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Multiple Advisors with Reputation*

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Abstract

This paper examines reputation, the belief of a decision maker about types of advisors, in a two period cheap talk model where the decision maker obtains messages from two advisors. The decision maker believes that an advisor can be one of two types - an advisor who is biased towards suggesting any particular advice (bad advisor) or an advisor who has the same preferences as the decision maker (good advisor). I assume that each advisor perfectly knows the type of the other advisor, but his signal about the state of the world is imperfect. Strong reputational concern makes the good advisor sometimes tell a lie in the first period regardless of the type of the other advisor. It is shown that the presence of the other advisor does affect the message sent by an advisor. The good advisor has a greater incentive to tell a lie when he knows that the other advisor is bad rather than good. If each type of advisor considers his second period sufficiently important, it is better for the decision maker to have a single advisor.

Keywords: Reputation; Cheap talk
JEL Classification: D82; D83

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Abstrakt

Tento článek zkoumá reputaci a předpoklady rozhodovatele o typech poradců v dvoufázovém modelu, kde komunikace před vlastní hrou je bez nákladů (tzv. cheap talk), a kde rozhodovatel obdrží návrhy od dvou poradců. Rozhodovatel věří, že poradce může být jedním ze dvou typů – buď poradce, jenž má tendenci navrhovat jakýkoli typ řešení (špatný poradce), nebo poradce, který má stejné preference jako rozhodovatel (dobrý poradce). Předpokládám, že každý poradce zná dokonale typ zbývajícího poradce, ale jeho signál o stavu světa je nedokonalý. Silný zájem uchovat si dobrou reputaci nutí dobrého poradce, aby někdy v prvním období lhal bez ohledu na typ druhého poradce. Je prokázáno, že přítomnost dalšího poradce ovlivňuje zprávu vyslanou poradcem. Dobrý poradce má silnější motivaci lhát jakmile ví, že druhý poradce je spíše špatný než dobrý poradce. Když každý typ poradce vnímá druhé období jako dostatečně vysoce důležité, je pro rozhodovatele lepší mít jen jednoho poradce.

1 Introduction

Many decisions are made after obtaining advice from others. Before going to a movie or buying a new computer, we usually seek advice from our friends, each of whom may have more private information than us. In many situations, there are some advisors who are biased towards suggesting a particular choice when information is conveyed by cheap talk. Car mechanics, for instance, usually point out serious problems with the car and suggest the replacement of expensive parts. Similarly, many movie makers hire reviewers to write and post positive reviews of their film on movie websites. Let's consider the specific example of a free consultation with a doctor. Assume the patient is uncertain about his health condition - either medicine is needed or surgery is needed. He therefore seeks advice from a doctor. Assume also that the patient is uncertain about the type of doctor - a good type or a bad type. The good doctor suggests appropriate advice to the patient whereas the bad doctor is biased towards suggesting surgery. Consider the case where the patient meets the good doctor. Assume that the good doctor observes the signal that surgery is needed. If the good doctor suggests surgery, the patient's belief that the doctor is of the bad type may be increased. If the good doctor considers his future payoff sufficiently important and the patient's belief that the doctor is of the bad type can decrease the future payoff of the doctor, he may suggest medicine to the patient. In this case, the patient loses information about his health condition. When the patient knows that he may lose information by having the advice of only one doctor, even if the doctor is of the good type, he may try to obtain additional advice from consulting an additional doctor.

When the patient meets the second doctor, he may or may not share the advice of the first doctor with the second doctor. Here, I assume that each doctor knows the presence of the other doctor, but does not know the advice given by her. For simplicity, it is also assumed that each doctor knows the type of the other doctor. In Cho (2006), I examine the case where the patient does tell the advice of the first doctor to the second doctor, which is sequential cheap talk.

Will the existence of another doctor change the advice of a doctor when the types of doctors are mutually known? If so, I can examine whether the existence of the other doctor affects the possibility that one doctor will tell the truth or lie. Although the patient receives additional information when he consults two doctors, the likelihood of being told a lie may increase because of the existence of the other doctor. Can the patient benefit if he takes advice from an additional doctor?

I consider a two period cheap talk model with two advisors to the decision maker. The decision maker believes that advisors can be of two types - good or bad. The good advisor is assumed to have exactly the same preferences as the decision maker, while the bad advisor has a payoff bias towards one of the two actions available to the decision maker. Each advisor observes a private signal regarding the state of the world, 0 or 1, and then sends a message (0 or

1) to the decision maker who does not have any prior information regarding the state of the world. The decision maker then chooses an action which can affect all players' payoffs. The state of the world is revealed after the decision maker takes an action. The decision maker then updates his belief about the type of each advisor, taking into account the message sent by each advisor and the real state of the world. The same stage game is repeated in the next period with the decision maker again consulting the same advisors.

The term "good reputation effect" refers to the bad advisor sending a truthful message after receiving a signal that the state of the world is one he is not biased towards. The term "bad reputation effect" involves the good advisor sending an untrue message after receiving a signal that the state of the world is one the bad advisor is biased towards. It is important to note that both of these effects arise from the reputational concern of an advisor to be perceived as a good advisor by the decision maker.

If each advisor has perfect information about the state of the world and it is common knowledge, I cannot determine the existence of a bad reputation effect. However, I show that both advisors have reputational incentives to send the message the bad advisor is not biased towards if the decision maker knows that each advisor has imperfect private information. In the example, suggesting a medicine can increase the reputation of each type of doctor even if the patient needs surgery. This is because the patient knows that each doctor has imperfect information regarding his health condition, and suggesting a medicine is a way for the good doctor to distinguish himself from the bad doctor.

If the bad advisor considers his second period sufficiently important, he tells the truth even if he suffers loss in current payoff when the signal is the one he is not biased towards. When the signal is the one the bad advisor is biased towards, the good advisor tells a lie if he considers his second period sufficiently important. There is greater (lesser) incentive for the good advisor to tell a lie when the other advisor is bad (good). By using numerical example, it is shown that the existence of the other advisor may reinforce the bad reputation effect. By comparing the expected payoff of the decision maker when he has two advisors with that when he has a single advisor, I find that it is better for the decision maker only to have a single advisor if each type of advisor considers his second period sufficiently important. If the decision maker is skeptical and so believes that each type of advisor considers the second period sufficiently important, i.e. if he believes each type of advisor sends the message the bad advisor is not biased towards in the first period, he is better off asking advice from a single advisor.

In cheap talk models, the question is whether informative equilibria from the sender (or the advisor) to the decision maker exist. [1] To make cheap talk informative, three necessary conditions are needed: The different sender

[1]Battaglini (2002, 2004) and Levy and Razin (2004) consider multidimensional cheap talk models. In Olszewski (2003), the decison maker also has private information regarding the state of the world. Park (2004) finds the conditions that make cheap talk informative in an infinitely repeated cheap talk game.

types have different preferences over the actions of the decision maker; the decision maker prefers the different actions depending on the sender's type; and the decision maker's preferences over actions cannot be completely opposed to the sender's (Gibbons, 1992). Crawford and Sobel (1982) examine the strategic information transmission from one advisor to the decision maker that satisfies these three conditions. By characterizing partially pooling equilibria, they find that more communication is possible when the preferences of the two players are more closely aligned, and that perfect information is conveyed when two players have the same preferences. Compared to the single advisor's cheap talk model, it is also possible to consider the welfare effect of the decision maker in multiple advisors' cheap talk model. Krishna and Morgan (2001) examine a one period cheap talk model with two advisors who send messages sequentially. By comparing the payoff of the decision maker when he has a single advisor with that of the decision maker when he has two advisors, they show that it is never beneficial to consult both advisors if both advisors are biased in the same direction. However, if the two advisors are biased in opposite directions, it is always beneficial to consult both advisors. Since they examine a one period model, each advisor has no reputational concern when he sends the message.

Advisors will most likely be concerned about their reputation if they are engaged in a repeated interaction with the decision maker.[2] Sobel (1985)

[2]Kreps and Wilson (1982) and Milgrom and Roberts (1982) developed a model of reputation where a long-run player meets a sequence of short-run players.

considers a finite cheap talk game where there is a single advisor who can be one of two types - enemy (an informed advisor with completely opposing interests to the decision maker) or friend (with identical interests to the decision maker), and finds that there is an incentive for an enemy to behave like a friend in order to increase his reputation.[3] Even if he considers the good reputation effect (as a term of my paper), the enemy is different than the bad advisor because the bad advisor is the person who is biased towards suggesting any particular strategy. As a bad advisor sometimes tells the truth to increase his reputation, a good advisor, who has the same preferences as the decision maker, sometimes tells a lie to increase his reputation in the case where there is one advisor to the decision maker. Morris (2001) considers a two period cheap talk model with one advisor having imperfect information regarding the state of the world. In equilibrium, an advisor has a reputational incentive to send the particular message the bad advisor is not biased towards in order to separate his type from the bad type regardless of the signal. Especially if an advisor considers the second period sufficiently important, no information is conveyed in the first period.[4] This paper extends Morris' model by considering multiple advisors, each of whom knows the type of the other advisor. Comparing the results, I can examine the effect of the presence

[3] In Ottaviani and Sorensen (2004), the real state of the world is revealed to the decision maker before the decision maker chooses his action. The action of the decision maker is evaluation of the advisor by comparing the message to the state of the world.

[4] Ely and Valimaki (2003) consider a model in which a long-lived mechanic interacts with a sequence of short-lived motorists. A bad reputation effect can emerge if motorists have imperfect information regarding the type of the mechanic.

of the other advisor and the welfare of the decision maker.

In section 2, using a two period cheap talk model, I examine the conditions required for the existence of both good and bad reputation effects when each advisor has imperfect information regarding the state of the world and has perfect information regarding the type of the other advisor. I examine the welfare effect of the decision maker in section 3.

2 Model

I consider a two period cheap talk model in which the decision maker has two advisors, each of whom knows the type of the other advisor. It is assumed that the decision maker does not know the state of the world and seeks the advice of both advisors. After receiving a private imperfect signal regarding the state of the world, both advisors simultaneously send a costless message to the decision maker.

The state of the world in period i is $\omega_i \in W = \{0,\ 1\}$ for $i = 1,\ 2$. Each state is equally likely, i.e. $P(\omega_i = 0) = \frac{1}{2} = P(\omega_i = 1)$. Advisor j (for $j = 1, 2$) observes signal S_i^j regarding the state of the world in period i. The decision maker does not know the type of advisors. The decision maker believes advisor j is good with probability λ_i^j in each period i. With probability $1 - \lambda_i^j$, the decision maker believes advisor j is bad. After observing the signal, advisor j sends the message m_i^j to the decision maker. After obtaining message from both advisors, the decision maker chooses his action

$a_i \in R$ which affects all players' payoffs. After the decision maker chooses his action, the state of the world in period i is revealed publicly. The message in the first period plays the additional role of changing the belief of the decision maker about the type of the advisor. After observing the state of the world in the first period, the decision maker updates his belief that advisor j is good, $\lambda_2^j = \lambda_2^j(\lambda_1^j, m_1^j, \omega_1)$, where λ_1^j is the prior belief of the decision maker that advisor j is good.

The utility function of both the decision maker and the good advisor is assumed to be $-(a_i - \omega_i)^2$ in each period i. Since it is assumed that the decision maker does not know the state of the world, the decision maker chooses the action a_i which is the probability that the state of the world is 1 given the message from both advisors. The utility function of the bad advisor is assumed to be a_i in each period i. The payoff obtained by the bad advisor in period i is greatest if the decision maker chooses action 1. Each type of advisor may put different weights on each period. The total utility of advisor j, if he is of the good type, is

$$-x_1^j(a_1 - \omega_1)^2 - x_2^j(a_2 - \omega_2)^2,$$

where x_1^j and x_2^j denote the weights on the payoffs in the first period and in the second period respectively. The total utility of advisor j, if he is of the bad type, is

$$y_1^j a_1 + y_2^j a_2,$$

where y_1^j and y_2^j denote the weights on the payoffs in the first period and in the second period respectively. Since both good and bad reputation effects are determined by the weight on the payoff in the first period, it is assumed that the sum of the weight in each period is 1.

I use backward induction to solve the model. I first solve for the action taken by the decision maker after receiving message from both advisors during the second period. Knowing the decision maker's action for each message in the second period, I am able to determine the value function in the second period for each type of advisor. While sending the message in the first period, the advisors consider not only their payoffs in the first period, but also the expected payoffs in the second period which are determined by the value function.

If each advisor observes a perfect signal regarding the state of the world and it is common knowledge, each advisor has no reputational incentive to tell a lie in the first period. When the state of the world in the first period is revealed as 1, I cannot guarantee that the updated belief of the decision maker that an advisor is good increases if an advisor sends message 0. This is because the decision maker will know that an advisor is a liar and will not believe what he says in the next period. Each advisor thus has an incentive to send message truthfully in the first period if he wants the decision maker to believe what he says in the next period. The bad reputation effect cannot be observed if each advisor has perfect information regarding the state of the world.

Consider the case in which advisor j observes an imperfect signal S_i^j in period i regarding the state of the world ω_i. Let γ denote the probability that the state of the world is the same as the signal received by each advisor, i.e. $\gamma = P(S_i = \omega_i)$. The signal is imperfect but informative, such that $\frac{1}{2} < \gamma < 1$. By applying different probability, γ_j, to each advisor, it is possible to examine how the ability to obtain the signal regarding the state of the world can change the results. In this paper, it is assumed for simplicity that each advisor has the same ability to obtain the signal regarding the state of the world.

2.1 Second Period

There is always a babbling equilibrium where the decision maker learns nothing about the type of the advisor and the state of the world in any cheap talk model.[5] Let's examine any informative equilibrium in the second period. Since this is the last period, each advisor does not consider his reputation. Advisor j, if he is of the good type, sends message $m_2^j = k$ when his signal in the second period is k for $k = 0$ or 1. If advisor j is of the bad type, he sends message $m_2^j = 1$ regardless of the signal.

In order to determine the value function in any informative equilibrium,

[5] Because messages have no direct effect on the advisors' payoffs, if the decision maker will ignore all messages then babbling is a best response for advisors; because messages have no direct effect on the decision maker's payoff, if advisors are babbling then the best response for the decision maker is to ignore all messages (Gibbons, 1992).

the probability that the state of the world in the second period is 1 given messages from the advisors must be calculated. As discussed, the decision maker knows that the bad advisor will never send message 0 in the second period. The probability that the state of the world is 1 given message 0 from both advisors in the second period is determined as

$$P^{2,1}_{0,0} = \frac{(1-\gamma)^2}{\gamma^2 + (1-\gamma)^2},$$

where $P^{i,1}_{m_i^1, m_i^2}$ represents the probability that the state of the world is 1 in period i given the message of the first advisor m_i^1 and the message of the second advisor m_i^2. Since the action of the decision maker is the probability that the state of the world is 1 given messages from both advisors, the decision maker chooses action $P^{2,1}_{0,0}$ when he receives message 0 from both advisors. Similarly, the action of the decision maker in each pair of messages in the second period is determined as

$$P^{2,1}_{0,1} = \frac{(1-\gamma)(1-\lambda_2^2+\gamma\lambda_2^2)}{1-(1-2\gamma+2\gamma^2)\lambda_2^2},$$

$$P^{2,1}_{1,0} = \frac{(1-\gamma)(1-\lambda_2^1+\gamma\lambda_2^1)}{1-(1-2\gamma+2\gamma^2)\lambda_2^1}$$

and

$$P^{2,1}_{1,1} = \frac{1-(1-\gamma)(\lambda_2^1+\lambda_2^2)+(1-\gamma)^2\lambda_2^1\lambda_2^2}{2-\lambda_2^1-\lambda_2^2+(1-2\gamma+2\gamma^2)\lambda_2^1\lambda_2^2}.$$

Suppose that the first advisor is of the bad type. The value function for the first advisor is either

$$v^1_{BG}[\lambda_2^1, \lambda_2^2] = y_2^1 a_2 = \frac{1}{2} y_2^1 (P^{2,1}_{1,0} + P^{2,1}_{1,1})$$

or

$$\upsilon^1_{BB}[\lambda^1_2, \lambda^2_2] = y^1_2 a_2 = y^1_2 P^{2,1}_{1,1}$$

depending on the type of the other advisor. The bad advisor sends message 1 regardless of the signal. The good advisor sends the message which is the signal he observes in each state of the world. In each state of the world, it is assumed that each advisor obtains the correct signal with probability γ. Similarly, the value function for the good advisor is either

$$\begin{aligned}
\upsilon^1_{GB}[\lambda^1_2, \lambda^2_2] &= -x^1_2(a_2 - \omega_2)^2 \\
&= -\frac{1}{2}x^1_2[\gamma\{(P^{2,1}_{0,1})^2 + (P^{2,1}_{1,1} - 1)^2\} \\
&\quad +(1-\gamma)\{(P^{2,1}_{1,1})^2 + (P^{2,1}_{0,1} - 1)^2\}]
\end{aligned}$$

or

$$\begin{aligned}
\upsilon^1_{GG}[\lambda^1_2, \lambda^2_2] &= -x^1_2(a_2 - \omega_2)^2 \\
&= -\frac{1}{2}x^1_2[\gamma^2\{(P^{2,1}_{0,0})^2 + (P^{2,1}_{1,1} - 1)^2\} \\
&\quad +\gamma(1-\gamma)\{(P^{2,1}_{0,1})^2 + (P^{2,1}_{1,0})^2 \\
&\quad +(P^{2,1}_{1,0} - 1)^2 + (P^{2,1}_{0,1} - 1)^2\} \\
&\quad +(1-\gamma)^2\{(P^{2,1}_{1,1})^2 + (P^{2,1}_{0,0} - 1)^2\}].
\end{aligned}$$

The value function for the good advisor is connected with not only the type of the other advisor but also the state of the world in the second period.

Irrespective of his type and the type of the other advisor, the value function for each advisor is increasing with the updated belief of the decision maker that the advisor is of the good type. If the message in the first period increases the updated belief of the decision maker that the advisor is good, it can also increase the value function in the second period.

2.2 First Period

In the first period, the payoff of the first advisor, if he is of the bad type, is either

$$y_1^1 a_1 + v_{BG}^1[\lambda_2^1(\lambda_1^1, m_1^1, \omega_1), \lambda_2^2(\lambda_1^2, m_1^2, \omega_1)]$$

or

$$y_1^1 a_1 + v_{BB}^1[\lambda_2^1(\lambda_1^1, m_1^1, \omega_1), \lambda_2^2(\lambda_1^2, m_1^2, \omega_1)]$$

depending on the type of the other advisor. The payoff of the first advisor, if he is of the good type, is either

$$-x_1^1(a_1 - \omega_1)^2 + v_{GB}^1[\lambda_2^1(\lambda_1^1, m_1^1, \omega_1), \lambda_2^2(\lambda_1^2, m_1^2, \omega_1)]$$

or

$$-x_1^1(a_1 - \omega_1)^2 + v_{GG}^1[\lambda_2^1(\lambda_1^1, m_1^1, \omega_1), \lambda_2^2(\lambda_1^2, m_1^2, \omega_1)]$$

depending on the type of the other advisor.

Suppose that a good advisor sometimes tells a lie in order not to be perceived as the bad advisor. The bad advisor also sometimes tells a lie, i.e. he has a payoff incentive to tell a lie if he observes signal 0 and has an incentive to tell a lie if he observes the signal 1 not to be perceived as advisor who always sends message 0. If advisor j is good, he sends message 0 when his signal is 0, and sends message 1 with probability z when the signal is 1.

If advisor j is bad, he sends message 1 with probability ν when his signal is 0 and sends message 1 with probability ρ when the signal is 1. I assume that the bad advisor sends message 1 more often than the good advisor[6].

By using Bayes' rule, the updated belief about the type of each advisor is determined. The updated belief of the decision maker that advisor j is good, when the decision maker receives message 1 and the real state of the world is revealed as 1, is

$$\lambda_2^j(\lambda_1^j, 1, 1) = \frac{\lambda_1^j \gamma z}{\lambda_1^j \gamma z + (1 - \lambda_1^j)\{\gamma\rho + (1-\gamma)\nu\}}.$$

If the decision maker receives message 1 from advisor j, he is uncertain about the type of the advisor. Advisor j observes signal 1 with probability γ since the state of the world is revealed as 1. The decision maker believes that advisor j, if the advisor is of the good type, sends message 1 with probability γz. He also believes that advisor j, if the advisor is of the bad type, sends message 1 with probability $\{\gamma\rho + (1-\gamma)\nu\}$. By using the same method, the updated belief about the type of advisor when the decision maker receives message 0 and the real state of the world is revealed as 1 is

$$\lambda_2^j(\lambda_1^j, 0, 1) = \frac{\lambda_1^j (1 - \gamma z)}{\lambda_1^j (1 - \gamma z) + (1 - \lambda_1^j)\{1 - \gamma\rho - (1-\gamma)\nu\}}.$$

Similarly, the updated belief about the type of advisor when the state of the

[6] In Appendix C, it is shown that the probability that the good advisor sends message 0 increases with the probability that the bad advisor sends message 1 if the signal is 1. If the signal is 0, the good advisor always sends message 0.

world is revealed as 0 given each message is

$$\lambda_2^j(\lambda_1^j, 0, 0) = \frac{\lambda_1^j\{1-(1-\gamma)z\}}{\lambda_1^j\{1-(1-\gamma)z\} + (1-\lambda_1^j)\{1-(1-\gamma)\rho - \gamma\nu\}}$$

and

$$\lambda_2^j(\lambda_1^j, 1, 0) = \frac{\lambda_1^j(1-\gamma)z}{\lambda_1^j(1-\gamma)z + (1-\lambda_1^j)\{(1-\gamma)\rho + \gamma\nu\}}.$$

Proposition 1 *Regardless of the state of the world in the first period, advisor j has a reputational incentive to announce 0 because*

$$\lambda_2^j(\lambda_1^j, 0, 0) > \lambda_1^j > \lambda_2^j(\lambda_1^j, 1, 0)$$

and

$$\lambda_2^j(\lambda_1^j, 0, 1) > \lambda_1^j > \lambda_2^j(\lambda_1^j, 1, 1).$$

Under the assumption that the bad advisor sends message 1 more often than the good advisor, each type of advisor has a reputational incentive to send message 0 in the first period. Even if the state of the world in the first period is revealed as 1, sending message 0 is the way to increase one's reputation. The decision maker knows that each advisor observes an imperfect signal regarding the state of the world. Thus, to send message 0 is a way to separate the good type from the bad type. In the earlier example, each type of doctor has a reputational incentive to suggest medicine regardless of the health condition of the patient. In Proposition 1, I will consider both good and bad reputation effects.

In order to consider the payoff of each advisor in the first period, the belief of the decision maker that the state of the world in the first period is 1 given each pair of messages is calculated. If the decision maker receives message 0 from both advisors, the probability that the state of the world is 1 is

$$P_{0,0}^{1,1} = \frac{Q_{0,0}^{1,1}}{Q_{0,0}^{1,0} + Q_{0,0}^{1,1}}$$

where $Q_{m_1^1,m_1^2}^{1,l}$ represents the conditional probability that the message of the first advisor is m_1^1 and the message of the second advisor is m_1^2 given that the state of the world in the first period is l, and

$$\begin{aligned} Q_{0,0}^{1,0} &= [\lambda_1^1\{1-(1-\gamma)z\} + (1-\lambda_1^1)\{1-(1-\gamma)\rho - \gamma\nu\}] \times \\ &\quad [\lambda_1^2\{1-(1-\gamma)z\} + (1-\lambda_1^2)\{1-(1-\gamma)\rho - \gamma\nu\}] \end{aligned}$$

and

$$\begin{aligned} Q_{0,0}^{1,1} &= [\lambda_1^1(1-\gamma z) + (1-\lambda_1^1)\{1-\gamma\rho - (1-\gamma)\nu\}] \times \\ &\quad [\lambda_1^2(1-\gamma z) + (1-\lambda_1^2)\{1-\gamma\rho - (1-\gamma)\nu\}]. \end{aligned}$$

The probability that both advisors send message 0 is calculated by considering the 32 possible cases. In each state of the world, the good advisor sends message 0 if his signal is 0, and sends message 0 with probability $1-z$ if the signal is 1. The bad advisor sends message 0 with probability $1-\nu$ if his signal is 0, and sends message 0 with probability $1-\rho$ if the signal is 1. The decision maker believes that each advisor may be either of the good or of the bad type. The decision maker chooses action $P_{0,0}^{1,1}$ when he receives message

0 from both advisors in the first period. Similarly, the action of the decision maker in each pair of messages in the first period is

$$P_{0,1}^{1,1} = \frac{Q_{0,1}^{1,1}}{Q_{0,1}^{1,0} + Q_{0,1}^{1,1}},$$

$$P_{1,0}^{1,1} = \frac{Q_{1,0}^{1,1}}{Q_{1,0}^{1,0} + Q_{1,0}^{1,1}}$$

and

$$P_{1,1}^{1,1} = \frac{Q_{1,1}^{1,1}}{Q_{1,1}^{1,0} + Q_{1,1}^{1,1}}$$

where

$$\begin{aligned} Q_{0,1}^{1,0} &= [\lambda_1^1\{1-(1-\gamma)z\} + (1-\lambda_1^1)\{1-(1-\gamma)\rho - \gamma\nu\}] \times \\ &\quad [\lambda_1^2(1-\gamma)z + (1-\lambda_1^2)\{(1-\gamma)\rho + \gamma\nu\}], \end{aligned}$$

$$\begin{aligned} Q_{0,1}^{1,1} &= [\lambda_1^1(1-\gamma z) + (1-\lambda_1^1)\{1-\gamma\rho - (1-\gamma)\nu\}] \times \\ &\quad [\lambda_1^2\gamma z + (1-\lambda_1^2)\{\gamma\rho + (1-\gamma)\nu\}], \end{aligned}$$

$$\begin{aligned} Q_{1,0}^{1,0} &= [\lambda_1^1(1-\gamma)z + (1-\lambda_1^1)\{(1-\gamma)\rho + \gamma\nu\}] \times \\ &\quad [\lambda_1^2\{1-(1-\gamma)z\} + (1-\lambda_1^2)\{1-(1-\gamma)\rho - \gamma\nu\}], \end{aligned}$$

$$\begin{aligned} Q_{1,0}^{1,1} &= [\lambda_1^1\gamma z + (1-\lambda_1^1)\{\gamma\rho + (1-\gamma)\nu\}] \times \\ &\quad [\lambda_1^2(1-\gamma z) + (1-\lambda_1^2)\{1-\gamma\rho - (1-\gamma)\nu\}], \end{aligned}$$

$$\begin{aligned} Q_{1,1}^{1,0} &= [\lambda_1^1(1-\gamma)z + (1-\lambda_1^1)\{(1-\gamma)\rho + \gamma\nu\}] \times \\ &\quad [\lambda_1^2(1-\gamma)z + (1-\lambda_1^2)\{(1-\gamma)\rho + \gamma\nu\}], \end{aligned}$$

and

$$\begin{aligned} Q^{1,1}_{1,1} &= [\lambda^1_1 \gamma z + (1-\lambda^1_1)\{\gamma\rho + (1-\gamma)\nu\}] \times \\ &\quad [\lambda^2_1 \gamma z + (1-\lambda^2_1)\{\gamma\rho + (1-\gamma)\nu\}]. \end{aligned}$$

Under the conditions that each advisor has imperfect information regarding the state of the world and has perfect information regarding the type of the other advisor, I first examine the existence of a good reputation effect. Let us consider the case in which the first advisor, who is of the bad type, observes signal 0 and knows that the second advisor is good. The first advisor knows that the second advisor sends message 0 with probability 1 if his signal is 0, or sends message 1 with probability z if his signal is 1. Since each advisor obtains the correct signal regarding the state of the world with probability γ, the belief of each advisor that the signal of the other advisor is the same as his signal is determined as $\frac{1}{2}$.

The bad advisor's total utility of telling the truth ($m^1_1 = 0$) when he observes signal 0 is

$$\begin{aligned} &\frac{1}{2}[y^1_1\{2P^{1,1}_{0,0} + z(P^{1,1}_{0,1} - P^{1,1}_{0,0})\} \\ &+\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1} \upsilon^1_{BG}[R_\epsilon \lambda^1_2(\lambda^1_1, 0, \epsilon), \\ &\frac{1}{2}\{R_\epsilon \lambda^2_2(\lambda^2_1, 0, \epsilon) + (1-R_\epsilon) z_\zeta \lambda^2_2(\lambda^2_1, \zeta, \epsilon)\}]] \end{aligned}$$

where $R_0 = \gamma$, $R_1 = 1-\gamma$, $z_0 = 1-z$ and $z_1 = z$. The bad advisor who sends message 0 believes that the other advisor also sends the message 0 if the signal of the other advisor is the same as his signal. He also believes that the other advisor sends message 0 with probability $1-z$ if the signal of the other advisor is different from his signal. When the real state of the world is revealed to be 1, i.e. when the bad advisor is misinformed, he needs to consider the cases in which the other advisor obtains the correct signal or is also misinformed. Similarly, the bad advisor needs to consider the case in which the other advisor obtains the correct signal or is misinformed when the real state of the world is revealed to be 0, i.e. when the bad advisor obtains the correct signal. The total utility to the bad advisor who observes signal 0 when he tells a lie ($m_1^1 = 1$) is

$$\frac{1}{2}[y_1^1\{2P_{1,0}^{1,1} + z(P_{1,1}^{1,1} - P_{1,0}^{1,1})\}$$
$$+\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}\upsilon_{BG}^1[R_\epsilon\lambda_2^1(\lambda_1^1, 1, \epsilon),$$
$$\frac{1}{2}\{R_\epsilon\lambda_2^2(\lambda_1^2, 0, \epsilon) + (1-R_\epsilon)z_\zeta\lambda_2^2(\lambda_1^2, \zeta, \epsilon)\}]].$$

Let's examine the strategic choice of the second advisor, who is of the good type, in order to consider equilibrium condition. Two different cases are examined depending on the signal the second advisor observes. The second advisor who observes signal 0 always sends message 0 because

$$-\frac{1}{2}x_1^2[\frac{1}{2}\gamma^2\sum_{\epsilon=0}^{1}(-1)^\epsilon(P_{\epsilon,0}^{1,1})^2$$
$$+\frac{1}{2}\gamma(1-\gamma)\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}z_j(-1)^\epsilon\{(P_{\epsilon,\zeta}^{1,1})^2 + (P_{\epsilon,\zeta}^{1,1} - 1)^2\}$$
$$+\frac{1}{2}(1-\gamma)^2\sum_{\epsilon=0}^{1}(-1)^\epsilon(P_{\epsilon,0}^{1,1} - 1)^2] + \upsilon_{BG}^2[\lambda_2^1, \lambda_2^2]$$

is greater than 0. Similarly, the probability that the second advisor, who observes signal 1, sends the message 0 is determined as $1 - z$.

If the bad advisor only considers his payoff in the first period, i.e. $y_1^1 = 1$ and $y_2^1 = 0$, then he will send message 1 after observing signal 0. Even if the bad advisor weights the two periods equally, i.e. $y_1^1 = \frac{1}{2} = y_2^1$, he will prefer to tell a lie. If the bad advisor only considers his second period payoff, i.e. $y_1^1 = 0$ and $y_2^1 = 1$, the payoff when he tells the truth is greater than the payoff when he tells a lie. Truth-telling is possible if y_2^1 is sufficiently large. The critical value of y_1^1 which guarantees the existence of the good reputation effect is calculated as a function of parameters.

Proposition 2 *There is a good reputation effect for the advisor who observes signal 0 and knows that the other advisor is good if he considers his second period sufficiently important (see Appendix A).*

If the bad advisor, knowing that the other advisor is good, strongly considers his reputation, then after observing signal 0 in the first period he sends message 0. The reputational concern that makes the bad advisor tell the truth is referred to the good reputation effect. In the example of doctor and patient, when a bad doctor observes the signal that the patient needs medicine, the strong reputational concern of the doctor not to be perceived as a bad doctor makes him suggest medicine even if he has loss in the current payoff.

Let us consider a numerical example to examine the relationship between the probability that the bad advisor sends the message truthfully if his signal is 0 and the prior belief of the decision maker about the type of each advisor, λ_1^1 and λ_1^2. If $\gamma = \frac{2}{3}$, $y_1^1 = \frac{1}{10}$ and $y_2^1 = \frac{9}{10}$, i.e. if the bad advisor considers his second period sufficiently important, the probability that the bad advisor sends the message truthfully if his signal is 0 is a function of λ_1^1, λ_1^2, ρ and z. In Morris' paper, when the prior belief of the decision maker regarding the type of advisor is either very low or very high, the probability that the advisor tells a lie after observing signal 0 is high. However, current paper shows that the belief of the decision maker about the type of the good advisor (the type of the other advisor) also can change the probability that the bad advisor tells a lie. If $\lambda_1^1 = \frac{1}{2} = \lambda_1^2$, the value of ν lies between 0 and 1. This in turn guarantees that the bad advisor tells the truth with a non-zero probability of $1-\nu$. Given $\lambda_1^1 = \frac{1}{2}$, the probability that the bad advisor tells the truth increases with λ_1^2, i.e. ν decreases with λ_1^2. Especially if the prior belief of the decision maker about the good advisor (λ_1^2) approaches 1, ν is at its lowest value. The probability that the bad advisor tells the truth in the first period is very high if the decision maker believes the other advisor to be good with a very high probability given that the prior belief of the decision maker that one advisor is good is $\frac{1}{2}$.

If the prior belief of the decision maker about the bad advisor is very high, i.e. if λ_1^1 is very high, the bad advisor is more likely to tell a lie if the prior belief of the decision maker about the type of the good advisor (λ_1^2)

approaches 0. The bad advisor is more likely to tell the truth if λ_1^2 approaches 1. This is because the reputation of the bad advisor cannot decrease a lot if the prior belief of the decision maker about him (λ_1^1) is very high. The bad advisor will have a greater fear of losing his reputation if the belief of the decision maker about the type of the good advisor (λ_1^2) is very high.

If the prior belief of the decision maker about the bad advisor, λ_1^1, is very low, the incentive to tell the truth increases when λ_1^2 is also very low. The incentive to tell a lie increases if λ_1^2 increases given very low λ_1^1. The bad advisor knows that it is very hard to increase his reputation if the prior belief of the decision maker about his type is very low. However, if the belief of the decision maker about the type of both advisors is very low, it is relatively easy for the bad advisor to increase his reputation. In each case, the presence of the other advisor (especially the belief of the decision maker about the type of the other advisor) plays an important role in determining the choice of message of one advisor.

Next is the case in which the first advisor, who is of the bad type, observes signal 0, and knows that the second advisor is also bad. Under the belief that the bad advisor sends message 1 with probability ν if the signal is 0, or sends message 1 with probability ρ if the signal is 1, the first advisor compares the total utility when he tells the truth with that when he tells a lie. There is a good reputation effect for the advisor who knows the other advisor is also bad if he puts a greater weight on the second period payoff (see Appendix B). It is shown that the area which guarantees the existence of a good reputation effect is bigger when the bad advisor faces a good advisor rather than a bad advisor. This is because the good advisor sends message 0 more often than does the bad advisor. The strong reputational concern not to be perceived as the bad advisor makes the bad advisor send message 0 more easily when he knows that the other advisor is of the good type rather than of the bad type.

In order to examine the existence of a bad reputation effect, let us consider the case in which the first advisor, who is of the good type, observes signal 1 and knows that the other advisor is of the bad type. The good advisor knows that the bad advisor sends message 1 with probability ν if the signal is 0, or sends message 1 with probability ρ if the signal is 1. The good advisor compares the total utility of telling the truth ($m_1^1 = 1$) with that of telling a lie ($m_1^1 = 0$).

The total utility to the good advisor from sending the message truthfully ($m_1^1 = 1$) is

$$\begin{aligned}
&\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}[[\rho_\epsilon\{\gamma^2(P_{1,\epsilon}^{1,1}-1)^2+(1-\gamma)^2(P_{1,\epsilon}^{1,1})^2\}\\
&+\gamma(1-\gamma)\nu_\epsilon\{(P_{1,\epsilon}^{1,1}-1)^2+(P_{1,\epsilon}^{1,1})^2\}](-\frac{1}{2}x_1^1)\\
&+\upsilon_{GB}^1[(1-R_\epsilon)\lambda_2^1(\lambda_1^1,1,\epsilon),\\
&\frac{1}{2}\{(1-R_\epsilon)\rho_\zeta\lambda_2^2(\lambda_1^2,\zeta,\epsilon)+R_\epsilon\nu_\zeta\lambda_2^2(\lambda_1^2,\zeta,\epsilon)\}]]
\end{aligned}$$

where $R_0 = \gamma$, $R_1 = 1-\gamma$, $\rho_0 = 1-\rho$, $\rho_1 = \rho$, $\nu_0 = 1-\nu$ and $\nu_1 = \nu$. The state of the world is equally likely and the belief of the good advisor that the other advisor obtains the same signal is $\frac{1}{2}$. In the case where the good advisor is misinformed, i.e. the real state of the world is revealed as 0, the good advisor needs to consider the case where the other advisor is also misinformed or obtains the correct signal. The good advisor also needs to consider the case in which the other advisor obtains the correct signal or is misinformed when he obtains the correct signal, i.e. when the real state of the world is 1. The total utility to the good advisor from telling a lie ($m_1^1 = 0$) is

$$\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}[[\rho_\epsilon\{\gamma^2(P_{0,\epsilon}^{1,1}-1)^2+(1-\gamma)^2(P_{0,\epsilon}^{1,1})^2\}$$
$$+\gamma(1-\gamma)\nu_\epsilon\{(P_{0,\epsilon}^{1,1}-1)^2+(P_{0,\epsilon}^{1,1})^2\}](-\frac{1}{2}x_1^1)$$
$$+\upsilon_{GB}^1[(1-R_\epsilon)\lambda_2^1(\lambda_1^1,0,\epsilon),$$
$$\frac{1}{2}\{(1-R_\epsilon)\rho_\zeta\lambda_2^2(\lambda_1^2,\zeta,\epsilon)+R_\epsilon\nu_\zeta\lambda_2^2(\lambda_1^2,\zeta,\epsilon)\}]].$$

Similarly, the strategic choice of the second advisor is determined by separating two different cases. The second advisor, who is of the bad type and observes the signal 0, sends message 0 if

$$\frac{1}{2}y_1^2\{(2-z)(P_{0,0}^{1,1}-P_{1,0}^{1,1})+z(P_{0,1}^{1,1}-P_{1,1}^{1,1})\}$$
$$+\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}(-1)^\zeta\upsilon_{GB}^2[R_\epsilon\lambda_2^1(\lambda_1^1,\zeta,\epsilon),$$
$$\frac{1}{2}\{R_\epsilon\lambda_2^2(\lambda_1^2,0,\epsilon)+(1-R_\epsilon)z_\zeta\lambda_2^2(\lambda_1^2,\zeta,\epsilon)\}]]$$

is greater than 0. The probability that the second advisor sends message 0 is determined as $1-\nu$. When the second advisor observes signal 1 in the first period, he sends message 0 if

$$\frac{1}{2}y_1^2\{(2-z)(P_{0,0}^{1,1}-P_{1,0}^{1,1})+z(P_{0,1}^{1,1}-P_{1,1}^{1,1})\}$$
$$+\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}(-1)^{\zeta}\upsilon_{GB}^2[(1-R_\epsilon)\lambda_2^1(\lambda_1^1,\zeta,\epsilon),$$
$$\frac{1}{2}\{(1-R_\epsilon)\lambda_2^2(\lambda_1^2,0,\epsilon)+R_\epsilon z_\zeta\lambda_2^2(\lambda_1^2,\zeta,\epsilon)\}]]$$

is greater than 0. When the state of the world is revealed as 0, the signal of the second advisor is wrong. The second advisor knows that the probability that the first advisor observes signal 1 is $\frac{1}{2}$. Since the payoff in the first period is not changed by the signal of the second advisor, the payoff in the first period is the same as in the previous equation. The probability that the second advisor sends message 0 is determined in the second case as $1-\rho$.

If the good advisor only cares about the first period payoff, i.e. $x_1^1 = 1$ and $x_2^1 = 0$, then the good advisor will send message truthfully ($m_1^1 = 1$) after observing signal 1. If the good advisor only cares about the second period, i.e. $x_1^1 = 0$ and $x_2^1 = 1$, then he tells a lie ($m_1^1 = 0$) to increase his reputation. Truth-telling is possible if x_2^1 is sufficiently large. The critical value of x_1^1 which guarantees the existence of the bad reputation effect is calculated as the function of the parameters.

Proposition 3 *There is a bad reputation effect for the advisor who observes signal 1 and knows that the other advisor is bad if he considers his second period sufficiently important (see Appendix C).*

The strong reputational concern of the good advisor who knows that the other advisor is bad makes him send message 0 in the first period after observing signal 1. Consider the example in which the good doctor who knows that the other doctor is of the bad type observes the signal that the patient requires surgery. If the good doctor does not want to be perceived as a bad type, he will suggest medicine to the patient, since he wants the patient to believe what he suggests next time.

If $y_1^2 = \frac{1}{2} = y_2^2$, i.e. if the second advisor weights the two periods equally, then the bad advisor always sends message 1 in the first period. The good advisor who observes signal 1 sends message 0 if he puts greater weight on the second period (if $x_1^1 < 0.2923$ for $\lambda_1^1 = \frac{1}{2} = \lambda_1^2$ and $\gamma = \frac{2}{3}$). In contrast to the example in Morris' paper, the existence of the other advisor may reinforce the bad reputation effect since the possibility of telling a lie is greater in the two-advisor model.

If the bad advisor sometimes sends message 0 (this happens for $y_1^2 = \frac{1}{10}$, $y_2^2 = \frac{9}{10}$ and $\lambda_1^1 = \frac{1}{2} = \lambda_1^2$), the area which guarantees the existence of the bad reputation effect is smaller than the area which guarantees the existence of the bad reputation effect when the bad advisor always sends message 1. This occurs because the area which guarantees the existence of the bad reputation effect (the critical value) increases with the probability that bad advisor sends message 1 if his signal is 1 (ρ). The good advisor has a greater incentive to tell a lie (i.e. he has a greater incentive to send message 0 after observing signal 1) in order to increase his reputation if the other bad advisor always sends message 1.

If the bad advisor always sends message 0 and the prior belief of the decision maker about the type of each advisor is $\frac{1}{2}$, i.e. if $y_1^2 = 0$, $y_2^2 = 1$ and $\lambda_1^1 = \frac{1}{2} = \lambda_1^2$, the good advisor also sends message 0 if he considers his second period sufficiently more important (if $x_1^1 < 0.2692$). This is a pooling equilibrium in the first period. In this case, if the real state of the world is revealed as 1 in the first period, the decision maker loses all information regarding the state of the world from having an additional advisor.

When the good advisor observes signal 1 and knows that the other advisor is also good (and will send message 0 if his signal is 0, or message 1 with probability z if the signal is 1), there is a bad reputation effect for the advisor if he puts greater weight on the second period payoff (see Appendix D). The area which guarantees the existence of a bad reputation effect is bigger when the good advisor faces a bad rather than a good advisor. Since the bad advisor sends message 1 more often than does the good advisor, the good advisor has a greater incentive to tell a lie when he knows that the other advisor is of the bad type in order to divorce himself from the bad type.

3 Welfare Effect

The decision maker tries to obtain additional information from an additional advisor. However, the presence of the other advisor may reinforce the bad reputation effect. To examine the welfare of the decision maker, I compare the payoff of the decision maker with one advisor and with two advisors. For simplicity, it is assumed that the decision maker believes that an advisor is good with probability $\frac{1}{2}$ before the first period starts.

In order to calculate the expected payoff of the decision maker when he has a single advisor, the action of the decision maker given each message is calculated in each period. Since the second period is the last period, the good advisor sends the message which is the same as his signal and the bad advisor always sends message 1. From Morris' paper, the probability that the state of the world is 1 in the second period given each message is

$$P_0^{2,1} = 1 - \gamma$$

and

$$P_1^{2,1} = \frac{1 - \lambda_2^1 + \lambda_2^1 \gamma}{2 - \lambda_2^1}$$

where $P_{m_i^1}^{i,1}$ represents the probability that the state of the world in period i is 1 given the message of one advisor m_i^1.

In the first period, the good advisor sends message 0 if his signal is 0 in the first period and sends message 1 with probability z if the signal is 1. The bad advisor sends message 1 with probability ρ if his signal is 1 and sends message 1 with probability ν if his signal is 0. The probability that the state of the world in the first period is 1 given the message of an advisor is

$$P_0^{1,1} = \frac{\lambda_1^1(1 - \gamma z) + (1 - \lambda_1^1)\{1 - \gamma\rho - (1 - \gamma)\nu\}}{\lambda_1^1(2 - z) + (1 - \lambda_1^1)(2 - \rho - \nu)}$$

and

$$P_1^{1,1} = \frac{\lambda_1^1 \gamma z + (1 - \lambda_1^1)\{\gamma\rho + (1 - \gamma)\nu\}}{\lambda_1^1 z + (1 - \lambda_1^1)(\rho + \nu)}.$$

I first separate each period payoff of the decision maker when he has two advisors or has one advisor, and then compare the total payoff of the decision maker. If each type of advisor considers his second period sufficiently more important, then each type of advisor sends message 0 in the first period regardless of his signal (for example, $y_2^2 = 1$ and $x_2^1 = 1$). The payoff of the decision maker when he has two advisors is

$$-\frac{1}{2}\{(P_{0,0}^{1,1})^2 + (P_{0,0}^{1,1} - 1)^2\}$$

and that when he has a single advisor is

$$-\frac{1}{2}\{(P_0^{1,1})^2 + (P_0^{1,1} - 1)^2\}.$$

The probability that the state of the world in the first period is 0 given that both advisors send message 0 is greater than the probability that the state of the world in the first period is 0 given that one advisor sends message 0, i.e. $P_{0,0}^{1,0} > P_0^{1,0}$ or $P_{0,0}^{1,1} < P_0^{1,1}$. If the state of the world is 0, $(P_{0,0}^{1,1})^2$ is less than $(P_0^{1,1})^2$. If the state of the world is 1, $(P_{0,0}^{1,1} - 1)^2$ is greater than $(P_0^{1,1} - 1)^2$. Since the welfare loss of having two advisors if the state of the world is 1 is a lot greater than that of having a single advisor if the state of the world in the first period is 1, compared with the welfare gain of having two advisors if the state of the world is 0, it is better for the decision maker to have a single advisor in the first period.

In the second period, the expected payoff of the decision maker from having two advisors is calculated by considering the cases in which both advisors are good, one advisor is good and the other advisor is bad, and both advisors are bad. The expected payoff of the decision maker when he has a single advisor in the second period is also calculated by considering the case in which the advisor is good or the advisor is bad (Appendix E). In the second period, the welfare loss from obtaining the wrong signal if the decision maker has two advisors is a lot greater than that if the decision maker has a single advisor, compared with the welfare gain from obtaining the correct signal. Having one good advisor is always better than having two advisors regardless of the types of both advisors. And to have at least one good advisor if the decision maker has two advisors is better than to have a single bad advisor. However, it is better for the decision maker to have a single bad advisor than two bad advisors. Since it is assumed that each advisor is good with probability $\frac{1}{2}$, a comparison of the total payoff of the decision maker between the two-advisor case and the single advisor case leads to the following proposition.

Proposition 4 *Under the condition that each type of advisor considers his second period sufficiently more important, the decision maker cannot benefit from taking the advice of an additional advisor.*

If a strong reputational concern makes each type of advisor send message 0 in the first period, the decision maker is better off having a single advisor than having two advisors. If the bad advisor always sends message 1 in the first period (for example, $y_1^2 = \frac{1}{2} = y_2^2$), and if the good advisor considers his second period sufficiently more important, it is better for the decision maker to have two advisors. If each type of the advisor sometimes sends message 1, the decision maker can benefit from taking the advice of an additional advisor. Except in the case where each type of advisor considers his second period sufficiently more important, it is better for the decision maker to have two advisors.

4 Conclusion

In this paper, I first characterize the conditions for the existence of both good and bad reputation effects when each advisor knows the type of the other advisor, and the advisors send their message simultaneously. In any informative equilibrium, regardless of the signal both advisors have a reputational incentive to send the message the bad advisor is not biased towards. By comparing the total payoff of telling the truth with that of telling a lie, I show that the bad advisor sometimes tells the truth to increase his reputation and show that a strong reputational concern makes the good advisor sometimes tell a lie regardless of the type of the other advisor. Moreover, a bad (good) reputation effect is more likely to emerge when the good (bad) advisor knows that the other advisor is bad (good) rather than good (bad). I then examine whether the decision maker is better off if he obtains information from an additional advisor. The expected payoff of the decision maker is lower with two advisors than with only one advisor if each type of advisor considers his second period sufficiently more important.

An obvious extension of the model is to analyze the case in which each advisor has imperfect information regarding the type of the other advisor. I have shown in this paper that the presence of another advisor can affect the message of an advisor when each advisor knows the other's type. If each advisor has imperfect information about the type of the other advisor, the strategic choice of an advisor may change, which may lead to different results about both good and bad reputation effects. Also, the results may be different if the game is repeated finitely. Especially when the good advisor observes the high signal continuously, the possibility of a bad reputation effect might increase because the reputational concern may increase with the period. The case in which advisor knows the type of the other advisor and send the message sequentially to the decision maker may lead to different results. When the decision maker asks for advice from an additional advisor, he informs the second advisor of the message sent by the first advisor. The second advisor adjusts his message by following the message of the first advisor. By examining the total payoff of the decision maker, it is possible to determine whether simultaneous advice or sequential advice is preferred by the decision maker. It is also possible to consider the case in which the state of the world is not realized. If the decision maker cannot observe the state of the world after choosing his action, he can determine the belief about the state of the world. In this case, I would expect that each advisor can adjust his message more easily compared to the case where the state of the world is revealed publicly.

Appendix A

The value function of the first advisor, if he is of the bad type, is calculated using the probability that the state of the world is 1 given his message is 1. The value function for the bad advisor is

$$\upsilon^1_{BG}[\lambda^1_2, \lambda^2_2] = y^1_2 a_2 = \frac{1}{2} y^1_2 (P^{2,1}_{1,0} + P^{2,1}_{1,1})$$

when the bad advisor knows that the other advisor is of the good type.

There is a good reputation effect for the bad advisor who observes signal 0 and knows that the other advisor is of the good type if

$$y^1_1(-\alpha_1) + \beta_1 y^1_2 + \eta_1 y^1_2 > 0$$

where $y^1_1 + y^1_2 = 1$

and $-\alpha_1 = \sum\limits_{\epsilon=0}^{1}(-1)^\epsilon\{(P^{1,1}_{\epsilon,0})(1-\frac{1}{2}z) + z(P^{1,1}_{\epsilon,1})\} < 0$,

$\beta_1 = \frac{1}{4}\{\sum\limits_{\epsilon=0}^{1}(-1)^\epsilon \frac{1-\gamma-\lambda^1_2(\lambda^1_1,\epsilon,1)(1-\gamma)^2}{1-\lambda^1_2(\lambda^1_1,\epsilon,1)(1-2\gamma+2\gamma^2)} + f_1(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)\} > 0$ and

$\eta_1 = \frac{1}{4}\{\sum\limits_{\epsilon=0}^{1}(-1)^\epsilon \frac{1-\gamma-\lambda^1_2(\lambda^1_1,\epsilon,0)(1-\gamma)^2}{1-\lambda^1_2(\lambda^1_1,\epsilon,0)(1-2\gamma+2\gamma^2)} + f_2(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)\} > 0.$

Here, $-\alpha_1$ explains the difference between the first advisor's first period payoff when he tells the truth and the payoff when he tells a lie. The bad advisor believes that the other advisor obtains signal 0 with probability $\frac{1}{2}$. He also believes that the good advisor who observes signal 0 sends message 0 if his signal is 0 and sends message 0 with probability $1 - z$ if the signal is 1, i.e. with probability $\frac{1}{2}(1-z)$ or $\frac{1}{2}$, the second advisor sends message 0. β_1 shows the difference between the first advisor's second period payoff which is determined by the value function of the bad advisor when he tells the truth and the payoff when he tells a lie in the case where the real state of the world is revealed as 1. If the first advisor believes that the second advisor observes signal 0, i.e. the first advisor believes that the second advisor's signal is the same as his signal, he knows that the updated belief of the decision maker about the type of the second advisor is $\frac{1}{2}(1-\gamma)\lambda_2^2(\lambda_1^2, 0, 1)$. This holds because the second advisor is misinformed when the real state of the world is 1. In this case, as the first advisor believes that the second advisor observes signal 1, the updated belief of the decision maker that the second advisor is good is $\frac{1}{2}\gamma\{z\lambda_2^2(\lambda_1^2, 1, 1) + (1-z)\lambda_2^2(\lambda_1^2, 0, 1)\}$. η_1 shows the difference of the first advisor's second period payoff between telling the truth and telling a lie in the case where the real state of the world is revealed as 0. Since the expression of β_1 and η_1 is so complicated, I use the functional expression $f_a(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)\}$ for $a = 1$ or 2 to show the remaining part of the difference between telling the truth and telling a lie when the state of the world is revealed as 1 or 0 respectively .

Under the condition that the second advisor who observes signal 0 sends message 0 and who observes signal 1 sends message 0 with probability $1-z$, the following condition is satisfied. If the bad advisor, who knows that the other advisor is of the good type, considers his second period sufficiently important, he sends the message he is not biased towards, i.e. if

$$y_1^1 < \frac{\beta_1 + \eta_1}{\alpha_1 + \beta_1 + \eta_1} = F_B(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu = 0) < \frac{1}{2},$$

the bad advisor who observes signal 0 sends the message 0. This is the case in which the decision maker obtains informative information even from the bad advisor.

Appendix B

The expressions in Appendix B are very similar to those in Appendix A. The value function of the bad advisor, if the first advisor is of the bad type, is

$$\upsilon^1_{BB}[\lambda^1_2, \lambda^2_2] = y^1_2 a_2 = y^1_2 P^{2,1}_{1,1}$$

when the bad advisor knows that the other advisor is also of the bad type. If the bad advisor considers his second period sufficiently more important, after observing signal 0 he sends message 0.

I compare the area which guarantees the existence of a good reputation effect when the bad advisor faces good advisor with the area which guarantees the existence of the good reputation effect when the bad advisor faces the other bad advisor, and find that the good reputation effect occurs more often when the bad advisor meets the good advisor.

Appendix C

If the first advisor is of the good type, the value function for the good advisor is

$$\begin{aligned}
& \upsilon^1_{GB}[\lambda^1_2, \lambda^2_2] \\
= \ & -x^1_2(a_2 - \omega_2)^2 \\
= \ & -\frac{1}{2}x^1_2[\gamma(P^{2,1}_{0,1})^2 + (1-\gamma)(P^{2,1}_{1,1})^2 \\
& +\gamma(P^{2,1}_{1,1} - 1)^2 + (1-\gamma)(P^{2,1}_{0,1} - 1)^2]
\end{aligned}$$

when he knows that the second advisor is of the bad type.

The good advisor tells a lie ($m_1^1 = 0$) to increase his reputation if

$$-x_1^1\alpha_3 + x_2^1\beta_3 + x_2^1\eta_3 > 0$$

where $x_1^1 + x_2^1 = 1$ and

$\alpha_3 = \frac{1}{2}\sum_{\epsilon=0}^{1}(P_{0,\epsilon}^{1,1} - P_{1,\epsilon}^{1,1})[\rho_\epsilon\{-2\gamma^2 + (1-2\gamma+2\gamma^2)(P_{0,\epsilon}^{1,1} + P_{1,\epsilon}^{1,1})\} + \nu_\epsilon(2\gamma^2 - 2\gamma)(P_{0,\epsilon}^{1,1} + P_{1,\epsilon}^{1,1} - 1)] > 0,$

$-\beta_3 = f_3(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu) = f_3[g(\lambda_2^1(\lambda_1^1, 0, 0)) - g(\lambda_2^1(\lambda_1^1, 1, 0))]$ and

$-\eta_3 = f_4(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu) = f_4[g(\lambda_2^1(\lambda_1^1, 0, 1)) - g(\lambda_2^1(\lambda_1^1, 1, 1))].$

α_3 explains the difference between the first advisor's first period payoff when he tells a lie and the payoff when he tells the truth. The first advisor believes that the second advisor has the same signal as him with probability $\frac{1}{2}$, i.e. $S_1^2 = 1$ with probability $\frac{1}{2}$. He also believes that the bad advisor sends message 0 with probability ρ if the signal of the second advisor is 1 and sends message 0 with probability ν if the signal is 0. Since the payoff of the good advisor is affected by both the action of the decision maker and the real state of the world, I need to distinguish the first period payoff as the case where the real state of the world is revealed as 0 or 1. $-\beta_3$ shows the difference between the first advisor's second period payoff which is determined by the value function of the first advisor when he tells a lie and the payoff when he tells the truth in the case where the real state of the world is revealed as 0. When the first advisor believes that the second advisor observes signal

1, the updated belief of the decision maker that the second advisor is good is $\frac{1}{2}(1-\gamma)\{\rho\lambda_2^2(\lambda_1^2, 1, 0) + (1-\rho)\lambda_2^2(\lambda_1^2, 0, 0)\}$. Similarly, if the good advisor believes that the second advisor observes signal 0, the updated belief of the decision maker that the second advisor is good is $\frac{1}{2}\gamma\{\nu\lambda_2^2(\lambda_1^2, 1, 0) + (1-\nu)\lambda_2^2(\lambda_1^2, 0, 0)\}$. $-\eta_3$ shows the difference between the first advisor's payoff when he tells a lie and the payoff when he tells the truth in the case where the real state of the world is revealed as 1.

Under the condition that the second advisor who observes the signal 0 sends message 0 with probability $1-\nu$ and who observes signal 1 sends message 0 with probability $1-\rho$, the following condition is satisfied. If the good advisor who faces bad advisor considers his second period sufficiently more important, he sends the message the bad advisor is not biased towards, i.e. if

$$x_1^1 < \frac{\beta_3 + \eta_3}{\alpha_3 + \beta_3 + \eta_3} = F_G(\lambda_1^1, \lambda_1^2, \gamma, z = 0, \rho, \nu) < \frac{1}{2},$$

the good advisor who observes the signal 1 sends the message 0.

The area which guarantees the existence of the bad reputation effect increases with the probability that the bad advisor sends message 1 if his signal is 1 because of $\frac{\partial F_G(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)}{\partial \rho} > 0$, which means that z decreases if ρ increases.

Appendix D

The expressions in Appendix D are very similar to those in Appendix C. When the first advisor knows that the second advisor is also of the good type, the value function of the good advisor is

$$\begin{aligned}
\upsilon^1_{GG}[\lambda^1_2, \lambda^2_2] &= -x^1_2(a_2-\omega_2)^2 \\
&= \frac{1}{2}x^1_2[\gamma^2\{(P^{2,1}_{0,0})^2+(P^{2,1}_{1,1}-1)^2\} \\
&\quad +\gamma(1-\gamma)\{(P^{2,1}_{0,1})^2+(P^{2,1}_{1,0})^2 \\
&\quad +(P^{2,1}_{1,0}-1)^2+(P^{2,1}_{0,1}-1)^2\} \\
&\quad +(1-\gamma)^2\{(P^{2,1}_{1,1})^2+(P^{2,1}_{0,0}-1)^2\}].
\end{aligned}$$

If the good advisor who knows that the other advisor is also of the good type considers his second period sufficiently more important, after observing signal 1 he sends message 0.

By comparing the area which guarantees the existence of the bad reputation effect in Appendix C with that in Appendix D, it is shown that the area which guarantees the existence of a bad reputation effect when the good advisor faces the other bad advisor is bigger than the area which guarantees the existence of a bad reputation effect when the good advisor faces the other good advisor.

Appendix E

Each type of advisor sends message 0 in the first period because of the strong reputational concern to be perceived as a good advisor. In the second period, if both advisors are good, the expected payoff of the decision maker is

$$E^{DM}_{GG} = -\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}[R_\epsilon R_\zeta(P^{2,1}_{\epsilon,\zeta})^2+(1-R_\epsilon)(1-R_\zeta)(P^{2,1}_{\epsilon,\zeta}-1)^2]$$

where $E^{DM}_{T_1T_2}$ represents the expected payoff of the decision maker if the first advisor is of the type T_1 and the second advisor is of the type T_2, and $R_0 = \gamma$ and $R_1 = 1 - \gamma$. This is because each good advisor sends the message which is the same as his signal and each advisor obtains the correct signal with probability γ. If the first advisor is good and the second advisor is bad, the expected payoff of the decision maker is

$$E^{DM}_{GB} = -\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}[R_\epsilon R_\zeta (P^{2,1}_{\epsilon,1})^2 + (1 - R_\epsilon)(1 - R_\zeta)(P^{2,1}_{\epsilon,1} - 1)^2],$$

since the bad advisor always sends message 1 regardless of his signal. If both advisors are bad, the payoff of the decision maker is

$$E^{DM}_{BB} = -\frac{1}{2}[(P^{2,1}_{1,1})^2 + (P^{2,1}_{1,1} - 1)^2].$$

Similarly, the payoff of the decision maker in the second period when he has a single advisor is calculated. If the decision maker has a good advisor, his expected payoff is

$$E^{DM}_{G} = -\frac{1}{2}\sum_{\epsilon=0}^{1}[R_\epsilon (P^{2,1}_{\epsilon})^2 + (1 - R_\epsilon)(P^{2,1}_{\epsilon} - 1)^2]$$

where $E^{DM}_{T_1}$ represents the expected payoff of the decision maker if the advisor is of the type T_1. The payoff of the decision maker when he has a bad advisor is

$$E^{DM}_{B} = -\frac{1}{2}[(P^{2,1}_{1})^2 + (P^{2,1}_{1} - 1)^2].$$

Since the decision maker believes both advisors are good, and since one advisor is good and the other advisor is bad, or both advisors are bad with the same probability, the payoff of the decision maker when he has two advisors is

$$-\frac{1}{2}[(P_{0,0}^{1,1})^2 + (P_{0,0}^{1,1} - 1)^2] + \frac{1}{3}[E_{GG}^{DM} + E_{GB}^{DM} + E_{BB}^{DM}].$$

The payoff of the decision maker when he has a single advisor is

$$\frac{1}{2}[-(P_0^{1,1})^2 - (P_0^{1,1} - 1)^2 + E_G^{DM} + E_B^{DM}].$$

Under the case in which each advisor considers his second period sufficiently more important, since the expected payoff of the decision maker when he has a single advisor is greater than that when he has two advisors, it is better for the decision maker to consult only a single advisor.

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