qquad + (1-\sigma_s)(\hat{Y}(\bar{n}_s, \bar{n}) + \beta\lambda(1-\mu)\cdot\hat{Y}(\bar{n}, \bar{n}))\Big) \\
&\qquad + \beta\lambda\mu\cdot\hat{W}_1\Big(\sum_s((1-\sigma_s)\cdot n_s), \sigma_1 n_1, \sigma_2 n_2, \ldots)\Big) - \beta^2\lambda^2\mu\hat{V}_o(\bar{n})\Big\} \\
&= \max_{\{\sigma_s\}}\Big\{\sum_s n_s\Big(\sigma_s(\hat{Y}(\bar{n}_s, \bar{n}_{s+1}) + \beta\lambda(1-\mu)\cdot\hat{Y}(\bar{n}_{s+1}, \bar{n}) + \beta\lambda\mu\hat{W}_1(\bar{n}_{s+1})) \\
&\qquad\qquad + (1-\sigma_s)(\hat{Y}(\bar{n}_s, \bar{n}) + \beta\lambda(1-\mu)\cdot\hat{Y}(\bar{n}, \bar{n}) + \beta\lambda\mu\hat{W}_1(\bar{n}))\Big) \\
&\qquad - \beta^2\lambda^2\mu\hat{V}_o(\bar{n})\Big\} \\
&= \max_{\{\sigma_s\}}\Big\{\sum_s n_s(\sigma_s\cdot\tilde{T}(\hat{W}_1)(\bar{n}_s, \bar{n}_{s+1}) + (1-\sigma_s)\cdot\tilde{T}(\hat{W}_1)(\bar{n}_s, \bar{n}))\Big\} \\
&= \sum_s(n_s\cdot\max\{\tilde{T}(\hat{W}_1)(\bar{n}_s, \bar{n}_{s+1}), \tilde{T}(\hat{W}_1)(\bar{n}_s, \bar{n})\})
\end{aligned}
\tag{A12}
$$

and

$$
\begin{aligned}
&T(\hat{W}_1)(n) \\
&= \max\Big\{\hat{W}_1(n), \max_{\tilde{n}}\{\tilde{T}(\hat{W}_1)(n, \tilde{n})\}\Big\} \\
&= \max\Big\{\sum_s(n_s\cdot\hat{W}_1(\bar{n}_s)), \sum_s(n_s\cdot\max\{\tilde{T}(\hat{W}_1)(\bar{n}_s, \bar{n}_{s+1}), \tilde{T}(\hat{W}_1)(\bar{n}_s, \bar{n})\})\Big\} \\
&\leq \sum_s(n_s\cdot T(\hat{W}_1)(\bar{n}_s)).
\end{aligned}
\tag{A13}
$$

This implies the identical adoption and matching behavior in updating $T(\hat{W}_1)$ if $n = \bar{n}_s$ for some s: $T^2(\hat{W}_1)(\bar{n}_s) = \max\{\hat{W}_1(\bar{n}_s), \tilde{T}(T(\hat{W}_1))(\bar{n}_s, \bar{n}_{s+1})), \tilde{T}(T(\hat{W}_1))(\bar{n}_s, \bar{n})\}$. Then,

$$
\begin{aligned}
&\max_{\tilde{n}}\{\tilde{T}(T(\hat{W}_1))(n,\tilde{n})\} \\
&\leq \max_{\{\sigma_s\}}\{\sum_s n_s(\sigma_s\cdot\tilde{T}(T(\hat{W}_1))(\bar{n}_s,\bar{n}_{s+1})+(1-\sigma_s)\cdot\tilde{T}(T(\hat{W}_1))(\bar{n}_s,\bar{n})\} \\
&=\sum_s(n_s\cdot\max\{\tilde{T}(T(\hat{W}_1))(\bar{n}_s,\bar{n}_{s+1}),\tilde{T}(T(\hat{W}_1))(\bar{n}_s,\bar{n})\})
\end{aligned}
\tag{A14}
$$

and

$$
\begin{aligned}
T^2(\hat{W}_1)(n) &= \max\left\{\hat{W}_1(n),\max_{\tilde{n}}\{\tilde{T}(T(\hat{W}_1))(n,\tilde{n})\}\right\} \\
&\leq \sum_s(n_s\cdot T^2(\hat{W}_1)(\bar{n}_s)).
\end{aligned}
\tag{A15}
$$

Repeating these steps, I have $T^{t+1}(\hat{W}_1)(\bar{n}_s) = \max\{\hat{W}_1(\bar{n}_s),\tilde{T}(T^t(\hat{W}_1))(\bar{n}_s,\bar{n}_{s+1})),$ $\tilde{T}(T^t(\hat{W}_1))(\bar{n}_s,\bar{n})\}$ for all $t \geq 0$; and $\hat{W}(\bar{n}_s) = \lim_{t\to\infty} T^t(\hat{W}_1)(\bar{n}_s) = \max\{\hat{W}_1(\bar{n}_s),$ $\lim_{t\to\infty}\tilde{T}(T^t(\hat{W}_1))(\bar{n}_s,\bar{n}_{s+1})), \lim_{t\to\infty}\tilde{T}(T^t(\hat{W}_1))(\bar{n}_s,\bar{n})\} = \max\{\hat{W}_1(\bar{n}_s),\hat{W}_2(\bar{n}_s,\bar{n}_{s+1}),$ $\hat{W}_2(\bar{n}_s,\bar{n})\}$.

[10] Consider Unconditional Growth Equilibrium in (9). From (4), (6), and (A3), $\hat{W}(n) = \hat{Y}(n,\bar{n})+\beta\lambda\hat{V}_o(\bar{n})$ so that $W_2(n,\tilde{n}) = \hat{Y}(n,\tilde{n})+\beta\lambda\hat{Y}(\tilde{n},\bar{n})$ for all $\tilde{n}$. I have $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s$ $\equiv \lim_{\epsilon\to 0}(\hat{W}_2(n,(\tilde{n}_0+\epsilon,\ldots,\tilde{n}_s-\epsilon,\ldots))-\hat{W}_2(n,\tilde{n}))/\epsilon = -(1-\phi)/\lambda^{s-1}+\beta\phi(\lambda-1/\lambda^{s-1}) \geq 0$ for all $s \geq 1$ if $\phi \geq 1/(1-\beta+\beta\lambda)$ so that $\max_{\tilde{n}}\{\hat{W}_2(n,\tilde{n})\} = \hat{W}_2(n,\bar{n})$. From (7) and (A3), I have $\hat{V}_o(\bar{n}_s) = \phi/\lambda^{s-1}$ and $\hat{W}(\bar{n}_s) = \hat{W}_2(\bar{n}_s,\bar{n}) = \phi/\lambda^{s-1}+\beta\lambda\phi$ for all s. From (A2), $W_1(\bar{n}_s) = (1+\beta\phi)/\lambda^{s-1} \leq \hat{W}_2(\bar{n}_s,\bar{n})$ for all s. This validates that $\hat{W}(\bar{n}_1) = \hat{W}_2(\bar{n}_1,\bar{n})$. Since $\sum_{s\leq u} n_s = 1$ for all u if $n = \bar{n}_1$ and $\sum_{s\leq u} n_s \leq 1$ for all u for all n, $\hat{W}_1(n) \leq$ $\max_{\tilde{n}}\{\hat{W}_2(n,\tilde{n})\} = \hat{W}_2(n,\bar{n})$ for all s (consult Step 8). This validates the equilibrium.

[11] Consider the following conjecture on the equilibrium when $\phi \leq 1/(1-\beta+\beta\lambda)$: $\hat{W}(\bar{n}_s) = \hat{W}_1(\bar{n}_s)$ for $s \leq u$; and $\hat{W}(\bar{n}_s) = \hat{W}_2(\bar{n}_s,\bar{n})$ for $s > u$, where $u \geq 1$. From (7), (A2)′, and (A3), I have $\hat{V}_o(\bar{n}) = (\phi+\mu(1-\phi))/(1-\beta\mu+\beta\lambda\mu)$; $\hat{W}_1(\bar{n}_s) = (1+\beta\phi(1-\mu))/(\lambda^{s-1}(1-\beta\mu))-\beta^2\lambda\mu(\phi+\mu(1-\phi))/((1-\beta\mu)(1-\beta\mu+\beta\lambda\mu))$; $\hat{W}_2(\bar{n}_s,\bar{n}_{s+1}) = (1+\beta\phi(1-\mu))/(\lambda^{s-1}(1-\beta\mu)) - \beta^2\lambda^2\mu(\phi+\mu(1-\phi))/((1-\beta\mu)(1-\beta\mu+\beta\lambda\mu))$ if $s \leq u-1$; $\hat{W}_2(\bar{n}_s,\bar{n}_{s+1}) =$

$(1+\beta\phi)/\lambda^{s-1}$ if $s \geq u$; and $\hat{W}_2(\bar{n}_s, \bar{n}) = \phi/\lambda^{s-1} + \beta\lambda(\phi+\mu(1-\phi))/(1-\beta\mu+\beta\lambda\mu)$. I have $\hat{W}_1(\bar{n}_s) \geq \hat{W}_2(\bar{n}_s, \bar{n}_{s+1})$ and $\hat{W}_1(\bar{n}_s) \geq \hat{W}_2(\bar{n}_s, \bar{n})$ if $\phi \leq \varphi(s) \equiv (1-\beta\mu+\beta\lambda\mu-\beta\lambda^s\mu)/((1-\beta)(1-\beta\mu+\beta\lambda\mu)+\beta\lambda^s(1-\mu))$; and $\hat{W}_2(\bar{n}_s, \bar{n}) \geq \hat{W}_1(\bar{n}_s)$ and $\hat{W}_2(\bar{n}_s, \bar{n}) \geq W_2(\bar{n}_s, \bar{n}_{s+1})$ if $\phi \geq \varphi(s)$. This validates the conjecture with $u = \max\{s|\phi \leq \varphi(s)\}$ (consult Step 9). In particular, $\hat{W}(\bar{n}_2) = \hat{W}_1(\bar{n}_2)$ if $\phi \leq \varphi(2)$. Since $\sum_{s\leq u} n_s$ when $n = (n_1, 1-n_1, \ldots)$ is greater than $\sum_{s\leq u} n_s$ when $n = \bar{n}_2$ for all u, $\hat{W}_1(n) \geq \max_{\tilde{n}}\{\hat{W}_2(n, \tilde{n})\}$ when $n = (n_1, 1-n_1, \ldots)$ if $\phi \leq \varphi(2)$ (consult Step 8). This validates Unconditional Stagnation Equilibrium 2 in (13).

[12] Now consider Conditional Growth Equilibrium 1 in (10). I have

- $\hat{V}_o(n) = \hat{V}_{gs}(n) \equiv \hat{Y}_0(n)$ if $n_1 \in [0, \hat{n}]$;
- $\hat{V}_o(n) = \hat{V}_{gg}(n)$ if $n_1 \in [\hat{n}, 1-\hat{n}]$, where $\hat{V}_{gg}$ is defined to be the solution to the equation: $\hat{V}_{gg}(n) = \hat{Y}_1(n) + \beta\lambda\mu(\hat{V}_{gg}(1-n_1, n_1, \ldots) - \hat{V}_o(\bar{n}))$;
- $\hat{V}_o(n) = \hat{V}_{gr}(n) \equiv \hat{Y}_1(n) + \beta\lambda\mu(\hat{V}_{gs}(1-n_1, n_1, \ldots) - \hat{V}_o(\bar{n}))$ if $n_1 \in [1-\hat{n}, \check{n}]$ or if $n_0 \in [\check{n}, 1]$ and $\hat{W}_{ss}(n) \equiv \hat{W}_1(n) < \hat{W}_{gr}(n) \equiv \hat{Y}(n, (1-n_1, n_1, \ldots)) + \beta\lambda\hat{V}_{gs}(1-n_1, n_1, \ldots)$;
- $\hat{V}_o(n) = \hat{V}_{ss}(n) \equiv (\hat{Y}_{stag}(n) - \beta\lambda\mu\hat{V}_o(\bar{n}))/(1-\beta\mu)$ otherwise.

In the above equations, $\hat{Y}_{stag}(n) \equiv \mu \cdot \hat{Y}(n, (0, n)) + (1-\mu) \cdot \hat{Y}(n, \bar{n})$; $\hat{Y}_0(n) \equiv \hat{Y}(n, \bar{n})$; $\hat{Y}_1(n) \equiv \mu \cdot \hat{Y}(n, (1-n_1, n_1, \ldots)) + (1-\mu) \cdot \hat{Y}(n, \bar{n})$; $\hat{n} = \beta^2\lambda\mu^2/((1-\beta\lambda\mu)(1-\beta\mu+\beta\lambda\mu))$; and $\check{n} = ((1-\beta\mu+\beta\lambda\mu-\beta^2\lambda^2\mu^2+\phi\beta^2\lambda^2\mu^2(1-\beta+\beta\lambda))/(1-\beta\mu+\beta\lambda\mu) - \phi(1-\beta+\beta\lambda^2))/(1-\beta\lambda\mu-\phi(1-\beta+\beta\lambda^2-\beta\lambda\mu(1-\beta+\beta\lambda)))$. I have $\hat{n} < 1/2$ given that $1-\beta\mu-\beta\lambda\mu > 0$. Let $d\hat{V}_z(n)/dn_s \equiv \lim_{\epsilon\to 0}(\hat{V}_z(n_1+\epsilon, \ldots, n_s-\epsilon, \ldots) - \hat{V}_z(n))/\epsilon$ for $s \geq 2$. I can derive:

- $d\hat{V}_{gs}(n)/dn_s = \phi(1-1/\lambda^{s-1})$;
- $d\hat{V}_{gg}(n)/dn_s = (\mu+\phi(1-\mu)+\beta\mu\phi)/(1+\beta\lambda\mu) - \phi/\lambda^{s-1}$;
- $d\hat{V}_{gr}(n)/dn_s = \mu+\phi(1-\mu) - \beta\mu\phi(\lambda-1) - \phi/\lambda^{s-1}$;

- $d\hat{V}_{ss}(n)/dn_s = (\mu + \phi(1-\mu))(1 - 1/\lambda^{s-1})/(1-\beta\mu)$.

I can show that $d\hat{V}_{gs}(n)/dn_s \leq d\hat{V}_{gg}(n)/dn_s \leq d\hat{V}_{gr}(n)/dn_s \leq d\hat{V}_{ss}(n)/dn_s$. I have $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_o(\tilde{n})/d\tilde{n}_s$, where $\psi(s) \equiv -(1-\phi)/\lambda^{s-1}$ so that:

- $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{gs}(\tilde{n})/d\tilde{n}_s$ if $\tilde{n}_0 \in [0, \hat{n}]$;
- $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{gg}(\tilde{n})/d\tilde{n}_s$ if $\tilde{n}_0 \in [\hat{n}, 1-\hat{n}]$;
- $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{gr}(\tilde{n})/d\tilde{n}_s$ if $\tilde{n}_0 \in [1-\hat{n}, \check{n}]$ or if $\tilde{n}_0 \in [\check{n}, 1]$ and $\hat{W}_{ss}(\tilde{n}) < \hat{W}_{gr}(\tilde{n})$;
- $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n})/d\tilde{n}_s$ otherwise.

Holding s, $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s$ is non-decreasing in $\tilde{n}_0$ so that $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s \geq -(1-\phi)/\lambda^{s-1} + \beta\lambda\phi(1-1/\lambda^s) \geq 0$ if $\phi \geq 1/(1-\beta+\beta\lambda^s)$. In particular, since $\phi \geq (1-\beta\lambda\mu)/(1-\beta+\beta\lambda-\beta\lambda\mu)$, I have $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s \geq 0$ for all $s \geq 2$. Since $\hat{W}(n)$ is increasing in n (consult Step 3), it follows that $\hat{W}(n) = \max\{\hat{W}_1(n), \hat{W}_2(n,(1-n_1,n_1,\ldots)), \hat{W}_2(n,\bar{n})\}$. Substituting $\hat{V}_o(\bar{n}) = (\phi + \mu(1-\phi))/(1-\beta\mu+\beta\lambda\mu)$ in $\hat{W}(n)$, I can show that:

- $\hat{W}_2(n,\bar{n}) \geq \hat{W}_1(n)$ and $\hat{W}_2(n,\bar{n}) \geq \hat{W}_2(n,(1-n_1,n_1,\ldots))$ if $n_1 \in [0,\hat{n}]$;
- $\hat{W}_2(n,(1-n_1,n_1,\ldots)) \geq \hat{W}_2(n,\bar{n})$ if $n_1 \in [\hat{n},1]$;
- $\hat{W}_2(n,(1-n_1,n_1,\ldots)) \geq \hat{W}_1(n)$ if $n_1 \in [\hat{n},\check{n}]$;
- $\hat{W}_1(n) \geq \hat{W}_2(n,(1-n_1,n_1,\ldots))$ if $n_1 \in [\check{n},1]$ and $n_1 + n_2 = 1$.

These properties of $\hat{W}_1$ and $\hat{W}_2$ imply (10).

[13] Now consider Conditional Growth Equilibrium 2 in (11). I have

- $\hat{V}_o(n) = \hat{V}_{gt}(n) \equiv \hat{Y}_1(n) + \beta\lambda\mu(\hat{V}_{ss}(1-n_1,n_1,\ldots) - \hat{V}_o(\bar{n}))$ if $n_1 \in [0, \min\{\check{n}, 1-\check{n}\}]$;
- $\hat{V}_o(n) = \hat{V}_{gg}(n)$ if $n_1 \in [1-\check{n}, \check{n}]$;
- $\hat{V}_o(n) = \hat{V}_{gu}(n) \equiv \hat{Y}_1(n) + \beta\lambda\mu(\hat{V}_{gt}(1-n_1,n_1,\ldots) - \hat{V}_o(\bar{n}))$ if $n_1 \in [\check{n},1]$ and $\hat{W}_{ss}(n) < \hat{W}_{gu}(n) \equiv \hat{Y}(n,(1-n_1,n_1,\ldots)) + \beta\lambda\hat{V}_{gt}(1-n_1,n_1,\ldots)$;

- $\hat{V}_o(n) = \hat{V}_{ss}(n)$ otherwise.

In the above equations, $\check{n} = (1-\phi+\beta\phi-\beta\lambda^2(\phi+\mu(1-\phi))(1+\beta^2\lambda\mu^2-\beta^2\lambda^2\mu^2)/(1-\beta\mu+\beta\lambda\mu))/((1-\beta\lambda\mu)(1-\phi+\beta\phi-\beta\lambda^2(\phi+\mu(1-\phi)))) \geq 1/2$ if $\check{\phi}_a \leq \phi \leq \hat{\phi}$; and $\check{n} = (1-\beta\mu-\phi(1-\beta)(1-\beta\mu)+\beta\lambda\mu(1-\phi+\beta\phi)-\beta\lambda^2(\phi+\mu(1-\phi)))/((1-\beta\mu+\beta\lambda\mu)(1-\phi+\beta\phi-\beta\lambda^2(\phi+\mu(1-\phi)))) \leq 1/2$ if $\tilde{\phi}_a \leq \phi \leq \check{\phi}_a$, where $\check{\phi}_a \equiv (1-\beta\mu-\beta\lambda\mu(1-\beta\lambda\mu)(\lambda-1))/((1-\beta)(1-\beta\mu+\beta\lambda\mu)+\beta\lambda^2(1-\mu)(1+\beta\mu-\beta\lambda\mu))$. I have $\tilde{\phi}_w < \check{\phi}_a < \hat{\phi}$ given that $1-\beta\mu-\beta\lambda\mu > 0$. I can derive:

- $d\hat{V}_{gt}(n)/dn_s = (\mu+\phi(1-\mu))(1-\beta\lambda\mu)/(1-\beta\mu) - \phi/\lambda^{s-1}$;
- $d\hat{V}_{gu}(n)/dn_s = (\mu+\phi(1-\mu))(1-\beta\lambda\mu(1-\beta\lambda\mu)/(1-\beta\mu)) + \beta\mu\phi - \phi/\lambda^{s-1}$.

I can show that $d\hat{V}_{gt}(n)/dn_s \leq d\hat{V}_{gg}(n)/dn_s \leq d\hat{V}_{gu}(n)/dn_s \leq d\hat{V}_{ss}(n)/dn_s$ if $\phi \geq (1-\beta\lambda^2\mu)/(1-\beta+\beta\lambda^2-\beta\lambda^2\mu)$ and that $\tilde{\phi}_a > (1-\beta\lambda^2\mu)/(1-\beta+\beta\lambda^2-\beta\lambda^2\mu)$. I have

- $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n})/d\tilde{n}_s$ if $\tilde{n}_0 \in [0, \min\{\check{n}, 1-\check{n}\}]$;
- $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{gg}(\tilde{n})/d\tilde{n}_s$ if $\tilde{n}_0 \in [1-\check{n}, \check{n}]$;
- $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{gu}(\tilde{n})/d\tilde{n}_s$ if $\tilde{n}_0 \in [\check{n}, 1]$ and $\hat{W}_{ss}(\tilde{n}) < \hat{W}_{gu}(\tilde{n})$;
- $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n})/d\tilde{n}_s$ otherwise.

Holding s, $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s$ is non-decreasing in $\tilde{n}_0$. Further, $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_1 \leq 0$ if $\phi \leq \hat{\phi}$; and $d\hat{W}_2(n,\tilde{n})/d\tilde{n}_s \geq 0$ for $s \geq 2$ if $\phi \geq \tilde{\phi}_w \equiv (1-\beta\mu-\beta\lambda^2\mu(1-\beta\lambda\mu))/((1-\beta)(1-\beta\mu)+\beta\lambda^2(1-\mu)(1-\beta\lambda\mu))$. Since $\hat{W}(n)$ is increasing in n (consult Step 3), it follows that $\hat{W}(n) = \max\{\hat{W}_1(n), \hat{W}_2(n,(1-n_1,n_1,\ldots))\}$. I can show that:

- $\hat{W}_2(n,(1-n_1,n_1,\ldots)) \geq \hat{W}_1(n)$ if $n_1 \in [0,\check{n}]$;
- $\hat{W}_1(n) \geq \hat{W}_2(n,(1-n_1,n_1,\ldots))$ if $n_1 \in [\check{n},1]$ and $n_1+n_2 = 1$.

These properties of $\hat{W}_1$ and $\hat{W}_2$ imply (11).

[14] Now consider Unconditional Stagnation Equilibrium 1 in (12). I have

- $\hat{V}_o(n) = \hat{V}_{gt}(n)$ if $n_2 \leq \zeta(n_1)$ and $\hat{W}_{ss}(n) < \hat{W}_{gt}(n)$;

- $\hat{V}_o(n) = \hat{V}_{go}(n) \equiv \hat{Y}_2(n) + \beta\lambda\mu(\hat{V}_{gt}(1-n_1-n_2, n_1, n_2, \ldots) - \hat{V}_o(\bar{n}))$ if $n_2 \geq \zeta(n_1)$ and $\hat{W}_{ss}(n) < \hat{W}_{go}(n) \equiv \hat{Y}(n, (1-n_1-n_2, n_1, n_2, \ldots)) + \beta\lambda\hat{V}_{gt}(1-n_1-n_2, n_1, n_2, \ldots)$;

- $\hat{V}_o(n) = \hat{V}_{ss}(n)$ otherwise.

In the above equations, $\hat{Y}_2(n) \equiv \mu \cdot \hat{Y}(n, (1-n_1-n_2, n_1, n_2, \ldots)) + (1-\mu) \cdot \hat{Y}(n, \bar{n})$; ζ is linear and decreasing in n_1; $\zeta(1-\breve{n}) = 0$; $\zeta(\breve{n}) = 1-\breve{n}$; $\breve{n} \equiv (1-\beta\mu)/(1-\beta\mu+\beta\lambda\mu) < 1-\breve{n}$; $\breve{n} > \breve{n}$; $\breve{n} = (1-\beta\mu-\phi(1-\beta)(1-\beta\mu)+\beta\lambda\mu(1-\phi+\beta\phi)-\beta\lambda^2(\phi+\mu(1-\phi)))/((1-\beta\mu+\beta\lambda\mu)(1-\phi+\beta\phi-\beta\lambda^2(\phi+\mu(1-\phi)))) < 1/2$ since $\phi \leq \tilde{\phi}_w < \check{\phi}_a$ given that $1-\beta\mu-\beta\lambda\mu > 0$. I can derive:

- $d\hat{V}_{go}(n)/dn_2 = (\mu + \phi(1-\mu) + \beta\mu\phi)(1-1/\lambda)$;

- $d\hat{V}_{go}(n)/dn_s = d\hat{V}_{gu}(n)/dn_s$ for $s \geq 3$.

I can show that $d\hat{V}_{gt}(n)/dn_s \leq d\hat{V}_{go}(n)/dn_s \leq d\hat{V}_{ss}(n)/dn_s$ for all s if $\phi \geq \tilde{\phi}_a$. I have

- $d\hat{W}_2(n, \tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n})/d\tilde{n}_s$ if $\tilde{n}_1 \leq \zeta(\tilde{n}_0)$ and $\hat{W}_{ss}(\tilde{n}) < \hat{W}_{gt}(\tilde{n})$;

- $d\hat{W}_2(n, \tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{go}(\tilde{n})/d\tilde{n}_s$ if $\tilde{n}_1 \geq \zeta(\tilde{n}_0)$ and $\hat{W}_{ss}(\tilde{n}) < \hat{W}_{go}(\tilde{n})$; and

- $d\hat{W}_2(n, \tilde{n})/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n})/d\tilde{n}_s$ otherwise.

Holding s, $\hat{W}_2(n, \tilde{n})$ is continuous in $\tilde{n}_0$, and $d\hat{W}_2(n, \tilde{n})/d\tilde{n}_s$ is non-decreasing in $\tilde{n}_0$. Further, $d\hat{W}_2(n, \tilde{n})/d\tilde{n}_1 \leq 0$ if $\phi \leq \hat{\phi}$; $d\hat{W}_2(n, \tilde{n})/d\tilde{n}_s \geq 0$ for $s \geq 3$ if $\phi \geq \breve{\phi} \equiv (1-\beta\mu-\beta\lambda^3\mu(1-\beta\lambda\mu))/((1-\beta)(1-\beta\mu)+\beta\lambda^3(1-\mu)(1-\beta\lambda\mu))$; and $\breve{\phi} \leq \tilde{\phi}_a$ if $1-\beta\mu-\beta^2\lambda^2\mu^2 > 0$. Then, $\max_{\tilde{n}}\{\hat{W}_2(n, \tilde{n})\} = \{\hat{W}_2(n, (1-n_1-n_2, n_1, n_2, \ldots)), \hat{W}_2(n, (1-n_1, n_1, \ldots))\}$. I can show that: $\hat{W}_2(n, (1-n_1-n_2, n_1, n_2, \ldots)) = \hat{W}_{go}(n) = \hat{W}_2(n, (1-n_1, n_1, \ldots))$ if $n_2 = \zeta(n_1)$. Then, for all n, $\hat{W}_2(n, (1-n_1-\tilde{n}_2, n_1, \tilde{n}_2, \ldots)) \geq \hat{W}_2(n, (1-n_1-\zeta(n_1), n_1, \zeta(n_1), \ldots)) = \hat{W}_2(n, (1-n_1, n_1, \ldots))$ if $\tilde{n}_2 \geq \zeta(n_1)$; and $\hat{W}_2(n, (1-n_1-\tilde{n}_2, n_1, \tilde{n}_2, \ldots)) \leq \hat{W}_2(n, (1-n_1-\zeta(n_1), n_1, \zeta(n_1), \ldots)) = \hat{W}_2(n, (1-n_1, n_1, \ldots))$ if $\tilde{n}_2 \leq \zeta(n_1)$. It follows that for all n,

- $\hat{W}_2(n, (1-n_1, n_1, \ldots)) = \hat{W}_{gt}(n) \geq \hat{W}_2(n, (1-n_1-n_2, n_1, n_2, \ldots))$ if $n_2 \leq \zeta(n_1)$; and

- $\hat{W}_2(n, (1 - n_1 - n_2, n_1, n_2, \ldots)) = \hat{W}_{go}(n) \geq \hat{W}_2(n, (1 - n_1, n_1, \ldots))$ if $n_2 \geq \zeta(n_1)$.

Further, I can show that:

- $\hat{W}_{gt}(n) \geq \hat{W}_{ss}(n)$ if $n_1 \leq \check{n}$; and
- $\hat{W}_{ss}(n) \geq \hat{W}_{gt}(n)$ if $n_1 \geq \check{n}$ and $n_1 + n_2 = 1$.

These properties of $\hat{W}_1$ and $\hat{W}_2$ imply (12).

Derivation of the Equilibrium in Section 3

[15] From $(4)'$ to $(7)'$, I have

$$\check{W}^i(n) = (1 - \max_j\{\check{q}^j(n)\}) \cdot \check{W}^i_1(n) + \max_j\{\check{q}^j(n)\} \cdot \max_{\tilde{n}^i}\{\check{W}^i_2(n, \tilde{n}^i)\}, \tag{A16}$$

where

$$\begin{aligned}\check{W}^i_1(n) = \check{Y}(n^i, (0, n^i)) + \beta\mu(\check{W}^i(n) - \beta\lambda\check{V}^i_o(\check{G}^{-i}(n, \bar{n}))\\ + \beta(1-\mu) \cdot \check{Y}(n^i, \bar{n});\end{aligned} \tag{A17}$$

and

$$\begin{aligned}\check{W}^i_2(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\mu(\check{W}^i(\check{G}^{-i}(n, \tilde{n}^i)) - \beta\lambda\check{V}^i_o(\check{G}^{-i}(\check{G}^{-i}(n, \tilde{n}^i), \bar{n})))\\ + \beta\lambda(1-\mu) \cdot \check{Y}(\tilde{n}^i, \bar{n}).\end{aligned} \tag{A18}$$

If $\max_j\{\check{q}^j(n)\} = 0$, $\check{W}^i(n) = \check{W}^i_1(n)$ and $\check{G}^{-i}(n, \bar{n}) = ((0, n^1), \ldots, (0, n^{i-1}), (0, \bar{n}), (0, n^{i+1}), \ldots, (0, n^I)))$ so that (A17) can be rewritten without a loss of generality:

$$\begin{aligned}\check{W}^i_1(n) = (\check{Y}(n^i, (0, n^i)) + \beta(1-\mu) \cdot \check{Y}(n^i, \bar{n})\\ - \beta^2\lambda\mu\check{V}^i_o((0, n^1), \ldots, (0, n^{i-1}), \bar{n}, (0, n^{i+1}), \ldots, (0, n^I)))/(1 - \beta\mu).\end{aligned} \tag{A17$'$}$$

[16] Consider Unconditional Growth World Equilibrium 1 in (14). From $(4)'$, $(6)'$, and (A18), $\check{W}^i(n) = \check{Y}(n^i, \bar{n}) + \beta\lambda\check{V}^i_o(\check{G}^{-i}(n, \bar{n}))$ so that $\check{W}^i_2(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{Y}(\tilde{n}^i, \bar{n})$ for all $\tilde{n}^i$. I have $d\check{W}^i_2(n, \tilde{n}^i)/d\tilde{n}^i_s \equiv \lim_{\epsilon\to 0}(\check{W}^i_2(n, (\tilde{n}^i_0 + \epsilon, ..., \tilde{n}^i_s - \epsilon, ...)) - \check{W}_2(n, \tilde{n}^i))/\epsilon = -(1-\phi)/\lambda^{s-1} + \beta\phi(\lambda - 1/\lambda^{s-1}) \geq 0$ for all $s \geq 1$ if $\phi \geq 1/(1-\beta+\beta\lambda)$. Since $\max_{j\neq i}\{\check{q}^j(n)\} = 1$, I have $\check{W}^i(n) = \max_{\tilde{n}^i}\{\check{W}^i_2(n, \tilde{n}^i)\} = \check{W}^i_2(n, \bar{n})$. This validates the equilibrium.

[17] Suppose that $\check{q}^i(n) = 0$ and $\check{W}^i(n) = \check{W}^i_1(n)$ for all i. Since $\check{V}^i_o(n) \geq \check{Y}(n^i, \bar{n})$, I have $\check{W}^i_1(n) \leq (\check{Y}(n^i, (0, n^i)) - \beta^2\lambda\mu\check{Y}(\bar{n}, \bar{n}) + \beta(1-\mu) \cdot \check{Y}(n^i, \bar{n}))/(1 - \beta\mu) = (\check{Y}(n^i, (0, n^i)) \cdot (1 + \beta\phi(1-\mu)) - \beta^2\lambda\mu\phi)/(1 - \beta\mu)$; and $\max_{\tilde{n}^i}\{\check{W}^i_2(n, \tilde{n}^i)\} \geq \check{W}^i_2(n, \bar{n}) = \check{Y}(n^i, \bar{n}) + \beta\lambda\mu(\check{W}^i(\check{G}^{-i}(n, \bar{n})) - \beta\lambda\check{V}_o(\check{G}^{-i}(\check{G}^{-i}(n, \bar{n}), \bar{n}))) + \beta\lambda(1-\mu) \cdot \check{Y}(\bar{n}, \bar{n}) \geq \check{Y}(n^i, \bar{n}) + \beta\lambda \cdot \check{Y}(\bar{n}, \bar{n}) = \check{Y}(n^i, (0, n^i)) \cdot \phi + \beta\lambda\phi$ so that $\max_{\tilde{n}^i}\{\check{W}^i_2(n, \tilde{n}^i)\} - \check{W}^i_1(n) = (\check{Y}(n^i, (0, n^i))(\phi(1-\beta) - 1) + \beta\lambda\phi)/(1 - \beta\mu) \geq (\phi(1 - \beta + \beta\lambda) - 1)/(1 - \beta\mu) > 0$ if $\phi > 1/(1 - \beta + \beta\lambda)$.

Therefore, an equilibrium in which every country stagnates for some n does not exist if $\phi > 1/(1-\beta+\beta\lambda)$.

[18] Now consider Unconditional Growth World Equilibrium 2 in (15). From (A16) and (A18), $\check{W}^i(n) = \check{Y}(n^i,(1-n_1^i,n_1^i,...)) + \beta\lambda\mu(\check{W}^i(\check{G}^{-i}(n,(1-n_1^i,n_1^i,...))) - \beta\lambda\check{V}_o^i(\check{G}^{-i}(n,\bar{n}) + \beta\lambda(1-\mu)\cdot\check{Y}((1-n_1^i,n_1^i,...),\bar{n})$ from which I have that $\check{W}_2^i(n,\tilde{n}^i)$ is independent of $\{n^j\}_{j\neq i}$; $\check{W}_2^i(n,\tilde{n}^i)$ is continuous in $\tilde{n}^i$; $d\check{W}_2^i(n,\tilde{n}^i)/d\tilde{n}_1^i = \psi(1) + \beta\lambda\mu\cdot d\hat{W}_{gg}(\tilde{n})/d\tilde{n}_1 < 0$ if $\phi < 1/(1-\beta+\beta\lambda)$; and $d\check{W}_2^i(n,\tilde{n}^i)/d\tilde{n}_s^i = \psi(1)+\beta\lambda\mu\cdot d\hat{W}_{gg}(\tilde{n})/d\tilde{n}_s > 0$ for all $s \geq 1$ if $\phi > \tilde{\phi}_w$. Since $\max_{j\neq i}\{\check{q}^j(n)\} = 1$, I have $\check{W}^i(n) = \max_{\tilde{n}^i}\{\check{W}_2^i(n,\tilde{n}^i)\} = \check{W}_2^i(n,(1-n_1^i,n_1^i,...))$. Further, $\check{q}^i(n) = 0$ is (trivially) optimal. This validates the equilibrium.

[19] Now consider Conditional Growth World Equilibrium 1 in (16). First, consider the case of $\hat{m} \leq 1/2$. I have $\check{V}_o^i(n) = \check{V}_{stag}(n^i)$ if $n_0^j = 1$ for all j; and $\check{V}_o^i(n) = \check{V}_{zz'}(n^i)$ if $n_0^j < 1$ for some j, where z characterizes the human capital distribution of country i and z' those of the other countries as follows:

For all z', subscript $\bar{l}z'$ means $n_1^i = 0$; lz' means $n_1^i \in (0,\hat{m}]$; cz' means $n_1^i \in [\hat{m},1-\hat{m}]$; xz' means $n_1^i \in [1-\hat{m},1-\hat{\rho}\hat{m}]$ and $n_1^j \notin [1-\hat{m},n_1^i]$ for all $j \neq i$; hz' means $n_1^i \in [1-\hat{m},1)$, not $x\tilde{h}$, and not xh; and $\bar{h}z'$ means $n_1^i = 1$.

For all z, subscript zg means $n_1^j \in [0,1-\hat{m}]$ for some $j \neq i$ and $n_1^j \in [\hat{m},1]$ for some $j \neq i$; $z\bar{h}$ means $n_1^j = 1$ all $j \neq i$; $z\bar{l}$ means $n_1^j = 0$ all $j \neq i$; $z\tilde{h}$ means $n_1^j \in [1-\hat{m},1]$ for all $j \neq i$ and $n_1^j \in [1-\hat{m},1-\hat{\rho}\hat{m}]$ for some $j \neq i$; $z\tilde{l}$ means $n_1^j \in [0,\hat{m}]$ for all $j \neq i$ and $n_1^j \in [\hat{\rho}\hat{m},\hat{m}]$ for some $j \neq i$; zh means $n_1^j \in [1-\hat{m},1]$ for all $j \neq i$, not $z\bar{h}$, and not $z\tilde{h}$; and zl means $n_1^j \in [0,\hat{m}]$ for all $j \neq i$, not $z\bar{l}$, and not $z\tilde{l}$.

Further, given zz', let subscript $\tilde{z}z'$ be defined as follows: $\tilde{z}z' = \bar{h}z'$ if $z = \bar{l}$, $\tilde{z}z' = hz'$ if $z = l$, $\tilde{z}z' = cz'$ if $z = c$, $\tilde{z}z' = lz'$ if $z = h$, and $\tilde{z}z' = \bar{l}z'$ if $z = \bar{h}$. Let $\check{Y}_{stag}(n^i) \equiv \mu\cdot\check{Y}(n^i,(0,n^i)) + (1-\mu)\cdot\check{Y}(n^i,\bar{n})$; $\check{Y}_0(n^i) \equiv \check{Y}(n^i,\bar{n})$; $\check{Y}_1(n^i) \equiv \mu\cdot\check{Y}(n^i,(1-n_1^i,n_1^i,\ldots)) + (1-\mu)\cdot\check{Y}(n^i,\bar{n})$; and $\check{Y}_x(n^i) \equiv \mu\cdot\check{Y}(n^i,(\hat{m},1-\hat{m},\ldots)) + (1-\mu)\cdot\check{Y}(n^i,\bar{n})$.

In the equations below, I underline the states of the world economy that are on the equilibrium path starting from some initial state of the world. I have

- $\check{V}_{\underline{stag}}(n^i) \equiv \check{Y}_{stag}(n^i) + \beta\mu(\check{V}_{stag}(n^i) - \lambda\check{V}_{\bar{h}\bar{l}}(\bar{n}))$;
- $\check{V}_{\underline{zg}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}g}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}g}(\bar{n}))$ for $z = \bar{l}, l, c, h, \bar{h}$;
- $\check{V}_{\underline{z\bar{h}}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\bar{l}}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\bar{l}}(\bar{n}))$ for $z = \bar{l}, l, c$;
- $\check{V}_{z\bar{h}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\bar{l}}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\bar{l}}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{\underline{z\bar{l}}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l$;
- $\check{V}_{\underline{z\bar{l}}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\bar{h}}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{stag}(\bar{n}))$ for $z = c, h, \bar{h}$;
- $\check{V}_{\underline{z\tilde{h}}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{l}}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{l}}(\bar{n}))$ for $z = \bar{l}, l, c$;
- $\check{V}_{x\tilde{h}}(n^i) \equiv \check{Y}_x(n^i) + \beta\lambda\mu(\check{V}_{c\tilde{l}}(\hat{m}, 1 - \hat{m}, \ldots) - \check{V}_{\bar{h}\tilde{l}}(\bar{n}))$;
- $\check{V}_{z\tilde{h}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}g}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}g}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{z\tilde{l}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l$;
- $\check{V}_{\underline{z\tilde{l}}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{h}}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{h}}(\bar{n}))$ for $z = c, h, \bar{h}$;
- $\check{V}_{\underline{zh}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}l}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}l}(\bar{n}))$ for $z = \bar{l}, l, c$;
- $\check{V}_{xh}(n^i) \equiv \check{Y}_x(n^i) + \beta\lambda\mu(\check{V}_{cl}(\hat{m}, 1 - \hat{m}, \ldots) - \check{V}_{\bar{h}l}(\bar{n}))$;
- $\check{V}_{zh}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}l}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}l}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{zl}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l$;
- $\check{V}_{\underline{zl}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}h}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}h}(\bar{n}))$ for $z = c, h, \bar{h}$.

Inspecting the equations, I have that $\check{V}_{zz'}(n^i)$ is the same across $\bar{l}g$, lg, cg, hg, $\bar{h}g$, $\bar{l}\bar{h}$, $l\bar{h}$, $c\bar{h}$, $\bar{l}\tilde{h}$, $l\tilde{h}$, $c\tilde{h}$, $h\tilde{h}$, $\bar{h}\tilde{h}$, $\bar{l}h$, lh, ch, $c\tilde{l}$, $h\tilde{l}$, $\bar{h}\tilde{l}$, cl, hl, $\bar{h}l$; $\check{V}_{zz'}(n^i)$ is the same across $c\bar{l}$, $h\bar{l}$, and

$\bar{h}\bar{l}$; $\check{V}_{zz'}(n^i)$ is the same across $\overline{ll}$, $l\bar{l}$, $\overline{l}\tilde{l}$, $l\tilde{l}$, $\bar{l}l$, and ll; $\check{V}_{zz'}(n^i)$ is the same across hh and $\bar{h}h$; and $\check{V}_{zz'}(n^i)$ is the same across xh and $x\tilde{h}$.

Let $d\check{V}_{zz'}(n^i)/dn_s \equiv \lim_{\epsilon\to 0}(\check{V}_{zz'}(n_1^i+\epsilon,\ldots,n_s^i-\epsilon,\ldots)-\check{V}_{zz'}(n^i))/\epsilon$ for $s\geq 2$. I can show that $d\hat{V}_{gs}(n^i)/dn_s = d\check{V}_{ll}(n^i)/dn_s = d\check{V}_{xh}(n^i)/dn_s < d\hat{V}_{gg}(n^i)/dn_s = d\check{V}_{lh}(n^i)/dn_s = d\check{V}_{h\bar{l}}(n^i)/dn_s < d\hat{V}_{gr}(n^i)/dn_s = d\check{V}_{hh}(n^i)/dn_s = d\check{V}_{h\bar{h}}(n^i)/dn_s < d\hat{V}_{ss}(n^i)/dn_s = d\check{V}_{stag}(n^i)/dn_s$.

I can show the following properties of $\check{V}$ that are useful for the reasoning in the subsequent paragraphs:

- $\check{V}_{ll}(\hat{m},1-\hat{m},\ldots)=\check{V}_{hl}(\hat{m},1-\hat{m},\ldots)$;
- $\check{V}_{lh}(1-\hat{m},\hat{m},\ldots)=\check{V}_{h\bar{h}}(1-\hat{m},\hat{m},\ldots)$;
- $\check{V}_{lh}(\bar{n})=\check{V}_{\bar{h}h}(\bar{n})$.

Now consider $(\check{q}^i(n),\check{g}^i(n))$ as an optimal response across various cases of $\{(\check{q}^j(n),\check{g}^j(n))\}_{j\neq i}$:

Case 1: If $\check{g}_0^j(n)\in[0,1-\hat{m}]$ for some $j\neq i$ and $\check{g}_0^j(n)\in[\hat{m},1]$ for some $j\neq i$, I have $\max_{j\neq i}\{\check{q}^j(n)\}=1$ and

- $\check{W}_2^i(n,\tilde{n}^i)=\check{Y}(n^i,\tilde{n}^i)+\beta\lambda\check{V}_{lh}(\tilde{n}^i)$ for all $\tilde{n}_0^i$.

I have $d\check{W}_2^i(n,\tilde{n}^i)/d\tilde{n}_s=\psi(s)+\beta\lambda\cdot d\hat{V}_{gg}(\tilde{n}^i)/d\tilde{n}_s^i>0$ for all $s\geq 2$, and $d\check{W}_2^i(n,\tilde{n}^i)/d\tilde{n}_1 < 0$. Then, $\check{g}^i(n)=(1-n_1^i,n_1^i,\ldots)$.

Case 2: If $\check{g}_0^j(n)=0$ for all $j\neq i$ and $\max_{j\neq i}\{\check{q}^j(n)\}=1$, I have

- $\check{W}_2^i(n,\tilde{n}^i)=\check{Y}(n^i,\tilde{n}^i)+\beta\lambda\check{V}_{ll}(\tilde{n}^i)$ if $\tilde{n}_0^i\in[0,\hat{m}]$;
- $\check{W}_2^i(n,\tilde{n}^i)=\check{Y}(n^i,\tilde{n}^i)+\beta\lambda\check{V}_{hl}(\tilde{n}^i)$ if $\tilde{n}_0^i\in[\hat{m},1]$.

I have that $\check{W}_2^i(n, \tilde{n}^i)$ is continuous in $\tilde{n}^i$; $d\check{W}_2^i(n, \tilde{n}^i)/d\tilde{n}_1 \leq \psi(1) + \beta\lambda \cdot d\hat{V}_{gg}(\tilde{n}^i)/d\tilde{n}_1^i < 0$; $d\check{W}_2^i(n, \tilde{n}^i)/d\tilde{n}_s \geq \psi(s) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n}^i)/d\tilde{n}_s^i > 0$ for $s \geq 2$ since $\phi > \tilde{\phi}_w$. Then, $\check{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$.

Case 3: If $\check{g}_0^j(n) = 0$ and $\check{q}^j(n) = 0$ for all $j \neq i$, I have $n_1^j = 1$ for all j. Using the result in Case 2, I have $\check{W}^i(n) = \max\{\check{W}_{stag}(\bar{n}), \check{W}_2^i(\bar{n}, (0, \bar{n}))\} = \max\{\check{W}_{stag}(\bar{n}), \check{Y}(\bar{n}, (0, \bar{n})) + \beta\lambda\check{V}_{ll}(0, \bar{n})\} = \check{W}_{stag}(\bar{n})$. Then, $\check{g}^i(n) = (0, \bar{n})$ and $\check{q}^i(n) = 0$.

Case 4: If $\check{g}_0^j(n) \in [0, \hat{m}]$ for all $j \neq i$ and $\check{g}_0^j(n) > 0$ for some $j \neq i$, I have $\max_{j \neq i}\{\check{q}^j(n)\} = 1$;

- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{ll}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [0, \hat{m}]$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{lh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [\hat{m}, 1]$.

If $1 - n_1^i \geq \hat{m}$, $\check{Y}(n^i, (1 - n_1^i, n_1^i, \ldots)) + \beta\lambda\check{V}_{lh}(1 - n_1^i, n_1^i, \ldots) = \max\{\check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{lh}(\tilde{n}^i)\} \geq \max\{\check{W}_2^i(n, \tilde{n}^i)\}$. If $1 - n_1^i \leq \hat{\rho}\hat{m}$, $\max_{\tilde{n}_0^i < \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\} = \check{Y}(n^i, (1 - n_1^i, n_1^i, \ldots)) + \beta\lambda\check{V}_{ll}(1 - n_1^i, n_1^i, \ldots) \geq \check{Y}(n^i, (\hat{m}, 1 - \hat{m}, \ldots)) + \beta\lambda\check{V}_{lh}(\hat{m}, 1 - \hat{m}, \ldots) \geq \max_{\tilde{n}_0^i \geq \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\}$. If $\hat{\rho}\hat{m} \leq 1 - n_1^i \leq \hat{m}$, $\max_{\tilde{n}_0^i < \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\} = \check{Y}(n^i, (1 - n_1^i, n_1^i, \ldots)) + \beta\lambda\check{V}_{ll}(1 - n_1^i, n_1^i, \ldots) \leq \check{Y}(n^i, (\hat{m}, 1 - \hat{m}, \ldots)) + \beta\lambda\check{V}_{lh}(\hat{m}, 1 - \hat{m}, \ldots) \geq \max_{\tilde{n}_0^i \geq \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\}$. Then, $\check{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$ if $1 - n_1^i \in [0, \hat{\rho}\hat{m}] \cup [\hat{m}, 1]$; and $\check{g}^i(n) = (\hat{m}, 1 - \hat{m}, \ldots)$ if $1 - n_1^i \in [\hat{\rho}\hat{m}, \hat{m}]$.

Case 5: If $\check{g}_0^j(n) = 1$ for all $j \neq i$, I have $\max_{j \neq i}\{\check{q}^j(n)\} = 1$;

- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{lh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [0, 1 - \hat{m}]$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{h\bar{h}}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [1 - \hat{m}, 1)$;
- $\check{W}_2^i(n, \bar{n}) = \check{Y}(n^i, \bar{n}) + \beta\lambda\check{V}_{stag}(\bar{n})$;

If $1 - n_1^i \leq 1 - \hat{m}$, $\max_{\tilde{n}_0^i \leq \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\} = \check{Y}(n^i, (1 - n_1^i, n_1^i, \ldots)) + \beta\lambda\check{V}_{lh}(1 - n_1^i, n_1^i, \ldots) \geq \check{Y}(n^i, \bar{n}) + \beta\lambda\check{V}_{stag}(\bar{n}) = \max_{\tilde{n}_0^i > \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\}$. If $1 - n_1^i \geq 1 - \hat{m}$, $\check{Y}(n^i, \bar{n}) + \beta\lambda\check{V}_{stag}(\bar{n}) \geq \check{Y}(n^i, (1 - n_1^i, n_1^i, \ldots)) + \beta\lambda\check{V}_{h\bar{h}}(1 - n_1^i, n_1^i, \ldots) \geq \max_{\tilde{n}_0^i < 1}\{\check{W}_2^i(n, \tilde{n}^i)\}$. Then, $\check{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$ if $1 - n_1^i \leq \hat{m}$; and $\check{g}^i(n) = \bar{n}$ if $1 - n_1^i \geq 1 - \hat{m}$.

Case 6: If $\breve{g}_0^j(n) \in [1-\hat{\rho}\hat{m}, 1]$ for all $j \neq i$ and $\breve{g}_0^j(n) < 1$ for some $j \neq i$, I have $\max_{j\neq i}\{\breve{q}^j(n)\} = 1$; $1 - n_1^i \leq 1 - \hat{m}$;

- $\breve{W}_2^i(n, \tilde{n}^i) = \breve{Y}(n^i, \tilde{n}^i) + \beta\lambda\breve{V}_{lh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [0, 1-\hat{m}]$;
- $\breve{W}_2^i(n, \tilde{n}^i) = \breve{Y}(n^i, \tilde{n}^i) + \beta\lambda\breve{V}_{xh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [1-\hat{\rho}\hat{m}, 1]$;
- $\breve{W}_2^i(n, \tilde{n}^i) = \breve{Y}(n^i, \tilde{n}^i) + \beta\lambda\breve{V}_{hh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [1-\hat{\rho}\hat{m}, 1]$.

I have $\breve{Y}(n^i, (1-n_1^i, n_1^i, \ldots)) + \beta\lambda\breve{V}_{lh}(1-n_1^i, n_1^i, \ldots) = \max\{\breve{Y}(n^i, \tilde{n}^i) + \beta\lambda\breve{V}_{lh}(\tilde{n}^i)\} \geq \max\{\breve{W}_2^i(n, \tilde{n}^i)\}$. Then, $\breve{g}^i(n) = (1-n_1^i, n_1^i, \ldots)$.

Case 7: If $\breve{g}_0^j(n) \in [1-\hat{m}, 1]$ for all $j \neq i$ and $\breve{g}_0^j(n) \in [1-\hat{m}, 1-\hat{\rho}\hat{m}]$ for some $j \neq i$, I have $\max_{j\neq i}\{\breve{q}^j(n)\} = 1$; $1 - n_1^i \leq 1 - \hat{m}$;

- $\breve{W}_2^i(n, \tilde{n}^i) = \breve{Y}(n^i, \tilde{n}^i) + \beta\lambda\breve{V}_{lh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [0, 1-\hat{m}]$;
- $\breve{W}_2^i(n, \tilde{n}^i) = \breve{Y}(n^i, \tilde{n}^i) + \beta\lambda(\vartheta(n, \tilde{n}^i)\cdot\breve{V}_{xh}(\tilde{n}^i) + (1-\vartheta(n, \tilde{n}^i))\cdot\breve{V}_{hh}(\tilde{n}^i))$ if $\tilde{n}_0^i \in [1-\hat{\rho}\hat{m}, 1]$;
- $\breve{W}_2^i(n, \tilde{n}^i) = \breve{Y}(n^i, \tilde{n}^i) + \beta\lambda\breve{V}_{lh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [1-\hat{\rho}\hat{m}, 1]$.

I have $\breve{Y}(n^i, (1-n_1^i, n_1^i, \ldots)) + \beta\lambda\breve{V}_{lh}(1-n_1^i, n_1^i, \ldots) = \max\{\breve{Y}(n^i, \tilde{n}^i) + \beta\lambda\breve{V}_{lh}(\tilde{n}^i)\} \geq \max\{\breve{W}_2^i(n, \tilde{n}^i)\}$. Then, $\breve{g}^i(n) = (1-n_1^i, n_1^i, \ldots)$.

The above properties of $(\breve{q}^i(n), \breve{g}^i(n))$ across various cases of $\{(\breve{q}^j(n), \breve{g}^j(n))\}_{j\neq i}$ imply (16) for the case of $\hat{m} \leq 1/2$.

[20] Now consider the case of $\hat{m} > 1/2$. I have that $\breve{V}_o^i(n) = \breve{V}_{stag}(n^i)$ if $n_0^j = 1$ for all j; and $\breve{V}_o^i(n) = \breve{V}_{zz'}(n^i)$ if $n_0^j < 1$ for some j, where z characterizes the human capital distribution of country i and z' those of the other countries as follows:

For all z', subscript $\bar{l}z'$ means $n_1^i = 0$; lz' means $n_1^i \in (0, 1-\hat{m}]$; xz' means $n_1^i \in [1-\hat{m}, 1-\hat{\rho}\hat{m}]$ and "chosen" for h; [comment: explain "chosen"] cz' means $n_1^i \in [1-\hat{m}, \hat{m}]$ and "not chosen" for h; hz' means $n_1^i \in [\hat{m}, 1)$ and "not chosen" for h; and $\bar{h}z'$ means $n_1^i = 1$.

For all z, subscript zg means $n_1^j \in [0, 1-\hat{m}]$ for some $j \neq i$ and $n_1^j \in [\hat{m}, 1]$ for some $j \neq i$; $z\bar{c}$ means $n_1^j \in [1-\hat{m}, \hat{\rho}\hat{m}]$ for all $j \neq i$, $n_1^j \in [1-\hat{m}, 1-\hat{\rho}\hat{m}]$ for some $j \neq i$, and $n_1^j \in [\hat{\rho}\hat{m}, \hat{m}]$ for some $j \neq i$; $z\tilde{c}c$ means $n_1^j \in [1-\hat{m}, \hat{\rho}\hat{m}]$ for all $j \neq i$ and $n_1^j \in [1-\hat{m}, 1-\hat{\rho}\hat{m}]$ for some $j \neq i$; $zc\tilde{c}$ means $n_1^j \in [1-\hat{\rho}\hat{m}, \hat{m}]$ for all $j \neq i$ and $n_1^j \in [\hat{\rho}\hat{m}, \hat{m}]$ for some $j \neq i$; zc means $n_1^j \in [1-\hat{\rho}\hat{m}, \hat{\rho}\hat{m}]$ for all $j \neq i$; $z\bar{h}$ means $n_1^j = 1$ all $j \neq i$; $z\bar{l}$ means $n_1^j = 0$ all $j \neq i$; $z\tilde{h}'$ means $I = 2$ and $n_1^j \in [\hat{m}, 1-\hat{\rho}\hat{m}]$ for $j \neq i$; $z\tilde{l}'$ means $I = 2$ and $n_1^j \in [\hat{\rho}\hat{m}, 1-\hat{m}]$ for $j \neq i$; $z\tilde{h}$ means $I \geq 3$, $n_1^j \in [1-\hat{m}, 1]$ for all $j \neq i$, $n_1^j \in [1-\hat{m}, 1-\hat{\rho}\hat{m}]$ for some $j \neq i$, not zc, and not $z\tilde{h}'$; $z\tilde{l}$ means $I \geq 3$, $n_1^j \in [0, \hat{m}]$ for all $j \neq i$, $n_1^j \in [\hat{\rho}\hat{m}, \hat{m}]$ for some $j \neq i$, not zc, and not $z\tilde{l}'$; zh means $n_1^j \in [1-\hat{m}, 1]$ for all $j \neq i$, not zc, not $z\bar{h}$, not $z\tilde{h}'$, and not $z\tilde{h}$; and zl means $n_1^j \in [0, \hat{m}]$ for all $j \neq i$, not zc, not $z\bar{l}$, not $z\tilde{l}'$, and not $z\tilde{l}$.

Further, given zz', let subscript $\tilde{z}z'$ be defined as follows: $\tilde{z}z' = \bar{h}z'$ if $z = \bar{l}$, $\tilde{z}z' = hz'$ if $z = l$, $\tilde{z}z' = cz'$ if $z = c$, $\tilde{z}z' = lz'$ if $z = h$, and $\tilde{z}z' = \bar{l}z'$ if $z = \bar{h}$.

In the equations below, I underline the states of the world economy that are on the equilibrium path starting from some initial state of the world as in [19]. I have

- $\check{V}_{\underline{stag}}(n^i) \equiv \check{Y}_{stag}(n^i) + \beta\mu(\check{V}_{stag}(n^i) - \lambda\check{V}_{\bar{h}\bar{l}}(\bar{n}))$;
- $\check{V}_{\underline{zg}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}g}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}g}(\bar{n}))$ for $z = \bar{l}, l, c, h, \bar{h}$;
- $\check{V}_{z\bar{c}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l$;
- $\check{V}_{c\bar{c}}(n^i) \equiv \check{Y}_0(n^i)$;
- $\check{V}_{z\bar{c}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{h}}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{h}}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{\underline{z\tilde{c}c}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l$;
- $\check{V}_{c\tilde{c}c}(n^i) \equiv \check{Y}_0(n^i)$;
- $\check{V}_{z\tilde{c}c}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}h}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}h}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{zc\tilde{c}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l$;

- $\check{V}_{cc\tilde{c}}(n^i) \equiv \check{Y}_0(n^i)$;
- $\check{V}_{zc\tilde{c}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{c}c}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{c}c}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{\underline{zc}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l$;
- $\check{V}_{cc}(n^i) \equiv \check{Y}_0(n^i)$;
- $\check{V}_{zc}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}c}(1-n_1^i, n_1^i, \ldots) - \check{V}_{hc}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{\underline{z\bar{h}}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\bar{l}}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\bar{l}}(\bar{n}))$ for $z = \bar{l}, l$;
- $\check{V}_{z\bar{h}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\bar{l}}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\bar{l}}(\bar{n}))$ for $z = c, h, \bar{h}$;
- $\check{V}_{\underline{z\bar{l}}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l, c$;
- $\check{V}_{\underline{z\bar{l}}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\bar{h}}(1-n_1^i, n_1^i, \ldots) - \check{V}_{stag}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{\underline{z\tilde{h}'}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{l}'}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{l}'}(\bar{n}))$ for $z = \bar{l}, l$;
- $\check{V}_{x\tilde{h}'}(n^i) \equiv \check{Y}_x(n^i) + \beta\lambda\mu(\check{V}_{h\tilde{l}'}(\hat{m}, 1-\hat{m}, \ldots) - \check{V}_{\bar{h}\tilde{l}'}(\bar{n}))$;
- $\check{V}_{z\tilde{h}'}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{h}'}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{h}'}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{\underline{z\tilde{l}'}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l$;
- $\check{V}_{c\tilde{l}'}(n^i) \equiv \check{Y}_0(n^i)$;
- $\check{V}_{\underline{z\tilde{l}'}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{h}'}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{h}'}(\bar{n}))$ for $z = h, \bar{h}$;
- $\check{V}_{\underline{z\tilde{h}}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{l}}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{l}}(\bar{n}))$ for $z = \bar{l}, l$;
- $\check{V}_{x\tilde{h}}(n^i) \equiv \check{Y}_x(n^i) + \beta\lambda\mu(\check{V}_{h\tilde{l}}(\hat{m}, 1-\hat{m}, \ldots) - \check{V}_{\bar{h}\tilde{l}}(\bar{n}))$;
- $\check{V}_{z\tilde{h}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}g}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}g}(\bar{n}))$ for $z = c, h, \bar{h}$;
- $\check{V}_{z\tilde{l}}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l, c$;
- $\check{V}_{\underline{z\tilde{l}}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}\tilde{h}}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}\tilde{h}}(\bar{n}))$ for $z = h, \bar{h}$;

- $\check{V}_{\underline{zh}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}l}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}l}(\bar{n}))$ for $z = \bar{l}, l$;

- $\check{V}_{xh}(n^i) \equiv \check{Y}_x(n^i) + \beta\lambda\mu(\check{V}_{hl}(\hat{m}, 1-\hat{m}, \ldots) - \check{V}_{\bar{h}l}(\bar{n}))$;

- $\check{V}_{zh}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}l}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}l}(\bar{n}))$ for $z = c, h, \bar{h}$;

- $\check{V}_{zl}(n^i) \equiv \check{Y}_0(n^i)$ for $z = \bar{l}, l, c$;

- $\check{V}_{\underline{zl}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}h}(1-n_1^i, n_1^i, \ldots) - \check{V}_{\bar{h}h}(\bar{n}))$ for $z = h, \bar{h}$;

Inspecting the equations, I have that $\check{V}_{zz'}(n^i)$ is the same across $\bar{l}g$, lg, cg, hg, $\bar{h}g$, $\bar{l}\bar{h}$, $l\bar{h}$, $\bar{l}\tilde{h}$, $l\tilde{h}$, $c\tilde{h}$, $h\tilde{h}$, $\bar{h}\tilde{h}$, $\bar{l}\tilde{h}'$, $l\tilde{h}'$, $h\tilde{h}'$, $\bar{h}\tilde{h}'$, $\bar{l}h$, lh, $h\tilde{l}$, $\bar{h}\tilde{l}$, $h\tilde{l}'$, $\bar{h}\tilde{l}'$, hl, $\bar{h}l$, $h\bar{c}$, $\bar{h}\bar{c}$, $h\tilde{c}c$, and $\bar{h}\tilde{c}c$; $\check{V}_{zz'}(n^i)$ is the same across $h\bar{l}$ and $\bar{h}\bar{l}$; $\check{V}_{zz'}(n^i)$ is the same across $\bar{l}\bar{l}$, $l\bar{l}$, $c\bar{l}$, $\bar{l}\tilde{l}$, $l\tilde{l}$, $c\tilde{l}$, $\bar{l}\tilde{l}'$, $l\tilde{l}'$, $c\tilde{l}'$, $\bar{l}l$, ll, cl, $\bar{l}\bar{c}$, $l\bar{c}$, $c\bar{c}$ $\bar{l}\tilde{c}c$, $l\tilde{c}c$, $c\tilde{c}c$ $\bar{l}c\tilde{c}$, $lc\tilde{c}$, $cc\tilde{c}$ $\bar{l}c$, lc, and cc; $\check{V}_{zz'}(n^i)$ is the same across ch, hh, $\bar{h}h$, $hc\tilde{c}$, $\bar{h}c\tilde{c}$, hc, and $\bar{h}c$; $\check{V}_{zz'}(n^i)$ is the same across $c\bar{h}$ and $h\bar{h}$; and $\check{V}_{zz'}(n^i)$ is the same across xh, $x\tilde{h}$, and $x\tilde{h}'$. Further, I have $d\hat{V}_{gs}(n^i)/dn_s = d\check{V}_{ll}(n^i)/dn_s = d\check{V}_{xh}(n^i)/dn_s < d\hat{V}_{gg}(n^i)/dn_s = d\check{V}_{lh}(n^i)/dn_s = d\check{V}_{h\bar{l}}(n^i)/dn_s < d\hat{V}_{gr}(n^i)/dn_s = d\check{V}_{hh}(n^i)/dn_s = d\check{V}_{h\bar{h}}(n^i)/dn_s < d\hat{V}_{ss}(n^i)/dn_s = d\check{V}_{stag}(n^i)/dn_s$.

I can show the following properties of $\check{V}$ that are useful for the reasoning in the subsequent paragraphs:

- $\check{V}_{ll}(\hat{m}, 1-\hat{m}, \ldots) = \check{V}_{hl}(\hat{m}, 1-\hat{m}, \ldots)$;

- $\check{V}_{lh}(1-\hat{m}, \hat{m}, \ldots) = \check{V}_{h\bar{h}}(1-\hat{m}, \hat{m}, \ldots)$;

- $\check{V}_{lh}(\bar{n}) = \check{V}_{\bar{h}h}(\bar{n})$.

Now consider $(\check{q}^i(n), \check{g}^i(n))$ as an optimal response across various cases of $\{(\check{q}^j(n), \check{g}^j(n))\}_{j \neq i}$.

Case 1: If $\check{g}_0^j(n) \in [0, 1-\hat{m}]$ for some $j \neq i$ and $\check{g}_0^j(n) \in [\hat{m}, 1]$ for some $j \neq i$, the same reasoning and results hold as in Case 1 in [19].

Case 2: If $\check{g}_0^j(n) = 0$ for all $j \neq i$ and $\max_{j \neq i}\{\check{q}^j(n)\} = 1$, the same reasoning and results hold as in Case 2 in [19].

Case 3: If $\check{g}_0^j(n) = 0$ and $\check{q}^j(n) = 0$ for all $j \neq i$, the same reasoning and results hold as in Case 3 in [19].

Case 4: If $\check{g}_0^j(n) \in [0, \hat{m}]$ for all $j \neq i$, $\check{g}_0^j(n) \in (0, \hat{m}]$ for some $j \neq i$, and $\check{g}_0^j(n) \in [0, 1-\hat{m}]$ for some $j \neq i$, the same reasoning and results hold as in Case 4 in [19].

Case 5: If $\check{g}_0^j(n) = 1$ for all $j \neq i$, the same reasoning and results hold as in Case 5 in [19].

Case 6: If $\check{g}_0^j(n) \in [1-\hat{\rho}\hat{m}, 1]$ for all $j \neq i$, $\check{g}_0^j(n) \in [1-\hat{\rho}\hat{m}, 1)$ for some $j \neq i$, and $\check{g}_0^j(n) \in [\hat{m}, 1]$ for some $j \neq i$, the same reasoning and results hold as in Case 6 in [19].

Case 7: If $\check{g}_0^j(n) \in [1-\hat{m}, 1]$ for all $j \neq i$, $\check{g}_0^j(n) \in [1-\hat{m}, 1-\hat{\rho}\hat{m}]$ for some $j \neq i$, and $\check{g}_0^j(n) \in [\hat{m}, 1]$ for some $j \neq i$, the same reasoning and results hold as in Case 7 in [19].

Case 8: If $\check{g}_0^j(n) \in [1-\hat{\rho}\hat{m}, \hat{\rho}\hat{m}]$ for all $j \neq i$, I have $\max_{j \neq i}\{\check{q}^j(n)\} = 1$; $1 - n_1^i \leq 1 - \hat{m}$;

- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{ll}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [0, \hat{m}]$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{hh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [1-\hat{m}, 1]$.

I have $\max_{\tilde{n}_0^i < \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\} = \check{Y}(n^i, (1-n_1^i, n_1^i, \ldots)) + \beta\lambda\check{V}_{ll}(1-n_1^i, n_1^i, \ldots) \geq \check{Y}(n^i, (\hat{m}, 1-\hat{m}, \ldots)) + \beta\lambda\check{V}_{lh}(\hat{m}, 1-\hat{m}, \ldots) = \max_{\tilde{n}_0^i \geq \hat{m}}\{\check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{lh}(\tilde{n}^i)\} \geq \max_{\tilde{n}_0^i \geq \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\}$. Then, $\check{g}^i(n) = (1-n_1^i, n_1^i, \ldots)$.

Case 9: If $\check{g}_0^j(n) \in [1-\hat{m}, \hat{\rho}\hat{m}]$ for all $j \neq i$ and $\check{g}_0^j(n) \in [1-\hat{m}, 1-\hat{\rho}\hat{m}]$ for some $j \neq i$, I have $\max_{j \neq i}\{\check{q}^j(n)\} = 1$; $1 - n_1^i \leq 1 - \hat{m}$;

- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{ll}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [0, \hat{m}]$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{lh}(\tilde{n}^i)$ if $\tilde{n}_0^i \in [1-\hat{m}, 1]$.

I have $\max_{\tilde{n}_0^i < \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\} = \check{Y}(n^i, (1-n_1^i, n_1^i, \ldots)) + \beta\lambda\check{V}_{ll}(1-n_1^i, n_1^i, \ldots) \geq \check{Y}(n^i, (\hat{m}, 1-\hat{m}, \ldots)) + \beta\lambda\check{V}_{lh}(\hat{m}, 1-\hat{m}, \ldots) = \max_{\tilde{n}_0^i \geq \hat{m}}\{\check{W}_2^i(n, \tilde{n}^i)\}$. Then, $\check{g}^i(n) = (1-n_1^i, n_1^i, \ldots)$.

The above properties of $(\check{q}^i(n), \check{g}^i(n))$ across various cases of $\{(\check{q}^j(n), \check{g}^j(n))\}_{j \neq i}$ imply (16) for the case of $\hat{m} > 1/2$.

[21] Now consider Conditional Growth Equilibrium 2 in (17). First, consider the case of $\check{m} \geq 1/2$. I have $\check{V}^i_o(n) = \check{V}_{stag}(n^i)$ if $\Lambda^j(n) \geq 0$ for all j, which requires that $n^j_0 \geq \check{m}$ for all j. The condition, $\Lambda^j(n) \geq 0$, is equivalent to $\check{W}_{stag}(n^j) \equiv \check{Y}(n^j, (0, n^j)) + \beta \check{V}_{stag}(n^j) \geq \check{W}_{hh}(n^j) \equiv \check{Y}(n^j, (1 - n^j_1, n^j_1, \ldots)) + \beta\lambda \check{V}_{ll}(1 - n^j_1, n^j_1, \ldots)$ for all j. The right-hand side of the inequality is the aggregate utility of country j from adopting technology while the other countries do not. The condition, $\Lambda^j(n) \geq 0$ or $\check{W}_{stag}(n^j) \geq \check{W}_{hh}(n^j)$, is also equivalent to $\check{V}_{stag}(n^j) \geq \check{V}_{hh}(n^j)$. Conversely, $\check{V}^i_o(n) = \check{V}_{zz'}(n^i)$ if $n^j_0 < \check{m}$ for some j or if $n^j_0 \geq \check{m}$ for all j and $\check{W}_{stag}(n^j) < \check{W}_{hh}(n^j)$ for some j, where z characterizes the human capital distribution of country i and z' those of the other countries as follows:

For all z', subscript lz' means $n^i_1 \in [0, 1 - \check{m}]$; cz' means $n^i_1 \in [1 - \check{m}, \check{m}]$; hz' means $n^i_1 \in [\check{m}, 1]$.

For all z, subscript zl means $n^j_1 \in [0, 1 - \check{m}]$ for all $j \neq i$; zc means $n^j_1 \in [0, \check{m}]$ for some $j \neq i$ and $n^j_1 \in [1 - \check{m}, 1]$ for some $j \neq i$; zh means $n^j_1 \in [\check{m}, 1]$ for all $j \neq i$.

Further, given zz', let subscript $\tilde{z}z'$ be defined as follows: $\tilde{z}z' = hz'$ if $z = l$, $\tilde{z}z' = cz'$ if $z = c$, and $\tilde{z}z' = lz'$ if $z = h$.

In the equations below, I underline the states of the world economy that are on the equilibrium path starting from some initial state of the world as in [19] and [20]. I have

- $\check{V}_{\underline{stag}}(n^i) \equiv \check{Y}_{stag}(n^i) + \beta\mu(\check{V}_{stag}(n^i) - \lambda \check{V}_{hl}(\bar{n}))$;
- $\check{V}_{\underline{zg}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}g}(1 - n^i_1, n^i_1, \ldots) - \check{V}_{hg}(\bar{n}))$ for $z = l, c, h$;
- $\check{V}_{\underline{zh}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}l}(1 - n^i_1, n^i_1, \ldots) - \check{V}_{hl}(\bar{n}))$ for $z = l, c$;
- $\check{V}_{hh}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{ll}(1 - n^i_1, n^i_1, \ldots) - \check{V}_{hl}(\bar{n}))$;

- $\check{V}_{\underline{ll}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{stag}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{stag}(\bar{n}))$;
- $\check{V}_{\underline{zl}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda\mu(\check{V}_{\tilde{z}h}(1 - n_1^i, n_1^i, \ldots) - \check{V}_{stag}(\bar{n}))$ for $z = c, h$.

Inspecting the equations, I have that $\check{V}_{zz'}(n^i)$ is the same across lh, ch, hg, cg, and lg; and $\check{V}_{zz'}(n^i)$ is the same across hl and cl. Further, I have $d\hat{V}_{gt}(n^i)/dn_s = d\check{V}_{ll}(n^i)/dn_s < d\hat{V}_{gg}(n^i)/dn_s = d\check{V}_{lh}(n^i)/dn_s = d\check{V}_{hl}(n^i)/dn_s = d\check{V}_{hg}(n^i)/dn_s < d\hat{V}_{gu}(n^i)/dn_s = d\check{V}_{hh}(n^i)/dn_s < d\hat{V}_{ss}(n^i)/dn_s = d\check{V}_{stag}(n^i)/dn_s$.

I can show the following properties of $\check{V}$ that are useful for the reasoning in the subsequent paragraphs:

- $\check{V}_{ll}(1 - \check{m}, \check{m}, \ldots) = \check{V}_{hl}(1 - \check{m}, \check{m}, \ldots)$;
- $\check{V}_{lh}(\check{m}, 1 - \check{m}, \ldots) = \check{V}_{hh}(\check{m}, 1 - \check{m}, \ldots) = \check{V}_{stag}(\check{m}, 1 - \check{m}, \ldots)$.

These properties imply that $\check{W}_2^i(n, \tilde{n}^i)$ is continuous in $\tilde{n}^i$.

Now consider $(\check{q}^i(n), \check{g}^i(n))$ as an optimal response across various cases of $\{(\check{q}^j(n), \check{g}^j(n))\}_{j \neq i}$.

Case 1: If $\check{g}_0^j(n) \in [0, \check{m}]$ for some $j \neq i$ and $\check{g}_0^j(n) \in [1 - \check{m}, 1]$ for some $j \neq i$, I have $\max_{j \neq i}\{\check{q}^j(n)\} = 1$ and

- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{lh}(\tilde{n}^i)$ for all $\tilde{n}_0$.

I have $d\check{W}_2^i(n, \tilde{n}^i)/d\tilde{n}_s = \psi(s) + \beta\lambda \cdot d\hat{V}_{gg}(\tilde{n}^i)/d\tilde{n}_s^i > 0$ for all $s \geq 2$; and $d\check{W}_2^i(n, \tilde{n}^i)/d\tilde{n}_1 < 0$. Then, $\check{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$.

Case 2: If $\check{g}_0^j(n) \in [0, 1 - \check{m}]$ for all $j \neq i$ and $\max_{j \neq i}\{\check{q}^j(n)\} = 1$, I have

- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{ll}(\tilde{n}^i)$ if $\tilde{n}_0 \in [0, 1 - \check{m}]$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{hl}(\tilde{n}^i)$ if $\tilde{n}_0 \in [1 - \check{m}, 1]$.

I have that $\check{W}_2^i(n,\tilde{n}^i)$ is continuous in $\tilde{n}^i$; $d\check{W}_2^i(n,\tilde{n}^i)/d\tilde{n}_1 \leq \psi(1) + \beta\lambda \cdot d\hat{V}_{gg}(\tilde{n}^i)/d\tilde{n}_1^i < 0$; $d\check{W}_2^i(n,\tilde{n}^i)/d\tilde{n}_s \geq \psi(s) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n}^i)/d\tilde{n}_s^i > 0$ for $s \geq 2$ since $\phi > \tilde{\phi}_w$. Then, $\check{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$.

Case 3: If $\check{g}_0^j(n) = 0$ and $\check{q}^j(n) = 0$ for all $j \neq i$, using the result in Case 2, I have $\check{W}^i(n) = \max\{\check{W}_{stag}(n^i), \check{W}_2^i(n,(1-n_1^i, n_1^i, \ldots))\}$. If $n_1^i \in [0, \check{m}]$, $\check{W}_{stag}(n^i) \leq W_2^i(n,(1-n_1^i, n_1^i, \ldots)) = \hat{W}_{lh}(n^i)$. If $n_1^i \in [\check{m}, 1]$, $\check{W}_2^i(n,(1-n_1^i, n_1^i, \ldots) = \hat{W}_{hh}(n^i)$. If $n_1^i \in [\check{m}, 1]$ and $n_1^i + n_2^i = 1$, $\check{W}_{stag}(n^i) \geq \check{W}_2^i(n,(1 - n_1^i, n_1^i, \ldots)) = \hat{W}_{hh}(n^i)$. Then, $\check{g}^i(n) = (0, n^i)$ and $\check{q}^i(n) = 0$ if $n_1^i \in [\check{m}, 1]$ and $\check{W}_{stag}(n^i) \geq \hat{W}_{hh}(n^i)$; and $\check{g}^i(n) = (1-n_1^i, n_1^i, \ldots)$ and $\check{q}^i(n) = 1$ otherwise. In particular, $\check{g}^i(n) = (0, n^i)$ and $\check{q}^i(n) = 0$ if $n_1^i \in [\check{m}, 1]$ and $n_1^i + n_2^i = 1$.

Case 4: If $\check{g}_0^j(n) \in [\check{m}, 1]$ for all $j \neq i$, I have $\max_{j\neq i}\{\check{q}^j(n)\} = 1$;

- $\check{W}_2^i(n,\tilde{n}^i) = \check{Y}(n^i,\tilde{n}^i) + \beta\lambda\check{V}_{lh}(\tilde{n}^i)$ if $\tilde{n}_0 \in [0, \check{m}]$;
- $\check{W}_2^i(n,\tilde{n}^i) = \check{Y}(n^i,\tilde{n}^i) + \beta\lambda\check{V}_{hh}(\tilde{n}^i)$ if $\tilde{n}_0 \in [\check{m}, 1]$ and $\check{W}_{stag}(\tilde{n}^i) < \check{W}_{hh}(\tilde{n}^i)$;
- $\check{W}_2^i(n,\tilde{n}^i) = \check{Y}(n^i,\tilde{n}^i) + \beta\lambda\check{V}_{stag}(\tilde{n}^i)$ if $\tilde{n}_0 \in [\check{m}, 1]$ and $\check{W}_{stag}(\tilde{n}^i) \geq \check{W}_{hh}(\tilde{n}^i)$.

Given n^i and $\tilde{n}^i$, let $\tilde{x} \equiv (\tilde{n}_0^i, \min\{n_1^i, 1 - \tilde{n}_0^i\}, \min\{n_2^i, \max\{0, \sum_{s\geq2} n_s^i - \tilde{n}_0^i\}\}, \min\{n_3^i, \max\{0, \sum_{s\geq3} n_s^i - \tilde{n}_0^i\}\}, \min\{n_4^i, \max\{0, \sum_{s\geq4} n_s^i - \tilde{n}_0^i\}\}, \ldots)$. I have $\check{W}_2^i(n,\tilde{n}^i) \leq \check{W}_2^i(n,\tilde{x}^i)$. I have $\check{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$ as the summary result of the following subcases:

Case 4-1: If $\tilde{n}_0^i > 1 - n_1^i$, I have $\tilde{x} = (\tilde{n}_0^i, 1 - \tilde{n}_0^i, \ldots)$ and $\check{W}_2^i(n,\tilde{n}^i) - \check{W}_2^i(n,(1 - n_1^i, n_1^i, \ldots)) \leq \check{W}_2^i(n,(\tilde{n}_0^i, 1 - \tilde{n}_0^i, \ldots)) - \check{W}_2^i(n,(1 - n_1^i, n_1^i, \ldots)) = \max\{0, \tilde{n}_0^i - \max\{\check{m}, 1 - n_1^i\}\} \cdot (\psi(1) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_1^i) + \max\{0, \min\{\tilde{n}_0^i, \check{m}\} - 1 + n_1^i\} \cdot (\psi(1) + \beta\lambda \cdot d\hat{V}_{gg}(\tilde{n}^i)/d\tilde{n}_1^i) < 0$.

Case 4-2: If $\tilde{n}_0^i < 1 - n_1^i$ and $\tilde{n}_0^i \leq \check{m}$, I have $\tilde{x}_1 = n_1^i$ and $\check{W}_2^i(n,(1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n,\tilde{n}^i) \geq \check{W}_2^i(n,(1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n,\tilde{x}) \geq \max\{0, 1 - n_1^i - \check{m}\} \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_1^i) + (\min\{\check{m}, 1 - n_1^i\} - \tilde{n}_0^i) \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{gg}(\tilde{n}^i)/d\tilde{n}_1^i) > 0$.

Case 4-3: If $\check{m} \leq \tilde{n}_0^i < 1 - n_1^i$ and $\check{W}_{stag}(\tilde{x}) \geq \check{W}_{hh}(\tilde{x})$, I have $\tilde{x}_1 = n_1^i$ and $\check{W}_2^i(n,(1-n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{n}^i) \geq \check{W}_2^i(n,(1-n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{x}) \geq (1 - n_1^i - \tilde{n}_0^i) \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_2^i) > 0$.

Case 4-4: If $\check{m} \leq \tilde{n}_0^i < 1 - n_1^i$ and $\check{W}_{stag}(\tilde{x}) < \check{W}_{hh}(\tilde{x})$, I have $\tilde{x}_1 = n_1^i$. Let $\tilde{y}$ be a human capital distribution that satisfies: $\check{W}_{stag}(\tilde{y}) = \check{W}_{hh}(\tilde{y})$, $\tilde{n}_0 < \tilde{y}_0 < 1-n_1^i$, $\tilde{y}_1 = n_1^i$, and $\tilde{n}_s^i \leq \tilde{y}_s \leq \tilde{x}_s$ for all $s \geq 2$. I have $\check{W}_2^i(n,(1-n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{n}^i) \geq \check{W}_2^i(n,(1-n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{x}) \geq (1-n_1^i-\tilde{y}_0)\cdot(\psi(2)+\beta\lambda\cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_2^i)+(\tilde{y}_0-\tilde{n}_0^i)\cdot(\psi(2)+\beta\lambda\cdot d\hat{V}_{gg}(\tilde{n}^i)/d\tilde{n}_2^i) > 0$.

The above properties of $(\check{q}^i(n), \check{g}^i(n))$ across various cases of $\{(\check{q}^j(n), \check{g}^j(n))\}_{j \neq i}$ imply (17) for the case of $\check{m} \geq 1/2$.

[22] Now consider the case of $\check{m} < 1/2$. As in [21], I have $\check{V}_o^i(n) = \check{V}_{stag}(n^i)$ if $\Lambda^j(n) \geq 0$ for all j, which requires that $n_0^j \geq \check{m}$ for all j. The condition, $\Lambda^j(n) \geq 0$, is equivalent to $\check{W}_{stag}(n^j) \equiv \check{Y}(n^j, (0, n^j)) + \beta\check{V}_{stag}(n^j) \geq \check{W}_{hh}(n^j) \equiv \check{Y}(n^j, (1-n_1^j, n_1^j, \ldots)) + \beta\lambda\check{V}_{ll}(1-n_1^j, n_1^j, \ldots)$ for all j. The condition, $\Lambda^j(n) \geq 0$ or $\check{W}_{stag}(n^j) \geq \check{W}_{hh}(n^j)$, is also equivalent to $\check{V}_{stag}(n^j) \geq \check{V}_{hh}(n^j)$. Conversely, $\check{V}_o^i(n) = \check{V}_{zz'}(n^i)$ if $n_0^j < \check{m}$ for some j or if $n_0^j \geq \check{m}$ for all j and $\check{W}_{stag}(n^j) < \check{W}_{hh}(n^j)$ for some j, where z characterizes the human capital distribution of country i and z' those of the other countries as follows:

For all z', subscript lz' means $n_1^i \in [0, \tilde{\rho}\check{m}]$; bz' means $n_1^i \in [\tilde{\rho}\check{m}, \check{m}]$; cz' means $n_1^i \in [\check{m}, 1-\check{m}]$; dz' means $n_1^i \in [1-\check{m}, 1-\tilde{\rho}\check{m}]$; and hz' means $n_1^i \in [1-\tilde{\rho}\check{m}, 1]$, where $\tilde{\rho} \equiv (1-\beta\mu)(1-\phi(1-\beta+\beta\lambda))/(1+\beta\lambda\mu)/(1-\beta\mu-\beta\lambda\mu+\beta^2\lambda^2\mu^2-\phi((1-\beta)(1-\beta\mu)+\beta\lambda(1-\mu)(1-\beta\lambda\mu)))$.

For all z, subscript zl means either $n_1^j \in [0, 1-\check{m}]$ for all $j \neq i$ and $n_1^k \in [0, \check{m}]$ and $n_1^l \in [0, \check{m}]$ for some $k, l \neq i$, or $n_1^j \in [0, 1-\check{m}]$ for all $j \neq i$ and $n_1^j \in [0, \tilde{\rho}\check{m}]$ for some $j \neq i$; zb means $n_1^k \in [\tilde{\rho}\check{m}, \check{m}]$ for some $k \neq i$ and $n_1^j \in [\check{m}, 1-\check{m}]$ for all $j \notin \{i, k\}$; zc means $n_1^j \in [\check{m}, 1-\check{m}]$ for all $j \neq i$; $z\tilde{c}$ means $n_1^j \in [0, \check{m}]$ for some $j \neq i$ and $n_1^j \in [1-\check{m}, 1]$ for some $j \neq i$; zd means $n_1^k \in [1-\check{m}, 1-\tilde{\rho}\check{m}]$ for some $k \neq i$ and $n_1^j \in [\check{m}, 1-\check{m}]$ for all $j \notin \{i,k\}$; and zh means either $n_1^j \in [\check{m},1]$ for all $j \neq i$ and $n_1^k \in [1-\check{m},1]$ and $n_1^l \in [1-\check{m},1]$ for some $k,l \neq i$, or $n_1^j \in [\check{m},1]$ for all $j \neq i$ and $n_1^j \in [1-\tilde{\rho}\check{m},1]$ for some $j \neq i$.

Further, given zz', let subscript $\tilde{z}z'$ be defined as follows: $\tilde{z}z' = hz'$ if $z = l$, $\tilde{z}z' = dz'$ if $z = b$, $\tilde{z}z' = cz'$ if $z = c$, $\tilde{z}z' = bz'$ if $z = d$, and $\tilde{z}z' = lz'$ if $z = h$. Let $\check{Y}_d(n^i) \equiv \check{Y}(n^i,(\check{m},1-\check{m},\ldots)) + \beta\lambda(1-\mu)\cdot\check{Y}((\check{m},1-\check{m},\ldots),\bar{n})$.

In the equations below, I underline the states of the world economy that are on the equilibrium path starting from some initial state of the world as in [19], [20], and [21]. I have

- $\check{V}_{\underline{stag}}(n^i) \equiv \check{Y}_{stag}(n^i) + \beta\mu(\check{V}_{stag}(n^i) - \lambda\check{V}_{hl}(\bar{n}))$;
- $\check{V}_{zg}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{\tilde{z}g}(1-n_1^i,n_1^i,\ldots) - \beta\lambda\check{V}_{hg}(\bar{n}))$ for $z = l,b,c,d,h$;
- $\check{V}_{\underline{zh}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{\tilde{z}l}(1-n_1^i,n_1^i,\ldots) - \check{V}_{hl}(\bar{n}))$ for $z = l,b$;
- $\check{V}_{zh}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{\tilde{z}l}(1-n_1^i,n_1^i,\ldots) - \check{V}_{hl}(\bar{n}))$ for $z = c,d,h$;
- $\check{V}_{\underline{zl}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{stag}(1-n_1^i,n_1^i,\ldots) - \check{V}_{stag}(\bar{n}))$ for $z = l,b,c$;
- $\check{V}_{\underline{zl}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{\tilde{z}h}(1-n_1^i,n_1^i,\ldots) - \check{V}_{stag}(\bar{n}))$ for $z = d,h$;
- $\check{V}_{\underline{zd}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{\tilde{z}b}(1-n_1^i,n_1^i,\ldots) - \check{V}_{hb}(\bar{n}))$ for $z = l,b$;
- $\check{V}_{cd}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{stag}(1-n_1^i,n_1^i,\ldots) - \check{V}_{stag}(\bar{n}))$;
- $\check{V}_{zd}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{\tilde{z}b}(1-n_1^i,n_1^i,\ldots) - \check{V}_{hb}(\bar{n}))$ for $z = d,h$;
- $\check{V}_{\underline{zb}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{stag}(1-n_1^i,n_1^i,\ldots) - \check{V}_{stag}(\bar{n}))$ for $z = l,b,c$;
- $\check{V}_{cb}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{stag}(1-n_1^i,n_1^i,\ldots) - \check{V}_{stag}(\bar{n}))$;
- $\check{V}_{\underline{zb}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{\tilde{z}d}(1-n_1^i,n_1^i,\ldots) - \check{V}_{hd}(\bar{n}))$ for $z = d,h$;

- $\check{V}_{\underline{lc}}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{stag}(1-n_1^i, n_1^i, \ldots) - \check{V}_{stag}(\bar{n}))$;
- $\check{V}_{zc}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{stag}(1-n_1^i, n_1^i, \ldots) - \check{V}_{stag}(\bar{n}))$ for $z = b, c$;
- $\check{V}_{dc}(n^i) \equiv \check{Y}_d(n^i) + \beta\lambda(\check{V}_{stag}(\check{m}, 1-\check{m}, \ldots) - \check{V}_{stag}(\bar{n}))$;
- $\check{V}_{hc}(n^i) \equiv \check{Y}_1(n^i) + \beta\lambda(\check{V}_{lc}(1-n_1^i, n_1^i, \ldots) - V_{stag}(\bar{n}))$;

Inspecting the equations, I have that $\check{V}_{zz'}(n^i)$ is the same across ll, bl, cl, lb, bb, cb, lc, bc, cc, and cd; $\check{V}_{zz'}(n^i)$ is the same across hh, dh, ch, hd, and dd; $\check{V}_{zz'}(n^i)$ is the same across lh, bh, ld, bd, hg, dg, cg, bg, and lg; and $\check{V}_{zz'}(n^i)$ is the same across hl, dl, hb, and db. Further, I have $d\hat{V}_{gs}(n^i)/dn_s = d\check{V}_{dl}(n^i)/dn_s \leq d\hat{V}_{gt}(n^i)/dn_s = d\check{V}_{ll}(n^i)/dn_s < d\hat{V}_{gg}(n^i)/dn_s = d\check{V}_{lh}(n^i)/dn_s = d\check{V}_{hl}(n^i)/dn_s = d\check{V}_{h\tilde{c}}(n^i)/dn_s < d\hat{V}_{gu}(n^i)/dn_s = d\check{V}_{hh}(n^i)/dn_s = d\check{V}_{hc}(n^i)/dn_s < d\hat{V}_{ss}(n^i)/dn_s = d\check{V}_{stag}(n^i)/dn_s$.

I can show the following properties of $\check{V}$ that are useful for the reasoning in the subsequent paragraphs:

- $\check{V}_{ll}(1-\check{m}, \check{m}, \ldots) = \check{V}_{hl}(1-\check{m}, \check{m}, \ldots)$;
- $\check{V}_{ll}(\tilde{\rho}\check{m}, 1-\tilde{\rho}\check{m}, \ldots) = \check{V}_{lh}(\check{m}, 1-\check{m}, \ldots) = \check{V}_{hh}(\check{m}, 1-\check{m}, \ldots) = \check{V}_{stag}(\check{m}, 1-\check{m}, \ldots)$;
- $\check{V}_{cc}(1-\check{m}, n_2^i, n_3^i, \ldots) = \check{V}_{dc}(1-\check{m}, n_2^i, n_3^i, \ldots)$ for any $(n_2^i, n_3^i, \ldots)$ with $\sum_{s\geq 2} n_s^i = 1-\check{m}$;
- $\check{V}_{dc}(1-\tilde{\rho}\check{m}, n_2^i, n_3^i, \ldots) = \check{V}_{hc}(1-\tilde{\rho}\check{m}, n_2^i, n_3^i, \ldots)$ for any $(n_2^i, n_3^i, \ldots)$ with $\sum_{s\geq 2} n_s^i = 1-\tilde{\rho}\check{m}$.

Now consider $(\check{q}^i(n), \check{g}^i(n))$ as an optimal response across various cases of $\{(\check{q}^j(n), \check{g}^j(n))\}_{j\neq i}$.

Case 1: If $\check{g}_0^j(n) \in [0, \check{m}]$ for some $j \neq i$ and $\check{g}_0^j(n) \in [1-\check{m}, 1]$ for some $j \neq i$, the same reasoning and results hold as in Case 1 in [21].

Case 2: If $\check{g}_0^j(n) \in [0, 1-\check{m}]$ for all $j \neq i$, $\check{g}_0^j(n) \in [0, \check{m}]$ for some $j \neq i$, and $\max_{j\neq i}\{\check{q}^j(n)\} = 1$, the same reasoning and results hold as in Case 2 in [21].

Case 3: If $\check{g}_0^j(n) = 0$ and $\check{q}^j(n) = 0$ for all $j \neq i$, the same reasoning and results hold as in Case 3 in [21].

Case 4: If $\check{g}_0^j(n) \in [\check{m}, 1]$ for all $j \neq i$ and $\check{g}_0^j(n) \in [1 - \tilde{\rho}\check{m}, 1]$ for some $j \neq i$, the same reasoning and results hold as in Case 4 in [21].

Case 5: If $\check{g}_0^j(n) \in [\check{m}, 1]$ for all $j \neq i$ and $\check{g}_0^j(n) \in [1 - \check{m}, 1]$ and $\check{g}_0^k(n) \in [1 - \check{m}, 1]$ for some $j, k \neq i$, $\{\check{q}^j(n)\}$ and $W_2^i(n, \tilde{n}^i)$ have the same properties as in Case 4 so that $\check{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$.

Case 6: If $\check{g}_0^k(n) \in [1 - \check{m}, 1 - \tilde{\rho}\check{m}]$ for some $k \neq i$ and $\check{g}_0^j(n) \in [\check{m}, 1 - \check{m}]$ for all $j \notin \{i, k\}$, I have $\max_{j\neq i}\{\check{q}^j(n)\} = 1$; $\check{W}_{stag}(\tilde{n}^i) \geq \check{W}_{hh}(\tilde{n}^i)$ for all $j \neq i$;

- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{lh}(\tilde{n}^i)$ if $\tilde{n}_0 \in [0, \check{m}]$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{ll}(\tilde{n}^i)$ if $\tilde{n}_0 \in [\check{m}, 1 - \check{m}]$ and $\check{W}_{stag}(\tilde{n}^i) < \check{W}_{hh}(\tilde{n}^i)$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{hh}(\tilde{n}^i)$ if $\tilde{n}_0 \in [1 - \check{m}, 1]$ and $\check{W}_{stag}(\tilde{n}^i) < \check{W}_{hh}(\tilde{n}^i)$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{stag}(\tilde{n}^i)$ if $\tilde{n}_0 \in [\check{m}, 1]$ and $\check{W}_{stag}(\tilde{n}^i) \geq \check{W}_{hh}^i(\tilde{n}^i)$.

For any $\tilde{n}^i$ with $\tilde{n}_0^i \geq \check{m}$ and $\check{W}_{stag}(\tilde{n}^i) < \check{W}_{hh}(\tilde{n}^i)$, $\check{W}_2^i(n, \tilde{n}^i) \leq \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{hh}(\tilde{n}^i)$. Then, the results in Case 4-1 to Case 4-4 hold so that $\check{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$.

Case 7: If $\check{g}_0^j(n) \in [\check{m}, 1 - \check{m}]$ for all $j \neq i$, I have $\max_{j\neq i}\{\check{q}^j(n)\} = 1$; $\check{W}_{stag}(\tilde{n}^i) \geq \check{W}_{hh}(\tilde{n}^i)$ for all $j \neq i$;

- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{ll}(\tilde{n}^i)$ if $\tilde{n}_0 \in [0, \check{m}]$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{ll}(\tilde{n}^i)$ if $\tilde{n}_0 \in [\check{m}, 1 - \check{m}]$ and $\check{W}_{stag}(\tilde{n}^i) < \check{W}_{hh}(\tilde{n}^i)$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{dc}(\tilde{n}^i)$ if $\tilde{n}_0 \in [1 - \check{m}, 1 - \tilde{\rho}\check{m}]$ and $\check{W}_{stag}(\tilde{n}^i) < \check{W}_{hh}(\tilde{n}^i)$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{V}_{hc}(\tilde{n}^i)$ if $\tilde{n}_0 \in [1 - \tilde{\rho}\check{m}, 1]$ and $\check{W}_{stag}(\tilde{n}^i) < \check{W}_{hh}(\tilde{n}^i)$;
- $\check{W}_2^i(n, \tilde{n}^i) = \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\mu(\check{W}_{stag}(\tilde{n}^i)$ if $\tilde{n}_0 \in [\check{m}, 1]$ and $\check{W}_{stag}(\tilde{n}^i) \geq \check{W}_{hh}(\tilde{n}^i)$.

For any $\tilde{n}^i$ with $\tilde{n}_0^i \geq \check{m}$ and $\check{W}_{stag}(\tilde{n}^i) < \check{W}_{hh}(\tilde{n}^i)$, $\check{W}_2^i(n, \tilde{n}^i) \leq \check{Y}(n^i, \tilde{n}^i) + \beta\lambda\check{W}_{hh}(\tilde{n}^i)$. Given n^i and $\tilde{n}^i$, let $\tilde{x}$ be defined as in Case 4 in [21]. I have $\breve{g}^i(n) = (1 - n_1^i, n_1^i, \ldots)$ if $1 - n_1^i \in [0, \tilde{\rho}\check{m}) \cup [\check{m}, 1]$; and $\breve{g}^i(n) = (\check{m}, 1 - \check{m}, \ldots)$ if $1 - n_1^i \in [\tilde{\rho}\check{m}, \check{m}]$ as the summary result of the following cases.

Case 7-1: If $1 - n_1^i \in [0, \tilde{\rho}\check{m}] \cup [\check{m}, 1]$, $\tilde{n}_0^i \in [0, \tilde{\rho}\check{m}] \cup [\check{m}, 1]$, and $\tilde{n}_0^i > 1 - n_1^i$, I have $\tilde{x} = (\tilde{n}_0^i, 1 - \tilde{n}_0^i, \ldots)$ and $\check{W}_2^i(n, \tilde{n}^i) - \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) \leq \check{W}_2^i(n, (\tilde{n}_0^i, 1 - \tilde{n}_0^i, \ldots)) - \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) = \max\{0, \tilde{n}_0^i - \max\{\check{m}, 1 - n_1^i\}\} \cdot (\psi(1) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_1^i) + \max\{0, \min\{\tilde{n}_0^i, \tilde{\rho}\check{m}\} - 1 + n_1^i\} \cdot (\psi(1) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n}^i)/d\tilde{n}_1^i) \leq 0$.

Case 7-2: If $1 - n_1^i \leq \tilde{\rho}\check{m}$, $\tilde{n}_0^i \in [\tilde{\rho}\check{m}, \check{m}]$, and $\tilde{n}_0^i > 1 - n_1^i$, I have $\tilde{x} = (\tilde{n}_0^i, 1 - \tilde{n}_0^i, \ldots)$ and $\check{W}_2^i(n, \tilde{n}^i) - \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) \leq \check{W}_2^i(n, (\tilde{n}_0^i, 1 - \tilde{n}_0^i, \ldots)) - \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) \leq \check{W}_2^i(n, (\check{m}, 1 - \check{m}, \ldots)) - \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) \leq 0$.

Case 7-3: If $1 - n_1^i \in [0, \tilde{\rho}\check{m}] \cup [\check{m}, 1]$, $\tilde{n}_0^i \leq \tilde{\rho}\check{m}$, and $\tilde{n}_0^i < 1 - n_1^i$, I have $\tilde{x}_1 = n_1^i$ and $\check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{n}^i) \geq \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{x}) \geq \max\{0, 1 - n_1^i - \check{m}\} \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_1^i) + (\min\{\tilde{\rho}\check{m}, 1 - n_1^i\} - \tilde{n}_0^i) \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n}^i)/d\tilde{n}_1^i) \geq 0$.

Case 7-4: If $1 - n_1^i \geq \check{m}$, $\tilde{n}_0^i \in [\tilde{\rho}\check{m}, \check{m}]$, and $\tilde{n}_0^i < 1 - n_1^i$, I have $\tilde{x}_1 = n_1^i$ and $\check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{n}^i) \geq \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{x}) \geq \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, (\check{m}, 1 - \check{m}, \ldots)) \geq 0$.

Case 7-5: If $\check{m} \leq \tilde{n}_0^i < 1 - n_1^i$ and $\check{W}_{stag}(\tilde{x}) \geq \check{W}_{hh}(\tilde{x})$, I have $\tilde{x}_1 = n_1^i$ and $\check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{n}^i) \geq \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{x}) \geq (1 - n_1^i - \tilde{n}_0^i) \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_2^i) > 0$.

Case 7-6: If $\check{m} \leq \tilde{n}_0^i < 1 - n_1^i$ and $\check{W}_{stag}(\tilde{x}) < \check{W}_{hh}(\tilde{x})$, I have $\tilde{x}_1 = n_1^i$. Let $\tilde{y}$ be defined as in Case 4-4 in [21]. I have $\check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{n}^i) \geq \check{W}_2^i(n, (1 - n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{x}) \geq (1 - n_1^i - \tilde{y}_0) \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_2^i) + (\tilde{y}_0 - \tilde{n}_0^i) \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n}^i)/d\tilde{n}_2^i) > 0$.

Case 7-7: If $1-n_1^i \in [\tilde{\rho}\check{m}, \check{m}]$ and $\tilde{n}_0^i > \check{m}$, I have $\tilde{x} = (\tilde{n}_0^i, 1-\tilde{n}_0^i, \ldots)$ and $\check{W}_2^i(n, \tilde{n}^i) - \check{W}_2^i(n, (\check{m}, 1-\check{m}, \ldots)) \leq \check{W}_2^i(n, (\tilde{n}_0^i, 1-\tilde{n}_0^i, \ldots)) - \check{W}_2^i(n, (\check{m}, 1-\check{m}, \ldots)) = (\tilde{n}_0^i - \check{m}) \cdot (\psi(1) + \beta\lambda \cdot d\hat{V}_{ss}(\tilde{n}^i)/d\tilde{n}_1^i) \leq 0$.

Case 7-8: If $1-n_1^i \in [\tilde{\rho}\check{m}, \check{m}]$ and $\tilde{n}_0^i \leq 1-n_1^i$, I have $\tilde{x}_1 = n_1^i$; $\check{W}_2^i(n, (1-n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{n}^i) \geq \check{W}_2^i(n, (1-n_1^i, n_1^i, \ldots)) - \check{W}_2^i(n, \tilde{x}) \geq (1-n_1^i-\tilde{n}_0^i) \cdot (\psi(2) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n}^i)/d\tilde{n}_1^i) \geq 0$; and $\check{W}_2^i(n, (\check{m}, 1-\check{m}, \ldots)) - \check{W}_2^i(n, (1-n_1^i, n_1^i, \ldots)) = (\check{Y}(n^j, (0, n^j)) - Y(n^j, (1-n_1^j, n_1^j, \ldots))) + \beta(\check{V}_{stag}(n^j) - \check{V}_{ll}(1-n_1^j, n_1^j, \ldots)) \geq 0$.

Case 7-9: If $1-n_1^i \in [\tilde{\rho}\check{m}, \check{m}]$ and $\tilde{n}_0^i \in [1-n_1^i, \check{m})$, I have $\tilde{x} = (\tilde{n}_0^i, 1-\tilde{n}_0^i, \ldots)$; $\check{W}_2^i(n, \tilde{n}^i) - \check{W}_2^i(n, (1-n_1^i, n_1^i, \ldots)) \leq \check{W}_2^i(n, (\tilde{n}_0^i, 1-\tilde{n}_0^i, \ldots)) - \check{W}_2^i(n, (1-n_1^i, n_1^i, \ldots)) = (\tilde{n}_0^i - 1 + n_1^i) \cdot (\psi(1) + \beta\lambda \cdot d\hat{V}_{gt}(\tilde{n}^i)/d\tilde{n}_1^i) \leq 0$; and $\check{W}_2^i(n, (\check{m}, 1-\check{m}, \ldots)) - \check{W}_2^i(n, (1-n_1^i, n_1^i, \ldots)) \geq 0$.

The above properties of $(\check{q}^i(n), \check{g}^i(n))$ across various cases of $\{(\check{q}^j(n), \check{g}^j(n))\}_{j \neq i}$ imply (17) for the case of $\check{m} < 1/2$.

Figure 1: An Illustration of The Parameter Space

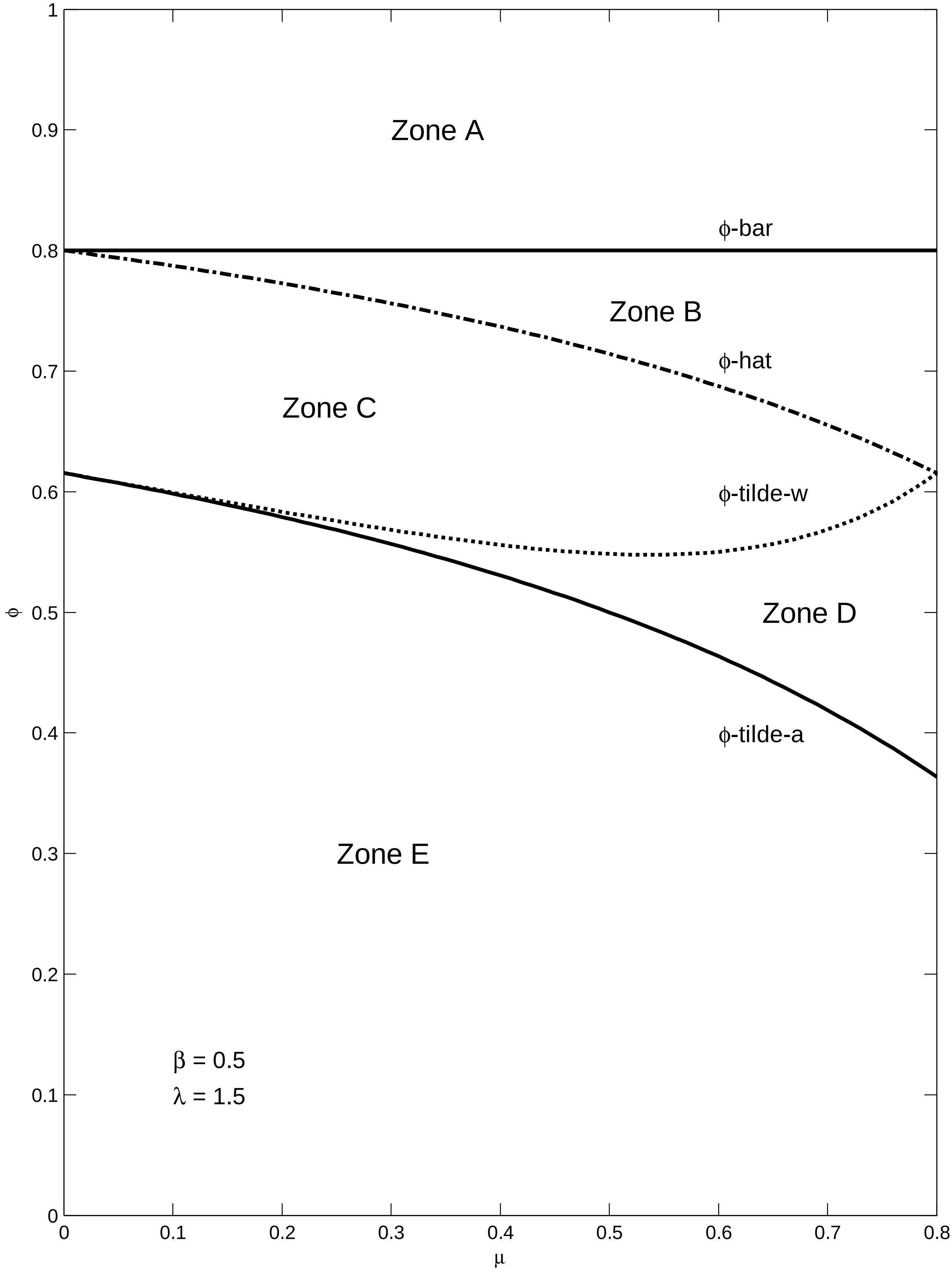
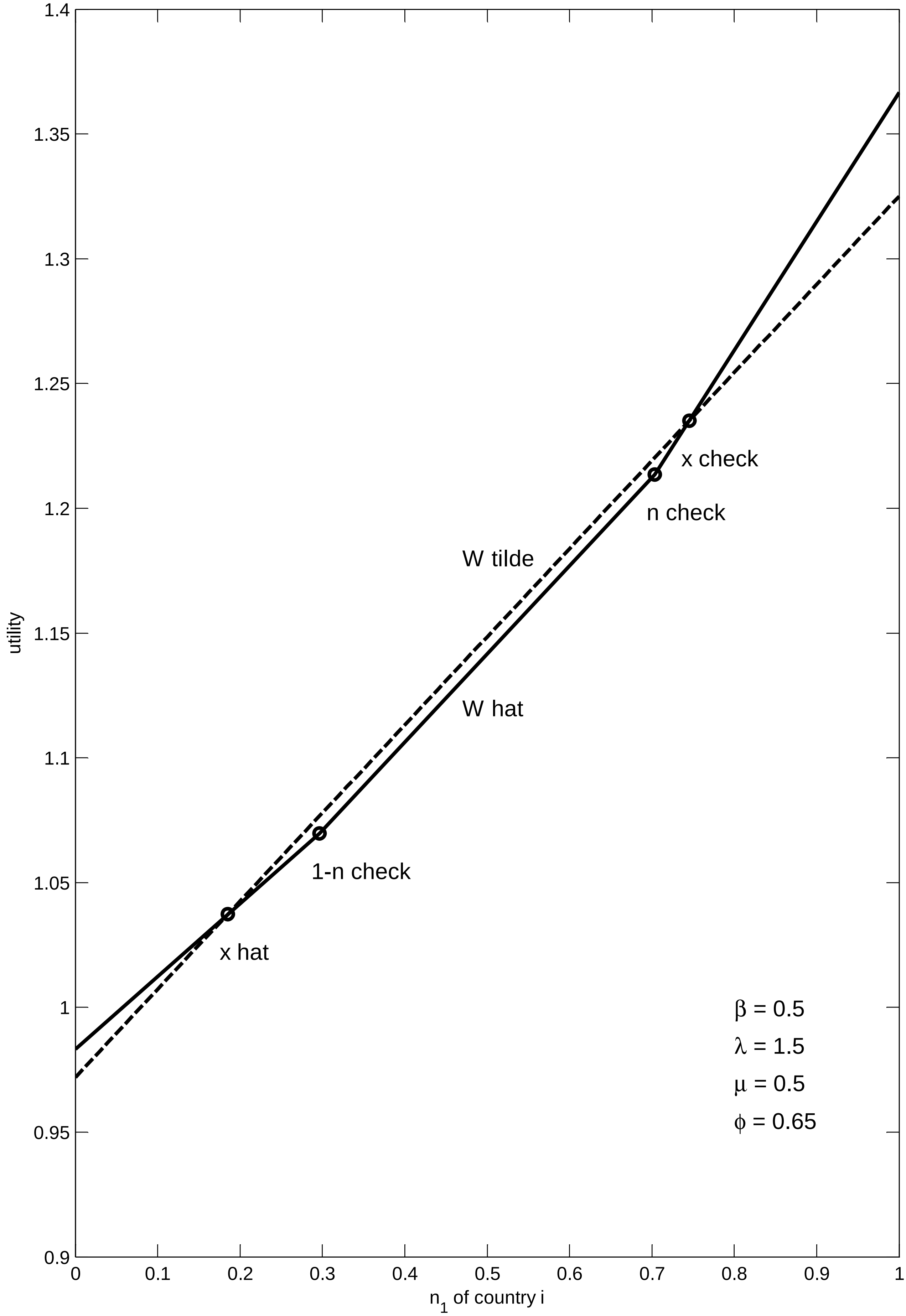


Figure 2: An Illustration of The Value Functions



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