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Junghun Cho

CERGE-EI

Charles University
Center for Economic Research and Graduate Education
Academy of Sciences of the Czech Republic
Economics Institute

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Sequential Cheap Talk from Advisors with Reputation*

Junghun Cho†

CERGE-EI‡

Abstract

I examine two-period sequential cheap talk in situations where the decision maker seeks advice from two advisors, each of whom knows the type of the other advisor. By considering the current payoff (which is determined by the message of each advisor) and the future payoff (which is connected with the reputation of each advisor), I examine conditions which guarantee the existence of both good and bad reputation effects. Compared to situations of simultaneous cheap talk, the decision maker loses information more easily if he seeks advice sequentially.

Abstrakt

Zkoumám dvoukolovou postupnou nezávaznou komunikaci v situaci, kdy rohodující se subjekt hledá radu od dvou poradců, kde každý z nich zná typ toho druhého. Na základě současného zisku (který je vypočítán na základě zprávy od obou poradců) a zisku budoucího (který je spojen s reputací každého z nich) zkoumám podmínky, které zaručují existenci efektu dobré a špatné pověsti. Porovnáním situací, kdy nezávazná komunikace je vedena současně, zjišťuji, že rohodující se ztrácí informaci spíše v případě, kdy získává radu od poradců postupně.

Keywords: Cheap Talk, Reputation
JEL classification: D82, D83

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† E-mail: Junghun.Cho@cerge-ei.cz
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1 Introduction

We want to choose the optimal action. If we do not have enough information about what will be the optimal choice, we sometimes ask advice from people. After obtaining information from other people, we usually seek advice from the same people again if we believe that advice is helpful. People who give advice, if they know they will meet us again and their payoffs are connected with our choices, try to make us believe what they will suggest.

Let us consider the example of free health consultations. A patient is uncertain whether medicine is needed or surgery is needed and seeks advice from one doctor. There are two uncertainties to the patient. First, the patient is uncertain about his health condition. Second, he is also uncertain about the type of the doctor. The patient believes that the doctor can be one of two types: good doctor or bad doctor. A good doctor has an incentive to give correct advice to the patient. A bad doctor is biased towards suggesting surgery.

Let us consider the case where the patient who needs surgery meets a good doctor. If the good doctor suggests surgery, the patient's belief that the doctor is of the bad type is increased. If the good doctor worries about his reputation (not to be perceived as bad doctor) he may suggest medicine to the patient. In this case, the patient loses information about his health condition. When the patient knows that he may lose information by having a single doctor, he may try to obtain additional advice from an additional doctor.

When the patient sees the second doctor, he may or may not share the advice of the first doctor with the second doctor. If the patient does not share the advice of the first doctor with the second doctor but mentions his earlier visit, we have a situation of simultaneous cheap talk. I have studied this situation in Cho (2006). In many cases when a patient sees the second doctor, he shares the advice of the first doctor with the second doctor. Since the doctors work in the same medical field, the doctors may know the type of the other doctor or may have beliefs about the type of the other doctor. For simplicity, I assume that each doctor knows the type of the other doctor and that the second doctor knows the advice of the first doctor.

In a two-period simultaneous game, Cho (2006) shows that the patient who seeks advice from two doctors sometimes loses information in the first period when each doctor worries about his reputation not to be perceived as a bad doctor. In this paper, I examine whether the loss of information is reduced by asking advice sequentially and ask if it is better for him to seek advice simultaneously or sequentially.

I examine four cases by considering each of the two types of advisor. In each period, each advisor observes an imperfect private signal about the state of the world. The decision maker reports the existence of the second advisor when he meets the first advisor. After receiving the message from the first advisor, the decision maker shares the message sent from the first advisor with the second advisor. Then, the decision maker chooses an action in each period. The state of the world is revealed after the action of the decision maker is chosen. Since the decision maker is uncertain about the preferences of each advisor, his action is affected not only by the messages from the two advisors but also by his belief about the type of each advisor.

By comparing the updated belief of the decision maker about the type of the advisor of each message in the first period, I show that the first advisor has an incentive to suggest the bad advisor's unbiased message in order to increase his reputation in the first period. Given the message of the first advisor, the second advisor can increase his reputation also by sending the bad advisor's unbiased message in the first period.

Let me now consider the case where the second advisor is of the bad type and observes the signal towards which the bad advisor is not biased. If the advisor sends a biased message, he can increase his current payoff. However, he loses his reputation by doing so. By considering the total payoff of the second advisor who knows the message sent by the first advisor, I find that the bad advisor sometimes tells the truth in the first period regardless of the message of the first advisor and the type of the first advisor. Since the bad advisor tells the truth even if he has a payoff loss in the first period to increase his reputation, it is called good reputation effect. The possibility of the existence of good reputation effect is greater (or less) in sequential cheap talk if the second advisor receives the message towards which the bad advisor is not biased (or biased) from the first advisor, compared to the simultaneous cheap talk.

Let me next consider the case where the second advisor is of the good type and observes the signal towards which the bad advisor is biased. By sending a truthful message, he obtains a high payoff in the current period. However, the probability of being perceived as the advisor who is biased towards suggesting the same message is increased by sending the truthful message. If the second advisor considers his second period sufficiently more important, he tells a lie in the first period regardless of both the type of the first advisor and the message of the first advisor. Such as the bad advisor sometimes tells the truth to increase his reputation, the good advisor sometimes tells a lie to make the decision maker believe what he will suggest in the next period (bad reputation effect). The possibility of telling a lie in sequential cheap talk is greater than in simultaneous cheap talk regardless of the message of the first advisor.

If the decision maker seeks advice from two advisors and believes that each advisor worries about his reputation, it is better for the decision maker to seek advice simultaneously. It is because the advisor who knows the message of the previous advisor can adjust his message more easily in sequential cheap talk and the possibility of telling a lie in sequential cheap talk is always greater than that in simultaneous cheap talk regardless of both the type and the message of the previous advisor.

Crawford and Sobel (1982) examine a one-period model where the advisor chooses the strategic message to send to the decision maker and show that more communication is possible if the preferences of players are more closely aligned. In cheap talk, there is always a babbling equilibrium[1] where the decision maker learns nothing from the advisor. Except babbling equilibrium, the question is to find an informative equilibrium[2]. Since Krishna and Morgan (2001) only consider one-period cheap talk where two advisors send the message sequentially, I extend their model by considering a two-period model in order to examine the reputation effect.

In repeated cheap talk, reputation plays an important role in the message of the advisor because the advisor meets the decision maker again. Sobel (1985) shows that the reputational concern not to be perceived as the person who has opposite payoff to the decision maker makes the advisor tell the truth in a finite cheap talk. Similar to the bad advisor (who has the opposite payoff to the decision maker or who is biased towards suggesting any particular advice) sometimes tells the truth to increase his reputation, the good advisor (who has the same preferences as the decision maker) sometimes tells a lie to increase his reputation if the decision maker seeks advice from one advisor (Morris, 2001)[3].

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[1] Since the message contains no information, the decision maker does not believe it when he makes the decision. From the perspective that the message will be ignored by the decision maker, no advice that improves the payoffs relative to babbling is delivered from the advisor (Gibbons, 1992).

[2] Krishina and Morgan (2001) extends Crawford and Sobel's model by adding one more advisor. Park (2005) examines an infinitely repeated cheap talk model where the advisor observes a perfect signal about the state of the world. Levi and Razin (2004) and Battaglini (2002, 2004) consider multidimensional cheap talk.

[3] Ely and Valimaki (2003) extend the models of Kreps and Wilson (1982) and Milgrom and

In the next section, using a two-period cheap talk model, I examine the existence of both good and bad reputation effects when each advisor has imperfect information regarding the state of the world and has perfect information regarding the type of the other advisor.

2 Model

I consider a two-period cheap talk model where the decision maker seeks the advice from two advisors sequentially. Since the method of conveying the message is cheap talk, the message of each advisor cannot enter into the utility function of either of the advisors or the decision maker.

Assumption 1. Each advisor knows the preferences of the other advisor but the decision maker does not know.

Before sending the message to the decision maker, each advisor knows both the presence of the other advisor and the type of the other advisor. The decision maker believes that the advisor can be one of two types: good or bad. In period i (for $i = 1, 2$), the decision maker believes that the advisor j (for $j = 1, 2$) is of the good type with probability λ_i^j. With probability $1 - \lambda_i^j$, the decision maker believes that the advisor j is of the bad type.

The state of the world in period i is $\omega_i \in W = \{0, 1\}$. In the example, the state of the world 0 is the case where the patient needs the medicine and the state of the world 1 is the case where the patient needs the surgery.

Assumption 2. Each state of the world is equally likely.

Roberts (1982), where a long lived player meets a sequence of short-run players and show that imperfect information about the type of short-run players can bring about an incentive to tell a lie to the good type advisor.

Assumption 3. Each advisor receives a signal about the state of the world. The signal is imperfect but informative.

In the simultaneous cheap talk model, Cho (2006) shows that the bad reputation effect may not occur if each advisor receives a perfect signal about the state of the world. The advisor j obtains the signal S_i^j which is the same as the state of the world with probability γ, i.e., $P(S_i^j = \omega_i) = \gamma$ and $\frac{1}{2} < \gamma < 1$.

The first advisor sends message m_i^1 to the decision maker in period i in accordance with his type and his signal S_i^1. Since the message of the first advisor m_i^1 is known to the second advisor, the second advisor sends the message m_i^2 in accordance with his type and the signal S_i^2, given the message of the first advisor.

Assumption 4. The second advisor believes that his signal is correct if there is a conflict between the signals of the two advisors.

In some cases, the second advisor knows the signal that the first advisor observes from the message of the first advisor. If the second advisor observes a signal which is different than that of the first advisor, he chooses the message in accordance with his signal. I will explain later in this paper why assumption 4 is needed for simplicity in sequential cheap talk.

The decision maker chooses an action $a_i^{DM} \in R$ which can affect all players' payoffs after receiving the messages from both advisors. In cheap talk, the message of each advisor cannot directly affect the action of the decision maker but can indirectly affect it by changing the belief of the decision maker about the state of the world. After payoffs are determined, the state of the world in period i (ω_i) is revealed publicly. Before starting the second period, the decision maker can update his belief about the type of each advisor by considering the messages of the advisors and the realized state of the world.

The preferences of advisors are explained by the utility function of each type of advisor. Since the good advisor is the person who has the same preferences as the decision maker, the utility function of the decision maker is assumed to be that of the good advisor, $-(a_i - \omega_i)^2$ in period i. Since the decision maker is uncertain about the state of the world, he chooses the action a_i as the belief of the decision maker that the state of the world in period i is 1 given messages from both advisors. Since the bad advisor is the person who is biased towards suggesting any particular message, the utility function of the bad advisor is assumed to be a_i in period i. The total payoff of advisor j, if he is of the good type, is

$$-x_1^j(a_1 - \omega_1)^2 - x_2^j(a_2 - \omega_2)^2,$$

where x_1^j and x_2^j denote the weights on the payoffs in the first period and in the second period respectively. The total payoff of advisor j, if he is of the bad type, is

$$y_1^j a_1 + y_2^j a_2,$$

where y_1^j and y_2^j denote the weights on the payoffs in the first period and in the second period respectively.

Assumption 5. $\sum_{i=1}^{2} x_i^j = 1$ and $\sum_{i=1}^{2} y_i^j = 1$ for advisor j in period i.

Since both good and bad reputation effects in equilibrium are determined by the weight on the payoff in the first period, it is assumed that the sum of the weight in each period is 1 for simplicity.

In each period, the message of each advisor depends on the type, the signal and the order of the advisor. Given messages from two advisors, the decision maker's action is determined. Then, I can determine the value function of each type of advisor which is the payoff of the advisor in each period. By comparing total payoff of telling a lie with that of telling the truth, I examine the existence of both good and bad reputation effects. I will use backward induction to examine the reputation effects.

2.1 Second Period

There is babbling equilibrium where the decision maker does not learn anything about the type of the advisor and the state of the world. Except babbling equilibrium, I examine the equilibrium where an informative message is conveyed in the second period. Since the second period is the last period, there is no reputational concern for advisors.

2.1.1 Messages of the Advisors

The message of the first advisor is determined by both the type and the signal of the first advisor. If the first advisor is of the good type, he sends the message 0 when he observes the signal 0. When the first advisor observes the signal 1, he sends the message 1 if he is of the good type. Regardless of the signal, the first advisor sends the message 1 if he is of the bad type.

The message of the second advisor depends on the type of the second advisor, the signal of the second advisor and the message of the first advisor. Let us consider the case where the second advisor is of the good type and receives the message 0 from the first advisor. The second advisor knows that the first advisor is of the good type and the signal of the first advisor is 0. If the signal of the second advisor is 0, he sends the message 0. But if the second advisor receives the signal 1, the second advisor is confused about the state of the world. The second advisor may have doubt about his signal because he knows that the first advisor observes the signal 0. In general, he may choose the message 0 or 1 randomly. For simplicity, I use the assumption that each advisor follows his signal. Thus, the second advisor sends the message 1 by assumption 4.

Now assume that the second advisor is of the good type and receives the message 1 from the first advisor. The second advisor knows that the first advisor is of the good type and his signal is 1, or knows that the second advisor is of the bad type regardless of the signal. If the signal of the second advisor is 0 when he knows that the first advisor is of the good type, he is confused about the state of the world. However, he sends the message 0 by assumption 4. If the second advisor observes the signal 1, he sends the message 1 to the decision maker. In the case where the second advisor is of the bad type, the second advisor sends the message 1 regardless of the signal.

By assumption 4, the second advisor, if he is of the good type, sends the message 0 regardless of the message of the first advisor when the signal of the second advisor is 0. The second advisor sends the message 1 regardless of the message of the first advisor when the signal of the second advisor is 1. Regardless of both the signal and the message of the first advisor, the second advisor sends the message 1 if he is of the bad type.

2.1.2 Action of the Decision Maker

In order to determine the action of the decision maker given messages from two advisors sequentially, the conditional probability that the second advisor sends the message 0 or 1 given the message of the first advisor as 0 or 1 is calculated. The conditional probability that the second advisor sends the message 0 given the message of the first advisor as 0 is

$$\begin{aligned}
P(m_2^2 &= 0|m_2^1 = 0) \\
&= \frac{\sum_{k=0}^{1} P(m_2^1 = 0 = m_2^2|\omega_2 = k)}{\sum_{k=0}^{1} P(m_2^1 = 0|\omega_2 = k)} \\
&= \frac{\gamma^2 + (1-\gamma)^2}{\lambda_2^1 + (1-\lambda_2^1\lambda_2^2)\{\gamma^2 + (1-\gamma)^2\}}.
\end{aligned}$$

The state of the world is equally likely. The denominator is determined given that the message of the second advisor is 0 or 1. The conditional probability that the second advisor sends the message 0 given the message of the first advisor as 1 is

$$\begin{aligned}
P(m_2^2 &= 0|m_2^1 = 1) \\
&= \frac{\sum_{k=0}^{1} P(m_2^1 = 1 \text{ and } m_2^2 = 0|\omega_2 = k)}{\sum_{k=0}^{1} P(m_2^1 = 1|\omega_2 = k)} \\
&= \frac{\lambda_2^2[1 - \{\gamma^2 + (1-\gamma)^2\}\lambda_2^1]}{2 - \lambda_2^1}.
\end{aligned}$$

Since the sum of the conditional probability is 1, the conditional probability that the second advisor sends the message 1 given the message of the first advisor is automatically determined.

If the decision maker receives the message 0 from the first advisor and then receives the message 0 from the second advisor, he believes that both advisors are of the good type. The decision maker also believes that the second advisor sends the message 0 with probability $P(m_2^2 = 0|m_2^1 = 0)$ because he knows that the second advisor knows the message of the first advisor. The probability that the state of the world is 1 given message 0 from two advisors sequentially is given by

$$P^{2,1}_{0,0} = \frac{1 - 2\gamma + \gamma^2}{1 - 2\gamma + 2\gamma^2}$$

where $P^{i,1}_{m^1_i, m^2_i}$ represents the probability that the state of the world in period i is 1 given the message of the first advisor m^1_i and the message of the second advisor m^2_i sequentially. The decision maker chooses action $P^{2,1}_{0,0}$ if he receives the message 0 from two advisors sequentially.

If the decision maker receives the message 0 from the first advisor and then receives the message 1 from the second advisor, he is certain that the first advisor is of the good type but is not sure whether the second advisor is of the good type or of the bad type. He will infer that the second advisor sends the message 1 with probability $P(m^2_2 = 1 | m^1_2 = 0)$. The belief of the decision maker that the state of the world in the second period is 1 given the message of the first advisor 0 and the message 1 from the second advisor becomes

$$P^{2,1}_{0,1} = \frac{(1-\gamma)\{1 - (1-\gamma)\lambda^2_2\}}{1 - \{\gamma^2 + (1-\gamma)^2\}\lambda^2_2}.$$

If the decision maker receives the message 0 and then receives the message 1, he chooses the action $P^{2,1}_{0,1}$ in the second period.

Similarly, the action of the decision maker in the second period ($P^{2,1}_{1,0}$ or $P^{2,1}_{1,1}$) given messages ($m^1_2 = 1$ and then $m^2_2 = 0$ or 1) from two advisors sequentially is determined as

$$P^{2,1}_{1,0} = \frac{(1-\gamma)\{1 - (1-\gamma)\lambda^1_2\}}{1 - \{\gamma^2 + (1-\gamma)^2\}\lambda^1_2}$$

and

$$P^{2,1}_{1,1} = \frac{1 - (1-\gamma)(\lambda^1_2 + \lambda^2_2) + (1-\gamma)^2\lambda^1_2\lambda^2_2}{2 - (\lambda^1_2 + \lambda^2_2) + \{\gamma^2 + (1-\gamma)^2\}\lambda^1_2\lambda^2_2}.$$

2.1.3 Payoff of the Advisor

If the action of the decision maker given messages sequentially is determined, the payoff of the second advisor in the second period is determined. This is the value function of the second advisor in accordance with the types of both the first advisor and the second advisor. If the value function of the second advisor is increasing with the updated belief of the decision maker that the second advisor is of the good type, the second advisor can adjust his message in the first period to increase his reputation.

Let us consider the case where the second bad advisor meets the other good advisor. Since the second advisor is the advisor who has a payoff incentive to suggest the message 1, the value function of the second advisor if the first advisor is of the good type is

$$v^2_{GB}[\lambda^1_2, \lambda^2_2] = y^2_2 a_2 = \frac{1}{2} y^2_2 (P^{2,1}_{0,1} + P^{2,1}_{1,1}).$$

The first advisor receives the correct signal with probability γ in each period. By assumption 2, the state of the world in the second period is 1 with probability $\frac{1}{2}$. So, the first advisor sends the message 0 with probability $\frac{1}{2}(\gamma + 1 - \gamma) = \frac{1}{2}$.

Next is the case where the second bad advisor knows that the first advisor is also of the bad type. Since both advisors send the message 1 regardless of the signal in the second period, the value function of the second advisor if the first advisor is of the bad type is

$$v^2_{BB}[\lambda^1_2, \lambda^2_2] = y^2_2 a_2 = y^2_2 P^{2,1}_{1,1}.$$

When the second advisor is of the good type, I need to consider two cases: the first advisor is of the bad type or of the good type. The value function of the second advisor if he knows that the first advisor is of the bad type is

$$
\begin{aligned}
\upsilon_{BG}^2[\lambda_2^1, \lambda_2^2] &= -x_2^2(a_2-\omega_2)^2 \\
&= -\frac{1}{2}x_2^2\sum_{k=0}^{1}[\gamma_k(P_{1,k}^{2,1})^2 + (1-\gamma_k)(P_{1,k}^{2,1}-1)^2]
\end{aligned}
$$

where $\gamma_0 = \gamma$ and $\sum_{k=0}^{1}\gamma_k = 1$. By assumption 2, the state of the world is equally likely. In each state of the world, the advisor receives the correct signal with probability γ. Since the second advisor has a payoff incentive to suggest the correct advice to the decision maker but the first advisor sends the message 1 regardless of his signal, the payoff of the second advisor is determined as $\upsilon_{BG}^2[\lambda_2^1, \lambda_2^2]$.

Similarly, the value function of the second advisor if he knows that the first advisor is also of the good type is

$$
\begin{aligned}
\upsilon_{GG}^2[\lambda_2^1, \lambda_2^2] &= -x_2^2(a_2-\omega_2)^2 \\
&= -\frac{1}{2}x_2^2\sum_{k=0}^{1}\sum_{i=0}^{1}[\gamma_k\gamma_l(P_{k,l}^{2,1})^2 \\
&\quad +(1-\gamma_k)(1-\gamma_l)(P_{k,l}^{2,1}-1)^2].
\end{aligned}
$$

In each state of the world, both advisors observe the correct signal with probability γ^2. One of two advisors observes the correct signal with probability $\gamma(1-\gamma)$. In this case, both advisors have a payoff incentive to suggest the correct advice to the decision maker. Similarly, the value function of the first advisor is determined in accordance with the type of the second advisor.

So far, I have examined, first, the message of each type of the first or the second advisor in the second period, second, the action of the decision maker in the second period given messages from two advisors sequentially, and third, the payoff of each type of advisor as a value function in the second period. As an important property in the second period, the value function of the first (or the second) advisor is increasing with the updated belief of the decision maker that the first (or the second) advisor is of the good type. If there is a message of an advisor which can increase the updated belief of the decision maker about his type in the first period, the message can also increase the payoff of the advisor in the second period.

2.2 First Period

In the first period, each advisor sends the message by considering both the first and the second period payoffs. If the second advisor is of the bad type, the total payoff of the second advisor is

$$\begin{aligned} & y_1^2 a_1 + \upsilon_{GB}^2[\lambda_2^1, \lambda_2^2] \\ = \; & y_1^2\{a_1 - \frac{1}{2}(P_{0,1}^{2,1} + P_{1,1}^{2,1})\} + \frac{1}{2}(P_{0,1}^{2,1} + P_{1,1}^{2,1}), \end{aligned}$$

when the second advisor knows that the other advisor is of the good type. If the second advisor knows that the first advisor is also of the bad type, the value function is changed from $\upsilon_{GB}^2[\lambda_2^1, \lambda_2^2] = \frac{1}{2}y_2^2(P_{0,1}^{2,1} + P_{1,1}^{2,1})$ to $\upsilon_{BB}^2[\lambda_2^1, \lambda_2^2] = y_2^2 P_{1,1}^{2,1} = (1-y_1^2)P_{1,1}^{2,1}$ in the first case. The total payoff of the second advisor, if he is of the good type, is

$$\begin{aligned} & -x_1^2(a_1 - \omega_1)^2 + \upsilon_{BG}^2[\lambda_2^1, \lambda_2^2] \\ = \; & x_1^2\{\frac{1}{2}\sum_{k=0}^{1}[\gamma_k(P_{1,k}^{2,1})^2 + (1-\gamma_k)(P_{1,k}^{2,1} - 1)^2] - (a_1 - \omega_1)^2\} \\ & -\frac{1}{2}\sum_{k=0}^{1}[\gamma_k(P_{1,k}^{2,1})^2 + (1-\gamma_k)(P_{1,k}^{2,1} - 1)^2], \end{aligned}$$

when the second advisor knows that the other advisor is of the bad type. The value function is changed from $v^2_{BG}[\lambda^1_2, \lambda^2_2] = -\frac{1}{2}x_2^2 \sum_{k=0}^{1}[\gamma_k (P^{2,1}_{1,k})^2 + (1-\gamma_k)(P^{2,1}_{1,k} - 1)^2]$ to $v^2_{GG}[\lambda^1_2, \lambda^2_2] = -\frac{1}{2}x_2^2 \sum_{k=0}^{1}\sum_{i=0}^{1}[\gamma_k \gamma_l (P^{2,1}_{k,l})^2 + (1-\gamma_k)(1-\gamma_l)(P^{2,1}_{k,l} - 1)^2]$ if the second advisor knows that the other advisor is also of the good type. Similarly, the total payoff of each type of first advisor is determined in accordance with the type of the second advisor.

2.2.1 Message of Each Advisor

Suppose that the good advisor sometimes tells a lie in the first period. If the signal is the one the bad advisor is biased towards, the good advisor sometimes sends the message the bad advisor is not biased towards in order not to be perceived as a bad advisor. In the model, I suppose that the good advisor sometimes sends the message 0 in the first period if he observes the signal 1. The bad advisor also sometimes tells a lie. If the signal is the one the bad advisor is not biased towards, the bad advisor sometimes sends the message he is biased towards in order to increase his current payoff. Also, the bad advisor may sometimes send the message he is not biased towards if his signal is the one he is biased towards. It is because the bad advisor wants to make the decision maker believe what he will suggest next time. In the model, the bad advisor sometimes sends the message 1 (or 0) if his signal is 0 (or 1).

By assumption 4, the message of each advisor is determined as follows. If the advisor j is of the good type, he sends the message 0 when he observes the signal 0 in the first period. If the signal in the first period is 1, the advisor j sends the message 1 with probability z. In the case where the decision maker believes that the advisor j is of the good type, the decision maker is uncertain about the state of the world if he receives the message 0 from the advisor j. It is because the decision maker believes that the signal of the advisor is 0, or believes that the signal of the advisor is 1 and the advisor sends the message 0 with probability $1-z$. Let us consider the case where the advisor j is of the bad type. If the advisor observes the signal 0, he sends the message 1 with probability ν. The advisor j sends the message 1 with probability ρ if his signal is 1.

2.2.2 Updated Belief of the Decision Maker about Type of the Advisor

Since the state of the world is revealed publicly at the end of the first period, the decision maker can update his belief that the advisor j is of the good type by considering the message of advisor j and the realized state of the world ω_1. Let us first consider the updated belief of the decision maker about the type of the first advisor when the state of the world is revealed as 0. If the decision maker receives the message 0 from the first advisor, the updated belief of the decision maker that the first advisor is of the good type is

$$\begin{aligned}&\lambda_2^1(\lambda_1^1, 0, 0)\\ =\ &\frac{\lambda_1^1(1-z+\gamma z)}{1-\rho+\gamma(\rho-\nu)+\lambda_1^1\{\rho-z+\gamma(z-\rho-\nu)\}}\end{aligned}$$

where $\lambda_2^j(\lambda_1^j, m_1^j, \omega_1)$ represents the updated belief of the decision maker about the type of the advisor j if the advisor j sends the message m_1^j and the state of the world in the first period is revealed as ω_1. When the first advisor sends the message, he considers his signal as well as his belief about the message of the second advisor. As one example, consider the case where each advisor knows that the other advisor is of the good type. When the first advisor who observes the signal 0, sends the message 0, he knows that the probability that the second advisor who observes the signal 0 and receives the message 0 from the first advisor sends the message 0 (or 1) is 1 (or 0). Similarly, the first advisor knows the probability that the second advisor who observes the signal 1 and receives the message 0 from the first advisor sends the message 0 (or 1) is $1-z$ (or z). Given these beliefs, the probability that the first advisor who observes the signal 0 sends the message 0 is 1 if both advisors are of the good type. Since the denominator explains all possible cases that the first advisor sends the message 0 if the state of the world is revealed as 0, I need to consider three more cases: 1. the first advisor who is of the good type observes the signal 1 sends the message 0 with probability $1-z$; 2. the first advisor who is of the bad type observes the signal 0 sends the message 0 with probability $1-\nu$; and 3. the first advisor sends the message 0 with probability $1-\rho$ if he is of the bad type and observes the signal 1 in the first period. Among those cases, the numerator explains the probability that the first advisor who is of the good type sends the message 0.

By using the same method, the updated belief of the decision maker that the first advisor is of the good type if the first advisor sends the message 1 and the real state of the world is revealed as 0 is

$$
\begin{aligned}
& \lambda_2^1(\lambda_1^1, 1, 0) \\
= \quad & \frac{\lambda_1^1(z-\gamma z)}{\rho-\gamma(\rho-\nu)+\lambda_1^1\{z-\rho+\gamma(\rho-z-\nu)\}}.
\end{aligned}
$$

In this case, the first advisor needs to consider the conditional probability that the second advisor sends the message 0 (or 1) given the type of the first or the second advisor, the signal of the second advisor, and the message 1 from the first advisor.

Next is the case where the state of the world is revealed as 1. The updated belief of the decision maker that the first advisor is of the good type if the first advisor sends the message 0 is

$$\begin{aligned}&\lambda_2^1(\lambda_1^1, 0, 1)\\ = \quad & \frac{\lambda_1^1(z-\gamma z)}{1-\nu+\gamma(\nu-\rho)+\lambda_1^1\{\nu+\gamma(\rho-z-\nu)\}}.\end{aligned}$$

The only difference between the first case and this case is that the state of the world is changed from 0 to 1. If the first advisor is of the good type, he sends the message 0 when he observes the wrong signal. The first advisor also sends the message 0 with probability $1-z$ when he observes the correct signal. When the first advisor sends the message to the decision maker, he also considers the conditional probability that the second advisor sends the message 0 (or 1) given the type of the first or the second advisor, the signal of the second advisor and the message 0 from the first advisor.

Similarly, the updated belief of the decision maker that the first advisor is of the good type if the first advisor sends the message 1 is

$$\begin{aligned}&\lambda_2^1(\lambda_1^1, 1, 1)\\ = \quad & \frac{\lambda_1^1\gamma z}{\nu+\gamma(\rho-\nu)+\lambda_1^1\{-\nu+\gamma(z+\nu-\rho)\}}.\end{aligned}$$

Proposition 1 *Regardless of the state of the world in the first period, the first advisor has reputational incentive to announce 0 because*

$$\lambda_2^1(\lambda_1^1, 0, 0) > \lambda_1^1 > \lambda_2^1(\lambda_1^1, 1, 0)$$

and

$$\lambda_2^1(\lambda_1^1, 0, 1) > \lambda_1^1 > \lambda_2^1(\lambda_1^1, 1, 1).$$

The first advisor (who knows that the second advisor can adjust his message in accordance with his message) sends the message 0 regardless of the signal to increase the updated belief of the decision maker about his type. In the example, to suggest the medicine is the way to increase the reputation of the first doctor regardless of the health condition of the patient. Specifically, even if the patient needs surgery, the first doctor suggests the medicine if he only considers the increase in his reputation. Since I have already shown that the payoff of the first advisor in the second period (or, the value function) is increased with the updated belief of the decision maker about the type of the first advisor, the way to increase the payoff in the second period is to suggest a message that the bad advisor is not biased towards.

Now, I examine the updated belief of the decision maker about the type of the second advisor. Since the fact that the second advisor knows the message sent by the first advisor before sending his message is common knowledge among players, the conditional probability that the second advisor sends the message 0 (or 1) given the message of the first advisor 0 (or 1) is calculated. If the second advisor receives the message 0 from the first advisor, the probability that the second advisor also sends the message 0 in the first period is

$$\begin{aligned} P(m_1^2 &= 0|m_1^1 = 0) \\ &= \frac{\sum_{k=0}^{1} Q_{0,0}^{1,k}}{\sum_{i=0}^{1}\sum_{j=0}^{1} Q_{0,i}^{1,j}} \end{aligned}$$

where $Q_{m_1^1,m_1^2}^{1,l}$ represents the conditional probability that the message of the first advisor is m_1^1 and the message of the second advisor is m_1^2 given that the state of the world in the first period is l, and

$$Q_{0,0}^{1,0} = \prod_{i=1}^{2}[\lambda_1^i\{1-(1-\gamma)z\} + (1-\lambda_1^i)\{1-(1-\gamma)\rho - \gamma\nu\}],$$

$$Q_{0,0}^{1,1} = \prod_{i=1}^{2}[\lambda_1^i(1-\gamma z) + (1-\lambda_1^i)\{1-\gamma\rho-(1-\gamma)\nu\}],$$

$$\begin{aligned} Q_{0,1}^{1,0} &= [\lambda_1^1\{1-(1-\gamma)z\} + (1-\lambda_1^1)\{1-(1-\gamma)\rho-\gamma\nu\}] \times \\ &\quad [\lambda_1^2(1-\gamma)z + (1-\lambda_1^2)\{(1-\gamma)\rho+\gamma\nu\}] \end{aligned}$$

and

$$\begin{aligned} Q_{0,1}^{1,1} &= [\lambda_1^1(1-\gamma z) + (1-\lambda_1^1)\{1-\gamma\rho-(1-\gamma)\nu\}] \times \\ &\quad [\lambda_1^2\gamma z + (1-\lambda_1^2)\{\gamma\rho+(1-\gamma)\nu\}]. \end{aligned}$$

The numerator shows the probability that both advisors send the message 0 if the state of the world in the first period is 0 or 1. The denominator explains the conditional probability that only the first advisor sends the message 0 given that the state of the world in the first period is 0 or 1.

Similarly, the conditional probability that the second advisor sends the message 0 given that the first advisor sends the message 1 is

$$\begin{aligned} P(m_1^2 &= 0|m_1^1 = 1) \\ &= \frac{\sum_{k=0}^{1} Q_{1,0}^{1,k}}{\sum_{i=0}^{1}\sum_{j=0}^{1} Q_{1,i}^{1,j}} \end{aligned}$$

where

$$\begin{aligned} Q_{1,0}^{1,0} &= [\lambda_1^1(1-\gamma)z + (1-\lambda_1^1)\{(1-\gamma)\rho+\gamma\nu\}] \times \\ &\quad [\lambda_1^2\{1-(1-\gamma)z\} + (1-\lambda_1^2)\{1-(1-\gamma)\rho-\gamma\nu\}], \end{aligned}$$

$$
\begin{aligned}
Q^{1,1}_{1,0} &= [\lambda^1_1\gamma z + (1-\lambda^1_1)\{\gamma\rho + (1-\gamma)\nu\}] \times \\
&\quad [\lambda^2_1(1-\gamma z) + (1-\lambda^2_1)\{1-\gamma\rho-(1-\gamma)\nu\}],
\end{aligned}
$$

$$
Q^{1,0}_{1,1} = \prod_{i=1}^{2}[\lambda^i_1(1-\gamma)z + (1-\lambda^i_1)\{(1-\gamma)\rho + \gamma\nu\}]
$$

and

$$
Q^{1,1}_{1,1} = \prod_{i=1}^{2}[\lambda^i_1\gamma z + (1-\lambda^i_1)\{\gamma\rho + (1-\gamma)\nu\}].
$$

The numerator explains the probability that the first advisor sends the message 1 and the second advisor sends the message 0 if the state of the world in the first period is 0 or 1. If the state of the world is 0 or 1, the denominator explains the probability that the first advisor sends the message 1 in the first period. Since the sum of the conditional probability that the advisor sends the message 0 or 1 given the message of the second advisor is 1, the probability that the second advisor sends the message 1 given the message of the first advisor (0 or 1) is determined automatically.

Since the second advisor knows the message of the first advisor, the updated belief of the decision maker that the second advisor is of the good type is calculated given the message of the first advisor. The first case is that the second advisor receives the message 0 from the first advisor in the first period. If the state of the world in the first period is revealed as 0, the updated belief of the decision maker about the type of the second advisor when the second advisor sends the message 0 is

$$
\begin{aligned}
&\lambda^2_2(\lambda^2_1, m^2_1 = 0, 0 | m^1_1 = 0) \\
= \;& \frac{\lambda^2_1\{1-(1-\gamma)z\}}{\lambda^2_1\{1-(1-\gamma)z\} + (1-\lambda^2_1)\{1-(1-\gamma)\rho-\gamma\nu\}}.
\end{aligned}
$$

The denominator explains all possible cases in which the second advisor sends the message 0. If the second advisor is of the good type, he sends the message 0 with probability $P(m_1^2 = 0|m_1^1 = 0)$ when he obtains the correct signal. When the second advisor receives the wrong signal, he sends the message 0 with probability $(1-z) \cdot P(m_1^2 = 0|m_1^1 = 0)$. Let us consider the case where the second advisor is of the bad type. If he obtains the correct signal, he sends the message 0 with probability $(1-\nu) \cdot P(m_1^2 = 0|m_1^1 = 0)$. Similarly, the second advisor sends the message 0 with probability $(1-\rho) \cdot P(m_1^2 = 0|m_1^1 = 0)$ if he receives the wrong signal. Among those cases, the numerator shows the cases that the second advisor sends the message 0 if he is of the good type.

By using the same method, the updated belief of the decision maker about the type of the second advisor when the second advisor sends the message 1 if the state of the world is revealed as 0 is

$$\begin{aligned} & \lambda_2^2(\lambda_1^2, m_1^2 = 1, 0|m_1^1 = 0) \\ = & \frac{\lambda_1^2(1-\gamma)z}{\lambda_1^2(1-\gamma)z + (1-\lambda_1^2)\{(1-\gamma)\rho + \gamma\nu\}}. \end{aligned}$$

In this case, the second advisor sends the message 1 with probability $P(m_1^2 = 1|m_1^1 = 0)$ in accordance with his type and the signal of the second advisor. It is easily shown that the updated belief of the decision maker that the second advisor sends the message 0 is greater than the updated belief of the decision maker that the second advisor sends the message 1 if the first advisor sends the message 0 in the first period.

Next, I consider the case where the state of the world is revealed as 1 and the message of the first advisor is given by 0. By comparing the updated belief of the decision maker about the type of the second advisor when the second advisor sends the message 0 ($\lambda_2^2(\lambda_1^2, m_1^2 = 0, 1|m_1^1 = 0)$) with that when the second advisor sends the message 1 ($\lambda_2^2(\lambda_1^2, m_1^2 = 1, 1|m_1^1 = 0)$), I conclude that the second advisor can increase the updated belief about his type by sending the message 0 in the first period.

Let us consider the case where the first advisor sends the message 1 and the state of the world is revealed as 1 in the first period. The updated belief of the decision maker that the second advisor is of the good type if the second advisor sends the message 0 is

$$\begin{aligned} & \lambda_2^2(\lambda_1^2, m_1^2 = 0, 1 | m_1^1 = 1) \\ = \quad & \frac{\lambda_1^2(1-\gamma z)}{\lambda_1^2(1-\gamma z) + (1-\lambda_1^2)\{1-\gamma\rho-(1-\gamma)\nu\}}. \end{aligned}$$

When the second advisor is of the good type, the second advisor sends the message 0 with probability $P(m_1^2 = 0|m_1^1 = 1)$ if he receives the wrong signal. If the second advisor observes the correct signal, he sends the message 0 with probability $(1-z) \cdot P(m_1^2 = 0|m_1^1 = 1)$. In the case where the second advisor is of the bad type, the second advisor sends the message 0 with probability $(1-\rho) \cdot P(m_1^2 = 0|m_1^1 = 1)$ if his signal is correct. Similarly, the second advisor sends the message 0 with probability $(1-\nu) \cdot P(m_1^2 = 0|m_1^1 = 0)$ if the signal of the second advisor is wrong.

Given that the message of the first advisor is 1, the updated belief of the decision maker that the second advisor is of the good type if the second advisor sends the message 1 is

$$\begin{aligned} & \lambda_2^2(\lambda_1^2, m_1^2 = 1, 1 | m_1^1 = 1) \\ = \quad & \frac{\lambda_1^2\gamma z}{\lambda_1^2\gamma z + (1-\lambda_1^2)\{\gamma\rho+(1-\gamma)\nu\}}. \end{aligned}$$

In order to calculate the updated belief of sending the message 1, the conditional probability that the second advisor sends the message 1 given the message of the first advisor as 1 is needed. Similarly, the updated belief of the decision maker about the type of the second advisor when the second advisor sends the message 0 or 1 if the state of the world is revealed as 0 and the message of the first advisor is 1 is determined.

Proposition 2 *Given the message of the first advisor, the second advisor has reputational incentive to announce 0 regardless of the state of the world because*

$$\lambda_2^2(\lambda_1^2, m_1^2 = 0, 0|m_1^1 = 0) > \lambda_1^2 > \lambda_2^2(\lambda_1^2, m_1^2 = 1, 0|m_1^1 = 0),$$

$$\lambda_2^2(\lambda_1^2, m_1^2 = 0, 1|m_1^1 = 0) > \lambda_1^2 > \lambda_2^2(\lambda_1^2, m_1^2 = 1, 1|m_1^1 = 0),$$

$$\lambda_2^2(\lambda_1^2, m_1^2 = 0, 0|m_1^1 = 1) > \lambda_1^2 > \lambda_2^2(\lambda_1^2, m_1^2 = 1, 0|m_1^1 = 1),$$

and

$$\lambda_2^2(\lambda_1^2, m_1^2 = 0, 1|m_1^1 = 1) > \lambda_1^2 > \lambda_2^2(\lambda_1^2, m_1^2 = 1, 1|m_1^1 = 1).$$

When the message of the first advisor is given as 0, to send the message 0 is a way to increase the reputation of the second advisor regardless of the state of the world. Also, a way to increase the reputation of the second advisor is to send the message 0 when the message of the first advisor is given as 1. In the example, the second doctor can increase his reputation by sending the advice that the medicine is needed even if he knows that the previous doctor recommends the surgery regardless of the health condition of the patient. Since the payoff of the advisor in the second period (or the value function of the advisor) is increased with the reputation of the second advisor, the second advisor sends the message 0 if he wants to increase the second period payoff.

2.2.3 Action of the Decision Maker

Given sequential messages from the two advisors in the first period, the decision maker chooses the action which can affect all players' first period payoff. If the decision maker receives the message 0 from the first advisor and then receives the message 0 from the second advisor who knows the message of the first advisor, the probability that the state of the world is 1 in the first period is

$$P_{0,0}^{1,1} = \frac{Q_{0,0}^{1,1}}{\sum_{k=0}^{1} Q_{0,0}^{1,k}}.$$

The denominator shows all possible cases that both advisors send the message 0 given each state of the world. From assumptions, it is known that each state of the world is equally likely and each advisor observes the correct signal with probability γ. In the first period, the advisor who sends the message 0 is either of the good type or of the bad type. If the advisor is of the good type, the advisor sends the message 0 when his signal is 0, and also sends the message 0 with probability $1-z$ when the signal is 1. Let us think about the message of the bad type advisor. If the signal is 0, the advisor sends the message 0 with probability $1-\nu$ and the advisor sends the message 0 with probability $1-\rho$ if his signal is 1. So, if the first advisor sends the message 0, this is one of four cases in each state of the world. The second advisor who sends the message 0 is also included in these cases. However, I need to apply that the second advisor sends the message 0 with $P(m_1^2 = 0|m_1^1 = 0)$ because the second advisor knows the message sent by the first advisor. Among those cases, the numerator explains the case where both advisors send the message 0 sequentially given that the state of the world in the first period is 1. The decision maker chooses the action $P_{0,0}^{1,1}$ if he receives the message 0 from both advisors sequentially.

If the decision maker receives the message 0 from the first advisor and then receives the message 1 from the second advisor, the probability that the state of the world in the first period is 1 is

$$P_{0,1}^{1,1} = \frac{Q_{0,1}^{1,1}}{\sum_{k=0}^{1} Q_{0,1}^{1,k}}.$$

If the decision maker receives the message 1 from the second advisor, he believes that one of the following cases is possible. The decision maker believes that the second advisor is either of the good type or of the bad type. It is because the good advisor who observes the signal 1 and receives the message 0 from the first advisor sends the message 1 with probability $z \cdot P(m_1^2 = 1|m_1^1 = 0)$. Also, the bad advisor who observes the signal 0 (or 1) and receives the message 0 from the first advisor sends the message 1 with probability $\nu \cdot P(m_1^2 = 1|m_1^1 = 0)$ (or $\rho \cdot P(m_1^2 = 1|m_1^1 = 0)$). The decision maker chooses the action $P_{0,1}^{1,1}$ if he receives the message 0 from the first advisor and then receives the message 1 from the second advisor.

By using the same method, the case where the decision maker receives the message 1 from the first advisor and then receives the message 0 (or 1) from the second advisor is considered. The action of the decision maker if he receives the message 1 and 0 sequentially is

$$P_{1,0}^{1,1} = \frac{Q_{1,0}^{1,1}}{\sum_{k=0}^{1} Q_{1,0}^{1,k}},$$

and that of the decision maker if he receives 1 from both advisors sequentially is

$$P_{1,1}^{1,1} = \frac{Q_{1,1}^{1,1}}{\sum_{k=0}^{1} Q_{1,1}^{1,k}}.$$

The action of the decision maker in the first period can determine the payoffs of all players in the first period. Since the total payoff is determined by considering the action of the decision maker in each period, I examine the existence of the good or the bad reputation effect from now on.

2.3 Reputation Effect

In this section, I determine the area which guarantees the existence of both the good and the bad reputation effect. Then, I ask if it is better for him to seek advice simultaneously or sequentially.

2.3.1 Good Reputation Effect

Let us consider the case where the second advisor is of the bad type and observes the signal 0 in the first period. Since each advisor knows the type of the other advisor and the second advisor knows the message sent by the first advisor, there are four cases the second advisor faces: 1. the first advisor is of the good type and sends the message 0 in the first period; 2. the first advisor is of the good type and his message is 1 in the first period; 3. the first advisor is of the bad type and sends the message 0 in the first period; and 4. the first advisor is of the bad type and his message is 1 in the first period. In each case, I first try to find when the second bad advisor tells the truth even if he has a loss in his current payoff given the message of the first advisor. Then, I examine whether there is an incentive for the first advisor to send the given message I analyzed before to find the equilibrium condition.

If the second advisor meets the other advisor who is of the good type and sends the message 0 in the first period, the total payoff of the second advisor who sends the message 0 in the first period is

$$y_1^2 P_{0,0}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 0)].$$

Since the second advisor receives the message 0 from the first advisor, the payoff of the second advisor in the first period is determined as $P_{0,0}^{1,1}$ if the second advisor also sends the message 0. The value function of the second advisor is determined by both the type of the first advisor and the updated belief of each advisor in each state of the world. The total payoff of the second advisor who sends the message 1 given the message of the first advisor as 0 is

$$y_1^2 P_{0,1}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 0)].$$

If the second advisor only cares about his first period payoff, i.e., $y_1^2 = 1$, he sends the message 1 in the first period. Also, the second advisor who only considers his second period payoff (i.e., $y_1^2 = 0$) sends the message 0 in the first period. So, I can determine the critical value of the weighted average in the first period as a function of parameters to guarantee the existence of the good reputation effect. In order to examine the equilibrium condition, the incentive of sending the message 0 by the first advisor who is of the good type is examined given the area where the second advisor tells the truth or tells a lie. Since the method of considering the equilibrium condition in the good reputation effect is the same as that in the bad reputation effect, I will explain the equilibrium condition in Appendix C. What I find here is that the bad advisor sometimes tells the truth even if he has loss in his current payoff in sequential cheap talk. After comparing the area which guarantees the existence of the good reputation effect when the second advisor does not know the message sent by the first advisor (Cho, 2006), I find that the possibility of the existence of the good reputation effect is greater in sequential cheap talk. It is because the advisor can adjust his message more easily if the advisor knows the message sent by the other advisor. If the bad advisor receives the message towards which he is not biased from the previous advisor, he can send the message he is not biased easily not to be perceived as the bad advisor.

Next is the case where the first advisor is of the good type and sends the message 1 in the first period. Since the only difference between this case and the first case is that the first advisor sends the message 1, the equations are easily changed. The second advisor who observes the signal 0 sends the message 0 if

$$y_1^2 P_{1,0}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 1)] >$$

$$y_1^2 P_{1,1}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 1)].$$

Since the message of the first advisor is fixed as 1, the first period payoff of the second advisor if he sends the message 0 (or 1) is determined as $P_{1,0}^{1,1}$ (or $P_{1,1}^{1,1}$). The value function is different from the first case even if the type of each advisor is the same because the updated beliefs of both the first and the second advisor are changed in accordance with the message of the first advisor. By considering the incentive for the first advisor to send the message 1 in the first period given each message of the second advisor, I find that there is still the good reputation effect when the second advisor who knows that the first advisor is of the good type receives the message 1 from the first advisor. However, the possibility of the existence of the good reputation effect is less in sequential cheap talk than in simultaneous cheap talk. This is because the possibility of being perceived as the bad advisor to the decision maker is less if the second advisor receives the message the bad advisor is biased towards from the first advisor. So, the bad advisor can easily choose to send the message he is biased towards to increase his current payoff.

Now, the case where the first advisor is of the bad type is considered. First is the case where the second bad advisor receives the message 0 from the first advisor. Since each advisor knows that the other advisor is of the bad type, the value function is changed from $\upsilon^2_{GB}[\lambda^1_2, \lambda^2_2] = \frac{1}{2} y^2_2 (P^{2,1}_{0,1} + P^{2,1}_{1,1})$ to $\upsilon^2_{BB}[\lambda^1_2, \lambda^2_2] = y^2_2 P^{2,1}_{1,1}$. The total payoff of the second advisor if he sends the message 0 is

$$y^2_1 P^{1,1}_{0,0} + \frac{1}{2} \sum_{\xi=0}^{1} \upsilon^2_{BB}[\lambda^1_2(\lambda^1_1, 0, \xi), \lambda^2_2(\lambda^2_1, 0, \xi | m^1_1 = 0)],$$

and that of the second advisor if he sends the message 1 is

$$y^2_1 P^{1,1}_{0,1} + \frac{1}{2} \sum_{\xi=0}^{1} \upsilon^2_{GB}[\lambda^1_2(\lambda^1_1, 0, \xi), \lambda^2_2(\lambda^2_1, 1, \xi | m^1_1 = 0)].$$

The payoff in the first period is the same as in the case where the first advisor is of the good type and sends the message 0 in the first period because it is connected not with the type of the advisor but with the action of the decision maker who does not know the type of the advisor. By considering equilibrium conditions, the existence of the good reputation effect is easily shown in this case. If the second advisor considers his second period sufficiently more important, the second advisor who knows that the first advisor is of the bad type and sends the message 0 in the first period, sends the message 0. When the second advisor knows that the first advisor sends the message 0 regardless of the type of the first advisor, the second advisor tells the truth more easily than in the case when the second advisor does not know the message sent by the first advisor.

Finally, the case where the first advisor is of the bad type and sends the message

1 is considered. The second advisor sends the message 0 if

$$y_1^2 P_{1,0}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BB}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 1)]$$

is greater than

$$y_1^2 P_{1,1}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BB}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 1)].$$

The payoff in the first period is determined given the message of the first advisor is 1. The payoff in the second period is calculated by the updated belief of the decision maker about the type of each advisor. There is the good reputation effect if the second advisor knows that the first advisor is of the bad type and sends the message 1 in the first period. When the second advisor knows that the message of the first advisor is 1 regardless of the type of the advisor, the probability that the second advisor tells the truth is greater in sequential cheap talk compared to the simultaneous cheap talk.

Proposition 3 *There is the good reputation effect for the advisor who observes the signal 0 and receives the message 0 from the other advisor if he considers his second period sufficiently more important (See Appendix A).*

Proposition 4 *There is the good reputation effect for the advisor who observes the signal 0 and receives the message 1 from the other advisor if he considers his second period sufficiently more important (See Appendix B).*

There is the good reputation effect in sequential cheap talk. Since the advisor can adjust his message more easily when he knows the message of the other advisor, the advisor can easily tell the truth if he receives the message he is not biased towards. If the advisor receives the message he is biased towards, the probability of telling the truth is less compared to the case where the advisor does not know the message sent by the other advisor. It is because the advisor is the person who does not want to be perceived as the bad advisor.

2.3.2 Bad Reputation Effect

Since the bad reputation effect means that the good advisor sometimes tells a lie to increase his reputation, I examine the case where the second advisor is of the good type and observes the signal 1 in the first period. Four cases are considered in accordance with the type of the first advisor and the message of the first advisor: 1. the first advisor is of the bad type and sends the message 0 in the first period; 2. the first advisor is of the bad type and his message is 1; 3. the first advisor is of the good type and sends the message 0 in the first period; and 4. the first advisor is of the good type and his message is 1 in the first period.

Let us consider the case where the second advisor who observes the signal 1 knows that the first advisor is of the bad type and knows that the message of the first advisor is 0. The total payoff of the second advisor if he sends the message 0 is

$$-\frac{1}{2}x_1^2\{(P_{0,0}^{1,1})^2 + (P_{0,0}^{1,1} - 1)^2\}$$

$$+\frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BG}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 0)].$$

Since the payoff of the good advisor in each period is affected by the state of the world, the assumption that each state of the world is equally likely is applied. In each state of the world, since both advisors send the message 0 sequentially in the first period, the payoff of the second advisor in the first period is $-\frac{1}{2}x_1^2\{(P_{0,0}^{1,1})^2 +$

$(P_{0,0}^{1,1}-1)^2\}$. The payoff of the second advisor in the second period is determined by the updated belief of both the first and the second advisor. The updated belief of the decision maker about the type of the first advisor is determined by $\lambda_2^1(\lambda_1^1, 0, \xi)$ given each state of the world and that of the second advisor is determined by the message of each advisor and the state of the world.

The total payoff of the second advisor if he sends the message 1 is

$$-\frac{1}{2}x_1^2\{(P_{0,1}^{1,1})^2+(P_{0,1}^{1,1}-1)^2\}$$

$$+\frac{1}{2}\sum_{\xi=0}^{1}\upsilon_{BG}^2[\lambda_2^1(\lambda_1^1,0,\xi),\lambda_2^2(\lambda_1^2,1,\xi|m_1^1=0)].$$

The payoff of the second advisor in the first period is changed from $-\frac{1}{2}x_1^2\{(P_{0,0}^{1,1})^2$ $+(P_{0,0}^{1,1}-1)^2\}$ to $-\frac{1}{2}x_1^2\{(P_{0,1}^{1,1})^2+(P_{0,1}^{1,1}-1)^2\}$ compared to the previous case because the message of the second advisor is changed from 0 to 1. In the case of the payoff of the second advisor in the second period, the updated belief of the decision maker about the type of the second advisor is changed to $\lambda_2^2(\lambda_1^2,1,\xi|m_1^1=0)$ because the message of the second advisor is 1 in this case.

If the second advisor only considers his first period (i.e., $x_1^2=1$), he tells the truth. But if the second advisor only considers his second period (i.e., $x_1^2=0$), he sends a message that the bad advisor is not biased towards. By comparing the total payoff of the second advisor in each message of the second advisor, what I find is that the second advisor sends the message 0 in the first period if he considers his second period sufficiently more important. Then, the incentive of the first advisor to send the message 0 is examined given the area where the second advisor sends the message 0 in the first period. I will explain the equilibrium condition in Appendix C. There is still the bad reputation effect in sequential cheap talk. Compared to the area which guarantees the existence of the bad reputation effect in the simultaneous cheap talk, the probability that the advisor sends the message 0 is greater if he knows that the first advisor is of the bad type and sends the message 0 in the first period. It is because the advisor who receives the message that the bad advisor is not biased towards follows the message of the previous advisor more easily compared to the case where he does not know the message of the first advisor.

The next is the case where the second advisor knows that the first advisor is of the bad type and sends the message 1 in the first period. The only difference between this case and the previous case is the message of the first advisor. The second advisor who observes the signal 1 sends the message 0 if

$$-\frac{1}{2}x_1^2\{(P_{1,0}^{1,1})^2 + (P_{1,0}^{1,1} - 1)^2\}$$

$$+\frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BG}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 1)]$$

is greater than

$$-\frac{1}{2}x_1^2\{(P_{1,1}^{1,1})^2 + (P_{1,1}^{1,1} - 1)^2\}$$

$$+\frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BG}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 1)].$$

The payoff of the second advisor in the first period is determined by the message from each advisor and the state of the world. So, the assumption that each state of the world is equally likely is applied. The payoff of the second advisor in the second period is determined by the updated belief of the decision maker about the type of each advisor. The updated belief of the decision maker that the second advisor is of the good type is determined by the message of each advisor and the realized state of the world. By considering the equilibrium condition, it is shown that there is the bad reputation effect if the second advisor considers his second period sufficiently more important. Compared to the simultaneous cheap talk, the possibility of the existence of the bad reputation effect is greater in sequential cheap talk. It is because the advisor wants to separate his type from the bad type more easily if he knows that the message of the other advisor is the one the bad advisor is biased towards.

If the second advisor knows that the first advisor is of the good type, the value function of the second advisor is changed from $\upsilon^2_{BG}[\lambda^1_2, \lambda^2_2]$ to $\upsilon^2_{GG}[\lambda^1_2, \lambda^2_2]$. The first period payoff of the second advisor if he sends the message 0 is

$$-\frac{1}{2}x_1^2\{(P^{1,1}_{0,0})^2 + (P^{1,1}_{0,0} - 1)^2\}.$$

This is the same as the payoff of the second advisor if he knows that the first advisor is of the bad type and sends the message 0 in the first period. It is because the payoff in the second period is only affected by the message of each advisor. The second period payoff of the second advisor if he sends the message 1 is

$$\frac{1}{2}\sum_{\xi=0}^{1} \upsilon^2_{GG}[\lambda^1_2(\lambda^1_1, 0, \xi), \lambda^2_2(\lambda^2_1, 0, \xi | m^1_1 = 0)].$$

The value function of the second advisor is affected by the type of the other advisor. Similarly, the total payoff of the second advisor if he sends the message 1 is calculated. What I find is that the second advisor tells a lie if he considers his second period sufficiently more important. Finally, the case where the first advisor is of the good type and sends the message 1 in the first period is considered. Since the message of the first advisor is changed, the payoff of each period is also changed. By considering the equilibrium condition, I also find that there is the bad reputation effect in this case.

Proposition 5 *There is the bad reputation effect for the advisor who observes the signal 1 and receives the message 0 from the other advisor if he considers his second period sufficiently more important (See Appendix C).*

Proposition 6 *There is the bad reputation effect for the advisor who observes the signal 1 and receives the message 1 from the other advisor if he considers his second period sufficiently more important (See Appendix D).*

There is the bad reputation effect in sequential cheap talk regardless of the message or the type of the previous advisor. Compared to the bad reputation effect in simultaneous cheap talk, the advisor tells a lie more easily if he knows the message of the previous advisor. If the advisor knows that the message of the previous advisor is the one the bad advisor is not biased towards, he is afraid of telling the truth because he is easily perceived as the bad type by doing so. If the advisor knows that the previous advisor sends the message the bad advisor is biased towards, it is better for him to tell a lie to separate his type from the bad type.

In the point of view of the decision maker, there are some chances to obtain the correct advice by seeking advice sequentially rather than simultaneously. However, the possibility of losing information by asking advice sequentially is greater than that in simultaneous cheap talk in many cases. It is because the second advisor can easily adjust his message in the case where he knows the message of the previous advisor. After obtaining the advice from the first advisor, it is better for the decision maker not to tell the message sent by the first advisor when he meets the second advisor.

3 Conclusions

After receiving advice from a doctor, the patient may seek more information by asking advice from a second doctor if he believes that the first doctor may sometimes give incorrect advice. When the patient sees the second doctor, he may share the advice of the previous doctor. By considering two period cheap talk where advisors send messages sequentially to the decision maker, I examine the conditions which guarantee the existence of both good and bad reputation effects. Also, I determine whether simultaneous advice or sequential advice is preferred by the decision maker.

Given any message of the first advisor, a second advisor who is of the bad type sometimes tells the truth in the first period. Regardless of the type of the first advisor, the decision maker benefits from seeking advice sequentially if he receives the message that the bad advisor is not biased towards from the first advisor. However, it is better to seek advice simultaneously if he receives the message that the bad advisor is biased towards from the first advisor.

There is also the bad reputation effect in sequential cheap talk. Given any message of the first advisor, a second advisor who is of the good type tells a lie in the first period if he considers his second period sufficiently more important. This incentive to tell a lie occurs in order to make the decision maker believe what the advisor will suggest the next time. Regardless of both the message of the first advisor and the type of the first advisor, it is better for the decision maker to consult advice simultaneously. It is because the advisor does not want to be perceived as the bad type and he wants to separate his type from the bad type more easily in sequential cheap talk.

Since the probability of receiving correct advice is less when seeking advice sequentially than when seeking advice simultaneously, it is beneficial for the decision maker to seek advice simultaneously if he seeks advice from two advisors. The analysis in the paper is simplified because of Assumption 4. In general, the second advisor chooses 0 or 1 randomly if there is a conflict of the signal with the first advisor. If the second advisor can choose whatever he wants, the message of the second advisor in each period is changed. When the message of the second advisor in the second period is changed, the value function of the second advisor is also changed.

For simplicity, the case where each advisor knows the type of the other advisor is considered. In real life, even if each advisor may know the existence of the other advisor, there are cases where each advisor may not know the type of the other advisor. In that case, the belief of each advisor about the type of the other advisor can be applied to examine the existence of both good and bad reputation effects. I will leave this case for future work.

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Appendix A

Let us consider the case where the first advisor is of the good type and sends the message 0 to the decision maker. Since the second advisor knows both the type of the first advisor and the message sent by the first advisor, the total payoff of the second advisor is determined by his message in the first period. The total payoff of the second advisor, if he is of the bad type, is

$$\begin{aligned}
& y_1^2 a_1 + v_{GB}^2[\lambda_2^1, \lambda_2^2] \\
= \quad & y_1^2 a_1 + \frac{1}{2} y_2^2 (P_{0,1}^{2,1} + P_{1,1}^{2,1}).
\end{aligned}$$

When the second advisor is of the bad type and receives the signal 0 in the first period, he sends the message 0 (i.e., there is the good reputation effect) if

$$y_1^2\{P_{0,0}^{1,1} - P_{0,1}^{1,1} + \frac{1}{2}(\beta_1 - \alpha_1)\} > \frac{1}{2}(\beta_1 - \alpha_1)$$

where $\alpha_1 = \frac{1}{2}[\sum\limits_{\epsilon=0}^{1} \frac{1-\gamma-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)(1-\gamma)^2}{1-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)(1-2\gamma+2\gamma^2)} + f_1(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)]$

$> \beta_1 = \frac{1}{2}[\sum\limits_{\epsilon=0}^{1} \frac{1-\gamma-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)(1-\gamma)^2}{1-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)(1-2\gamma+2\gamma^2)} + f_2(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)].$

α_1 explains the value function of the second advisor when he meets the other good advisor if the second advisor sends the message 0 in the first period given the message of the first advisor as 0. The first part shows the probability that the state of the world in the second period is 1 given that the message of the first advisor is 0 and then the message of the second advisor is 1. The next expression $f_1(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)$ shows the conditional probability that the state of the world in the second period is 1 given message 0 from both advisors sequentially. Here, the updated belief of the decision maker about the type of the first advisor is $\lambda_2^1(\lambda_1^1, 0, 0)$ or $\lambda_2^1(\lambda_1^1, 0, 1)$ in accordance with the state of the world. The updated belief of the decision maker that the second advisor is of the good type is $\lambda_2^2(\lambda_1^2, 0, 0|m_1^1 = 0)$ or $\lambda_2^2(\lambda_1^2, 0, 1|m_1^1 = 0)$ in accordance with the state of the world.

Only different thing in β_1 is that the second advisor sends the message 1 in the first period. So, the updated belief of the decision maker about the type of the first advisor is the same as that in α_1. But, the updated belief of the decision maker that the second advisor is of the good type is changed to $\lambda_2^2(\lambda_1^2, 1, 0|m_1^1 = 0)$ or $\lambda_2^2(\lambda_1^2, 1, 1|m_1^1 = 0)$ in accordance with the state of the world. The expression $f_2(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)$ shows the conditional probability that the state of the world in the second period is 1 given that the message of the first advisor is 0 and the message of the second advisor is 1 in the first period.

If the bad advisor who meets the other good advisor and receives the message 0 from previous advisor considers his second period sufficiently more important, he sends the message he is not biased towards, i.e., if

$$y_1^2 < \frac{\frac{1}{2}(\alpha_1 - \beta_1)}{P_{0,1}^{1,1} - P_{0,0}^{1,1} + \frac{1}{2}(\alpha_1 - \beta_1)} = F_B^1(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu) < \frac{1}{2},$$

the bad advisor who observes the signal 0 sends the message 0 in the first period.

By using the same method, the case where the first advisor is of the bad type and sends the message 0 is considered. Since the payoff in the first period is only affected by the action of the decision maker who does not know the type of each advisor, the payoff in the first period is the same as in the previous case. However, the type of the first advisor is changed, the value function of the second advisor is changed to $\upsilon^2_{BB}[\lambda^1_2, \lambda^2_2] = y^2_2 P^{2,1}_{1,1}$.

When the second advisor is of the bad type and receives the signal 0 in the first period, he sends the message 0 if

$$y^2_1(P^{1,1}_{0,0} - P^{1,1}_{0,1} + \beta_2 - \alpha_2) > \beta_2 - \alpha_2$$

where $\alpha_2 = \frac{1}{2}\sum\limits_{\epsilon=0}^{1} \frac{1-(1-\gamma)(\lambda^1_2(\lambda^1_1,0,\epsilon)+\lambda^2_2(\lambda^2_1,0,\epsilon|m^1_1=0))+(1-\gamma)^2\lambda^1_2\lambda^2_2}{2-\lambda^1_2(\lambda^1_1,0,\epsilon)-\lambda^2_2(\lambda^2_1,0,\epsilon|m^1_1=0)+(1-2\gamma+2\gamma^2)\lambda^1_2\lambda^2_2}$
$> \beta_2 = \frac{1}{2}\sum\limits_{\epsilon=0}^{1} \frac{1-(1-\gamma)(\lambda^1_2(\lambda^1_1,0,\epsilon)+\lambda^2_2(\lambda^2_1,1,\epsilon|m^1_1=0))+(1-\gamma)^2\lambda^1_2\lambda^2_2}{2-\lambda^1_2(\lambda^1_1,0,\epsilon)-\lambda^2_2(\lambda^2_1,1,\epsilon|m^1_1=0)+(1-2\gamma+2\gamma^2)\lambda^1_2\lambda^2_2}$.

α_2 (or β_2) explains the value function of the second advisor if the first advisor is of the bad type when the second advisor sends the message 0 (or 1) in the first period. The updated belief of the decision maker about the type of the first advisor is calculated in each period given the message of the first advisor is 0. The updated belief of the decision maker that the second advisor is of the good type is determined by the message of both the first and the second advisor in each state of the world.

If the second advisor considers his future payoff sufficiently more important, the advisor who observes the signal 0 sends the message 0 in the first period, i.e., if

$$y^1_1 < \frac{\frac{1}{2}(\alpha_2 - \beta_2)}{P^{1,1}_{0,1} - P^{1,1}_{0,0} + \frac{1}{2}(\alpha_2 - \beta_2)} = F^2_B(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu) < \frac{1}{2},$$

there is the good reputation effect.

Appendix B

Let us first consider the case where the first advisor is of the good type and sends the message 1 in the first period. The payoff of the second advisor in the first period is changed to $P^{1,1}_{1,0}$ (or $P^{1,1}_{1,1}$) in accordance with the message of the second advisor. If the second advisor considers his second period sufficiently more important, i.e., if

$$y_1^2 < \frac{\frac{1}{2}(\alpha_3 - \beta_3)}{P^{1,1}_{1,1} - P^{1,1}_{1,0} + \frac{1}{2}(\alpha_3 - \beta_3)} = F^3_B(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu) < \frac{1}{2},$$

where

$\alpha_3 = \frac{1}{2}[\sum\limits_{\epsilon=0}^{1} \frac{1-\gamma-\lambda^2_2(\lambda^2_1,0,\epsilon|m^1_1=1)(1-\gamma)^2}{1-\lambda^2_2(\lambda^2_1,0,\epsilon|m^1_1=1)(1-2\gamma+2\gamma^2)} + f_3(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)]$

$> \beta_3 = \frac{1}{2}[\sum\limits_{\epsilon=0}^{1} \frac{1-\gamma-\lambda^2_2(\lambda^2_1,1,\epsilon|m^1_1=1)(1-\gamma)^2}{1-\lambda^2_2(\lambda^2_1,1,\epsilon|m^1_1=1)(1-2\gamma+2\gamma^2)} + f_4(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)]$, the second advisor sends the message 0 in the first period.

The first expression in α_3 is different from the first case in Appendix A because the updated belief of the decision maker about the type of the second advisor is changed by the message sent by the first advisor. $f_3(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)$ shows the conditional probability that the state of the world in the second period is 1 given that the message of the first advisor is 1 and that of the second advisor is 0. The expression β_3 explains the value function of the second advisor if he sends the message 1 in the first period.

Similarly, the existence of the good reputation effect is easily shown in the case where the first advisor is of the bad type and sends the message 1 in the first period.

Appendix C

Let us consider the case where the first advisor is of the bad type and sends the message 0 in the first period. The total payoff of the second advisor, if he is of the good type, is

$$\begin{aligned}
& -x_1^2(a_1-\omega_1)^2+\upsilon_{BG}^2[\lambda_2^1,\lambda_2^2] \\
= & -x_1^2(a_1-\omega_1)^2-\frac{1}{2}x_2^2\sum_{k=0}^{1}[\gamma_k(P_{1,k}^{2,1})^2+(1-\gamma_k)(P_{1,k}^{2,1}-1)^2].
\end{aligned}$$

The second advisor, who observes the signal 1, sends the message 0 in the first period if

$$-\frac{1}{2}x_1^2\sum_{\epsilon=0}^{1}\{(P_{0,\epsilon}^{1,1})^2+(P_{0,\epsilon}^{1,1}-1)^2\}-\frac{1}{2}(1-x_1^2)(\alpha_4-\beta_4)>0$$

where $\alpha_4=\frac{1}{2}[\sum_{\epsilon=0}^{1}\gamma_\epsilon\{(\frac{1-\gamma-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)(1-\gamma)^2}{1-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)(1-2\gamma+2\gamma^2)}-\epsilon)^2+(f_5(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)-1+\epsilon)^2\}]$ and $\beta_4=\frac{1}{2}[\sum_{\epsilon=0}^{1}\gamma_\epsilon\{(\frac{1-\gamma-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)(1-\gamma)^2}{1-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)(1-2\gamma+2\gamma^2)}-\epsilon)^2+(f_6(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)-1+\epsilon)^2\}]$.

Since the first period payoff is affected by both the action of the decision maker and the state of the world in the first period, the assumption that each state of the world is equally likely is applied. In each state of the world, α_4 explains the value function of the second advisor if both advisors send the message 0 in the first period. β_4 explains the value function of the second advisor if the first advisor sends the message 0 and the message of the second advisor is 1 in the first period in each state of the world. $f_5(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)$ and $f_5(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)$ show the probability that the state of the world in the second period is 1 given message 1 from both advisors if the updated belief of the decision maker about the type of the second advisor is $\lambda_2^2(\lambda_1^2,0,\epsilon)$ and $\lambda_2^2(\lambda_1^2,1,\epsilon)$ respectively.

Let us consider the equilibrium condition by examining the incentive of the first advisor to send the message 0. Given the area which guarantees the existence of the bad reputation effect, the first advisor who is of the bad type and observes the signal 0 sends the message 0 if

$$\frac{1}{2}y_1^1\{(2-z)(P_{0,0}^{1,1}-P_{1,0}^{1,1})+z(P_{0,1}^{1,1}-P_{1,1}^{1,1})\}$$
$$+\frac{1}{2}\sum_{k=0}^{1}\sum_{\epsilon=0}^{1}(-1)^k\upsilon_{BG}^1[\gamma_\epsilon\lambda_2^1(\lambda_1^1,k,\epsilon),$$
$$\frac{1}{2}\{\gamma_\epsilon\lambda_2^2(\lambda_1^2,0,\epsilon)+(1-\gamma_\epsilon)z_k\lambda_2^2(\lambda_1^2,k,\epsilon)\}]$$

is greater than 0 where $z_0 = 1-z$ and $z_1 = z$. By using Bayes' rule, the probability that the second advisor observes the same signal to the first advisor is $\frac{1}{2}$. Each advisor observes the correct signal with probability γ. The first advisor knows that the second advisor sends the message 0 if the signal is 0 and sends the message 0 with probability $1-z$ if the second advisor observes the signal 1. The first advisor who observes the signal 0 sends the message 0 if he considers his second period sufficiently more important, i.e., if $y_1^1 < y_1^* = f_7(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)$, the first advisor sends the message 0 in the first period.

When the first advisor is of the bad type and observes the signal 1 in the first period, he sends the message 0 if

$$\frac{1}{2}y_1^1\{(2-z)(P_{0,0}^{1,1}-P_{1,0}^{1,1})+z(P_{0,1}^{1,1}-P_{1,1}^{1,1})\}$$
$$+\frac{1}{2}\sum_{k=0}^{1}\sum_{\epsilon=0}^{1}(-1)^k\upsilon_{BG}^1[(1-\gamma_\epsilon)\lambda_2^1(\lambda_1^1,k,\epsilon),$$
$$\frac{1}{2}\{(1-\gamma_\epsilon)\lambda_2^2(\lambda_1^2,0,\epsilon)+\gamma_\epsilon z_k\lambda_2^2(\lambda_1^2,k,\epsilon)\}]$$

is greater than 0. When the state of the world is revealed as 0, the signal of the first advisor is wrong. The first advisor knows that probability that the second advisor observes the signal 1 is $\frac{1}{2}$. Since the payoff in the first period is not changed by the signal of the first advisor, the payoff in this equation is the same as that in the previous case. It is easily shown that the first advisor who observes the signal 1 sends the message 0 in the first period.

In equilibrium, the second advisor, who observes the signal 1 and receives the message 0 from the first advisor, tells a lie if he considers his second period payoff sufficiently more important, i.e., if

$$x_1^2 < \frac{\alpha_4 - \beta_4}{(P_{0,\epsilon}^{1,1})^2 + (P_{0,\epsilon}^{1,1} - 1)^2 + \alpha_4 - \beta_4},$$

the second advisor sends the message 0 in the first period.

If the second advisor knows that the first advisor is of the good type and receives the message 0 in the first period, the payoff of the second advisor in the first period is the same as previous case. However, the payoff of the second advisor in the second period is changed from $v_{BG}^2[\lambda_2^1, \lambda_2^2] = -\frac{1}{2}x_2^2 \sum_{k=0}^{1}[\gamma_k (P_{1,k}^{2,1})^2 + (1-\gamma_k)(P_{1,k}^{2,1} - 1)^2]$ to $v_{GG}^2[\lambda_2^1, \lambda_2^2] = -\frac{1}{2}x_2^2 \sum_{k=0}^{1}\sum_{i=0}^{1}[\gamma_k \gamma_l (P_{k,l}^{2,1})^2 + (1-\gamma_k)(1-\gamma_l)(P_{k,l}^{2,1} - 1)^2]$. By comparing the total payoff of the second advisor of each message, it is shown that the second advisor who observes the signal 1 sends the message 0 if he considers his second period sufficiently more important. Let us consider the incentive of the first advisor to send the message 0 under the conditions where the second advisor sometimes tells a lie in the first period. Two cases are examined: the first advisor who observes the signal 0 sends the message 0 if the first advisor considers his second period sufficiently more important; the first advisor who observes the signal 1 sends the message 0 if the second period is sufficiently more important to the first advisor.

The first advisor who observes the signal 0 sends the message 0 if

$$
\begin{aligned}
&-\frac{1}{2}x_1^1[\frac{1}{2}\gamma^2\sum_{i=0}^{1}(-1)^i(P_{i,0}^{1,1})^2 \\
&+\frac{1}{2}\gamma(1-\gamma)\{(1-z)\sum_{i=0}^{1}((-1)^i(P_{i,0}^{1,1})^2+(-1)^i(P_{i,0}^{1,1}-1)^2)) \\
&+z\sum_{i=0}^{1}((-1)^i(P_{i,1}^{1,1})^2+(-1)^i(P_{i,1}^{1,1}-1)^2))\} \\
&+\frac{1}{2}(1-\gamma)^2\sum_{i=0}^{1}(-1)^i(P_{i,0}^{1,1}-1)^2]+\upsilon_{GG}^2[\lambda_2^1,\lambda_2^2]
\end{aligned}
$$

is greater than 0. The payoff of the first advisor in the second period is determined by the updated belief of the decision maker in each message.

Appendix D

If the second advisor who observes the signal 1 meets the other advisor who is of the bad type and receives the message 1 from the first advisor in the first period, he sends the message 0 if

$$
\begin{aligned}
&-\frac{1}{2}x_1^2\sum_{\epsilon=0}^{1}\{(P_{1,\epsilon}^{1,1})^2+(P_{1,\epsilon}^{1,1}-1)^2\} \\
&+\frac{1}{2}\sum_{\epsilon=0}^{1}\upsilon_{BG}^2[\lambda_2^1(\lambda_1^1,1,\epsilon),\lambda_2^2(\lambda_1^2,0,\epsilon)] \\
&-\frac{1}{2}\sum_{\epsilon=0}^{1}\upsilon_{BG}^2[\lambda_2^1(\lambda_1^1,1,\epsilon),\lambda_2^2(\lambda_1^2,1,\epsilon)]
\end{aligned}
$$

is greater than 0. By considering the incentive of the first advisor to send the message 1 in the first period, it is shown that the second advisor tells a lie if he considers his second period sufficiently more important. In the case where the first advisor is of the good type and sends the message 1 in the first period, the value function of the second advisor is changed from $\upsilon_{BG}^2[\lambda_2^1,\lambda_2^2]$ to $\upsilon_{GG}^2[\lambda_2^1,\lambda_2^2]$.

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CERGE-EI
P.O.BOX 882
Politických vězňů 7
111 21 Praha 1
Czech Republic
http://www.cerge-ei.cz