

Securitization under Asymmetric Information over the Business Cycle

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Abstract

The paper studies the efficiency of financial intermediation through securitization with asymmetric information about the quality of securitized loans. In this theoretical model I show that, in general, by providing reputation based implicit recourse the issuer can credibly signal the loan quality. However, in boom stages of the business cycle, information on loan quality remains private and lower quality loans accumulate on balance sheets. This deepens the subsequent downturn. The longer the duration of a boom, the deeper the fall of output in the subsequent recession will be. I present empirical evidence from loan-level data consistent with this result. In recessions the model also produces amplification of adverse selection problems on re-sale markets for securitized loans. This is especially severe after a prolonged boom period and when securitized loans of high quality are no longer traded. Finally, the model suggests that the newly proposed regulation requiring higher explicit risk-retention by the originators of loans could adversely affect both quantity and quality of investment in the economy.

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Contents

1	Introduction	3
2	Literature review	5
2.1	Securitization and implicit recourse	5
2.2	Financial intermediation imperfections, information frictions and business cycles	8
3	Model	9
3.1	Investment projects	9
3.2	Case with no financial frictions - first best	11
3.3	Cases with frictions and without implicit recourse	13
3.4	Implicit recourse and reputation equilibrium case	17
4	Dynamics and numerical examples	27
5	Extensions	29
5.1	Endogenizing the skin in the game	29
5.2	Skin in the game as a policy parameter	31
5.3	Adverse selection on re-sale markets	31
6	Empirical analysis	33
6.1	Hypotheses	34
6.2	Data description	35
6.3	Panel regression results	35
7	Conclusions	39
8	Appendix	44
8.1	Proofs	44
8.2	Derivation of firms' policy functions	59
8.3	Numerical solutions of the stochastic dynamic system	65

1 Introduction

Securitization recently attracted a lot of criticism due to its role in the late 2000's financial crisis (e.g. Bernanke, 2010). Securitization and in general a market-based system of financial intermediation significantly grew in importance in the decades preceding the crisis (Adrian and Shin, 2009). The late 2000's financial crisis led to intensified research into the problematic aspects of securitization. New research is often very critical about securitization such as Shleifer and Vishny (2010), who argue that securitization creates systemic risk and inefficiencies in financial intermediation. Currently regulation of the financial sector including securitization practices is being redrafted and strengthened on national as well as international level. The agency problems related to securitization to which most of the criticism points are, however, not new and securitization design contained tools such as tranche retention schemes or implicit recourse that were supposed to limit these negative aspects of securitization. The question is whether these tools were efficient in the period before the late 2000's financial crisis.

In this paper I show that reputation concerns can allow sponsors of securitized products to credibly signal the quality of the loans by providing implicit recourse and thus limit the problem of asymmetric information. Implicit recourse is an implicit support provided by the issuer of securitized products to the holders of these assets. This support is not contractual and is enforced in a reputation equilibrium¹. Typically there are both pooling and separating equilibria in this signaling game, from which by applying Intuitive Criterion refinement I can select a unique separating equilibrium in which the information about loan quality is transferred and thus the outcome is efficient. However, there are limits to the degree of commitment based on reputation and therefore also to the efficiency of implicit recourse in eliminating the problem of asymmetric information. Following the empirical evidence in Bloom (2009) and Bloom et al. (2011) who find that second moments of firms' TFP in the economy are countercyclical, in this model the relative difference in the projects' (loans') productivity is also countercyclical. As a result it turns out that even though in the steady state provision of implicit recourse helps to achieve a separating equilibrium, in boom stages of the business cycle separation equilibrium would require too high implicit recourse which cannot be enforced through reputation. Therefore, in boom stages of business cycles there are only pooling equilibria, in which the information about the quality of

¹For the review of empirical evidence on implicit recourse, description of its types and discussion of its role in the securitization process I would like to refer the reader to the literature review.

loans remains private and the allocation of investment is inefficient. This has only very moderate effects as long as the economy stays in the boom, where relative difference in loans productivity is low. But the accumulated inefficiency becomes more pronounced in the subsequent downturn of the economy, which is thus amplified. Also the longer is the boom, the larger is the share of lower quality loans on the balance sheets and the deeper will be the subsequent downturn.

Results of this paper have also implications for the related macro-prudential policy which requires higher explicit risk-retention for the originators of the securitized products (such as the section 941 of the Dodd-Frank reform). In this model higher then equilibrium explicit risk-retention, such as the practice of keeping larger fraction of issued loans on the balance sheet of the issuer, limits financial intermediation ability of the issuer. Since higher explicit risk-retention restricts the supply of loans, through the general equilibrium effect this increases the equilibrium prices of securitized assets and makes securitization more profitable. Higher prices make even the securitization of lower quality loans profitable. Therefore, the result of this policy can lower both the quantity as well as the quality of the investment in the economy.

In an extension of the model I also introduce asymmetric information between sellers and buyers of securitized loans on the re-sale market. The model then produces adverse selection which is amplified in a recession. The negative impact on the adverse selection on the market price depends on the share of low quality investment on the balance sheets. Therefore, adverse selection is especially severe in a recession following a prolonged boom period. When a price on resale markets falls low enough, even firms in need of liquidity would find it unprofitable to sell high quality loans for low market price in order to finance new investment opportunities. Then the securitized loans of high quality stop being traded on the re-sale markets altogether.

In an empirical part of the paper I test hypotheses from the theoretical model on the level of securitization deals using the data for residential mortgage backed securities issued in Europe. Lagged credit support provided to holders of securitized assets is found to have a positive relation to the loan quality, which is in line with the signaling hypothesis. Also this effect is smaller or even overturned for assets that have been issued in a boom stage of the business cycle. This is in line with the higher likelihood of a pooling equilibrium in a boom which is derived in the theoretical model. The results are especially strong for deals issued in the UK, however, are statistically insignificant for deals issued in Spain. The difference could be explained by a large differences in regulatory framework and practice of securitization.

The mechanism presented in this paper can contribute to the understanding of the recent financial crisis since it can replicate some of the securitization market outcomes we could have witnessed prior and during the recent financial crisis. In the period preceding the crisis many inefficient investments, whose exact quality was unknown, were undertaken. While this was not a problem as long as the economy was performing well, these low quality loans and their large amount in the economy contributed to the depth of the financial crisis. Also during the crisis the markets for securitized products have been severely strained. The paper also points to some unexpected effects of the newly proposed regulation.

The paper is organized in the following way. Chapter 2 reviews the related literature. Chapter 3 introduces the set-up of the model and shows the basic behavior of the model, the effect of assumed financial frictions and the effect of implicit recourse. For analytical tractability this chapter focuses on steady state with only idiosyncratic stochasticity and where the aggregate variables are deterministic. Chapter 4 shows the numerical results of the full-fledged model with aggregate stochasticity and focuses on the switching between the separating and pooling equilibria over the business cycle. Chapter 5 develops extensions of the model in particular discusses the policy implications of the model and embeds the adverse selection on re-sale markets. Chapter 6 contains the empirical testing of hypotheses derived in the theoretical model.

2 Literature review

My research is broadly related to several strands of literature. In this chapter I would like to focus on research related to securitization with implicit recourse and to financial intermediation imperfections, information frictions and business cycles.

2.1 Securitization and implicit recourse

Securitization is the process of selling cash flows related to the loans issued by the originator (often called the sponsor). The sale of loans is effectuated in a legally separated entity called a special purpose vehicle (SPV) or special purpose entity (SPE). The entity purchases the right to the cash flows with resources obtained by issuing securities in the capital market. The sponsor and the SPV are “bankruptcy remote” and the sale of loans is officially considered to be complete, i.e., the sponsor should transfer all the risks to the buyers of newly emitted securities. Loans are pooled in a portfolio,

which is then usually divided into several tranches ordered by seniority which have a different exposure to risk. Before the crisis securitization was perceived mainly as a means of dispersing credit risk and allocating it to less risk-averse investors who would be compensated by higher returns, while highly risk-averse investors could invest into the most senior tranches with high ratings. Due to the role of securitization played in the late 2000's financial crisis (e.g. Bernanke 2010) securitization attracted a lot of criticism and the attention of researchers turned more to the set of agency problems present at different stages of the securitization process (Shin, 2009). A detailed review of those agency conflicts has been compiled, for instance, by Paligorova (2009).

Gorton and Pennacchi (1995) were among the first to point to moral hazard problems related to securitization and to address the issue why securitization takes place despite them. Moral hazard problems stem from the fact that if the risk is transferred with the loan from the originator of the loan to the investor, the bank has a reduced incentive to monitor borrowers to increase loan quality. Gorton and Pennacchi (1995) argue that before the 1980s securitization was very limited. In the 1980s several regulatory changes took place that effectively increased the cost of deposit funding. One key factor was the imposition of a binding credit requirement for commercial banks.² Banks could avoid increased capital requirements by securitization, which moved some of the risky assets off their balance sheet. This view that an important reason for securitization is regulatory arbitrage is shared by many economists (e.g. Gorton and Pennacchi, 1995, Gertler and Kiyotaki, 2010, Gorton and Metrick, 2010). Calomiris and Mason (2004) present some evidence suggesting regulatory arbitrage is effectuated by securitizing banks rather to increase efficiency of contracting in the situation where capital requirements are unreasonably high than to abuse the safety net. The moral hazard problems and agency problems in general were then alleviated by the practice of keeping part of the loan in the portfolio on the balance sheet of the originator. Fender and Mitchell (2009) study different tranche retention design and their effect on incentives. But any loan sale, partial or complete, results in lower incentives to monitor borrowers, which of course affects the price investors are willing to pay for the securitized loan. Loan originators have thus incentive to provide implicit recourse.

Implicit recourse is a particular form of implicit support provided by the issuers of securitized products to the holders of these assets. They represent a certain guarantee

²"In 1981 regulators announced explicit capital requirements for the first time in U.S. banking history: all banks and bank holding companies were required to hold primary capital of at least 5.5 percent of assets by June 1985." (Gorton and Metrick, 2010, p. 10)

on the quality of the loan. The guarantee cannot be explicit since then it would have to abide to regulations and the loan would have to be kept on the balance sheet of the bank. Nevertheless, much evidence suggests that implicit recourse was frequently used during the securitization process (“As the saying goes, the only securitization without recourse is the last.” (Mason and Rosner, 2007, p. 38)). Gorton and Souleles (2006) show in a theoretical model that this mutually implicit collusion between investors and originators of the loans can be an equilibrium result in a repeated game due to the reputation concerns of the originator who wants to pursue securitization in the future at favorable conditions. Several empirical studies documented concrete cases of implicit recourse or showed indirect evidence of its presence. Higgins and Mason (2004) study 17 discrete recourse events that were directed to an increase in the quality of receivables sponsored by 10 different credit-card banks. The forms of the support provided were for instance adding higher quality accounts to the pool of receivables, removing lower quality accounts, increasing the discount on new receivables, increasing credit enhancement, waiving servicing fee, etc. Higgins and Mason (2004) argue that implicit recourse increases sponsors’ stock prices in the short and long run following the recourse. It also improves their long-run operating performance. Recourse may help to signal investors that shocks that made recourse necessary are only transitory.

Another example showing that the risks were not fully transferred during securitization to the SPV is given by Brunnermeier (2009), who argues that when the SPV was subject to liquidity problems which arise from a maturity mismatch between SPV’s assets and liabilities and a sudden reduced interest in the instruments emitted by the SPV, the sponsor would grant credit lines to it.

In my model I will concentrate on the relationship between investors and banks, where the latter have better information about the quality of loans, and I will show that due to reputation concerns bank has an incentive to signal this quality. This follows the suggestion by Higgins and Mason (2004) that implicit recourse is used to as a signaling tool.

The implications of securitization with tools similar to implicit recourse were recently studied in Ordogñez (2012) who argues that unregulated banking disciplined only by reputation forces may be efficient due to saving on regulatory and bankruptcy costs, but it seems to be more fragile.

2.2 Financial intermediation imperfections, information frictions and business cycles

In the current financial crisis we could have witnessed important disruptions of financial intermediation. It became clear that frictions in the financial sector are important and should not be omitted from macroeconomic models. The classical papers that endogenize financial frictions on the side of borrowers include Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997). These papers introduce an agency problem between borrowers and lenders. The resulting endogenous amplification of the effects of the shocks in the economy is denoted as the “financial accelerator”. Some of the recent macroeconomic models with financial frictions directly incorporate securitization. Brunnermeier and Sannikov (2011) find that securitization enables to share idiosyncratic risks but may be amplifying the systemic risk.

In this paper I will refer often to Kiyotaki and Moore (2012) model of monetary economy with differences in liquidity among different asset classes. Their model features borrowing and re-saleability constraints and stochastic uninsurable arrival of idiosyncratic investment shocks among the market participants. Some of the assumptions such as logarithmic utility function and constant returns to scale on the individual firm level while decreasing returns at the aggregate level simplify the aggregation across heterogeneous agents in the economy and allow for a relatively tractable treatment. Therefore similar framework is used in other papers such as Kurlat (2011) who studies the sale of projects under asymmetric information and shows how this could lead to the lemons problem and potential market shutdowns.

My model is also related to research about the degree of asymmetric information over the business cycle. While some researchers argue that booms are associated with higher degree of trading and therefore more learning (Veldkamp, 2005), others argue that information may be lost in boom periods of business cycles. Gorton and Ordoñez (2012) present a model where assets with unknown value can serve as a collateral for borrowing. In booms none of the parties has the incentive to verify the the value of the asset, the economy saves on information acquisition costs and enjoys a “bliss-full ignorance” equilibrium, while in periods with low aggregate productivity lenders have incentives to verify the value of collateral, which leads to underinvestment. In my model higher productivity will be also associated with less public information but this would create inefficiencies.

3 Model

To allow for maximum tractability the set-up of the model is rather simple. The economy contains a continuum of financial firms which have stochastic investment opportunities. The problem in this model is to transfer resources from firms without investment opportunities or with low quality investment opportunities to firms with the best investment opportunities. The transfer of funds is possible through securitization which is modeled as a sale of cash flows from the funded projects.³

3.1 Investment projects

There are three types of projects available to financial firms and the allocation of firms to projects is stochastic through an i.i.d. shock:

- $(1 - \pi)$ share of firms don't have access to new investment projects,
- $\pi\mu$ share of firms have access to high quality projects with high gross profit per unit of capital $r_t^h = A_t^h K_t^{\alpha-1}$,
- $\pi(1 - \mu)$ share of firms have access to low quality projects with low gross profit per unit of capital $r_t^l = A_t^l K_t^{\alpha-1}$.

This shock cannot be insured.

Assumption: I assume that the relative difference in high quality and low quality projects is countercyclical:

$$\frac{\partial}{\partial A_t} \frac{A_t^h - A_t^l}{A_t^l} < 0, \tag{3.1}$$

where A_t is the aggregate component of the total factor productivity (TFP) of the projects. In this model this assumption is satisfied due to additive combination of the time varying aggregate component A_t and constant type-specific component of TFP Δ^h and Δ^l resp.: $A_t^h = A_t + \Delta^h$ and $A_t^l = A_t + \Delta^l$.⁴ This assumption is inspired by

³To keep the model simple I do not model alternative means of transferring funds like debt in this paper. Kuncl (2013) presents an extension of this model, where among others different types of debt such as deposits or interbank loans are considered, and replicates the main qualitative results of this paper.

⁴An alternative to additive combination of aggregate and type-specific productivity is maybe a more standard multiplicative combination. Then Δ^h and Δ^l have to be functions of A_t to satisfy the assumption (3.1). Then the qualitative results of the paper remain unchanged. Kuncl (2013) uses the multiplicative functional form and reproduces the results.

the empirical evidence on countercyclical cross-sectional variance in TFP of US firm in Bloom (2009) and Bloom et al. (2011).⁵

Some of the basic features of the model are inspired by Kiyotaki and Moore (2012). Similarly to Kiyotaki and Moore (2012), agents are subject to an i.i.d. investment shock, and face constant returns to scale, i.e., they take r_t^h , resp. r_t^l , as given, however, on the aggregate level there are decreasing returns to scale:

$$Y_t = r_t^h H_t + r_t^l L_t = \left(A_t^h \frac{H_t}{K_t} + A_t^l \frac{L_t}{K_t} \right) K_t^\alpha$$

where $K_t = H_t + L_t$ and H_t (L_t) are aggregate holdings of high (low) quality capital.⁶

Two **core frictions** are assumed in the model:

- Investing firms selling securitized loans have to keep a “*skin in the game*” - $(1 - \theta)$ fraction of the investment. This means they can sell only θ fraction of the current investment and the rest have to be financed from their own resources. For simplicity θ is taken throughout most of the paper as a parameter. But in chapter 5 this friction is endogenized by the existence of a moral hazard problem.
- There is an *asymmetry of information* about the above described allocation of investment opportunities among firms. Each firm knows the type of the project it is assigned to in the current period, but it is not aware of the allocation of projects among other firms.

The second friction is motivated by the reality of the securitization market and by the mentioned criticism of securitization, which takes the asymmetric information as the source of most of the agency problems (for details see the literature review). The first friction can be also observed in reality, but the main reason why I include it in this otherwise simple model is that despite competition among financial firms, a binding “skin in the game” constraint increases equilibrium prices above the costs of investment, and therefore makes the securitization process profitable. Only when securitization is profitable, there exists a reputation equilibrium with implicit recourse, where loosing

⁵Models in Bloom (2009) and Bloom et al. (2011) assume time-varying variance of idiosyncratic TFP shocks and show that higher variance leads to recession, which they also document empirically on the firm level data. I assume a weaker version. While second moments of cross-sectional productivity remain constant in my model, the relative difference in projects’ productivity is countercyclical, as in the mentioned models.

⁶Kiyotaki and Moore (2012) obtain this result by including labor in the production function and requiring a competitive wage to be paid to workers in order to run a project. Here for simplicity I omit the workers from the model, but use the mentioned result by assumption.

reputation is costly. (I assume that it is possible to commit to not buying securitized assets from a particular firm if the related incentive compatibility constraint holds, but it is not possible to prevent a particular firm from buying securitized assets from others, i.e., a threat of complete autarky is not possible. I believe this assumption corresponds to the reality of securitization markets.)

Each financial firm maximizes the expected discounted utility from future consumption stream:

$$\max_{c_t^j, i_t^j, h_{t+1}^j, l_{t+1}^j, z_{t+1}^j} \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^j),$$

where $u(c_t) = \log(c_t)$ and firms with access to high quality projects, low quality projects and without access to new projects are denoted by superscripts $j = \{h, l, z\}$, respectively. ⁷Firms use stochastic revenues from projects financed in the past to consume c_t^j , invest into new project if they can i_t^j or buy high (low) quality securitized loans on the market h_{t+1}^j (l_{t+1}^j) for the price q^h (q^l).

To demonstrate the effect of the core frictions in the model, I will first briefly show in the next subchapters the behavior and solution of the model without frictions, then I will successively introduce a binding skin in the game and the asymmetric information. Then I will show that when both frictions are binding, there exists a reputation equilibrium, where implicit recourse can signal the loan quality and result in a separating equilibrium, where the inefficiency related to asymmetric information is eliminated.

To show the results analytically, I will, in the next subchapters, mostly refer to the case with constant aggregate productivity $A_t = A$. In the next chapter I report numerical results from the fully stochastic case.

3.2 Case with no financial frictions - first best

If none of the two frictions are present, i.e., project allocation is public information and the “skin in the game” constraint is not binding, only firms with high investment opportunities will invest, securitize loans and sell them to firms with low or unproductive investment opportunities. The budget constraints of individual firms with different investment opportunities are:

⁷Note that these superscripts refer to individual firms of this type, while value of variables might differ within each group, the policy functions remain the same. The superscripts refer to firms’ types in period t even when they appear over the variables with subscript $t+1$ (since these are control variables chosen in time t).

$$\begin{aligned}
c_t^h + i_t^h + (h_{t+1}^h - i_t^h) q_t^h &= h_t(r_t^h + \lambda q_t^h), \\
c_t^l + h_{t+1}^l q_t^h &= h_t(r_t^h + \lambda q_t^h), \\
c_t^z + h_{t+1}^z q_t^h &= h_t(r_t^h + \lambda q_t^h),
\end{aligned}$$

where i^h are new investments into high quality projects and λ is the share of capital (projects) left after depreciation. Similarly to Kiyotaki and Moore (2012), I assume that subjective discount factor exceeds the share of capital left after depreciation: $\beta > \lambda$. This regularity assumption makes the model well-behaved.

Because of competition among firms with high investment opportunities, the price of loans will equal the unit costs of issuing the loan, $q^h = 1$. The amount of investment and the allocation is first best.

Since utility is logarithmic and budget constraints are linear in individual holdings of assets, the policy functions will be also linear in individual holdings of assets. With logarithmic utility all firms will always consume a constant fraction of their current wealth (for derivation see the appendix 8.2.):

$$c_t^j = (1 - \beta) h_t (r_t^h + \lambda q_t^h) \quad \forall j \in \{h, l, z\}.$$

Linear policy functions and i.i.d. investment opportunities enable easy aggregation. Application of the law of large numbers implies that the aggregate quantities and prices do not depend on the distribution of wealth across individual firms. In this case aggregate level of high quality assets H does not depend on the wealth distribution, therefore so does not r and neither the price q^h .

The law of motion for capital is $K_{t+1} = \lambda K_t + I_t$ ⁸. Goods markets clear, $Y_t = C_t + I_t$.

Combining the aggregate consumption function, the goods market clearing condition and the law of motion for capital we obtain⁹:

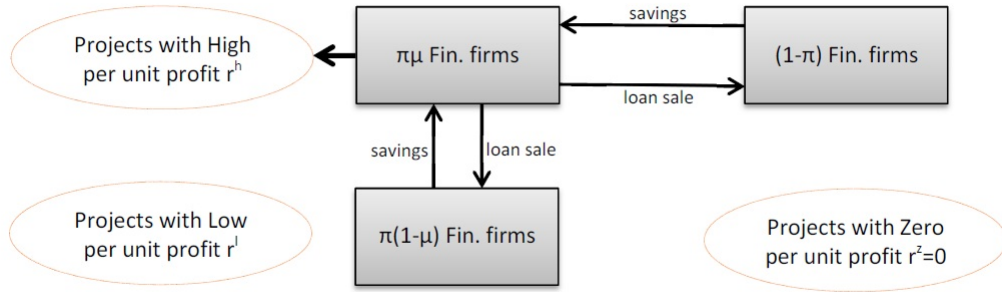
$$r^h + \lambda = \frac{1}{\beta}. \tag{3.2}$$

The current period return plus the value of non-depreciated assets is equal the time preference rate, therefore the amount of investment is indeed first best.

⁸Similar laws hold for both types of capital (low quality and high quality): $H_{t+1} = \lambda H_t + I_t^h$, $L_{t+1} = \lambda L_t + I_t^l$.

⁹For details see the appendix 8.1.1.

Figure 3.1. First best case



Note: In the first best case only firms with access to projects with high profit per unit of capital invest and they sell some of these projects to remaining firms.

3.3 Cases with frictions and without implicit recourse

3.3.1 Introducing the “skin in the game” constraint

In this chapter I show that a binding “skin in the game” constraint (only θ fraction of new loans can be sold) increases the equilibrium prices above the replacement rate, which makes securitization profitable. As already mentioned only when securitization is profitable a reputation equilibrium can exist. The “skin in the game” constraint is also a usual practice observed in securitization contracts in the form of tranche retention schemes. This constraint can be motivated and endogenized by a moral hazard problem, which is derived in chapter 5. Chapter 5 also discusses some potential policy implications of making θ a policy parameter, as is the case e.g. in the Dodd-Frank Act. However, in most of the exposition of the model in this chapter I will assume for simplicity a constant θ .

By lowering θ we limit the capacity of firms with access to high quality projects to issue new investments. When this capacity is lower than the demand for new investments at the zero-profit price $q^h = 1$, then the “skin in the game” constraint becomes binding and the price has to increase above the unit costs of investment to clear the market. Securitization becomes profitable.

If the “skin in the game” is binding in equilibrium for firms with access to high quality projects, i.e., their holdings of newly issued assets represent $(1 - \theta)$ fraction of their investment $h_{t+1}^h = (1 - \theta) i_t^h$, we can rewrite their budget constraint to:

$$c_t^h + \frac{(1 - \theta \hat{q}_t^h)}{(1 - \theta)} h_{t+1}^h = h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l), \quad (3.3)$$

where market price for securitized loans \hat{q}_t^h depends on the information sets of market participants (see below)¹⁰. Combining these two equations and the consumption function we can find the level of investment of the constrained firm with access to high quality projects:

$$i_t^h = \frac{\beta (h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l))}{(1 - \theta \hat{q}_t^h)}. \quad (3.4)$$

All policy functions are again linear, therefore can be easily aggregated and as the appendix 8.1.2. shows we can obtain the following proposition.

Proposition 1. *If skin in the game is sufficiently large to be binding, i.e., θ is sufficiently low to satisfy*

$$\theta < 1 - \frac{\pi\mu}{1 - \lambda},$$

then in the deterministic steady state:

- (i) *the price of high quality assets q^h exceeds 1;*
- (ii) *the steady state level of output and capital is lower than in the first best case.*

The above proposition is analogue to Claim 1 in Kiyotaki and Moore (2012), but here it does not suffice for the full characterization of the model's steady state (see Proposition 2).

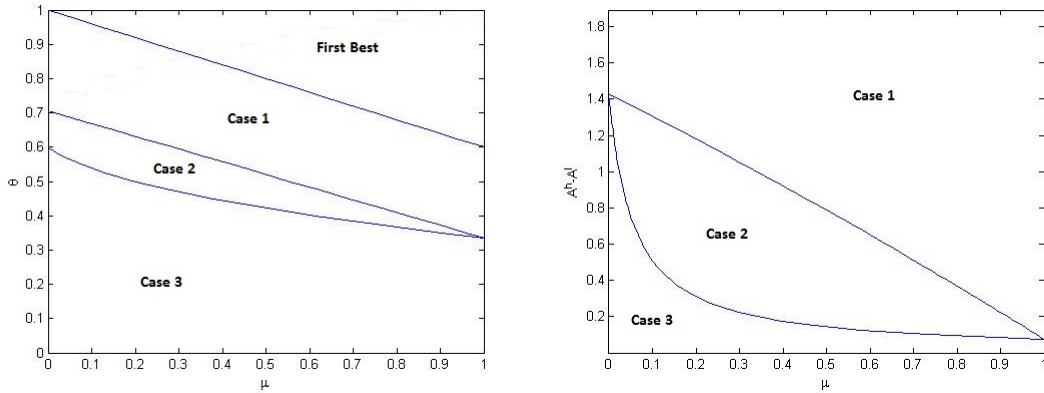
Proposition 2. *Suppose that the condition from Proposition 1 holds, then depending on parameter values deterministic steady state is characterized by one of the following cases:*

Case 1: only firms with access to high quality projects issue credit and securitize ($q^l < 1$);

Case 2: firms with access to low quality loans use mixed strategy and issue credit with probability φ , ($q^l = 1$);

¹⁰I will show below that for a subset of parameters firms with access to low quality projects will be investing and securitizing loans in equilibrium too. They may also face binding skin in the game constraint, i.e., $l_{t+1}^l = (1 - \theta) i_t^l$.

Figure 3.2. Type of deterministic steady state depending on selected parameter values



Case 3: all firms with access to high and low quality projects issue credit and securitize ($q^l > 1$).

The above cases are ranked from the least restricted ($q^l < 1$), where output and capital levels are relatively the closest to first best case, to the most restricted ($q^l > 1$), where output and capital is the lowest:

$$Y_{FB} > Y_H > Y_M > Y_B,$$

$$K_{FB} > K_H > K_M > K_B,$$

where subscript FB denotes first-best case, subscript H denotes Case 1 with only high projects financed, subscript M denotes Case 2 with mixed strategy of firms with access to low quality investment and subscript B denotes Case 3 where both firms with access to low and high quality projects issue credit to the limit of the skin in the game.

Proofs of the above propositions are in the appendix (8.1.2. and 8.1.3.).

The figure 3.2. shows the effect of selected parameter values on the type of the steady state. On the left plot we can see that lowering θ or μ moves the steady state from unrestricted first-best case to more restricted cases. The right plot shows that lowering the difference in productivity of the two types makes it more likely that low quality projects would be financed in the steady state.

3.3.2 Introducing asymmetric information

In this subchapter I will describe the consequences of the introduction of asymmetric information about the allocation of investment opportunities among firms. Since the returns are observed and values of Δ^h, Δ^l, A_t are public information, the uncertainty about the quality of financed projects is resolved in this model in the period following its issuance and sale. I also focus on the effect of asymmetric information between issuers of securitized assets and their first buyers, therefore at this point I do not consider asymmetric information on re-sale markets.¹¹

Unless the difference in qualities is very large, firms with access to low quality projects will mimic high quality firms. Since it is not possible to distinguish between the projects, saving firms, which want to maximally diversify their portfolio, would invest into both high and low quality projects in the rate corresponding to the probabilities of their arrival. This means that μ fraction of investment is allocated into high quality and $1 - \mu$ fraction to low quality projects.

Proposition 3. *Compared to public information case the allocation of capital is generally less efficient (more in favor of low quality projects), therefore, the capital is less productive and in the steady state the amount of capital and output is lower.*

For proof see the appendix 8.1.4.

The public information case will be equal to the private information case only if the difference in the qualities is large enough. The low quality firm will mimic firms with high quality investment opportunities as long as the return from doing so exceeds the return from buying high quality assets:

$$R \mid \text{mimicking} > R \mid \text{buying high loans}$$

As shown in the appendix 8.1.5. in the steady state this condition implies

$$\frac{A^h}{A^l} < \frac{(1 - \pi)(1 - \lambda)(1 - \theta)}{\pi\lambda + (1 - \lambda)\theta\pi}.$$

¹¹I assume that past projects are not anonymous, therefore, the quality of all existing projects becomes public information in the period following their securitization. In chapter 5.3., I relax this assumption and show that, if there exist asymmetric information in general between the buyer and seller on the re-sale markets, there can be partial markets shutdowns as in Kurlat (2011).

Therefore, the ratio of the high and low productivity of loans should be sufficiently low. Note also that increasing the “skin in the game”, i.e., lowering θ will only increase the upper bound for the ratio of qualities in the above condition, and therefore make mimicking more likely. This result is driven by the general equilibrium effect. Lower θ increases the prices in the economy, and therefore makes mimicking more profitable.

Proposition 4. *Under private information, increasing the “skin in the game”, i.e., lowering θ makes pooling equilibrium, in which firms with low quality investment opportunities mimic firms with high quality investment opportunities, more likely.*

3.4 Implicit recourse and reputation equilibrium case

3.4.1 Introducing implicit recourse

Proposition 3 implies that the outcome of private information case is generally inefficient compared to public information case. Firms with high quality investment opportunities have incentives to distinguish themselves from low quality investment firms. However, by Proposition 4 we can see that retaining higher “skin in the game” does not lead to a separating equilibrium.

It turns out that by providing **implicit recourse**, the firm with high quality investment opportunities can distinguish themselves without restricting their investment potential. Under this strategy issuing firm promises minimum gross profit per unit of capital r_t^G to the buyers of securitized loans and should the true gross profits in the following period fall below this minimum, the issuing firm would reimburse the difference. This promise is not enforced by any explicit contract, rather it is a result of a collusion between issuers of loans and their buyers. Implicit recourse can be enforced in a reputation equilibrium, where securitizing firms try to keep reputation of sticking to the promise and firms buying securitized projects enforce this promise by punishing the issuing firms in case of default on the implicit recourse. As mentioned earlier I assume a trigger strategy punishment that prevents a firm without reputation to sell securitized assets on the market. The punishment has to be credible, therefore in this reputation equilibrium buyers of securitized products with implicit support are trying to keep reputation of being “tough investors”, i.e., always punishing firms that did not full-fill the promise.

At this point it is convenient to write the problem recursively:

$$V^{ND}(\bar{s}, w - cir; \bar{S}) = \pi (\mu V^{ND,h}(\bar{s}, w - cir; \bar{S}) + (1 - \mu) V^{ND,l}(\bar{s}, w - cir; \bar{S})) + (1 - \pi) V^{ND,z}(\bar{s}, w - cir; \bar{S}), \quad (3.5)$$

$$\begin{aligned} V^D(\bar{s}, w; \bar{S}) &= \pi (\mu V^{D,h}(\bar{s}, w; \bar{S}) + (1 - \mu) V^{D,l}(\bar{s}, w; \bar{S})) + (1 - \pi) V^{D,z}(\bar{s}, w; \bar{S}), \quad (3.6) \\ V^{ND,j}(\bar{s}, w; \bar{S}) &= \max_{c,i,h',l',r\{G\}'} [\log(c) + \beta E [\max(V^{ND}(\bar{s}', w' - cir'; \bar{S}'), V^D(\bar{s}', w'; \bar{S}'))] \beta], \quad (3.7) \\ V^{D,j}(\bar{s}, w; \bar{S}) &= \max_{c,i,h',l'} [\log(c) + \beta EV^D(\bar{s}', w'; \bar{S}')], \quad (3.8) \end{aligned}$$

where V^{ND} (V^D) are the value functions for the firm, that never defaulted (has already defaulted) on implicit recourse. w is individual wealth before deducting costs of implicit recourse cir , $\bar{s} = \{h, l, h^p, l^p\}$ is a vector of other individual state variables, where P, S superscripts denote assets sold in the previous period on the primary market, which potentially bear implicit guarantee, or on the secondary market, respectively. $\bar{S} = \{K, \omega, A\}$ is a vector of aggregate state variables and r_t^G is the promised minimum return provided as implicit recourse. The equations (3.5) and (3.6) show the investment shock that takes place after the realization of aggregate productivity shock and decision on (non)default on implicit recourse from previous period. After the investment shock firms with assigned investment opportunities choose optimally the level of consumption, investment into new projects and potential securitization of their cash flows with implicit recourse or into securitized loans. This problem is described by the equations (3.7) and (3.8) for firms with reputation of having never defaulted and without this reputation respectively.

The above problem is constrained by the budget constraints which take the following form for investing firms for which “skin in the game” constraint is binding (e.g. in case of firms with high investment opportunities):

$$c_t^h + \frac{(1 - \theta q_t^{\hat{G},h})}{(1 - \theta)} h_{t+1}^h + cir_t = h_t^S (r_t^h + \lambda q_t^h) + l_t^S (r_t^l + \lambda q_t^l) + h_t^P (r_t^{\hat{G},h} + \lambda q_t^h) + l_t^P (r_t^{\hat{G},l} + \lambda q_t^l),$$

where $r_t^{\hat{G},h}$ is the return received from securitized assets with implicit recourse conditional on potential default, and $q_t^{\hat{G},j}$ is the price of securitized loans of type j , depending on the information structure.

The incentive compatible constraints, which have to be satisfied in equilibrium for the existence of reputation based implicit recourse are the following:

$$V^{ND}(\bar{s}, w - cir; \bar{S}) > V^D(\bar{s}, w; \bar{S}) \quad (3.9)$$

$$V^P(\bar{s}; \bar{S}) > V^{NP}(\bar{s}; \bar{S}) \quad (3.10)$$

where V^P, V^{NP} are the value functions for the firm, that always punished for default on implicit recourse, and failed to punish for default, respectively. The condition 3.9 determines the level of implicit recourse that can be credibly provided given the trigger strategy rule, i.e., it is not defaulted upon. The trigger punishment strategy has to be credible, therefore the saving firm which observes default on implicit recourse has to be better off punishing the investing firm that defaulted rather than not punishing it. This corresponds to the condition 3.10¹².

Definition 1. A recursive competitive equilibrium consists of prices $\{q^h(\bar{S}), q^l(\bar{S}), q^{G,h}(\bar{S}), q^{G,l}(\bar{S}), q^G(\bar{S})\}$ and gross profit per unit of capital $\{r^h(\bar{S}), r^l(\bar{S})\}$, individual decision rules $\{c^j(\bar{s}; \bar{S}), h^{j'}(\bar{s}; \bar{S}), l^{j'}(\bar{s}; \bar{S}), r^{G,h'}(\bar{s}; \bar{S}), r^{G,l'}(\bar{s}; \bar{S})\}$, value functions $\{V^{ND}(\bar{s}; \bar{S}), V^{ND,j}(\bar{s}; \bar{S}), V^D(\bar{s}; \bar{S}), V^{D,j}(\bar{s}; \bar{S}), V^{NP}(\bar{s}; \bar{S}), V^P(\bar{s}; \bar{S})\}$ and law of motion for $\bar{S} = \{K, \omega, A, \Sigma\}$ such that: (i) $\{c^j(\bar{s}; \bar{S}), h^{j'}(\bar{s}; \bar{S}), l^{j'}(\bar{s}; \bar{S}), z^{j'}(\bar{s}; \bar{S}), r^{G,h'}(\bar{s}; \bar{S}), r^{G,l'}(\bar{s}; \bar{S})\}$ and $\{V^{ND}(\bar{s}; \bar{S}), V^{ND,j}(\bar{s}; \bar{S}), V^D(\bar{s}; \bar{S}), V^{D,j}(\bar{s}; \bar{S}), V^{NP}(\bar{s}; \bar{S}), V^P(\bar{s}; \bar{S})\}$ solve the each firms' problem given the available information set and taking $\{q^h(\bar{S}), q^l(\bar{S}), q^{G,h}(\bar{S}), q^{G,l}(\bar{S}), q^G(\bar{S})\}$, $\{r^h(\bar{S}), r^l(\bar{S})\}$, and law of motion for $\bar{S} = \{K, \omega, A\}$ as given; (ii) both primary and secondary (re-sale) markets for high and low quality loans and good markets clear and (iii) the law of motion for $\bar{S} = \{K, \omega, A\}$ is consistent with the individual firms' decisions.

Since the asymmetry of information concerns the quality of the loan and not the aggregate productivity, guaranteeing the aggregate productivity where issuers of both types of projects have the same advantage is not efficient if investor in high quality projects wants to distinguish himself from the low type. I allow the implicit contract to guarantee relative return instead of absolute, which is more efficient:

$$r_{t+1}^G(A_{t+1}) = (A_{t+1} + G_t) K_{t+1}^{\alpha-1}$$

The costs of implicit recourse are then given by:

$$cir_{t+1} = \theta i_t K_{t+1}^{\alpha-1} \left(G_t - \Delta_t^{h/l} \right)$$

¹²I show that this condition holds in the appendix 8.1.6.

Since the uncertainty about project quality lasts only for one period in this set-up, for simplicity and tractability I will also restrict the guarantee to the performance of the loans to one period after the issuance.

3.4.2 Public information case with implicit recourse

Although one might think that public information case is uninteresting, it is an important benchmark. If firms could coordinate, they wouldn't be providing implicit recourse in this case, where it does not serve as a tool that would distinguish the firm type. However, due to competition firms tend to out-bet each other.

If the promises would be credible, the optimal level of implicit recourse will be determined by the following F.O.C. (note that individual firm ignores the effects of this choice on aggregate variables):

$$\frac{\partial V^{ND}}{\partial G^j} = \frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial (w' - cir')}{\partial G^j} = 0.$$

I show in the appendix 8.1.7. that this condition implies that $q^j = 1$, which means that as far as there are positive profits from securitization, the competition will drive the level of implicit recourse so high that profits from securitization are zero. However, when profits from securitization are zero, the punishment has zero costs, and the original non-defaulting incentive compatibility constraint (3.9) is not satisfied. This leads us to the following conclusion.

Proposition 5. *As long as the implicit recourse is credible, firms find it optimal to increase it up to the level, where $q^j = 1$. So the level of implicit recourse is defined by the maximum, which can be sustained by the no-default condition (3.9).*

For details on the derivation see the appendix 8.1.7. The steady state in this case is characterized by the following propositions.

Proposition 6. *Suppose that the condition from Proposition 1 holds, then depending on parameter values deterministic steady state is characterized by one of the following cases:*

Case 1: only firms with access to high quality projects issue credit, securitize loans and provide implicit recourse G_{cred}^h ($q^h > 1$, $q^l < 1$, $G_{cred}^h \geq \Delta^h$);

Case 2: firms with access to high quality projects issue credit, securitize loans and provide implicit recourse G_{cred}^h , firms with access to low quality projects use a mixed

strategy and issue credit with probability φ and provide implicit recourse equal to the type quality ($q^h > 1$, $q^l = 1$, $G_{cred}^h \geq \Delta^h$, $G_{cred}^l = \Delta^l$);

Case 3: all firms with access to high and low quality projects issue credit, securitize and provide implicit recourse ($q^h > 1$, $q^l > 1$, $G_{cred}^h \geq \Delta^h$, $G_{cred}^l \geq \Delta^l$).

As I will discuss later the model in the next chapter is calibrated such that the steady state will be characterized by Case 1.

Proposition 7. *Compared to the public information case without implicit recourse, the amount of capital and output is higher, the allocation of capital is in favoring high quality projects, and the wealth is less concentrated at the firms with investment opportunities. This holds in all cases except when the provided implicit recourse has no value ($G_{cred}^h = \Delta^h$), and the two cases are identical.*

3.4.3 Implicit recourse as a signal of loan quality

Finally, we will analyze the case of interest, where both main constraints bind and where implicit recourse can signal the type of the investment opportunity. Due to signaling there is a multiplicity of Perfect Bayesian Equilibria, generally both pooling and separating. I use the Intuitive Criterion (Cho and Kreps, 1997) as a refinement to eliminate the dominated equilibria with unreasonable out of equilibrium beliefs.

Pooling Equilibria: In pooling equilibria both firms choose to provide the same level of implicit recourse given beliefs of investors. Under no aggregate stochasticity there are several candidates for the pooling Perfect Bayesian Equilibria (PBE):

Case 1: Firms with access to both high and low quality projects select $G^* = G_{cred,p}^l$, where $G_{cred,p}^l$ is the maximum implicit recourse, that can be provided by firms with low quality assets under pooling. Investors' out of equilibrium beliefs could be, for instance, the following: when observing implicit recourse $G > G_{cred,p}^l$, then $Pr(j = h) = 0$ and when observing $G < G_{cred,p}^l$ then $Pr(j = h) \in (0, 1)$. In this equilibrium none of the firms defaults. None of the firms has incentive to unilaterally decrease implicit recourse or increase it.

Note that choosing $G < G_{cred,p}^l$ is not an equilibrium since both types will have incentives to increase implicit recourse to $G = G_{cred,p}^l$ due to competition, no matter what are the beliefs of investors, since both types would fulfill the implicit recourse in this interval.

Case 2: Firms with access to both high and low quality projects select G s.t.:

$$G_{lb,p} \leq G^* \leq \min(G_{minsep}, G_{cred,p}^h).$$

Investors' out of equilibrium beliefs can be for instance the following: when observing implicit recourse $G > G^*$ then $Pr(j = h) = 0$ and when observing implicit recourse $G < G^*$ then $Pr(j = h) \leq \mu$.

G_{minsep} is the minimum level of implicit recourse which the low types would not mimic under any beliefs (see derivation in the appendix 8.1.9.). $G_{lb,p}$ is the lower bound on G , where firms with high quality investments do not have incentives to deviate to $G_{cred,p}^l$. The fact that for G such that $G_{cred,p}^l < G < G_{lb,p}$, both types have incentives to decrease implicit recourse to $G = G_{cred,p}^l$, is due to equilibrium defaults on implicit recourse of firms with low investment, which bring investors lower utility, than when $G = G_{cred,p}^l$. And this negative effect on price together with potentially higher costs of higher implicit recourse (when $G > \Delta^h$) outweighs the positive effect of higher implicit recourse on the price.

Separating Equilibria: There is potentially a continuum of separating equilibria, where firms with access to low quality investments save and buy securitized assets from firms with high investment opportunities. Firms with access to high quality investments invest, securitize and provide implicit recourse $G^* \in (G_{minsep}, G_{cred,s}^h)$, where G_{minsep} is the minimum implicit recourse which prevents mimicking by firms with low investment opportunities, and $G_{cred,s}^h$ is the maximum level of implicit recourse that can be promised credibly in a separating equilibrium. Investors' out of equilibrium beliefs could be for instance the following: for an observed G s.t. $G^* < G < G_{cred}^h$ they believe $Pr(j = h) = 0$.

Application of Intuitive Criterion: If a separating equilibrium exists, then all pooling equilibria are dominated, and therefore fail the Intuitive Criterion. In particular, due to competition among firms with access to high quality investments, Intuitive Criterion selects only one separating equilibrium, where firms with access to high quality investments invest, securitize and provide the maximum credible implicit recourse $G^* = G_{cred,s}^h$. So after application of Intuitive Criterion, there is either one unique separating equilibrium left, or one or multiple pooling equilibria.

The condition for the existence of a separating equilibrium:

By Proposition 5 we know that firms have incentives to unilaterally increase the provided implicit recourse up to the maximum credible level. But then, if low quality firms are already at the maximum credible level, where cost of defaulting and keeping

Figure 3.3. Case where Intuitive Criterion selects unique Separating Equilibrium

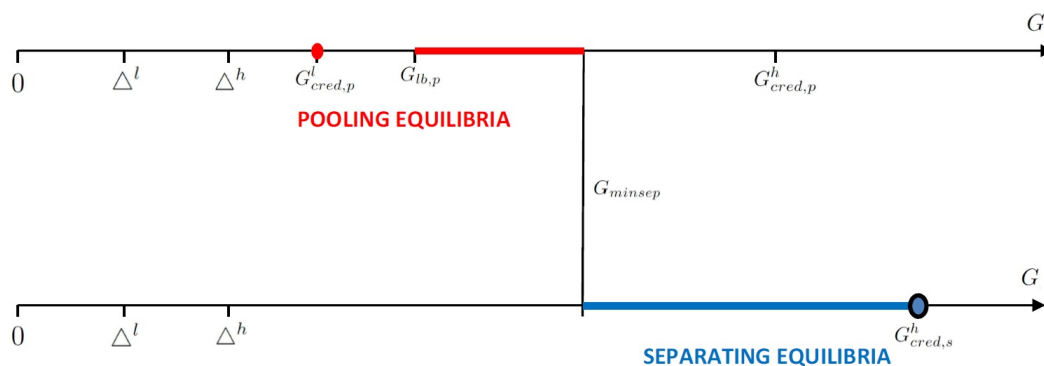
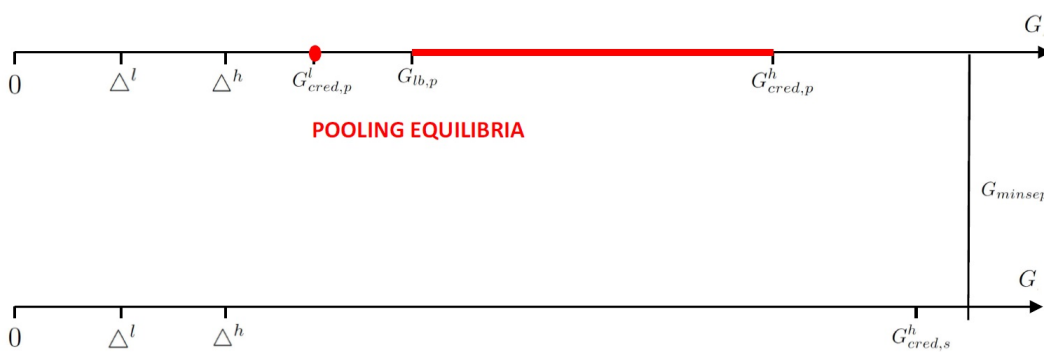


Figure 3.4. Case where there is no Separating Equilibrium



the implicit recourse is equalized, they are better off if they increase the implicit recourse without increasing the cost further, but potentially getting benefits from being mistaken for a firm with high quality investment. Therefore, there cannot exist a separating equilibrium, in which firms with low quality investment will provide a different level of implicit recourse. Firms with low quality investments always prefer mimicking firms with high quality investments to providing a lower implicit recourse and disclosing their quality.

Therefore, separation can take place only when the costs of mimicking become so large that investing into high quality assets is preferred. Under deterministic case this condition can be expressed analytically. The implicit recourse G have to be high enough to satisfy:

$$V^l \mid \text{mimicking} < V^l \mid \text{buying high loans} \quad (3.11)$$

When a firm with access to low quality investment mimics, it can decide it is optimal to default on the promise, which would make the the value function under mimicking even larger. Therefore, it turns out that a necessary condition for a separation equilibrium is

$$\frac{R^l \mid \text{mimicking \& non - defaulting}}{\frac{1-\theta q^{h,IR}}{1-\theta}} < \frac{R^l \mid \text{buying high loans}}{q^h}$$

$$\frac{(A + \Delta^l - \frac{\theta}{1-\theta} \max(G - \Delta^l, 0)) r + \lambda q^h}{\frac{1-\theta q^{h,IR}}{1-\theta}} < \frac{(A + \Delta^h) r + \lambda q^h}{q^h}$$

which reduces under no default condition (under default this condition is still necessary for separation but no longer sufficient) to the following equation:

$$(r^l + \lambda q^h) (q^l - 1) < \lambda (q^l - q^h). \quad (3.12)$$

Since RHS of the equation is always negative this implies that a necessary condition for separation is $q^l < 1$.

When defaulting on the implicit recourse is optimal, which is the case since in a separation equilibrium $G^* > G_{cred,s}^l$, the condition 3.11 can be simplified to (for details see the appendix 8.1.8.):

$$\frac{A^h - A^l}{A^h} > \frac{(q^h - 1) (1 - \lambda)}{\pi \mu [(1 - \lambda) + (1 - \beta) \lambda q^h]} \frac{(1 - \theta B q^h)}{(1 - \theta B) q^h}, \quad (3.13)$$

where $B \equiv \frac{q^G}{q^h} = \frac{r^G + \lambda q^h}{r^h + \lambda q^h}$ is the price premium for the equilibrium implicit guarantee. The equation (3.13) as derived in the appendix 8.1.8. assumes that promise is at the maximum credible level $G_{cred,s}^h$, i.e., characterizes the unique separation equilibrium selected by the Intuitive Criterion. Note that when implicit recourse is at the maximum $G_{cred,s}^h$, separation is the most likely, therefore this condition is equivalent to the existence of any separating equilibrium. There may be other separating equilibria with $G \in (G_{minsep}, G_{cred,s}^h)$, which however fail to satisfy the Intuitive Criterion. Derivation of G_{minsep} is sketched in the appendix 8.1.9.

This brings us to one of the main findings in this paper.

Proposition 8. *Under asymmetric information separating equilibrium is possible in the deterministic steady state only for low enough levels of aggregate productivity not exceeding the threshold level \bar{A} . A necessary condition for existence of a separating equilibrium is that $q^l < 1$. In this separating equilibrium, firms with low quality investment projects save and buy securitized assets from firms with high investment opportunities.*

Proof: The second part of the Proposition 8 comes directly from condition 3.12. The threshold level for productivity can be derived from 3.13. In particular, the maximum level of aggregate productivity, for which separation is still possible, is:

$$\bar{A} = \frac{(\Delta^h - \Delta^l) \pi \mu [(1 - \lambda) + (1 - \beta) \lambda q^h]}{(q^h - 1)(1 - \lambda)} \frac{(1 - \theta B) q^h}{(1 - \theta B q^h)} - \Delta^h.$$

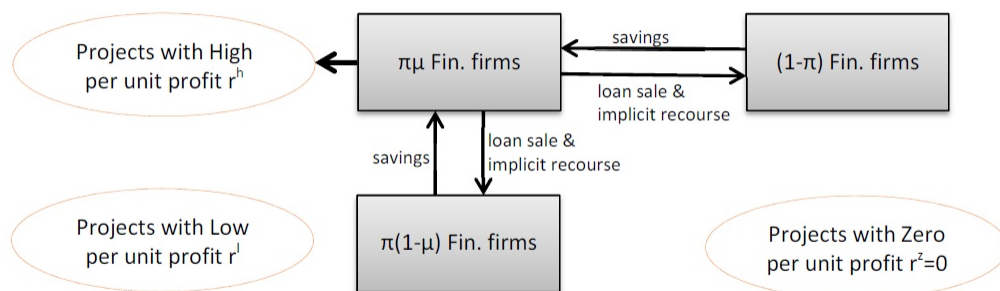
Crucially, as I show in the appendix 8.1.8., in a separation equilibrium both q^h and B and therefore also the whole RHS of 3.13 are independent of the realizations of aggregate productivity A , and are uniquely determined by the intensity of frictions and the punishment for default on implicit recourse.

Separating steady state is more efficient from aggregate perspective, since level of capital and output are higher due to resources being allocated better. Pooling is less efficient, since the allocation of capital is not favoring high quality projects.

Uniqueness of pooling equilibrium:

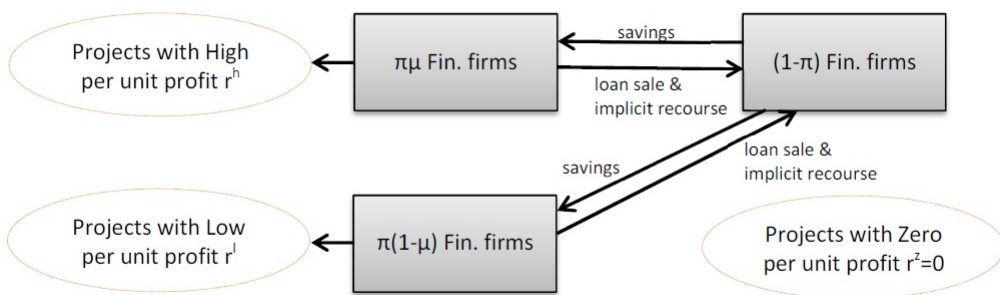
When a separating equilibrium does not exist, there is generally a continuum of pooling equilibria. However, it turns out that for a large set of parameter space there is only one pooling equilibrium with $G^* = G_{cred,p}^l$, independent on a specific form of out of equilibrium beliefs. I calibrate the model to have only one pooling equilibrium. The advantage of this is besides having a unique equilibrium, that punishment is never triggered in equilibrium. It still provides the disciplining role, but the dynamic results

Figure 3.5. Private information case with implicit recourse: Separating equilibrium



In the separating equilibrium the implicit recourse provided by the firms with access to high quality projects is high enough so that it is not profitable for firms with access to low quality projects to mimic them. They are better off buying the high quality projects.

Figure 3.6. Private information case with implicit recourse: Pooling equilibrium



In the pooling equilibrium both firms with access to high and low quality projects provide the same level of implicit recourse. They are indistinguishable and therefore both firms invest into projects and sell them to firms with no investment opportunities.

are not influenced by exercise of a particular punishment rule.

To obtain such an equilibrium, in general I have to find values of parameters such that $G_{lb,p} > G_{cred,p}^h$, i.e., the minimum level of implicit recourse for which it pays off to provide recourse higher than $G_{cred,p}^l$ is not credible in equilibrium, since it exceeds $G_{cred,p}^h$. It turns out that when $\mu < 1/q^h$, this condition is satisfied. For details see the appendix 8.1.9.

4 Dynamics and numerical examples

In this chapter I show the solution of the fully stochastic version of the model with asymmetric information, binding skin in the game and implicit recourse.. The allocation of projects to firms is still driven by an i.i.d. shock. The aggregate productivity is the following stochastic process:

$$\log A_t = (1 - \rho) \log \bar{A} + \rho \log A_{t-1} + u_t.$$

For simplicity I assume that u_t has a binomial distribution. With probability $p = 0.5$: $u_t = \epsilon$ and with probability $(1 - p)$: $u_t = -\epsilon$.¹³

In the analysis of the dynamic properties of the model I focus on the switching between the separating and pooling equilibria over the business cycle. Even though in the steady state there is a separating equilibrium, when the aggregate productivity increases and the economy is in boom stage of the business cycle, the separating equilibrium is no longer sustainable, and the economy is in the pooling equilibrium, where both type of firms provide the same level of implicit support and both invest. This follows directly from Proposition 8. The intuition behind the result is the following. As the aggregate productivity increases the relative difference in productivity of the two nonzero profit project types is reduced. Therefore, a higher implicit recourse is needed to satisfy the separation condition (3.12). Intuitively, following Proposition 8, the condition says that $q^l < 1$ is necessary for separation, but in boom even the quality of low type projects is relatively high, and therefore one has to provide high implicit recourse to drive the prices of low projects below one. At some point the level of implicit recourse required to achieve separation exceeds the incentive compatible limits, and the economy switches to the pooling equilibrium.

¹³This assumption simplifies the solution but is not crucial for the results, and the solution can be generalized with $u \sim N(0, \sigma)$.

Since the model is rather abstract and simple the purpose of this numerical example is only to illustrate the switching mechanism, which is the main contribution of this paper. I used the following somewhat arbitrary parameters: $\alpha = 0.5$, $\beta = 0.95$, $\mu = 0.8$, $\pi = 0.1$, $\lambda = 0.75$, $\theta = 0.6$, $\epsilon = 0.05$, $\rho = 0.9$, $\bar{A} = 2.4$, $\Delta^h = 1$, $\Delta^l = 0$. But as long as the assumptions of the model presented in the previous chapter hold, the qualitative results of the paper do not depend on particular parameter values. Kuncl (2013) embeds this mechanism into a richer environment and does a proper calibration.

In the Figure 4.1 I show how the economy behaves in a particular episode of two positive shocks followed by three negative productivity shocks.¹⁴ The point of this exercise is to show the switch from separating equilibrium to pooling and back and its effects on the output. On the graph, I report for comparison impulse responses¹⁵ of the constrained model under private information and with implicit recourse provision as well as the efficient first-best case. Note that the graph depicts deviations from each model's steady state. Only the share of high quality assets on the balance sheets (ω) is showed in absolute value. So even though on the graph both first-best and constrained case start at the same point, the first-best case is characterized by higher absolute levels of steady state output and capital.

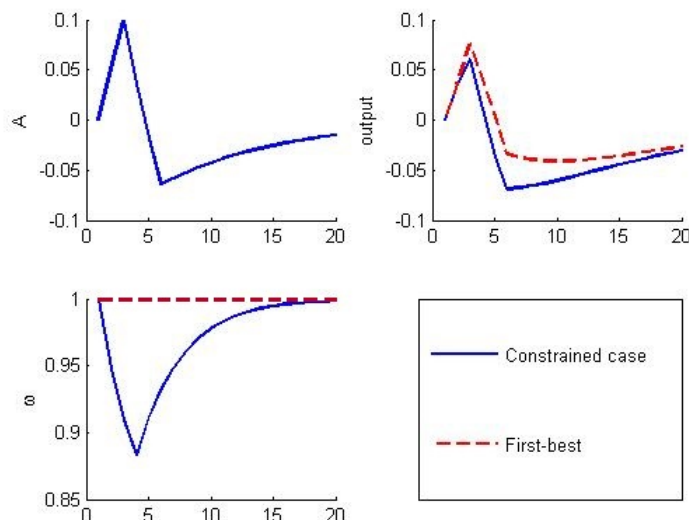
You can see on the figure that as the constrained economy moves to the boom stage of the business cycle, the separating equilibrium changes to pooling equilibrium, i.e., ω decreases, while the share of high quality projects(ω) remains constant in the first best case at 100%. Lower share of high quality projects in the constrained case slows slightly the growth of output and accumulation of capital, but the effect is small, since in boom stage the difference in the two qualities is rather small. But the inefficiency in allocation of capital keeps accumulating. As the economy exogenously moves to a recession with higher difference in qualities, one can see that the accumulated inefficiency in the allocation of capital is more pronounced. Therefore, booms have almost the same relative size in constrained and first-best case, but busts following a boom stage are much deeper in constrained case.

Figure 4.2. shows the result directly following from the switching property of the model - the fact that the the longer is the boom period preceding the recession, the larger are the inefficiencies accumulated in the pooling equilibrium and the larger is the

¹⁴Recall that u_t has a binomial distribution so the size of the shock is limited.

¹⁵The impulse responses start from a steady state to which they converge after a long period of zero productivity shocks; then I introduce the described sequence of productivity shock, after which the shocks are zero again.

Figure 4.1. Impulse responses



difference in the depth of a recession compared to the first best case (recession gap).

5 Extensions

5.1 Endogenizing the skin in the game

So far the “skin in the game” (or equivalently the share of loans which could be sold, θ) was taken as an exogenous parameter. In this chapter I will sketch a simple moral hazard problem, which would try to justify the existence of the “skin in the game”.

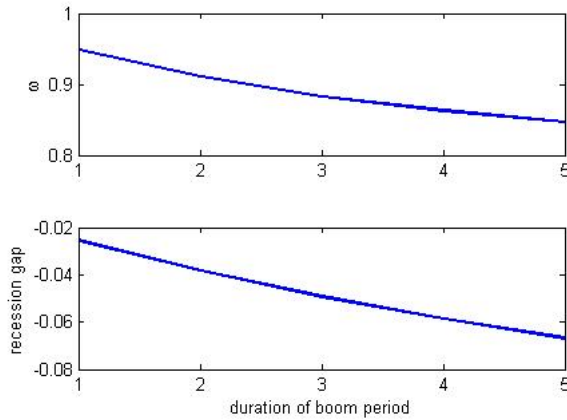
Consider that firms can divert funds from the sale of current period loans needed to cover the unit investment costs. This cannot be immediately verified. To eliminate this problem investors require the issuing firms to retain a sufficiently large “skin in the game” ($1 - \theta$), i.e., to finance a fraction $1 - \theta$ of funds in the project from own resources. The incentive compatible constraint then points down a sufficiently high θ that prevents this moral hazard problem¹⁶:

$$V^D(w\beta R' \mid \text{diverting funds}) \leq V^{ND}(w\beta R' \mid \text{investing properly}),$$

where return from diverting funds is $R' \mid \text{diverting funds} = \left(\frac{\theta q^{IR}}{1-\theta}\right)^x$, with x being the

¹⁶It is intuitive to assume that if a firm would divert funds, other firms will use the same punishment tools as for the case of implicit recourse default.

Figure 4.2. The longer the boom stage, the deeper the subsequent recession



number of times the individual recycles the returns from this operation to issue and sell new “castles-in-the-air” projects. Since I do not restrict the practice of sequential issuance of loans, which is technically needed even under proper investing, the ICC will always fail unless $\theta q^{IR} < (1 - \theta)$, which translates to

$$\theta < \frac{1}{q^{IR} + 1}. \quad (5.1)$$

Thus, the higher the sale price of loans (q^{IR}), the higher skin in the game ($1 - \theta$) is required to prevent the mentioned moral hazard problem.

Note that in this version of the model I have two sources of asymmetric information. First is the potential diversion of resources needed to make investment properly, which cannot be immediately observed. The “skin in the game” is found to be an efficient tool to prevent this behavior, while the loss of reputation and subsequent punishment is not so efficient. The second source of information asymmetry is the unobserved allocation of investment opportunities among firms. In this case by Proposition 4 the “skin in the game” is not an efficient tool, while the reputation based implicit support can overcome the related inefficiencies.

Even with endogenous skin in the game, the main qualitative result of the paper, which is the endogenous switching between the pooling and separating equilibrium, remains unchanged (for details see the appendix 8.1.10.).

17

¹⁷Also note that the assumption of moral hazard problem is absolutely essential since without it the solution would be first best even under asymmetric information. Under first best, securitization is not profitable, therefore firms with access to low quality investment do not have any incentives to mimic

5.2 Skin in the game as a policy parameter

The skin in the game can be considered as a potential policy parameter. For instance the section 941 of Dodd-Frank Reform already requires minimum retention of 5%.

If, as in this model, the skin in the game is determined endogenously by a moral hazard problem, and securitization is the only means of financial intermediation, policy which tries to increase the skin in the game beyond the endogenously determined value would not improve the efficiency of financial intermediation. The reasons are twofold.

First, higher skin in the game increases the profits from securitization and lowers the aggregate quantity of investment (this follows from Proposition 1 and 2). Second, higher profits also make issuance and sale of loans profitable even for firms with lower quality projects, which would otherwise be buyers of high quality projects (this holds both in the symmetric information case from Proposition 2 as well under asymmetric information since pooling equilibrium is more likely see Proposition 4 and Proposition 8). Therefore, both quantity as well as quality of investment is lower with higher skin in the game than with the one determined by the market.

Unlike in some other models of securitization such as Gorton and Pennacchi (1995) my model does not feature continuous monitoring or effort level. I only have an option of funds diversion which is observed only with a time lag. On high level of abstraction this can be understood as the analogy to costly monitoring in Gorton and Pennacchi (1995), where the level of monitoring would take only two values (no monitoring or full monitoring). This moral hazard problem indeed points down the optimum level of skin in the game. But as long as everyone is rational, not only there is no reason to increase the skin in the game above the level determined by the equilibrium, but increasing the skin in the game would have negative effects on the economy as described above¹⁸.

5.3 Adverse selection on re-sale markets

So far we have considered the asymmetry of information between the originators of securitized assets and buyers of these assets. In this section I extend the asymmetry of information also to the re-sale market. In particular I assume that the holder of the

firms with high quality investments. Therefore, neither reputation equilibria nor implicit recourse would take place.

¹⁸It can be argued that this model is too simplistic to give policy recommendations. That is why I reproduce the above results in a richer framework with debt as well as deposit financing and study the optimal mix of macro-prudential policy in Kuncl (2013).

asset can learn the quality of the underlying asset, while the buyer cannot. This leads to the typical adverse selection on the re-sale market.

The new result in this paper comes from the interaction of the adverse selection on re-sale markets with the switching between pooling and separating equilibria. The severity of the adverse selection on the secondary markets depends on the difference in qualities but as well on the share of low quality assets on the balance sheets. Therefore, intuitively the adverse selection is more important in a recession than in a boom. But also the longer is the boom period which precedes the recession, the larger is the share of low quality loans on the market and the more acute the adverse selection issue becomes. If adverse selection is strong enough, securitized loans of high quality stop being traded on the re-sale markets altogether, which deepens further the recession.

The motivation for including this section are the problems we could have witnessed on the securitization markets during the late 2000's financial crisis.

The assumption of asymmetric information on re-sale markets has the following impact on the model behavior. First, when an asset is re-sold, there is a unique price which is independent on the quality of this asset q_t^s . If an asset is not re-sold, the owner who know its quality will value high quality asset q_t^h and low quality asset q_t^l , but this is not the market price. Second, prices depend on the share of high quality assets on the re-sale market¹⁹. Holders of assets find out their quality and sell all low quality assets. Unlike original issuers in the period when investment is made, they no longer have the technology to provide implicit recourse. High assets on the market are sold only by firms with investment opportunities which are in the need for liquidity.

Therefore, the share of high quality assets on the re-sale market is

$$f_t^h = \frac{\pi\mu\omega_t}{\pi\mu + (1 - \pi\mu)(1 - \omega_t)}$$

in case of a separating equilibrium and

$$f_t^h = \frac{\pi\omega_t}{\pi + (1 - \pi)(1 - \omega_t)}$$

in case of a pooling equilibrium.

If due to the adverse selection the price of assets on the re-sale market drops low enough, even firms which sell assets due to liquidity reasons will stop selling high quality assets. The price is so low that the return from taking advantage of the investment

¹⁹See appendix 8.1.11. for details.

opportunity would not compensate for the cost of selling valuable asset at a low market price. In a deterministic steady state this situation takes place if:

$$R^h > q^s \frac{R^h - \theta R^G}{1 - \theta q^{IR}},$$

where $R^h = r_{t+1}^h + \lambda \pi \mu q_{t+1}^s + \lambda (1 - \pi \mu) q_{t+1}^h$ and $R^G = r_{t+1}^G + \lambda \pi \mu q_{t+1}^s + \lambda (1 - \pi \mu) q_{t+1}^h$. As shown in the appendix 8.1.11. this condition implies that the share of high quality assets traded on the re-sale market has to be low enough to satisfy:

$$f^h < 1 - \frac{q^h - 1}{(q^h - q^l)(1 - \theta B)}.$$

If this conditions would be satisfied, there would not be complete market shutdowns since low quality assets would be still sold at a fair price, but the volume of sales would greatly diminish by the absence of high quality assets and the level of overall investment in the economy would be also significantly lower.

6 Empirical analysis

The main results of the theoretical model is the prediction that providing implicit support can signal the quality of the underlying loans and the prediction that this signaling is less efficient in boom stages of the business cycle. This section presents empirical tests of these hypotheses. The results are in line with the model predictions.

Due to the implicit nature of the reputation based support there is no data which would measure directly the level of implicit support. However, when the implicit support is activated for instance in periods of lower than expected returns from the securitized products, it can be observed and often appears in the data.²⁰ Even using the data on support provided by the originator (credit enhancement) when it is actually explicitly provided, we can test the hypotheses contained in the theoretical model.

The empirical literature on the relationship between credit enhancements and the

²⁰As an anecdotal evidence let me cite the example reported originally by Mandel et al. (2012) on the increase in credit enhancement by Chase Issuance Trust. The originator of the securitized assets increases credit enhancement on both future issuance as well as all outstanding securitized products. Note that they had no contractual obligation to provide higher credit enhancement on loans products issued in the past, so this is a typical case of implicit support that appears in the data only at the time when the implicit support is activated. Fitch: Chase Increases Credit Enhancement in Credit Card Issuance Trust (CHAIT),” <http://www.reuters.com/article/2009/05/12/idUS260368+12-May-2009+BW20090512>.

quality of the loans (typically approximated by the delinquencies on the collateral) is limited. The most relevant paper is the work by Mandel et al. (2012), where the authors test the signaling and the buffer hypotheses of credit enhancement (credit protection provided to holders of the securitized assets). The signaling hypothesis, which is already described in this paper, predicts a negative correlation of credit enhancements and delinquencies on the collateral. According to the buffer hypothesis credit enhancement does not serve as a signal of high quality of collateral but is rather provided as a buffer against observable risk. In this case securitized assets with poor quality of collateral will need higher credit enhancement.

6.1 Hypotheses

I perform two tests: first tests the signaling hypothesis (with the alternative being the buffer hypothesis) and the second tests the hypothesis of lower efficiency of signaling (switching to pooling equilibria) when loans are issued in boom periods of the business cycle.

H1: Credit enhancement signals the quality of collateral If the signaling hypothesis is correct, then more support would be positively correlated with the quality of the securitized products. Therefore, this hypothesis would suggest a negative effect of lagged credit enhancements on the delinquency rates of the collateral. If the relationship is opposite then the buffer effect dominates.

H2: For loans issued in the boom stage of the business cycle a pooling equilibrium is more likely, therefore signaling is less efficient If the signaling is less strong for assets originated in the boom period of the business cycle as predicted by the model due to higher likelihood of the pooling equilibrium, the positive correlation between credit enhancements and quality of collateral should be smaller or even turn negative for this particular subset of products. I construct a dummy for securitized products issued in boom stage of the business cycle. This hypothesis would suggest that an interaction term of lagged credit enhancements with the dummy for deals issued in the boom should have a positive effect (an increase) on delinquency rates of the collateral.

6.2 Data description

I use the database Performance Data Services (PDS) provided by Moody's, which contains the data on delinquency rate of collateral in the pool as well as on the credit enhancement provided to back securitized products. I have access the part of the database which covers Residential Mortgage Backed Securities (RMBS) issued in Europe²¹.

As a proxy for quality of collateral (mortgage loans) which backs the securitized products I use 90plus delinquency rate which is defined as the amount of receivables that are 90 or more days past due divided by the original collateral balance. The support provided to securitized products is captured by credit enhancement which is the amount of credit protection available to the holders of securitized assets in the form of subordination, overcollateralization, reserve funds, letters of credit, spread accounts, cash collateral accounts and other non-guaranteed funds. The data is available for individual tranches.

Since the quality of collateral is available only on the level of the pool, I need to aggregate credit enhancement data. I aggregate on the level of deals. A deal is typically backed by a pool of collateral and consists of several tranches. I drop the observations where more pools back the same deal or more deals are backed by the same pool of loans since I do not have information needed to do proper aggregation. The data on credit enhancement is available on tranche level, therefore I compute a weighted average. Credit enhancement is expressed as total amount of credit protection as a fraction of current pool balance. I winsorize both delinquency rates and the credit enhancement rate at the 2.5%-level to account for data errors and limit the effect of potential outliers.

The real output data for the respective countries are obtained from Eurostat. I construct the output gap using the Hodrick-Prescott filter with the smoothing parameter 1600.

6.3 Panel regression results

I run the following fix effect regression:

$$\begin{aligned} \text{DelinquencyRate}_{i,t} = & \alpha_i + \alpha_t + \beta \text{CERatio}_{i,t-1} + \gamma \text{CERatio}_{i,t-1} \times D\{\text{boom}\}_{i,t} \\ & + \delta \text{CERatio}_{i,t-1} \times D\{\text{originated in boom}\}_{i,t} + \iota \text{Deal age}_{i,t} + \kappa \text{Output gap}_{i,t} + \varepsilon_{i,t} \end{aligned}$$

²¹I would like to thank the European Central Bank for providing me with the access to this part of the PDS database.

on data with quarterly frequency, where $CERatio_{i,t-1}$ is the ratio of total credit enhancement to current pool balance lagged one period in time²²; $D\{boom\}$ is the dummy variable for boom period in the country of issuance; $D\{originated\ in\ boom\}$ is the dummy variable for deals issued in a boom period of the respective country; $Deal\ age$ is the number of quarters since the closing date of the deal; and $Output\ gap = \ln(GDP) - \ln(GDP_{HP})$, where GDP_{HP} is the smoothed level of respective real Gross Domestic Product obtain by HP filter.

The table 1 shows the results for the three largest European countries by securitization activity for residential mortgage loans: the United Kingdom (UK), Netherlands (NL), Spain and Italy. I show results for the whole subset and for UK and Spain separately. I use fixed effects for deals and time and report Huber-White robust standard errors. Standard errors are clustered by deals. I report the results on the maximum sample period, but also on the period without the recent crisis. The results are consistent for both periods. I also checked the results when initial periods with relatively few observations are excluded and the results are still consistent. Although I do not claim that the found relationship are necessarily causal, I still find that analyzing the magnitude of the found relationship is interesting and informative.

For the whole sample of three countries (UK, NL, Spain and Italy) the results are in line with the signaling hypothesis (coefficient of $CERatio$ is significantly negative), and also in line with the hypothesis, that signaling in case of loans issued in periods of boom is much weaker (coefficient of $CERatio \times D\{originated\ in\ boom\}$ is significantly positive). Finally, the coefficient of $CERatio \times D\{boom\}$ is significantly negative. This would suggest that the signaling effect is stronger in the boom period for all loans irrespective of the time of issuance. However, I would offer a slightly different interpretation. Following the model presented in the previous chapters, since the guaranteed minimum return is not conditional on the state of the economy, implicit support is most likely to be activated and therefore appear in the data in a recession. The lower the quality of the asset the higher the support (additional credit enhancement) needed to keep to the expected implicit obligation. This is an analogue to the buffer effect mentioned in Mandel et al. (2012). Both signaling and buffer effect are likely to operate all

²²Note that I use the variable credit enhancement lagged by one quarter. This is because contemporaneous correlation between credit enhancements and loan quality could be positive due to trigger of some implicit support in times of temporary distress. However, this does not contradict the signaling hypothesis. In fact it is a part of the signaling story developed in this model. On the other hand if the signaling hypothesis is correct then the lagged credit enhancement should be negatively correlated with current quality of the collateral.

the time. But in recession the buffer effect might be stronger that is why the effect of credit enhancements on delinquencies is less negative.

I also analyzed selected countries individually. UK and Spain had the highest number of observations, so I report these results. In UK the results are qualitatively the same as for the whole sample. However, in Spain the credit enhancement has no significant effect on delinquencies. I believe that this result is due to a very different regulation of securitization in both countries. Unlike in other countries in Spain the regulator treated off-balance sheet assets (i.e. all securitized products) in the same way as if they would remain on the balance sheet.²³ Therefore, the securitization practice in Spain was very different from other countries. Securitization wasn't used to transfer risk, but rather to obtain more liquidity. Consistent with this Almazan et al. (2013) reports that securitization in Spain was used mainly by small banks which had problems to obtain debt financing. Following the evidence from Almazan et al. (2013) in Spain securitization was not related to adverse selection problems which was so typical for practice in other countries. As a result credit enhancement did not serve as a signaling tool. Consistently with this I cannot find any significant relationship between credit enhancements on the delinquencies on the collateral in Spain.

To conclude, the results of the panel regressions are consistent with the signaling hypothesis as well as the lower efficiency of the signaling for loans issued in a boom period for countries, where securitization was related to a transfer of risk, such as the United Kingdom. However, in countries, such as Spain, where the risk primarily remained on the balance sheet of the originators, no significant relationship between credit enhancement and the quality of loans is found.

²³ See Acharya and Schnabl (2009) for detailed description of the regulatory practice in different countries.

Table 1. Panel Regression Results (Dependent variable: Delinquency rate) ^a

Countries	UK, NL, Spain, Italy	UK, NL, Spain, Italy	UK	UK	Spain	Spain
Time period	1998q3-2013q2	1998q3-2007q2	2000q2-2013q2	2000q2-2007q2	1998q3-2013q2	1998q3-2007q2
$CERatio(-1)$	-0.0191 [4.40]***	-0.0107 [2.17]**	-0.0212 [3.86]***	-0.0118 [2.41]**	0.0031 [0.70]	0.0058 [1.22]
$CERatio(-1) \times$	-0.0039 [4.65]***	-0.0061 [2.79]***	-0.0033 [1.91]*	-0.0052 [1.68]*	-0.0014 [0.67]	-0.0015 [1.72]*
D_{boom}	0.0115 [2.31]**	0.0200 [3.63]***	0.0144 [2.31]**	0.0301 [4.83]***	-0.0034 [0.52]	-0.0047 [0.99]
$Deal\ age$	0.0032 [1.57]	-0.0027 [0.33]	-0.042 [4.12]***	-0.068 [5.23]***	0.003 [1.47]	-0.004 [1.88]*
$Output\ gap$	0.15 [0.03]	-5.13 [0.45]	omitted ^b	omitted ^b	omitted ^b	omitted ^b
Observations	15826	4664	4210	1184	5717	1707
Number of deals	747	399	197	129	227	122
$R^2(w/b/o)^c$	0.13/0.19/0.13	0.14/0.08/0.10	0.28/0.04/0.00	0.32/0.00/0.00	0.12/0.01/0.01	0.12/0.04/0.06

^a Robust t-statistics appear in brackets. Time dummies are not reported. Variables are defined in text. ***/**/* - Statistically significant at 1/5/10 percent level.

^b Ouput gap for individual country varies only over time, so cannot be included due to time fixed effects.

^c Reports R^2 within/between/overall.

7 Conclusions

In this paper I show that in general reputation concerns allow sponsors of securitized products to signal the quality of the loans by providing implicit recourse and thus they limit the problem of private information typical for securitization. However, there are limits to the efficiency of this particular reputation based tools, which become more pronounced in boom stages of the business cycles. The costs of sufficiently high implicit recourse that would avoid mimicking by firms with investment projects of lower quality exceed the limit which can still be credibly promised. In the resulting pooling equilibrium the information about the quality of loans is lost and the investment allocation becomes more inefficient. Due to this mechanism large inefficiencies in the allocation of capital can be accumulated in the boom stage of the business cycle. The accumulated inefficiencies can then amplify the subsequent downturn of the economy. Also the longer is the duration of the boom stage of the business cycle the deeper will be the fall of output in the recession.

Results of this paper have also implications for the related macro-prudential policy, which requires higher explicit risk-retention (skin in the game). In this model such requirements restrict the supply of loans and through the general equilibrium effect make securitization more profitable. As a result this regulation lowers both the quantity as well as quality (higher likelihood of pooling equilibria) of the investment in the economy.

In an extension of the model I introduce asymmetric information also on the resale market for securitized loans. The model predicts amplified adverse selection in a recession especially if the recession is preceded by long period of boom. If the adverse selection is severe enough high quality securitized loans stop being traded altogether.

In an empirical part I test hypotheses from the theoretical model on the level of securitization deals using the data for residential mortgage backed securities issued in Europe. Lagged credit support provided to holders of securitized assets is found to have a positive relation to the loan quality, which is in line with the signaling hypothesis. Also this effect is smaller or even overturned for assets that have been issued in a boom stage of the business cycle. This is in line with higher likelihood of a pooling equilibrium in a boom which is derived in the theoretical model. The results are especially strong for deals issued in the UK, however, are not statistically significant for deals issued in Spain. The difference could be explained by a large differences in regulatory framework and practice of securitization.

The mechanism presented in this paper can contribute to the our understanding of the recent financial crisis since it describes well securitization markets experience prior and during the recent financial crisis. In the period preceding the crisis many inefficient investments whose exact quality was unknown were undertaken. While this was not a problem as long as the economy was performing well these low quality loans and their large amount in the economy contributed to the depth of the financial crisis. During the crisis the markets for securities products have been severely strained. The paper also points to some unexpected effects of the newly proposed regulation.

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8 Appendix

8.1 Proofs

8.1.1 First best case

Due to logarithmic utility firms always consume $1 - \beta$ fraction of their wealth: $c = (1 - \beta) h (r^h + \lambda)$. This policy function is linear, so it is trivial to aggregate it across the continuum of firms to obtain the equation describing the evolution of aggregate variables: $C = (1 - \beta) H (r^h + \lambda)$.

From the market clearing condition we know that $I = Y - C = Hr^h - C$. And from the law of motion for capital we know that in the steady state $I = (1 - \lambda)H$. Combining these two conditions we obtain:

$$Hr^h - C = (1 - \lambda)H.$$

Substituting there for aggregate consumption we get:

$$\begin{aligned} Hr^h - (1 - \beta)H(r^h + \lambda) &= (1 - \lambda)H, \\ r^h + \lambda &= \frac{1}{\beta}. \end{aligned}$$

8.1.2 Proof of Proposition 1

In the first best allocation $q^h = 1$. Should the skin in the game be binding the $q^h > 1$. Let's consider the least restrictive case where still only the firm with access to high quality loans is issuing credit and securitizes these loans and the skin in the game is not high enough to allow firm with access to low quality investment opportunities to profitably issue loans $q^l < 1$.

Under binding "skin in the game" constraint the aggregate investment into higher quality project will be (obtained as an aggregation of eq. 3.4):

$$I_t^H = \pi\mu \frac{\beta (H_t ((A_t + \Delta^h) K_t^{\alpha-1} + \lambda q_t^h) + L_t ((A_t + \Delta^l) K_t^{\alpha-1} + \lambda q_t^l))}{(1 - \theta q_t^h)}. \quad (8.1)$$

Prices of particular assets are determined from Euler equations of saving firms. In equilibrium these firms are indifferent between investing in high or low quality projects:

$$E_t \left[\frac{\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}}{\left(\omega_{t+1} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1 - \omega_{t+1}) \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right)} \right] = 1 \quad (8.2)$$

$$E_t \left[\frac{\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}}{\left(\omega_{t+1} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1 - \omega_{t+1}) \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right)} \right] = 1, \quad (8.3)$$

where ω_t is the share of high quality projects in the overall assets in the economy $\omega_t = H_t/K_t$. The derivation of these conditions can be found in the appendix 8.2.

Finally goods market clearing condition has to hold too:

$$Y_t = C_t + I_t. \quad (8.4)$$

Steady state conditions (8.1, combination of 8.2 and 8.3, 8.4) in the steady state become the following:

$$(1 - \lambda)(1 - \theta q^h) = \pi \mu \beta (r^h + \lambda q^h)$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l}$$

$$r^h = (1 - \lambda) + (1 - \beta)(r^h + \lambda q^h).$$

Combining these equations we can obtain

$$q_H^h = \frac{(1 - \lambda)(1 - \pi \mu)}{(1 - \lambda)\theta + \pi \mu \lambda}$$

$$K_H = \left[\frac{(1 - \lambda) + \frac{(1 - \beta)\lambda(1 - \lambda)(1 - \pi \mu)}{(1 - \lambda)\theta + \pi \mu \lambda}}{\beta A^h} \right]^{\frac{1}{\alpha - 1}}.$$

As long as $q^h = 1$, we would obtain $K_H = \left[\frac{1}{A^h} \left(\frac{1}{\beta} - \lambda \right) \right]^{\frac{1}{\alpha - 1}}$ which is the first best optimal level of capital (compare with (3.2)). If $(1 - \lambda)(1 - \pi \mu) > (1 - \lambda)\theta + \pi \mu \lambda$ then $q^h > 1$. Deterministic steady state level of capital is then lower than in the first best case:

$$K_H = \left[\frac{(1 - \lambda) + (1 - \beta)\lambda q_H^h}{\beta A^h} \right]^{\frac{1}{\alpha - 1}} < \left[\frac{(1 - \lambda) + (1 - \beta)\lambda}{\beta A^h} \right]^{\frac{1}{\alpha - 1}} = K_{FB}.$$

8.1.3 Proof of Proposition 2

Proposition 2 claims that there are three possible types of steady state depending on the parameter values. In the proof of Proposition 1 above I described already the

least restricted case where only firm with access to high quality projects will be issuing and securitizing loans. By continuing to tighten the skin in the game constraint we will increase the price of low quality asset to 1 ($q^l = 1$). At this point the firms with access to low quality loans will be indifferent between buying high quality securitized assets or issue and securitize their own loans. Credit to low quality projects counterweights the effect of tightening skin in the game constraint and therefore the price stay at the same levels ($q^l = 1$, $q^h = A^h/A^l$). For an interval of θ there will be an steady state in which firms with access to low quality investment will play a mixed strategy when giving credit with probability φ . As θ decreases (skin in the game rises), φ increases all the way up to 1, where a third type of steady state takes place. In this firms with access to both high and low quality projects will be all issuing credit and securitizing always.

Steady state conditions are the following:

$$(1 - \lambda) (1 - \theta q^h) \omega = \pi \mu \beta (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)) \quad (8.5)$$

$$(1 - \lambda) (1 - \theta q^l) (1 - \omega) = \pi (1 - \mu) \varphi \beta (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)) \quad (8.6)$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l} \quad (8.7)$$

$$q^l = 1 \quad (8.8)$$

$$\omega r^h + (1 - \omega) r^l = (1 - \lambda) + (1 - \beta) (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)). \quad (8.9)$$

Let's define

$$q \equiv \frac{q^h}{A^h} = \frac{q^l}{A^l} \quad (8.10)$$

and

$$D \equiv \omega A^h + (1 - \omega) A^l. \quad (8.11)$$

Using (8.10), (8.11) and combining equations (8.5), (8.6) and (8.7):

$$(1 - \lambda) (1 - \theta q D) = \pi (\mu + \varphi (1 - \mu)) \beta D (K^{\alpha-1} + \lambda q)$$

$$(1 - \lambda) - \pi (\mu + \varphi (1 - \mu)) \beta D K^{\alpha-1} = q D [(1 - \lambda) \theta + \pi (\mu + \varphi (1 - \mu)) \beta \lambda] \quad (8.12)$$

We can also rewrite (8.9):

$$\beta DK^{\alpha-1} = 1 - \lambda + (1 - \beta) D\lambda q \quad (8.13)$$

Combining (8.12), (8.13) we get

$$q_M = \frac{(1 - \lambda) (1 - \pi (\mu + \varphi (1 - \mu)))}{(1 - \lambda) \theta + \pi (\mu + \varphi (1 - \mu)) \lambda} \frac{1}{D} \quad (8.14)$$

Substituting (8.14) back into (8.13) we get:

$$K_M = \left[\frac{(1 - \lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda}}{\beta D} \right]^{\frac{1}{\alpha-1}} \quad (8.15)$$

Deterministic steady state is defined by:

$$(1 - \lambda) (1 - \theta q^h) \omega = \pi \mu \beta (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)) \quad (8.16)$$

$$(1 - \lambda) (1 - \theta q^l) (1 - \omega) = \pi (1 - \mu) \beta (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)) \quad (8.17)$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l} \quad (8.18)$$

$$\omega r^h + (1 - \omega) r^l = (1 - \lambda) + (1 - \beta) (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)) . \quad (8.19)$$

Using (8.10), (8.11) and combining equations (8.16), (8.17) and (8.18):

$$(1 - \lambda) (1 - \theta q D) = \pi \beta D (K^{\alpha-1} + \lambda q)$$

$$(1 - \lambda) - \pi \beta DK^{\alpha-1} = q D [(1 - \lambda) \theta + \pi \beta \lambda] \quad (8.20)$$

We can also rewrite (8.19):

$$\beta DK^{\alpha-1} = 1 - \lambda + (1 - \beta) D\lambda q \quad (8.21)$$

Combining (8.20), (8.21) we get

$$q_B = \frac{(1 - \lambda) (1 - \pi)}{(1 - \lambda) \theta + \pi \lambda} \frac{1}{D} \quad (8.22)$$

Substituting (8.22) back into (8.21) we get:

$$K_B = \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta + \pi\lambda}}{\beta D} \right]^{\frac{1}{\alpha-1}} \quad (8.23)$$

Second part of proposition claims that $K_H > K_M > K_B$.

To show this lets first focus on the in the brackets part of the formulas for capital:

Since in Case 1 $q_H^l < 1$ then $q_H^h < \frac{A^h}{A^l}$. And since $q_M^l = 1$ then $\frac{(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta + \pi(\mu+\varphi(1-\mu))\lambda} = \frac{D_M}{A^l}$. The following inequality then holds

$$\frac{(1-\lambda) + (1-\beta)\lambda q_H^h}{\beta A^h} < \frac{(1-\lambda)}{\beta A^h} + (1-\beta)\lambda \frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_M} + (1-\beta)\lambda \frac{1}{\beta A^l} = \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta + \pi(\mu+\varphi(1-\mu))\lambda}}{\beta D_M}.$$

This implies that

$$K_H = \left[\frac{(1-\lambda) + (1-\beta)\lambda q_H^h}{\beta A^h} \right]^{\frac{1}{\alpha-1}} > \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta + \pi(\mu+\varphi(1-\mu))\lambda}}{\beta D_M} \right]^{\frac{1}{\alpha-1}} = K_M.$$

Similarly we can show that $K_P > K_B$. Since $w_B < w_P$ then $D_B < D_P$. Also $q_B^l > 1$ then $\frac{(1-\lambda)(1-\pi)}{(1-\lambda)\theta + \pi\lambda} > \frac{D_B}{A^l}$. This implies that

$$\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta + \pi(\mu+\varphi(1-\mu))\lambda}}{\beta D_M} = \frac{(1-\lambda)}{\beta D_M} + (1-\beta)\lambda \frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_B} + (1-\beta)\lambda \frac{1}{\beta A^l} < \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta + \pi\lambda}}{\beta D_B},$$

$$K_M = \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta + \pi(\mu+\varphi(1-\mu))\lambda}}{\beta D_M} \right]^{\frac{1}{\alpha-1}} > \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta + \pi\lambda}}{\beta D_B} \right]^{\frac{1}{\alpha-1}} = K_B$$

8.1.4 Proof of Proposition 3

Even when “skin in the game” constraint is not binding enough to influence aggregate quantities and prices, the capital and output levels are lower than in the first best case due to the inefficient allocation of capital. When the “skin in the game” constraint is not binding average return on investment in the economy equals

$$\bar{r} = \mu r^h + (1-\mu)r^l = \frac{1}{\beta} - \lambda.$$

The level of capital K_P is determined by:

$$K_P = \left[\frac{1}{\mu A^h + (1-\mu)A^l} \left(\frac{1}{\beta} - \lambda \right) \right]^{\frac{1}{\alpha-1}} < \left[\frac{1}{A^h} \left(\frac{1}{\beta} - \lambda \right) \right]^{\frac{1}{\alpha-1}} = K_{FB}.$$

Suppose $(1-\pi)(1-\lambda) > \pi\lambda + (1-\lambda)\theta$, in which case the skin in the game constraint starts to bind in this case of private information. The deterministic steady state conditions then collapse to the two following equations in (K, q) :

$$(1-\lambda)(1-\theta q) = \pi\beta(\mu r^h + (1-\mu)r^l + \lambda q),$$

$$\mu r^h + (1-\mu)r^l = (1-\lambda) + (1-\beta)(\mu r^h + (1-\mu)r^l + \lambda q),$$

where $q = \mu q^h + (1-\mu)q^l$. From this we can easily derive:

$$q = \frac{(1-\pi)(1-\lambda)}{\pi\lambda + (1-\lambda)\theta} \quad (8.24)$$

$$K = \left[\frac{(1-\lambda) + (1-\beta)\lambda q}{\beta(\mu A^h + (1-\mu)A^l)} \right]^{\frac{1}{\alpha-1}}.$$

In the proof of Proposition 1 and 2 we already proved that $K_{FB} > K_H > K_M > K_B$. To prove Proposition 3 it suffices to prove that $K_B > K_{private}$, where $K_{private}$ is the level of capital under private information about the allocation of investment opportunities. To obtain $K_B > K_{private}$, we need:

$$K_B^{\alpha-1} < K_{private}^{\alpha-1}$$

$$\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta + \pi\lambda}}{\beta(\omega A^h + (1-\omega)A^l)} < \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta + \pi\lambda}}{\beta(\mu\Delta A^h + (1-\mu)A^l)}$$

$$\omega > \mu.$$

Writing equations (8.16) and (8.17) in a ratio we obtain:

$$\frac{(1-\lambda)(1-\theta q^h)\omega}{(1-\lambda)(1-\theta q^l)(1-\omega)} = \frac{\pi\mu\beta(\omega(r^h + \lambda q^h) + (1-\omega)(r^l + \lambda q^l))}{\pi(1-\mu)\beta(\omega(r^h + \lambda q^h) + (1-\omega)(r^l + \lambda q^l))}.$$

Since $q^h > q^l$ we can obtain:

$$\frac{\omega}{(1-\omega)} = \frac{(1-\theta q^l)}{(1-\theta q^h)} \frac{\mu}{(1-\mu)} > \frac{\mu}{(1-\mu)},$$

and this implies that $\omega > \mu$.

8.1.5 Proof of proposition 4

Under private information case, firms with low quality investment opportunities prefer to mimic firms with high quality investment opportunities if:

$$\begin{aligned}
 R \mid \text{mimicking} &> R \mid \text{buying high loans}, \\
 \frac{r^l + \lambda q^l}{\frac{1-\theta q}{1-\theta}} &> \frac{r^h + \lambda q^h}{q}, \\
 \frac{(1-\theta)q}{1-\theta q} &> \frac{q^h}{q^l}
 \end{aligned}$$

Substituting for q from (8.24) and using $\frac{A^h}{q^h} = \frac{A^l}{q^l}$, we get

$$\frac{A^h}{A^l} < \frac{(1-\pi)(1-\lambda)(1-\theta)}{\pi\lambda + (1-\lambda)\theta\pi}.$$

8.1.6 Credibility of the trigger punishment strategy

A necessary condition for the existence of the reputation equilibrium in which implicit recourse is being provided is the credibility of the punishment rule. The saving firm which observes default on the implicit recourse has to be prefer punishing the defaulting firm to non-punishing even ex-post. This condition is expressed in condition (3.10). I will express analytically both elements of that inequality in the case of the separating deterministic steady state, where level of aggregate TFP is constant. In fully stochastic version this can be solved numerically. Following the same steps as in the appendix 8.1.9. we can find that the value function of the firm that always punished and therefore has a reputation of being a “tough investor” is:

$$V^P(w) = \frac{\log[(1-\beta)w]}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2} (\pi\mu \log(R^{h,IR}) + (1-\pi\mu) \log(R^s)),$$

and the value function of the firm that failed to punish and therefore lost reputation of being a “tough investor” is:

$$V^{NP}(w) = \frac{\log[(1-\beta)w]}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2} (\pi\mu \log(R^{h,IR}) + (1-\pi\mu) \log(R^{s,NP})).$$

If a firm loses the reputation of being “tough investor”, other firms will expect that this firm will never punish in the future and as a consequence they will never provide implicit support to this firm anymore. So when the firms which have lost their

reputation of being “tough investors” invest in the assets with implicit support issued in the primary market, their return is $R^{s,NP} = \frac{r^h + \lambda q^h}{q^{h,IR}}$. While firms with reputation of “tough investors” have return $R^{s,NP} = \frac{r^G + \lambda q^h}{q^{h,IR}}$. If these firms without reputation of being “tough investors” buy assets without implicit recourse on the secondary (re-sale) markets, they are also in a disadvantageous position. Selling firms with reputation of being “tough investors” sell a high quality assets to firms with reputation for q^h , but they know that the firm without reputation has the outside option only buying on the primary market, so they will be willing to buy this asset even for the price $q^{h,IR}$. The price for which a high quality asset is sold on the secondary market to the firms without reputation is somewhere on the interval $q^{h,NP} \in (q^h, q^{h,IR})$ depending on the bargaining power of sellers and buyers. Unless all the bargaining power is on the side of firms without reputation, which I rule out by assumption, $q^{h,NP} > q^h$. This implies that $R^{s,NP} < R^s$ and therefore saving firms are better off punishing and the equation (3.10) is satisfied.

It is well known that trigger strategies are often not renegotiation-proof. While in this paper I do not address this problem in detail and rule out renegotiation by assumption, it can be shown that for large set of parameter space and relative bargaining power of different agents in the economy renegotiation is not optimal. Therefore, trigger strategy will be robust even in the case when renegotiation is allowed.

Suppose one firm decides to default on the implicit support (which is the case that is relevant for the ICC for non-defaulting 3.9), firms that decide whether to punish this firm will face lower return if they buy from firms with reputation $R^{s,NP}$ as shown above, but may negotiate with the defaulted firm better terms and buy from them the assets for a better (lower) than the market price $q^{h,RN} < q^h$, giving it a return $R^{s,RN} > R^s$. However, those benefits from renegotiation are limited by the fact that the defaulted firm would be selling the assets only with probability $\pi\mu$ and the quantity of assets the firm can sell is limited and proportional to its equity. Even if quantity of the assets sold by the defaulted firm is large enough, renegotiation would not be optimal as long as

$$R^s > \pi\mu R^{s,RN} + (1 - \pi\mu) R^{s,NP}.$$

This depends on the prices $q^h, q^{h,NP}, q^{h,RN}$, which themselves depend on the relative bargaining power of different agents in the economy.

8.1.7 Proofs of proposition 5

I claimed that if the implicit recourse would be credible, the optimal level of promise would mean $q^j = 1$ and therefore zero profit for securitizing firms. The relevant F.O.C. can be transformed in the following way (Let’s consider F.O.C. for firms with high quality investment opportunities. The remaining would not invest at all.):

$$\frac{\partial V^{ND}}{\partial \Delta^{G,j}} = \frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial (w' - cir')}{\partial G^j} = 0.$$

$$\begin{aligned} \frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial}{\partial G^j} \frac{(1 - \theta) \beta w (r^{j'} + \lambda q^j) - \theta \beta w K^{\alpha-1} (G^j - \Delta^j)}{1 - \theta q^{G,j}} &= 0 \\ \frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial}{\partial G^j} \frac{\beta w (r^{j'} + \lambda q^j - \theta (r^{Gj'} + \lambda q^j))}{1 - \theta q^{G,j}} &= 0. \end{aligned}$$

After substituting in this case with constant aggregate productivity $q^{G,j} = \frac{r^{Gj'} + \lambda q^j}{r^{j'} + \lambda q^j} q^j$ this condition implies that

$$\frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial}{\partial G^j} \frac{\beta w (r^{j'} + \lambda q^j) \left(1 - \theta \frac{q^{G,j}}{q^j}\right)}{1 - \theta q^{G,j}} = 0,$$

and since $\frac{\partial V^{ND'}}{\partial (w' - cir')} > 0$, $\frac{\partial q^{G,j}}{\partial G^j} > 0$ the above condition simplifies to

$$\frac{\partial}{\partial q^{G,j}} \frac{\left(1 - \theta \frac{q^{G,j}}{q^j}\right)}{1 - \theta q^{G,j}} = \frac{\theta (q^j - 1)}{q^j (1 - \theta q^{G,j})^2} = 0.$$

This implies $q^j = 1$.

Note that for when the level of G satisfies this condition, return from investing and securitizing is equal to the return from investing but not securitizing, i.e., securitization does not increase the return:

$$\begin{aligned} R \mid \text{investing \& securitizing} &= R \mid \text{investing} \\ \frac{(1 - \theta) \left[(A' + \Delta^j - \frac{\theta}{1 - \theta} (G^j - \Delta^j)) K^{\alpha-1} + \lambda q^j \right]}{1 - \theta \frac{(A+G)K^{\alpha-1} + \lambda q^j}{(A+\Delta^j)K^{\alpha-1} + \lambda q^j} q^j} &= \frac{(A' + \Delta^j) K^{\alpha-1} + \lambda q^j}{1} \end{aligned}$$

Since $q^j = 1$ we get:

$$(1 - \theta) ((A' + \Delta^j) K^{\alpha-1} + \lambda) - \theta (G^j - \Delta^j) K^{\alpha-1} = ((A' + \Delta^j) K^{\alpha-1} + \lambda) - \theta ((A + G^j) K^{\alpha-1} + \lambda),$$

which always holds.

8.1.8 Proof of Proposition 8

To complete the proof of Proposition 8 sketched in the main text I need to proof that in a separating equilibrium q^h is independent of the level of aggregate productivity A and show the derivation of equation (3.13).

Under separation steady state conditions are the following:

$$(1 - \lambda) (1 - \theta q^{h,IR}) = \pi \mu \beta (r^h + \lambda q^h) \quad (8.25)$$

$$r^h = (1 - \lambda) + (1 - \beta) (r^h + \lambda q^h) \quad (8.26)$$

$$\frac{(A + \Delta^G) K^{\alpha-1} + \lambda q^h}{q^{h,IR}} = \frac{(A + \Delta^h) K^{\alpha-1} + \lambda q^h}{q^h} \quad (8.27)$$

$$V^{ND} (w' - cir') = V^D (w') \quad (8.28)$$

Using the following property given by the logarithmic utility function:

$$\begin{aligned} V(w) &= \log((1 - \beta) w) + \beta \log((1 - \beta) \beta R w) + \beta^2 \log((1 - \beta) \beta^2 R^2 w) + \beta^3 \log((1 - \beta) \beta^3 R^3 w) \dots \\ &= \frac{1}{1 - \beta} \log(w) + \log((1 - \beta)) + \beta \log((1 - \beta) \beta R) + \beta^2 \log((1 - \beta) \beta^2 R^2) + \beta^3 \log((1 - \beta) \beta^3 R^3) \dots \\ &= \frac{1}{1 - \beta} \log(w) + V(1), \end{aligned}$$

we can transform the no-default condition expressed in eq. (8.28) in the following way:

$$\begin{aligned} V^D (w') &= V^D \left(w \beta \frac{(1 - \theta) (r^h + \lambda q^h)}{(1 - \theta q^{h,IR})} \right) = V^D (w) + \frac{1}{1 - \beta} \log \left(\beta \frac{(1 - \theta) (r^h + \lambda q^h)}{(1 - \theta q^{h,IR})} \right) \\ V^{ND} (w' - cir') &= V^{ND} \left(w \beta \frac{(1 - \theta) (r^h + \lambda q^h - \frac{\theta}{1 - \theta} (G - \Delta^h) K^{\alpha-1})}{(1 - \theta q^{h,IR})} \right) \\ &= V^{ND} (w) + \frac{1}{1 - \beta} \log \left(\beta \frac{(1 - \theta) (r^h + \lambda q^h - \frac{\theta}{1 - \theta} (G - \Delta^h) K^{\alpha-1})}{(1 - \theta q^{h,IR})} \right) \end{aligned}$$

For simplicity lets express value functions separately from the individual wealth in the following way, which is easy to do given the log utility: $V(w) = V(1) + \frac{1}{1 - \beta} \log(w)$. And we can find solutions for value functions with wealth normalized to unity which we can denote simply $V = V(1)$.

$$\begin{aligned} V^{ND} &= \log(1 - \beta) + \beta \left(\pi \mu V^{ND} (\beta R^{h,IR}) + \pi (1 - \mu) V^{ND} (\beta R^l) + (1 - \pi) V^{ND} (\beta R^z) \right) \\ &= \log(1 - \beta) + \beta \left(\frac{\pi \mu \log(\beta R^{h,IR})}{1 - \beta} + \pi (1 - \mu) \frac{\log(\beta R^l)}{1 - \beta} + (1 - \pi) \frac{\log(\beta R^z)}{1 - \beta} + V^{ND} \right) \\ &= \frac{\log(1 - \beta)}{1 - \beta} + \frac{\beta \log(\beta)}{(1 - \beta)^2} + \frac{\beta}{(1 - \beta)^2} \left(\pi \mu \log(R^{h,IR}) + \pi (1 - \mu) \log(R^l) + (1 - \pi) \log(R^z) \right) \end{aligned}$$

$$\begin{aligned} V^D &= \log(1 - \beta) + \beta \left(\pi \mu V^D (\beta R^{h,D}) + \pi (1 - \mu) V^D (\beta R^l) + (1 - \pi) V^D (\beta R^z) \right) \\ &= \log(1 - \beta) + \beta \left(\frac{\pi \mu \log(\beta R^{h,D})}{1 - \beta} + \pi (1 - \mu) \frac{\log(\beta R^l)}{1 - \beta} + (1 - \pi) \frac{\log(\beta R^z)}{1 - \beta} + V^D \right) \\ &= \frac{\log(1 - \beta)}{1 - \beta} + \frac{\beta \log(\beta)}{(1 - \beta)^2} + \frac{\beta}{(1 - \beta)^2} \left(\pi \mu \log(R^{h,D}) + \pi (1 - \mu) \log(R^l) + (1 - \pi) \log(R^z) \right) \end{aligned}$$

Substituting the above derived conditions into the no-default condition (eq. 8.28) and canceling the terms equal for both value functions we obtain:

$$\log \left(\beta (1 - \theta) \left(r^h + \lambda q^h - \frac{\theta}{1 - \theta} (G - \Delta^h) \right) \right) + \frac{\beta \pi \mu}{1 - \beta} \log (R^{h,IR}) = \log \left(\beta (1 - \theta) (r^h + \lambda q^h) \right) + \frac{\beta \pi \mu}{1 - \beta} \log (R^{h,D}),$$

where LHS shows the utility from consumption when the wealth is reduced by repayment of implicit recourse and the future discounted benefit of having good reputation. The RHS then shows higher immediate utility from saving on implicit recourse, but the future utility is lower since the firm cannot longer issue and sell new loans. This equation can further be simplified using (8.27) and substituting for the returns:

$$\begin{aligned} \log \left(\frac{r^h + \lambda q^h - \theta (r^G + \lambda q^h)}{(1 - \theta) (r^h + \lambda q^h)} \right) &= -\frac{\beta \pi \mu}{1 - \beta} \log \left(\frac{R^{h,IR}}{R^{h,D}} \right) \\ &= -\frac{\beta \pi \mu}{1 - \beta} \log \left(\frac{(1 - \theta) \left(r^h + \lambda q^h - \frac{\theta}{1 - \theta} (G - \Delta^h) \right)}{(1 - \theta q_t^{h,IR})} \frac{1}{(r^h + \lambda q^h)} \right) \\ &= -\frac{\beta \pi \mu}{1 - \beta} \log \left(\frac{r^h + \lambda q^h - \theta (r^G + \lambda q^h)}{r^h + \lambda q^h - \theta q^h (r^G + \lambda q^h)} \right) \end{aligned}$$

Now for let's denote the price premium for the equilibrium implicit guarantee $B \equiv \frac{q^G}{q^h} = \frac{r^G + \lambda q^h}{r^h + \lambda q^h}$, then we can express the above equation as follows:

$$\log \left(\frac{1 - \theta B}{1 - \theta} \right) = \frac{\beta \pi \mu}{1 - \beta} \log \left(\frac{1 - \theta q^h B}{1 - \theta B} \right), \quad (8.29)$$

which is an equation in two unknown endogenous variables (B, q^h) depending on time preference parameters β and parameters defining the strength of the financing frictions (π, μ, θ) .

We can express a second steady state condition in two endogenous variables (B, q^h) combining two remaining conditions for the steady state (8.25, 8.26):

$$(1 - \lambda) (1 - \theta B q^h) = \pi \mu (1 - \lambda + \lambda q^h). \quad (8.30)$$

Combining the two equations (8.29, 8.30) we can obtain the solution to both the price of the high quality asset q^h and the price premium for the equilibrium implicit guarantee B . Crucially the solution does not depend on the level of aggregate productivity A . Which is one step we needed to show to complete the proof of Proposition 8.

Second step is the derivation of equation 3.13. Similarly as with the condition 8.28 we can transform the following condition for separation (eq. 3.11):

$$\begin{aligned}
V^l (\text{mimicking \& default}) &< V^l (\text{buying high loans}) \\
\log \left(\frac{\beta (1 - \theta) (r^l + \lambda q^h)}{(1 - \theta q^{h,IR})} \right) + \frac{\beta \pi \mu}{1 - \beta} \log (R^{h,D}) &< \log \left(\beta \frac{(r^h + \lambda q^h)}{q^h} \right) + \beta \pi \mu \log (R^{h,IR}) \\
-\frac{\beta \pi \mu}{1 - \beta} \log \left(\frac{R^{h,IR}}{R^{h,D}} \right) &< \log \left(\frac{(1 - \theta q^{h,IR})}{(r^l + \lambda q^h) (1 - \theta)} \frac{(r^h + \lambda q^h)}{q^h} \right)
\end{aligned}$$

Using the equation 8.29 and the the preceding transformations we can replace RHS to get:

$$\begin{aligned}
\log \left(\frac{1 - \theta B}{1 - \theta} \right) &< \log \left(\frac{(1 - \theta B q^h)}{(r^l + \lambda q^h) (1 - \theta)} \frac{(r^h + \lambda q^h)}{q^h} \right) \\
\frac{(r^l + \lambda q^h)}{(r^h + \lambda q^h)} = 1 - \frac{(A^h - A^l) K^{\alpha-1}}{(r^h + \lambda q^h)} &< \frac{(1 - \theta B q^h)}{(1 - \theta B) q^h} \\
\frac{(A^h - A^l) K^{\alpha-1}}{(r^h + \lambda q^h)} &> 1 - \frac{(1 - \theta B q^h)}{(1 - \theta B) q^h} = \frac{(q^h - 1)}{(1 - \theta B) q^h}
\end{aligned}$$

From market clearing condition (8.26) we can substitute for $K^{\alpha-1} = ((1-\lambda)+(1-\beta)\lambda q^h)/\beta A^h$ and from steady state level of investment condition (8.25) we can substitute for $r^h + \lambda q^h = (1-\lambda)(1-\theta B q^h)/\pi \mu \beta$. Then we get the equation (3.13):

$$\frac{A^h - A^l}{A^h} > \frac{(q^h - 1) (1 - \lambda)}{\pi \mu [(1 - \lambda) + (1 - \beta) \lambda q^h]} \frac{(1 - \theta B q^h)}{(1 - \theta B) q^h}.$$

8.1.9 Other derivations from subchapter 3.4.3

Conditions for the minimum level of implicit recourse needed for separation G_{minsep} :

At G_{minsep} , firms with low quality investments are indifferent between mimicking and separating:

$$\begin{aligned}
V^l | \text{mimicking \& default} &= V^l | \text{buying high loans} \\
\log \left(\frac{\beta (1 - \theta) (r^l + \lambda q^h)}{(1 - \theta q^{h,IR})} \right) + \beta \pi \mu \log (R^{h,D}) &= \log \left(\beta \frac{(r^h + \lambda q^h)}{q^h} \right) + \beta \pi \mu \log (R^{h,IR}) \\
-\beta \pi \mu \log \left(\frac{1 - \theta B_{min}}{1 - \theta} \right) &= \log \left(\frac{(1 - \theta B_{min} q^h)}{(r^l + \lambda q^h) (1 - \theta)} \frac{(r^h + \lambda q^h)}{q^h} \right) \quad (8.31)
\end{aligned}$$

Combining equation (8.31 with the following equilibrium investment condition

$$(1 - \lambda) (1 - \theta B_{min} q^h) = \pi \mu (1 - \lambda + \lambda q^h), \quad (8.32)$$

where $B_{min} \equiv \frac{q^G}{q^h} = \frac{(A + G_{minsep}) K^{\alpha-1} + \lambda q^h}{r^h + \lambda q^h}$, we can obtain the $\{G_{minsep}, q^h, B_{min}\}$.

Conditions for a unique pooling equilibrium:

A necessary condition for firms to have incentives to increase G above $G_{cred,p}^l$ is that it must be considered as profitable to at least individually deviate above $G_{cred,p}^l$. The following condition should, therefore, be satisfied:

$$\frac{\partial V^{ND}}{\partial G} = \frac{\partial V^{ND}}{\partial R^{h,IR}} \frac{\partial R^{h,IR}}{\partial G} > 0$$

Since $\frac{\partial V^{ND}}{\partial R^{h,IR}} > 0$, this becomes:

$$\frac{\partial R^{h,IR}}{\partial G} = \frac{\partial}{\partial G} \frac{\left((A + \Delta^h - \frac{\theta}{1-\theta} (G - \Delta^h)) K^{\alpha-1} + \lambda q^h \right) (1 - \theta)}{1 - \theta \frac{(A + \mu G + (1-\mu)\Delta^l) K^{\alpha-1} + \lambda(\mu q^h + (1-\mu)q^l)}{(A + \Delta^h) K^{\alpha-1} + \lambda q^h} q^h} > 0$$

Taking the derivative we obtain:

$$\begin{aligned} -\theta K^{\alpha-1} \left(1 - \theta \frac{(A + \mu G + (1-\mu)\Delta^l) K^{\alpha-1} + \lambda(\mu q^h + (1-\mu)q^l)}{(A + \Delta^h) K^{\alpha-1} + \lambda q^h} q^h \right) &+ \frac{\theta \mu q^h K^{\alpha-1}}{(A + \Delta^h) K^{\alpha-1} + \lambda q^h} \left((A + \Delta^h - \frac{\theta}{1-\theta} (G - \Delta^h)) K^{\alpha-1} + \lambda q^h \right) (1 - \theta) > 0 \\ \left((A + \Delta^h - \frac{\theta}{1-\theta} (G - \Delta^h)) K^{\alpha-1} + \lambda q^h \right) (1 - \theta) \mu q^h &> \left((A + \Delta^h) K^{\alpha-1} + \lambda q^h - \theta (A + \mu G + (1-\mu)\Delta^l) K^{\alpha-1} + \lambda(\mu q^h + (1-\mu)q^l) \right) q^h \\ (\mu q^h - 1) (r^h + \lambda q^h) &> \theta q^h (A K^{\alpha-1} (\mu - 1) - (1-\mu)\Delta^l K^{\alpha-1} - (1-\mu)\lambda q^l) \\ (\mu q^h - 1) (r^h + \lambda q^h) &> \theta q^h (\mu - 1) (r^l + \lambda q^l). \end{aligned} \quad (8.33)$$

As long as $(\mu q^h - 1) > 0$ the condition (8.33) always holds since $\mu < 1$. When $(\mu q^h - 1) < 0$, then we get

$$(r^h + \lambda q^h) < \theta \frac{q^h (\mu - 1)}{(\mu q^h - 1)} (r^l + \lambda q^l),$$

which is never satisfied.

Therefore, a sufficient condition for a unique pooling equilibrium is $\mu < 1/q^h$.

8.1.10 Endogenizing the skin in the game

If we endogenize the skin in the game with the moral hazard problem described in Chapter 5 we obtain the incentive compatible constraint (5.1). In this subchapter I would like to show briefly that the main result concerning the provision of implicit recourse and the endogenous switching between the pooling and separating equilibrium hold.

First, firms have incentive to provide implicit support up to the level $q^j = 1$. The proof of the equivalent Proposition 5 as discussed in chapter 8.1.7. boils down to

showing that

$$\frac{\partial}{\partial q^{G,j}} \frac{\left(1 - \theta \frac{q^{G,j}}{q^j}\right)}{1 - \theta q^{G,j}} = \frac{(q^j - 1)}{q^j (1 - \theta q^{G,j})^2} \frac{\partial \theta q^{G,j}}{\partial q^{G,j}} = 0.$$

Since $\frac{\partial \theta q^{G,j}}{\partial q^{G,j}} = \frac{\partial}{\partial q^{G,j}} \frac{q^{G,j}}{q^{G,j+1}} = \frac{1}{(q^{G,j+1})^2} > 0$, the above condition corresponds again to $q^j = 1$.

The separating equilibrium in the deterministic steady state is defined by:

$$\theta < \frac{1}{q^{IR} + 1}.$$

$$\log\left(\frac{1 - \theta B}{1 - \theta}\right) = \frac{\beta \pi \mu}{1 - \beta} \log\left(\frac{1 - \theta q^h B}{1 - \theta B}\right), \quad (8.34)$$

$$\begin{aligned} (1 - \lambda)(1 - \theta B q^h) &= \pi \mu (1 - \lambda + \lambda q^h) \\ \log\left(\frac{1 - \theta B}{1 - \theta}\right) &= \frac{\beta \pi \mu}{1 - \beta} \log\left(\frac{1 - \theta q^h B}{1 - \theta B}\right) \\ \theta &= \frac{1}{B q^h + 1}. \end{aligned}$$

Which simplifies to two equations in which are independent on the level of TFP A :

$$\begin{aligned} (1 - \lambda) \left(\frac{1}{B q^h + 1}\right) &= \pi \mu (1 - \lambda + \lambda q^h) \\ \log\left(\frac{B(q^h - 1) + 1}{B q^h}\right) &= \frac{\beta \pi \mu}{1 - \beta} \log\left(\frac{1}{B(q^h - 1) + 1}\right). \end{aligned}$$

The conditions for the existence of a separating equilibrium (3.13) becomes:

$$\frac{A^h - A^l}{A^h} > \frac{(q^h - 1)(1 - \lambda)}{q^h \pi \mu [(1 - \lambda) + (1 - \beta) \lambda q^h] (B(q^h - 1) + 1)}.$$

8.1.11 Adverse selection on re-sale markets

We derive the pricing conditions from the F.O.C. of saving firms. In the case of a separating equilibrium they are the following. The value of a high quality asset q_t^h reflects the expected return next period and the value of the asset next period which

is q_{t+1}^h if the firm has no investment opportunities and keeps the asset on the balance sheet or q_{t+1}^s if the firm has an investment opportunity and sells the asset:

$$E_t \left[\frac{1}{\Xi_{t+1}} \frac{r_{t+1}^h + \lambda \pi \mu q_{t+1}^s + \lambda (1 - \pi \mu) q_{t+1}^h}{q_t^h} \right] = 1.$$

The value of the low quality asset reflects the expected next period return and the expected next period resale price since low assets are always sold on the re-sale market.

$$E_t \left[\frac{1}{\Xi_{t+1}} \frac{r_{t+1}^l + \lambda q_{t+1}^s}{q_t^l} \right] = 1.$$

The price of the newly issued asset with implicit support in a separating equilibrium and the price of an asset sold on the re-sale market satisfy the following:

$$E_t \left[\frac{1}{\Xi_{t+1}} \frac{r_{t+1}^G + \lambda f_t^h (\pi \mu q_{t+1}^s + \lambda (1 - \pi \mu) q_{t+1}^h)}{q_t^G} \right] = 1,$$

$$E_t \left[\frac{1}{\Xi_{t+1}} \frac{f_t^h r_{t+1}^h + (1 - f_t^h) r_{t+1}^l + \lambda f_t^h (\pi \mu q_{t+1}^s + \lambda (1 - \pi \mu) q_{t+1}^h) + \lambda (1 - f_t^h) q_{t+1}^s}{q_t^s} \right] = 1,$$

where

$$\begin{aligned} \Xi_{t+1} = & I_t \frac{r_{t+1}^G + \lambda q_{t+1}^s}{q_t^G} + \lambda K_t [(\pi \mu + (1 - \pi \mu)(1 - \omega_t)) \frac{f_t^h r_{t+1}^h + (1 - f_t^h) r_{t+1}^l + \lambda q_{t+1}^s}{q_t^s} \\ & + (1 - \pi \mu) \omega_t \frac{r_{t+1}^h + \lambda \pi \mu q_{t+1}^s + \lambda (1 - \pi \mu) q_{t+1}^h}{q_t^h}]. \end{aligned}$$

Also note that $q_t^s = f_t^h q_t^h + (1 - f_t^h) q_t^l$.

For investing firms to prefer keeping their high quality loans to selling them and investing such obtained liquidity the following condition has to be satisfied in the deterministic steady state:

$$R^h > q^s \frac{R^h - \theta R^G}{1 - \theta q^s R^G},$$

where $R^h = r_{t+1}^h + \lambda \pi \mu q_{t+1}^s + \lambda (1 - \pi \mu) q_{t+1}^h$ and $R^l = r_{t+1}^l + \lambda \pi \mu q_{t+1}^s + \lambda (1 - \pi \mu) q_{t+1}^l$. This can be transformed as follows:

$$\begin{aligned} R^h - \theta q^h R^G &> q^s R^h - \theta q^s R^G \\ R^h (1 - q^s) &> \theta R^G (q^h - q^s). \end{aligned}$$

Substituting $q^s = f^h q^h + (1 - f^h) q^l$ and $B = R^G/R^h$ we get

$$\begin{aligned}
1 - f^h q^h - (1 - f^h) q^l &> \theta B (1 - f^h) (q^h - q^l) \\
\frac{1 - f^h q^h}{1 - f^h} &> \theta B q^h + (1 - \theta B) q^l \\
f^h (q^l - q^h) (1 - \theta B) &> \theta B q^h - 1 + (1 - \theta B) q^l \\
f^h &< 1 - \frac{q^h - 1}{(q^h - q^l) (1 - \theta B)}.
\end{aligned}$$

8.2 Derivation of firms' policy functions

8.2.1 Case without implicit recourse

Individual firm maximizes

$$\max_{c_t^j, h_{t+1}^j, l_{t+1}^j, z_{t+1}^j} \sum_{s=0}^{\infty} \beta^s \log(c_{t+s}^j)$$

subject to the following borrowing constraints

$$c_t^h + i_t^h + (h_{t+1}^h - i_t^h) q_t^h + l_{t+1}^h q_t^l + z_{t+1}^h q_t^z = h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l)$$

$$c_t^l + i_t^l + (l_{t+1}^l - i_t^l) q_t^l + h_{t+1}^l q_t^h + z_{t+1}^l q_t^z = h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l)$$

$$c_t^z + i_t^z + (z_{t+1}^z - i_t^z) q_t^z + h_{t+1}^z q_t^h + l_{t+1}^z q_t^l = h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l),$$

and subject to "skin in the game" constraints:

$$h_{t+1}^h \geq (1 - \theta) i_t^h, \quad l_{t+1}^l \geq (1 - \theta) i_t^l.$$

When the skin in the game constraint are binding all constraints together can be written as follows (in the case where the constraint is binding for firms with access to both high and low quality investment opportunities):

$$c_t^s + h_{t+1}^s q_t^h + l_{t+1}^s q_t^l = h_t^s (r_t^h + \lambda q_t^h) + l_t^s (r_t^l + \lambda q_t^l)$$

$$c_t^h + \frac{(1 - \theta q_t^h)}{(1 - \theta)} h_{t+1}^h = h_t^h (r_t^h + \lambda q_t^h) + l_t^h (r_t^l + \lambda q_t^l)$$

$$c_t^l + \frac{(1 - \theta q_t^l)}{(1 - \theta)} l_{t+1}^l = h_t^l (r_t^h + \lambda q_t^h) + l_t^l (r_t^l + \lambda q_t^l).$$

The problem can be written into a recursive formulation:

$$V(l, h; K, \omega, A) = \pi (\mu V^h(l, h; K, \omega, A) + (1 - \mu) V^l(l, h; K, \omega, A)) + (1 - \pi) V^s(l, h; K, \omega, A),$$

where for $i = \{h, l, s\}$:

$$V^i(l, h; K, \omega, A) = \max_{c, h', l'} [\log(c) + \beta EV(l', h'; K, \omega, A)]$$

subject to the respective borrowing constraint stipulated above.

From first order conditions we can obtain the following Euler equations:

$$E_t \left[\beta \frac{c_t^s}{c_{t+1}^s} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} \right] = 1 \quad (8.35)$$

$$E_t \left[\beta \frac{c_t^l}{c_{t+1}^l} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right] = 1 \quad (8.36)$$

$$E_t \left[\beta \frac{c_t^h}{c_{t+1}^h} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{\frac{(1 - \theta q_t^h)}{(1 - \theta)}} \right] = 1 \quad (8.37)$$

$$E_t \left[\beta \frac{c_t^l}{c_{t+1}^l} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{\frac{(1 - \theta q_t^l)}{(1 - \theta)}} \right] = 1 \quad (8.38)$$

We guess and verify that

$$c_t^s = (1 - \beta) (h_t^s (r_t^h + \lambda q_t^h) + l_t^s (r_t^l + \lambda q_t^l))$$

$$c_t^h = (1 - \beta) (h_t^h (r_t^h + \lambda q_t^h) + l_t^h (r_t^l + \lambda q_t^l))$$

$$c_t^l = (1 - \beta) (h_t^l (r_t^h + \lambda q_t^h) + l_t^l (r_t^l + \lambda q_t^l))$$

$$h_{t+1}^h = \frac{\beta (h_t^h (r_t^h + \lambda q_t^h) + l_t^h (r_t^l + \lambda q_t^l))}{\frac{(1 - \theta q_t^h)}{(1 - \theta)}}$$

$$l_{t+1}^h = 0$$

$$l_{t+1}^l = \frac{\beta (h_t^l(r_t^h + \lambda q_t^h) + l_t^l(r_t^l + \lambda q_t^l))}{\frac{(1-\theta q_t^l)}{(1-\theta)}}$$

$$h_{t+1}^l = 0$$

$$h_{t+1}^s = \frac{\zeta \beta (h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l))}{q_t^h}$$

$$l_{t+1}^s = \frac{(1-\zeta) \beta (h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l))}{q_t^l}$$

$$c_{t+1}^s = (1-\beta) (h_{t+1}^s(r_{t+1}^h + \lambda q_{t+1}^h) + l_{t+1}^s(r_{t+1}^l + \lambda q_{t+1}^l))$$

$$c_{t+1}^h = (1-\beta) (h_{t+1}^h(r_{t+1}^h + \lambda q_{t+1}^h))$$

$$c_{t+1}^l = (1-\beta) (l_{t+1}^l(r_{t+1}^l + \lambda q_{t+1}^l))$$

Using these guesses and substituting in equations (8.37) and (8.38) we can see that these conditions always hold.

The remaining Euler equations (8.35) and (8.36) can be rewritten into:

$$E_t \left[\frac{\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}}{\zeta \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1-\zeta) \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}} \right] = 1$$

$$E_t \left[\frac{\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}}{\zeta \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1-\zeta) \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}} \right] = 1.$$

The allocation of saving firms (those with zero-profit projects) between high and low investment projects have to satisfy the market clearing conditions on both markets for high and low projects. From $H_{t+1} = \lambda H_t + I_t^h$, $L_{t+1} = \lambda L_t + I_t^l$ after substituting $H_{t+1} = H_{t+1}^h + H_{t+1}^s$, $L_{t+1} = L_{t+1}^h + L_{t+1}^s$ and $H_{t+1}^h = (1-\theta) I_t^h$, $L_{t+1}^l = (1-\theta) I_t^l$

$$H_{t+1}^s = \frac{\theta}{(1-\theta)} H_{t+1}^h + \lambda H_t$$

$$L_{t+1}^s = \frac{\theta}{(1-\theta)} L_{t+1}^h + \lambda L_t,$$

which can be rewritten as

$$\frac{\zeta (1 - \pi) \beta (H_t^s(r_t^h + \lambda q_t^h) + L_t^s(r_t^l + \lambda q_t^l))}{q_t^h} = \theta \pi \mu \frac{\beta (H_t^h(r_t^h + \lambda q_t^h) + L_t^h(r_t^l + \lambda q_t^l))}{(1 - \theta q_t^h)} + \lambda H_t$$

$$\frac{(1 - \zeta) (1 - \pi) \beta (H_t^s(r_t^h + \lambda q_t^h) + L_t^s(r_t^l + \lambda q_t^l))}{q_t^l} = \theta \pi (1 - \mu) \frac{\beta (H_t^h(r_t^h + \lambda q_t^h) + L_t^h(r_t^l + \lambda q_t^l))}{(1 - \theta q_t^l)} + \lambda L_t$$

And the goods market clears too $Y_t = C_t + I_t$.

8.2.2 Case with implicit recourse

The problem with implicit recourse and potential default on it is better written in a recursive formulation:

$$V^{ND}(\bar{s}, w - cir; \bar{S}) = \pi (\mu V^{ND,h}(\bar{s}, w - cir; \bar{S}) + (1 - \mu) V^{ND,l}(\bar{s}, w - cir; \bar{S})) + (1 - \pi) V^{ND,z}(\bar{s}, w - cir; \bar{S})$$

$$V^D(\bar{s}, w; \bar{S}) = \pi (\mu V^{D,h}(\bar{s}, w; \bar{S}) + (1 - \mu) V^{D,l}(\bar{s}, w; \bar{S})) + (1 - \pi) V^{D,z}(\bar{s}, w; \bar{S})$$

$$V^{ND,j}(\bar{s}, w; \bar{S}) = \max_{c,i,h',l',r\{G\}'} [\log(c) + \beta E [\max(V^{ND}(\bar{s}', w' - cir'; \bar{S}'), V^D(\bar{s}', w'; \bar{S}'))]]$$

$$V^{D,j}(\bar{s}, w; \bar{S}) = \max_{c,i,h',l'} [\log(c) + \beta E V^D(\bar{s}', w'; \bar{S}')]]$$

subject to the budget constraints which take the following form for investing firms for which “skin in the game” constraint is binding (e.g. in case of firms with high investment opportunities):

$$c_t^h + \frac{(1 - \theta q_t^{\hat{G},h})}{(1 - \theta)} h_{t+1}^h + cir_t = h_t^S(r_t^h + \lambda q_t^h) + l_t^S(r_t^l + \lambda q_t^l) + h_t^P(r_t^{\hat{G},h} + \lambda q_t^h) + l_t^P(r_t^{\hat{G},l} + \lambda q_t^l).$$

The incentive compatible constraints for non-defaulting are the following:

$$V^{ND}(\bar{s}, w - cir; \bar{S}) > V^D(\bar{s}, w; \bar{S}),$$

$$V^P(\bar{s}; \bar{S}) > V^{NP}(\bar{s}; \bar{S}),$$

where V^{ND}, V^D, V^P, V^{NP} are the value functions if firm, never defaulted, when firm defaulted, when firm always punished a default on a promise on gross profits and when firm failed to punished respectively. w is individual wealth level before deducting cir , which are costs of providing implicit recourse, $\bar{s} = \{h, l, h^P, l^P\}$ is a vector of other

individual state variables, where P, S superscripts denote assets sold in the previous period on the primary market which potentially bear implicit guarantee or on the secondary market respectively, $\bar{S} = \{K, \omega, A\}$ is a vector of aggregate state variables, $r_t^{\hat{G},h}$ is the return received from securitized assets with implicit recourse conditional on potential default and $q_t^{\hat{G},j}$ is the price of securitized loans of type j depending on the information structure. Costs of implicit recourse are given by:

$$cir' = \theta i (K')^{\alpha-1} (G - \Delta^{h/l})$$

From first order conditions we can obtain the following Euler equations:

$$E_t \left[\beta \frac{c_t^s}{c_{t+1}^s} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} \right] = 1 \quad (8.39)$$

$$E_t \left[\beta \frac{c_t^s}{c_{t+1}^s} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right] = 1 \quad (8.40)$$

$$E_t \left[\beta \frac{c_t^s}{c_{t+1}^s} \frac{(A + \max(G_t^h, \Delta^h)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}} \right] = 1 \quad (8.41)$$

$$E_t \left[\beta \frac{c_t^s}{c_{t+1}^s} \frac{(A + \max(G_t^l, \Delta^l)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}} \right] = 1 \quad (8.42)$$

$$E_t \left[\beta \frac{c_t^h}{c_{t+1}^h} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{\frac{(1-\theta q_t^{G,h})}{(1-\theta)}} \right] = 1 \quad (8.43)$$

$$E_t \left[\beta \frac{c_t^l}{c_{t+1}^l} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{\frac{(1-\theta q_t^{G,l})}{(1-\theta)}} \right] = 1. \quad (8.44)$$

Equations (8.41) and (8.42) hold if non-default conditions are satisfied, i.e., $G^h \leq \Delta^{Gcred,h}$ and $G^l \leq \Delta^{Gcred,l}$. If these conditions are not satisfied then investor while taking expectations have to take into account the respective probability of default.

We guess and verify the following policy functions. Note that here I report the general the policy functions for the **pooling equilibrium**, where $G^l = G^h$ and firms with access to low quality projects also invest.

$$c_t^s = (1 - \beta) (h_t^s (r_t^h + \lambda q_t^h) + l_t^s (r_t^l + \lambda q_t^l))$$

$$c_t^h = (1 - \beta) (h_t^h (r_t^h + \lambda q_t^h) + l_t^h (r_t^l + \lambda q_t^l))$$

$$c_t^l = (1 - \beta) (h_t^l (r_t^h + \lambda q_t^h) + l_t^l (r_t^l + \lambda q_t^l))$$

$$h_{t+1}^h = \frac{\beta (h_t^h(r_t^h + \lambda q_t^h) + l_t^h(r_t^l + \lambda q_t^l))}{\frac{(1-\theta q_t^G)}{(1-\theta)}}$$

$$l_{t+1}^h = 0$$

$$l_{t+1}^l = \frac{\beta (h_t^l(r_t^h + \lambda q_t^h) + l_t^l(r_t^l + \lambda q_t^l))}{\frac{(1-\theta q_t^G)}{(1-\theta)}}$$

$$h_{t+1}^l = 0$$

$$h_{t+1}^s = \frac{\zeta^h \beta (h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l))}{q_t^h}$$

$$l_{t+1}^s = \frac{\zeta^l \beta (h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l))}{q_t^l}$$

$$h_{t+1}^{p,s} = \frac{\zeta^{h^p} \beta (h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l))}{q_t^G}$$

$$l_{t+1}^{p,s} = \frac{\zeta^{l^p} \beta (h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l))}{q_t^G}$$

$$c_{t+1}^s = (1 - \beta) (h_{t+1}^s(r_{t+1}^h + \lambda q_{t+1}^h) + l_{t+1}^s(r_{t+1}^l + \lambda q_{t+1}^l))$$

$$c_{t+1}^h = (1 - \beta) (h_{t+1}^h(r_{t+1}^h + \lambda q_{t+1}^h))$$

$$c_{t+1}^l = (1 - \beta) (l_{t+1}^l(r_{t+1}^l + \lambda q_{t+1}^l)),$$

where $\zeta^h + \zeta^l + \zeta^{h^p} + \zeta^{l^p} = 1$.

Using these guesses and substituting in equations (8.43) and (8.44) we can see that these conditions always hold.

The remaining Euler equations (8.39), (8.40), (8.41) and (8.42) can be rewritten into:

$$E_t \left[\frac{\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}}{\zeta^h \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^l \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} + \zeta^{h^p} \frac{(A + \max(G_t^h, \Delta^h)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}} + \zeta^{l^p} \frac{(A + \max(G_t^l, \Delta^l)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}}} \right] = 1$$

$$E_t \left[\frac{\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}}{\zeta^h \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^l \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} + \zeta^{hP} \frac{(A + \max(G_t^h, \Delta^h)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}} + \zeta^{lP} \frac{(A + \max(G_t^l, \Delta^l)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}}} \right] = 1.$$

$$E_t \left[\frac{\frac{(A + \max(G_t^h, \Delta^h)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}}}{\zeta^h \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^l \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} + \zeta^{hP} \frac{(A + \max(G_t^h, \Delta^h)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}} + \zeta^{lP} \frac{(A + \max(G_t^l, \Delta^l)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}}} \right] = 1$$

$$E_t \left[\frac{\frac{(A + \max(G_t^l, \Delta^l)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}}}{\zeta^h \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^l \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} + \zeta^{hP} \frac{(A + \max(G_t^h, \Delta^h)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}} + \zeta^{lP} \frac{(A + \max(G_t^l, \Delta^l)) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}}} \right] = 1$$

The allocation of saving firms (those with zero-profit projects) between high and low investment projects have to satisfy the market clearing conditions on both primary and secondary markets for high and low projects.

$$\lambda H_t = \zeta^h \beta (1 - \pi) (H_t^s (r_t^h + \lambda q_t^h) + L_t^s (r_t^l + \lambda q_t^l))$$

$$\lambda L_t = \zeta^l \beta (1 - \pi) (H_t^s (r_t^h + \lambda q_t^h) + L_t^s (r_t^l + \lambda q_t^l))$$

$$\theta \pi \mu \frac{\beta (H_t^h (r_t^h + \lambda q_t^h) + L_t^h (r_t^l + \lambda q_t^l))}{(1 - \theta q_t^G)} = \frac{\zeta^{hp} (1 - \pi) \beta (H_t^s (r_t^h + \lambda q_t^h) + L_t^s (r_t^l + \lambda q_t^l))}{q_t^G}$$

$$\theta \pi (1 - \mu) \frac{\beta (H_t^h (r_t^h + \lambda q_t^h) + L_t^h (r_t^l + \lambda q_t^l))}{(1 - \theta q_t^G)} = \frac{\zeta^{hl} (1 - \pi) \beta (H_t^s (r_t^h + \lambda q_t^h) + L_t^s (r_t^l + \lambda q_t^l))}{q_t^G}$$

And the goods market clears too $Y_t = C_t + I_t$.

8.3 Numerical solutions of the stochastic dynamic system

To solve the fully stochastic dynamic model I use numerical approximation methods. Since depending on the state variables the economy is switching between separating and pooling equilibrium I am using global approximation methods. In particular I look for the values of the following functions:

$$\begin{aligned}
q_t^h &= \Gamma_1 (A_t, K_t, \omega_t) \\
q_t^l &= \Gamma_2 (A_t, K_t, \omega_t) \\
V^D &= \Gamma_4 (A_t, K_t, \omega_t) \\
V^{ND} &= \Gamma_5 (A_t, K_t, \omega_t)
\end{aligned}$$

I construct a grid for the three aggregate states A , K , ω and start with the guess equal to the steady-state values for prices and zero for value functions. Then I iterate using the set of equilibrium conditions to find the updated values of $(\Gamma_1, \dots, \Gamma_5)$ until the updated values are close to the previous guesses.

$$\begin{aligned}
&\left(q_t^h (iter) - q_t^h (iter - 1) \right)^2 + \left(q_t^l (iter) - q_t^l (iter - 1) \right)^2 \\
&+ \left(V^{ND} (iter) - V^{ND,h} (iter - 1) \right)^2 + \left(V^D (iter) - V^D (iter - 1) \right)^2 > \varepsilon.
\end{aligned}$$

During iteration at each point of the grid it is evaluated whether the economy is in separating or pooling equilibrium. The points out of grid are obtained through trilinear interpolation.