Smooth transition vector error-correction (STVEC) models: An application to real exchange rates*

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Abstract

This paper is devoted to studying econometric models of smooth transition characterized by continuously changing parameter regimes. The process of specifying, estimating and evaluating smooth transition regression (STR) models is discussed. The first attempts at extending nonlinear STR techniques to vector autoregressive (VAR) models have emerged in the last few years. This paper proposes an augmented specification procedure for STVAR models that allows for different transition variables and different types of transition functions in different equations of the system. As an application of the described modelling approach, a three-variable linear vector error-correction model (VECM) of the components of the real exchange rate between Slovenia and Slovakia, namely the consumer price indices and the nominal exchange rate between the currencies of both countries is investigated. The closely related question of the validity of purchasing power parity (PPP) is also discussed.

Keywords: smooth transition vector autoregressive models, smooth transition vector error-correction models, real exchange rate, purchasing power parity.

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1 INTRODUCTION AND LITERATURE OVERVIEW

1.1 About Exchange Rates and Purchasing Power Parity

The theory of purchasing power parity became particularly interesting after the introduction of flexible exchange rate regimes in the 1970s. Since then there is a number of theoretical as well as empirical studies dealing with the phenomena of purchasing power parity.

The purchasing power parity theory suggests that exchange rate system should provide a mechanism, which would enable a basket of goods being purchased in both analyzed countries to cost the same amount of money when recalculated in one currency.

Considering different views on how the process of economic transformation since the beginning of the nineties and its effects on reforming countries' price mechanisms are compatible with rigorous assumptions of the theory of PPP (see Brada 1998), there is an obvious need for further empirical evaluation to supply clear-cut evidence on macroeconomic forces that govern the exchange rate behavior in transition economies. Because the majority of transition countries have undergone several phases of economic restructuring, these most likely also triggered shifts in their equilibrium real exchange rates. This suggests that, when comparing developed market economies with those still under economic reforms, the degree of a country's similarity, especially in terms of trade pattern, level of development and the structure of relative prices, could importantly affect the assessment of PPP.

The general model of testing for PPP can be specified in the following form:

$$s_t = \alpha_0 + \alpha_1 p_t - \alpha_2 p_t^* + \xi_t, \tag{1}$$

where s_t stands for nominal exchange rates, defined as the price of foreign currency in the units of domestic currency; p_t denotes domestic price index and p_t^* foreign price index; while ξ_t stands for the error term showing deviations from PPP. All the variables are given in logarithmic form. In the strictest version of PPP, there are the following assumptions: $\alpha_0=0$, $\alpha_1=\alpha_2=1$. The symmetry restriction applies such that α_1 and α_2 are equal, whereas the limitation of α_1 and α_2 being equal to one is called the proportionality restriction (Froot and Rogoff 1995).

According to Boršič (2003, 2005) and Boršič and Bekő (2007), we show results based on monthly data series for Slovenia from January 1992 December 2001, when the euro was put into circulation. Primary data included monthly averages of nominal exchange rates and consumer price indices gathered from the central bank. The exchange rate has been defined as tolar (SIT) cost of a unit of foreign currency. Consumer price indices used in this study for Slovenia refer to January 1992.

The traditional empirical analysis starts off with the most restrictive version of equation (1), $\alpha_1=\alpha_2=1$, that is, with testing the properties of real exchange rates. In the context of relative PPP, the movements in nominal exchange rates are expected to compensate for price level shifts. Thus, real exchange rates should be constant over the long run and their time series should be stationary (Parikh and Wakerly 2000).

Results of the augmented Dickey-Fuller test are given in Table 1. The figures show that the four time series of the real exchange rates the tolar are integrated of order one, which means we cannot reject the hypothesis of the presence of the unit root. Thus, the ADF tests confirm the non-stationarity in the observed time series.

Table 1: Results of the ADF Test for Real Exchange Rates of the Slovenian tolar

Variable	Level		First difference	
	AIC	t-statistic	AIC	t-statistic
LRATSSIT	-0.6400_6	0.0867_3	-2.9538_6	-2.9538_6
LRDEMSIT	-0.8162_6	0.0138_3	-3.2579_6	-3.2579_6
LRFRFSIT	-0.6003_6	-0.3864_2	-2.8850_6	-2.8850_6
LRITLSIT	-0.8123_4	-0.8123_4	-5.7593_3	-5.3557_4

Notes: L stands for logarithm, R for real; the next three letters (ATS, DEM, FRF, ITL) represent the currencies of Austria, Germany, France and Italy, respectively, while the last two letters (CZK, SIT) denote the currencies of the Czech Republic and Slovenia, respectively. Critical values: -3.4890 (1%), -2.8870 (5%) and -2.5802 (10%). The subscripts indicate the time lag used in the test.

Source: Boršič and Bekő 2007

Relaxing the proportionality condition in equation (1) allows us to test if nominal exchange rates and relative prices are cointegrated. PPP holds if the presence of long-run equilibrium relation is confirmed. In the case of searching for cointegration among two variables Engle-Granger test is an appropriate one.

Results of the Engle-Granger test for Slovenia are presented in Table 2, which show the estimated equations for individual pairs of countries. The first column contains the independent variable. In all cases the choice of the independent variable does not influence the results of cointegration tests. In the following columns, there are constants, estimated coefficients of independent variables, R^2, CDRW statistics and ADF statistics for residuals. There are no t-statistics, since the time series are nonstationary and, as tests show, noncointegrated. Consequently, the estimated t-statistics are not reliable.

Table 2 presents the results for Slovenia. It can be concluded that the series are not cointegrated. In the case of Italy CRDW statistics are above the critical value but the ADF statistics are below the critical values. Thus, there are no signs of cointegration among the observed variables and the validity of purchasing power parity cannot be accepted.

Table 2: Results of Engle-Granger test for Slovenia

Independent variable	Constant	Coefficient	R^2	CRDW	ADF
Austria					
LATSSIT	-0.7484	1.9708	0.84	0.0344	-2.0751
LCPISA	0.5040	0.4242	0.84	0.0257	-1.3594
Germany					
LDEMSIT	0.2090	0.8948	0.84	0.0341	-2.4138
LCPISN	-1.0108	0.9363	0.84	0.0256	-1.6412
France					
LFRFSIT	-0.2091	1.3560	0.87	0.0478	-2.8212
LCPISF	0.2791	0.6449	0.87	0.0387	-1.8946
Italy					
LITLSIT	-0.8956	1.9997	0.92	0.2360	-2.7893
LCPISIT	0.5032	0.4592	0.92	0.2116	-2.4965

Notes: L stands for logarithm, the next three letters (ATS, DEM, FRF, ITL) represent the currencies of Austria, Germany, France and Italy, respectively, while the last two letters (SIT) denote the currency of Slovenia. Critical values for Engle-Granger test: -3.73 (1%), -3.17 (5%) and -2.91 (10%). Critical values for CRDW test: 0.455 (1%), 0.282 (5%) and 0.209 (10%).

Source: Boršič 2003.

Relaxing the symmetry assumption in equation (1) enables testing for cointegration among nominal exchange rates, domestic price indices and foreign price indices. Johansen cointegration test is appropriate for testing for cointegration among three variables. The results of the test are shown in the Table 3, which clearly shows that there is no evidence for purchasing power parity in the observed countries. Even when there is evidence of existing cointegration among the variables in question, there is a violation of signs in the cointegrating coefficients. In terms of equation (1), we are looking for a normalized cointegrating vector with positive coefficients α_1 and α_2.

Table 3: Results of the Johansen Cointegration Test for Slovenia

Number of cointegrating equations		Slovenia		
		Statistic [1,2]		
Austria$_1$		α_1=-0.7075(0.3732) $-\alpha_2$=-1.8258 (2.7453)		
H_0:	r=0 r≤1 r≤2	LR_{tr}	**61.4312 13.6185 *4.7704	
H_0:	r=0 r=1 r=2	LR_{max}	**47.8127 8.8481 *4.7704	
Germany$_2$		α_1=-0.7090 (0.3012) $-\alpha_2$=2.2350 (2.4313)		α_1=-0.4601 (0.0461) $-\alpha_2$=0.0000
H_0:	r=0 r≤1 r≤2	LR_{tr}	**42.2441 **20.7690 2.7287	
H_0:	r=0 r=1 r=2	LR_{max}	*21.4751 *18.0402 2.7287	
France$_1$		α_1=-0.3307 (0.3878) $-\alpha_2$=-4.4198 (3.5580)		
H_0:	r=0 r≤1 r≤2	LR_{tr}	25.9711 6.7105 0.7913	
H_0:	r=0 r=1 r=2	LR_{max}	19.2606 5.9192 0.7913	
Italy$_2$		α_1=-1.1967 (0.4644) $-\alpha_2$=0.9779 (1.8231)		
H_0:	r=0 r≤1 r≤2	LR_{tr}	**37.7476 12.3176 *4.4377	
H_0:	r=0 r=1 r=2	LR_{max}	*25.4300 7.8798 *4.4377	

Notes: ** (*) denotes rejection of the null hypothesis at the 1% (5%) significance level, respectively; figures in parentheses are standard errors. [1]Critical values for LR_{tr} at the 5% level are 29.68 (r=0), 15.41 (r≤1), and 3.76 (r≤2); and at the 1% level are 35.65 (r=0), 20.04 (r≤1), and 6.65 (r≤2). [2]Critical values for LR_{max} at the 5% level are 20.97 (r=0), 14.07 (r=1), and 3.76 (r=2); and at the 1% level are 25.52 (r=0), 18.63 (r=1), and 6.65 (r=2).

Source: Boršič and Bekő 2007

The values of LR_{tr} and LR_{max} statistics show that there is cointegration among the nominal exchange rates and consumer price indices in comparison to Austria, Germany and Italy. In all three cases the coefficients of domestic prices are proven to be statistically significantly different from zero, while for coefficients of foreign prices the standard errors are too high to conclude the same. The signs of the estimated cointegrating coefficients are again not in accordance with PPP. Only the coefficient of Austrian consumer prices tends to have the right sign. In case of France, there is no proof of cointegration either. In addition, the estimated coefficients are statistically insignificant and only the coefficient of French consumer prices has a sign corresponding to the PPP theory.

The discussed results of the time series unit root tests and cointegration tests in searching for evidence of purchasing power parity in Slovenia and Czech Republic failed to find any support for the theory. The limitations of the usage of time series analysis are: long-run data unavailability, nonstationarity and low power of time series unit root tests.

Thus, the next step is to test for unit roots by panel data techniques, which are supposed to have higher power that time series unit root tests. Table 4 presents results of such tests. Despite the fact that panel unit root tests are supposed to have a higher power that time series unit root tests, the results show no evidence of PPP validity for the observed countries in the selected period.

Table 4: Summary of Panel Unit Root Tests for Slovenia

Method	Statistic	Prob.
Null: unit root (assumes common unit root process)		
Levin, Lin, Chu	0.45054	0.6738
Breitung	-0.08462	0.4663
Null: unit root (assumes individual unit root process)		
Im, Pesaran, Shin	1.76657	0.9613
ADF-Fisher	1.49442	0.9928
PP-Fisher	1.05497	0.9979
Null: no unit root (assumes common unit root process)		
Hadri	10.4444	0.0000

1.2 Nonlinearities in Real Exchange Rates: Overview of Recent Literature

Since the real exchange rate in logarithmic form may be viewed as a measure of the deviation from PPP, the question of mean reversion in the real exchange rate is closely related to the issue of validity of purchasing power parity. In order to circumvent the low power problem of conventional unit root tests, the validity of PPP is usually investigated through long-span studies or panel unit root studies. Sarno and Taylor (2002) point out the disadvantages of both of the mentioned approaches. As far as the long-span studies are concerned, the long samples required to generate a reasonable level of power with univariate unit root tests may be unavailable for many currencies. Panel studies, on the other hand, impose the null hypothesis that all of the series under observation are generated by unit root processes implying that the probability of rejection of the null hypothesis may be quite high when as few as just one of the series is stationary. For this reason, Sarno and Taylor develop a smooth transition autoregressive (STAR) model to study the behaviour of the real exchange rate. In their model, the real exchange rate in the logarithmic form is explained by its lagged values. It is shown that the four major real dollar exchange rates are becoming increasingly mean reverting with the absolute size of the deviation from equilibrium, which is consistent with the recent theoretical literature on the nature of the real exchange rate dynamics in the presence of the international arbitrage costs.

Traditional empirical analyses of purchasing power parity validity and its deviations are based on linear framework and mostly suggest that the long run equilibrium is constant. Moreover, these analyses suggest that real exchange rate dynamics should be explained by linear autoregressive process with continuous and constant speed of adjustment, not taking into account the size of deviations from purchasing power parity (Sarno and Taylor 2002). Using linear framework for a nonlinear dataset, the rejection of a unit root as a null hypothesis is more likely (Taylor 2006), while the assumption of constant speed of adjustment implies downward bias of the results.

Taylor (2006) presents three potential reasons of nonlinearities in real exchange rates:

- frictions due to transport costs, tariffs or non-tariff barriers;
- interaction of heterogeneous agents in the foreign exchange market at the micro-structural level;
- influence of official intervention in the foreign exchange market.

Sarno and Taylor (2002), Sarno (2003) and Taylor (2006) provide an overview of nonlinear exchange rate models and assess their contribution to explaining the behaviour of the exchange rates. Below we present some studies of real exchange rate dynamics providing evidence in support of the nonlinearity in exchange rate processes.

Taylor, Peel, and Sarno (2001) take into account nonlinearly mean reverting models of real exchange rates and transaction costs in international arbitrage. By means of Monte Carlo simulations, the authors show that in this case the half lives of real exchange rates imply the fastest adjustment process in comparison to other techniques.

Baum, Barkoulas and Caglayan. (2001) apply ESTAR models to deviations from purchasing power parity obtained by Johansen cointegration method. They find evidence that mean reverting process of deviations varies nonlinearly with the size of disequilibrium.

On the basis of eleven Asian countries Liew, Chong and Lim (2003) show that the behaviour of real exchange rates is better presented by nonlinear STAR models than by linear autoregressive models.

Guerra (2003) tests nonlinear adjustment towards purchasing power parity by estimating an ESTAR model for Swiss frank-German mark rate in the period of 1960-1998. The results imply that mean reversion is rapid for the whole period as well as for the post-Bretton-Woods period.

Paya, Venetis, and Peel (2003) take into consideration two different approaches in solving the purchasing power parity puzzles: nonlinear adjustment of real exchange rates induced by transaction costs and non-constant real exchange rate equilibrium induced by different productivity growth rates. Consequently, the real exchange rate can be described as symmetric, nonlinear dynamics. Additionally, the authors show that the estimated half-lives of the shocks are much shorter than those obtained by linear models.

ESTAR models have also been used to forecast the behaviour of real exchange rates. Kilian and Taylor (2003) find evidence of exchange rate predictability in 2 to 3 years given ESTAR real exchange rate dynamics.

Liew, Bahrumshah and Lim (2004) find support of ESTAR nonlinear mean reverting adjustment process of nominal Singapure dollar-US dollar rate towards relative consumer prices.

Due to the lack of correct size of stationarity for PPP within linear tests, Paya and Peel (2005) employ Monte Carlo experiments to show that nonlinear tests provide support for PPP. They also apply ESTAR models to data from high inflation countries and provide further evidence in support of PPP.

Peel and Venetis (2005) present some theoretical limitations of ESTAR models and propose a new linear model consistent with rational expectations, while ESTAR model assumes adaptive expectations. Authors show fast adjustment speeds implied by their model using post-1973 monthly real exchange rate data.

Sollis (2005) uses univariate smooth transition models to test for unit roots under the alternative hypothesis of stationarity around a gradually changing deterministic trend function. They reject the null hypothesis of a unit root for real exchange rates of some countries in comparison to the US dollar.

Leon and Najarian (2005) check the stationarity of PPP deviations in the presence of nonlinearity and symmetry of adjustment towards PPP from above and below. Alternative nonlinear models including STAR models provide evidence of mean reversion and asymmetric adjustment dynamics.

One of the relatively rare papers examining purchasing power parity deviations in Central European countries is Arghyrou, Boinet, and Martin (2005). The authors analyze the data from Czech Republic, Hungary, Poland, Slovakia and Slovenia. Among other results it is shown that the short run dynamics of the real exchange rates displays nonlinear and asymmetric behaviour while the speed of adjustment depends on the size and sign of the deviation.

Lahtinen (2006) uses a STAR model on the basis of US dollar-euro exchange rate. The author distinguishes between the sudden and smooth adjustment to the long-run equilibrium and argues that the adjustment for the data under observation is sudden.

Rapach and Wohar (2006) study the performance of nonlinear models of the US Dollar real exchange rate monthly data in the post-Bretton Woods period and point out that nonlinear models (ESTAR and threshold models) provide more accurate point forecasts at long horizons for some countries.

2 SMOOTH TRANSITION REGRESSION

Many elements of economic theory mention the idea that the economy behaves differently if values of certain variables lie in one region rather than in another, or, in other words, follow different regimes. The first attempt at modelling such phenomena is represented by discrete switching models, where a finite number of different regimes is assumed. The central tool of this class of models is the so-called switching variable that can be either observable or unobservable.

As smooth transition between regimes is often more convenient and realistic than just the sudden switches, several scientists proposed a generalization of discrete switching models of the following form:

$$y_t = x_t'\varphi + \left(x_t'\theta\right)\cdot G(\gamma, c; s_t) + u_t, \quad t = 1, 2, \ldots, T, \tag{2}$$

where $\varphi = (\varphi_0, \varphi_1, \ldots, \varphi_p)'$ and $\theta = (\theta_0, \theta_1, \ldots, \theta_p)'$ are the parameter vectors, x_t is the vector of explanatory variables containing lags of the endogenous variable and the exogenous variables, (i.e. $x_t = (1, x_{t1}, \ldots, x_{tp})' = (1, y_{t-1}, \ldots, y_{t-m}, z_{t1}, \ldots, z_{tn})'$), whereas u_t denotes a sequence of independent identically distributed errors. G stands for a continuous transition function usually bounded between 0 and 1. Because of this property not only the two extreme states can be explained by the model, but also a continuum of states that lie between those two extremes. The slope parameter $\gamma > 0$ is an indicator of the speed of transition between 0 and 1, whereas the threshold parameter c points to where the transition takes place. The transition variable s_t is usually one of the explanatory variables or the time trend.

The most popular functional forms of the transition function are as follows:

- LSTR1 Model: $G_1(\gamma, c; s_t) = \dfrac{1}{1 + e^{-\gamma(s_t - c)}}$,

- LSTR2 Model: $G_2(\gamma, c_1, c_2; s_t) = \dfrac{1}{1 + e^{-\gamma(s_t - c_1)(s_t - c_2)}}$

  This is a non-monotonous transition function that is particularly useful in case of reswitching.

- ESTR Model: $G_3(\gamma, c; s_t) = 1 - e^{-\gamma(s_t - c)^2}$

  The function is symmetric about c and very similar to the LSTR2 case with $c_1 = c_2$. Therefore it is sometimes difficult to distinguish between an ESTR and an LSTR2 model.

2.1 Testing Linearity against STR

Let us start by defining a more convenient notation: $G_i^* = G_i - 0.5$ for i = 1, 2 and $G_3^* = G_3$. Obviously, $G_i^* = 0$ for $\gamma = 0$. The null hypothesis of linearity for model (2) can be expressed as $H_0 : \gamma = 0$ against $H_1 : \gamma > 0$ or as $H_0' : \theta = 0$ against $H_1' : \theta \neq 0$. This indicates an identification problem, since the model is identified under the alternative but not identified under the null hypothesis. Namely, the parameters c and θ are nuisance parameters that are not present in the model under H_0 and whose values do not affect the value of the log - likelihood. Consequently, the likelihood ratio test, the Lagrange multiplier and the Wald test do not have their standard asymptotic distributions under the null hypothesis and one cannot use these tests for a consistent estimation of the parameters c and θ. To overcome this problem, Luukkonen, Saikkonen and Teräsvirta (1998) replaced the transition function with its Taylor approximation of a suitable order. Let us write the first order Taylor approximation around $\gamma = 0$ for the logistic transition function G_1^* as a polynomial in the transition variable s_t:

$$T_1 = a_0 + a_1 s_t + R_1(\gamma, c; s_t) . \tag{3}$$

After replacing G_1^* by T_1 in equation (2), one obtains

$$y_t = x_t' b_0 + (x_t' s_t) b_1 + u_t^* , \tag{4}$$

where b_0 and b_1 are (p+1)-dimensional column vectors of parameters. The null hypothesis of linearity can be tested as $H_0'' : b_1 = 0$ against $H_1'' : b_1 \neq 0$ with a straightforward Lagrange multiplier test. The test statistic is asymptotically χ^2-distributed with p+1 degrees of freedom. We have to emphasize that auxiliary regression (4) is suitable only if the transition variable s_t is not an element of the vector x_t. Otherwise, the variable s_t appears twice on the right-hand side of equation (4). The problem is solved by substituting x_t with $\tilde{x}_t = (x_{t1}, \ldots, x_{tp})'$ in the second term of (4).

To avoid dealing with low power in some special cases, the third order Taylor polynomial is applied. This leads to the following auxiliary regression:

$$y_t = x_t'b_0 + (x_t's_t)b_1 + (x_t's_t^2)b_2 + (x_t's_t^3)b_3 + u_t^* . \quad (5)$$

Under the null hypothesis of linearity, the parameter vectors b_1, b_2 and b_3 are jointly tested to zero. F-version of the linearity test is usually preferred because of its better small sample properties. Comprehensive discussion on these issues is given in Teräsvirta (1998) and in Luukkonen, Saikkonen and Teräsvirta (1998).

2.2 Model Specification

The choice of the transition variable is not straightforward, since the underlying economic theory often gives no clues as to which variable should be taken for the transition variable under the alternative. Teräsvirta (1998) suggests testing the null hypothesis of linearity for each of the possible transition variables in turn. The candidates for the transition variable are usually the explanatory variables and the time trend. If the null is rejected for more than one variable, the variable with the strongest rejection of linearity (i.e. with the lowest p-value) is chosen for the transition variable. This intuitive and heuristic procedure can be justified by observing that the test is most powerful when the alternative hypothesis is correctly specified, and this is achieved for the "right" transition variable. It has to be emphasized that one cannot control the overall significance level of the linearity test for this heuristic procedure, since several individual tests have to be performed.

If the transition variable has already been decided upon, the next step in the modeling process consists of choosing the transition function. The decision rule is based on a sequence of nested hypotheses that test for the order of the polynomial in auxiliary regression (5):

$$\begin{aligned} &H_{04} : b_3 = 0 \\ &H_{03} : b_2 = 0 \mid b_3 = 0 \\ &H_{02} : b_1 = 0 \mid b_2 = b_3 = 0 . \end{aligned} \quad (6)$$

The 3 hypotheses are tested with a sequence of F-tests named F4, F3 and F2, respectively. If the rejection of the hypothesis H_{03} is the strongest, Teräsvirta (1998) advises choosing the

LSTR2 or the ESTR model. In the practice, one usually chooses the LSTR2 model and additionally tests the hypothesis $c_1 = c_2$ after estimation. If it cannot be rejected, it seems better to select the LSTR2 model, otherwise ESTR should be selected. In case of the strongest rejection of the hypotheses H_{04} or H_{02}, LSTR1 is chosen as the appropriate model. This heuristic decision rule is based on expressing the parameter vectors b_1, b_2 and b_3 from auxiliary regression (5) as functions of the parameters γ, c (or c_1 and c_2) and θ and the first three partial derivatives of the transition function G_i^* at the point $\gamma = 0$.

Teräsvirta (1998) conducted a series of simulation experiments to investigate the properties of the proposed heuristic specification strategy for choosing the transition variable and the transition function. The study was conducted for smooth transition autoregressive (STAR) models in the univariate setting. Different types of STAR models were examined and their parameters were varied. The "true" transition variable was the lagged endogenous variable y_{t-d}, where the delay parameter d ran from 1 to 5. For each d the linearity test was performed for every possible transition variable in turn (i.e. for $y_{t-1}, y_{t-2}, \ldots, y_{t-5}$) and the variable with the lowest p-value was chosen. The empirical size of the overall linearity test was 3 to 4 % when the nominal size was 5 %. The results of the simulation study justified the heuristic specification procedure and also showed that the power of the linearity test is better for higher γ values and for lower values of the delay parameter d. The decision rule for choosing the type of the transition function was tested for distinguishing between LSTAR1 and ESTAR models. It works best when the number of observations of the transition variable that lie below c is about the same as the number of observations above c. The performance of the rule improves with the sample size.

2.3 Estimation of STR models

The specified STR model is usually estimated with nonlinear least squares or with maximum likelihood estimation under the assumption of normally distributed errors. Both methods are equivalent in this case. Nonlinear optimization procedures are used to maximize the log-likelihood or to minimize the sum of squared residuals. The applied STR models from the next sections are estimated with the Gauss package or with EViews. Several nonlinear optimization algorithms are available in Gauss. For example, the Newton - Raphson algorithm, the Broyden - Fletcher - Goldfarb – Shanno (BFGS) algorithm, the steepest descent algorithm and the Davidon - Fletcher - Powell (DFP) algorithm are all implemented in the Gauss library Optmum.

An additional remark should be made on the slope parameter γ of the transition function. The magnitude of the parameter γ depends on the magnitude of the transition variable s_t and is therefore not scale-free. The numerical optimization is more stable if the exponent of the transition function is standardized prior to optimization. In other words, it is advisable to divide γ by the sample standard deviation (in case of LSTR1 models) or by the sample variance (for ESTR and LSTR2 models) of the transition variable. In this way the magnitude of the slope parameter is brought closer to the magnitude of other parameters.

2.4 Misspecification Tests

The misspecification tests were first developed by Eitrheim and Teräsvirta (1996) for univariate time series, i.e. for smooth transition autoregressive (STAR) models, but the generalization to STR models is straightforward. Three tests have to be developed especially for the STAR models, namely the test of no remaining nonlinearity, the test of no error autocorrelation and the parameter constancy test. For a detailed derivation of these tests, see Eitrheim and Teräsvirta (1996) and Lin and Teräsvirta (1994). Other tests, like the LM test of no autoregressive conditional heteroscedasticity of Engle and of McLeod and Li, and the Lomnicki-Jarque-Berra test of the normal distribution of errors are performed in the same way as in the linear setting.

3 SYSTEMS OF EQUATIONS

From recent studies of univariate models one has learned that there is much to be gained by allowing a nonlinear specification. Representations of asymmetric reactions, structural changes and other phenomena of economic development can be fruitfully investigated by nonlinear modeling techniques. As many issues in economics require the specification of several relationships, techniques to handle nonlinear features in systems are required. Only during the recent years such methods have appeared in the literature. Most of the work has been done in the nonlinear VAR framework.

Anderson and Vahid (1998) devised a procedure for detecting common nonlinear components in a multivariate system of variables. The common non-linearities approach is based on the canonical correlations technique and can help us interpret the relationships between different economic variables. The specification and estimation of the system of equations is also simplified, since the existence of common nonlinearities reduces the dimension of nonlinear components in the system and enables parsimony. This is particularly important in empirical investigations involving economic time series of shorter length. Namely, most of the macroeconomic indicators are published on a quarterly basis.

Weise (1999), van Dijk (2001) and Camacho (2004) extended the STR modeling approach developed by Teräsvirta and coworkers to vector autoregressive models of smooth transition. Their STR specification is limited to the case where the transition between different parameter regimes is governed by the same transition variable and the same type of transition function in every equation of the system. They argue that since the economic practice imposes common nonlinear features, all equations share the same switching regime. But this argument is not convincing, since such a conclusion cannot be derived from economic theory, while applied econometric studies analyzing nonlinear systems are scarce. For this reason we shall try to extend their approach by allowing different smooth transition functional forms in different equations. The proposed augmented specification procedure is explained in section 3.2.

3.1 Smooth Transition Approach to Vector Autoregressive Models

Whereas there has been extensive research in the field of univariate nonlinear modeling, the statistical theory of multivariate nonlinear modeling has yet to be developed. The first attempts at extending nonlinear smooth transition regression techniques to a multivariate setting can be found in Weise (1999), van Dijk (2001) and Camacho (2004). Similarly, multivariate Markov – switching models are treated in Krolzig (1997) and multivariate threshold models in Tsay (1998). Van Dijk (2001) applies the STVAR modeling approach to study the intraday spots and futures prices of the FTSE100 index, whereas Camacho (2004) examines the nonlinear forecasting power of the composite index of leading indicators to predict both output growth and the business - cycle phases of the US economy. Since all three studies are similar, while the most comprehensive description of the methodological approach is given by Camacho (2004), we shall start with a short review of his work.

3.1.1 Specification and Estimation

Camacho (2004) considers a 2 - dimensional smooth transition vector autoregressive (STVAR) model

$$\begin{aligned} y_t &= \varphi_y' X_t + \left(\theta_y' X_t\right) G_y(s_{yt}) + u_{yt} \\ x_t &= \varphi_x' X_t + \left(\theta_x' X_t\right) G_x(s_{xt}) + u_{xt}, \end{aligned} \tag{7}$$

where $X_t = (1, y_{t-1}, x_{t-1}, \ldots, y_{t-p}, x_{t-p})' = (1, \tilde{X}_t)'$, $\varphi_x, \varphi_y, \theta_x, \theta_y$ are the corresponding parameter vectors and $U_t = (u_{yt}, u_{xt})' \sim N(0, \Omega)$ is a vector series of serially uncorrelated errors. The difference $D_{it} = s_{it} - c_i$, $i = x, y$, in the exponent of the transition function G_i is called the switching expression. The letters y_t and x_t are used for the two variables in the autoregressive system, since the smooth transition approach is applied to the rate of growth of the US GDP and the rate of growth of the US composite index of leading indicators, respectively. The discussion is restricted to the case of $s_{xt} = s_{yt} = s_t$ and $G_x = G_y$, where the same transition variable and the same transition function is used in both equations.

After the linear VAR has been specified, the linearity test is applied. The problems with nuisance parameters are solved with suitable Taylor series expansions, as usually. The auxiliary regression to be performed in case the transition variable s_t belongs to X_t is

$$\begin{aligned} y_t &= \eta_{y0}' X_t + \sum_{h=1}^{3} \eta_{yh}' \tilde{X}_t s_t^h + v_{yt} \\ x_t &= \eta_{x0}' X_t + \sum_{h=1}^{3} \eta_{xh}' \tilde{X}_t s_t^h + v_{xt} \end{aligned} \tag{8}$$

and the null hypothesis of linearity reads as

$$H_0 : \eta_{i1} = \eta_{i2} = \eta_{i3} = 0, \qquad i = x, y. \tag{9}$$

Consequently, the null hypothesis can be tested with the Lagrange multiplier test.

If the null hypothesis of linearity is rejected in favor of the alternative smooth transition vector autoregressive model, one has to decide which transition function to use. The decision is based on the sequence of nested hypotheses tests described in section 2.2. The parameters of the specified model are estimated with the maximum likelihood estimator under the assumption of normally distributed errors:

$$U_t = (u_{yt}, u_{xt})' \sim N(0, \Omega). \tag{10}$$

3.1.2 Testing the Model Adequacy

As proposed by Eitrheim and Teräsvirta (1994), three tests are performed in order to check for the adequacy of the estimated model, namely the test of no error autocorrelation, the test of no remaining nonlinearity and the parameter constancy test. The multivariate generalizations of the three tests were developed by Camacho (2004).

3.2 Smooth Transition Vector Autoregressive Models with Different Transition Variables and Transition Functions

As already mentioned, Weise (1999), van Dijk (2001) and Camacho (2004) all assume the same transition variable and the same type of the transition function in every equation of a smooth transition vector autoregressive model, with the interpretation that the economic practice imposes common nonlinear features. But this argument is not convincing, since such a conclusion cannot be derived from economic theory, while applied econometric studies analyzing nonlinear systems are scarce. For this reason we shall try to extend the work of Camacho by allowing different smooth transition functional forms in different equations.

When performing the system linearity test Camacho postulates a two-variable linear VAR(p) model under the null hypothesis and the smooth transition vector autoregressive model (7) with the same transition variable $s_{xt} = s_{yt} = s_t$ and the same type of transition function $G_x = G_y = G$ under the alternative. After estimating the auxiliary regression

$$
\begin{aligned}
y_t &= X_t'\eta_{y0} + \left(\tilde{X}_t' s_t\right)\eta_{y1} + \left(\tilde{X}_t' s_t^2\right)\eta_{y2} + \left(\tilde{X}_t' s_t^3\right)\eta_{y3} + v_{yt} \\
x_t &= X_t'\eta_{x0} + \left(\tilde{X}_t' s_t\right)\eta_{x1} + \left(\tilde{X}_t' s_t^2\right)\eta_{x2} + \left(\tilde{X}_t' s_t^3\right)\eta_{x3} + v_{xt},
\end{aligned}
\tag{11}
$$

the null hypothesis

$$
H_0 : \eta_{i1} = \eta_{i2} = \eta_{i3} = 0, \qquad i = x, y \tag{12}
$$

(with the alternative of at least one of the coefficient vectors different from zero) is tested. The equations in (11) are estimated as a system with the method of maximum likelihood. Single equation estimators would also be consistent, although not efficient. Null hypothesis (12) can be tested with the LM test.

If the restriction $s_{xt} = s_{yt} = s_t$ is not imposed, the system linearity test can be performed by testing the same null hypothesis, this time based on the auxiliary regression allowing for different transition variables:

$$
\begin{aligned}
y_t &= X_t'\eta_{y0} + \left(\tilde{X}_t' s_{yt}\right)\eta_{y1} + \left(\tilde{X}_t' s_{yt}^2\right)\eta_{y2} + \left(\tilde{X}_t' s_{yt}^3\right)\eta_{y3} + v_{yt} \\
x_t &= X_t'\eta_{x0} + \left(\tilde{X}_t' s_{xt}\right)\eta_{x1} + \left(\tilde{X}_t' s_{xt}^2\right)\eta_{x2} + \left(\tilde{X}_t' s_{xt}^3\right)\eta_{x3} + v_{xt}.
\end{aligned}
\tag{13}
$$

The system linearity test will be rejected if at least one of the relationships under observation is nonlinear, or more specifically, is characterized by smooth transition between parameter regimes. It is reasonable to believe that situations with only one of the equations being nonlinear can occur in the economic practice. Estimating both equations with the smooth transition specification would be inefficient in this case. To solve this problem, single equation linearity tests based on the system estimates of auxiliary regression (13) may be applied. For example, to develop the single equation linearity test for the first equation, the null hypothesis

$$
H_0 : \eta_{y1} = \eta_{y2} = \eta_{y3} = 0, \tag{14}
$$

should be verified. Testing such a null hypothesis corresponds to imposing the model

$$
\begin{aligned}
y_t &= \varphi_y' X_t + u_{yt} \\
x_t &= \varphi_x' X_t + \left(\theta_x' X_t\right) G_x(s_{xt}) + u_{xt}
\end{aligned}
\tag{15}
$$

under the null and model (7) with the STR specification in both equations under the alternative. The heuristic procedure for selecting the transition variable(s) can be derived similarly to the one explained by Teräsvirta (1998) in the univariate setting:

Perform the system linearity test for each of the pairs of the possible transition variables in turn.

1. Carry out single equation linearity tests for each of the pairs of transition variables that reject the system linearity test.
2. (i) If there are pairs of transition variables for which the single equation tests reject the null hypothesis of linearity for both equations, choose the pair with the strongest rejection of the system linearity test.

   (ii) If for each of the pairs rejecting the null of system linearity only one of the single equation linearity tests is rejected, choose the pair of transition variables with the strongest rejection of the single equation test. Specify the corresponding equation as a smooth transition regression, while specifying the other equation as linear.

After the transition variables (or variable) have been chosen, the decision about the type of the transition function should be made. Camacho proposes a straightforward generalization of the sequence of nested hypotheses from section 2.2, namely

$$\begin{aligned} &H_{04}: \eta_{y3} = \eta_{x3} = 0, \\ &H_{03}: \eta_{y2} = \eta_{x2} = 0 \mid \eta_{y3} = \eta_{x3} = 0, \\ &H_{03}: \eta_{y1} = \eta_{x1} = 0 \mid \eta_{y2} = \eta_{x2} = \eta_{y3} = \eta_{x3} = 0, \end{aligned} \tag{16}$$

which can be tested with a sequence of F-tests. The coefficient vectors η_{ij} refer to auxiliary regression (11). The decision rule determines the type of the transition function depending on the nested hypothesis with the strongest rejection (see section 2.2 for details). Note that the same functional form is selected for both equations.

If the assumption of the same type of the transition function in both equations, namely $G_x = G_y = G$, is also relaxed, the transition function is chosen for each equation separately. In case of the first equation, the regressions in (13) are estimated with a system estimation method and the following sequence of nested hypotheses is verified:

$$\begin{aligned} &H_{04}: \eta_{y3} = 0, \\ &H_{03}: \eta_{y2} = 0 \mid \eta_{y3} = 0, \\ &H_{02}: \eta_{y1} = 0 \mid \eta_{y2} = \eta_{y3} = 0. \end{aligned} \tag{17}$$

When the second equation has a linear specification, auxiliary regression (13) should be modified accordingly, with the coefficient vectors η_{x1}, η_{x2} and η_{x3} set to zero prior to estimation.

An alternative specification procedure would follow the suggestion of Camacho and specify a model for each pair of the transition variables rejecting the null hypothesis of system linearity. The final model can be selected on the basis of forecasting power or any other measure of the model adequacy. Of course this strategy is more time consuming, especially if the null hypothesis of system linearity is rejected for several pairs of transition variables. Our approach extends the modeling cycle developed by Camacho (2004), since the set of models considered for smooth transition specification includes all of the models studied by Camacho.

The model adequacy tests for the proposed augmented specification procedure, namely the parameter constancy test, the test of no error autocorrelation and the test of no remaining nonlinearity are straightforward generalizations of the tests developed by Camacho (2004).

To illustrate the proposed heuristic procedure, we shall apply it to the data analyzed by Camacho and obtained from his web site (www.um.es/econometria/maximo). The y_t and x_t time series denoting the rate of growth of the US GDP and the rate of growth of the US composite index of leading indicators, respectively, include quarterly observations from 1959:1 to 2002:1.

Let us first derive the final model of Camacho. Using the information criteria, a linear VAR(1) model is specified and subjected to the system linearity tests. In addition to y_{t-1} and x_{t-1}, the variables y_{t-2} and x_{t-2} are also regarded as candidates for the transition variable. The results are given in the corresponding rows of Table 5. As the null of system linearity is rejected in all four cases, Camacho estimates four smooth transition autoregressive models. The type of the transition functions for each of the transition variables is selected using the decision rule explained on page 19 and is the same for both equations. The final model is decided upon on the basis of predictive accuracy (see Camacho (2004) for the description of the employed measures of predictive accuracy). The maximum likelihood estimates of the final model given below are slightly different from those in the paper by Camacho:

$$y_t = 0.603 \cdot x_{t-1} + 0.900 \cdot \left(\frac{1}{1+e^{-1.840(y_{t-2}-0.116)}} \right) \qquad (18)$$

(0.092) (0.200) (1.359) (0.393)

$$x_t = 0.415 - 0.364 \cdot y_{t-1} + 0.507 \cdot x_{t-1} + (-0.308 + 0.324 \cdot y_{t-1}) \cdot \left(\frac{1}{1+e^{-4.413(y_{t-2}-0.373)}} \right)$$

(0.126) (0.154) (0.060) (0.252) (0.213) (10.100) (0.343)

Standard errors of the parameter estimates are given in brackets. As pointed out by Teräsvirta (1998), precise joint estimation of the slope parameter and the threshold can be problematic, since it requires a high number of observations in the close neighborhood of the threshold parameter c.

When the restrictions regarding the transition variable and the type of the transition function are omitted, the linearity tests have to be carried out for every pair of the possible transition variables. The results of the system linearity tests as well as single equation tests are given in Table 5.

It can be observed that the single equation linearity test is strongly rejected for every pair of the transition variables when testing the second equation, while this holds true only for the pair x_{t-2}, y_{t-1} in case of the first equation. Thus the rejection of system linearity is due mainly to the nonlinear features in the second relation. Following the previously explained heuristic procedure, we choose the transition variables x_{t-2} and y_{t-1} for the first and second equation, respectively.

Next, the question of the type of the transition function in each of the equations has to be investigated. The sequence of nested hypotheses (17) is performed with the results given in Table 6. The F2 test yields the lowest p-value for both equations thus indicating the LSTR1 transition function in both cases.

Table 5: Linearity Test Results (p-values)

Tvar in 1. and 2. eq.	LINEARITY TESTS (p-values)		
	System test	First eq. test	Second eq. test
x_{t-1}, x_{t-1}	0.0020	0.4137	0.0017
x_{t-1}, x_{t-2}	0.0091	0.1882	0.0103
x_{t-1}, y_{t-1}	0.0039	0.1475	0.0038
x_{t-1}, y_{t-1}	0.0076	0.1332	0.0083
x_{t-2}, x_{t-1}	0.0003	0.0774	0.0004
x_{t-2}, x_{t-2}	0.0037	0.0755	0.0082
x_{t-2}, y_{t-1}	0.0008	0.0300	0.0014
x_{t-2}, y_{t-1}	0.0033	0.0565	0.0073
y_{t-1}, x_{t-1}	0.0041	0.6775	0.0003
y_{t-1}, x_{t-2}	0.0553	0.8199	0.0075
y_{t-1}, y_{t-1}	0.0315	0.8003	0.0035
y_{t-1}, y_{t-1}	0.0529	0.7331	0.0070
y_{t-2}, x_{t-1}	0.0018	0.3793	0.0005
y_{t-2}, x_{t-2}	0.0119	0.2449	0.0054
y_{t-2}, y_{t-1}	0.0057	0.2106	0.0022
y_{t-2}, y_{t-1}	0.0114	0.2005	0.0052

Table 6: Tests for Choosing the Type of Transition Function (p-values)

Tvar in 1. and 2. eq.	NESTED TESTS (p-values)					
	First equation			Second equation		
	F4	F3	F2	F4	F3	F2
x_{t-2}, y_{t-1}	0.2250	0.3676	0.0112	0.6859	0.0370	0.0008

The maximum likelihood estimates of the specified smooth transition vector autoregressive model are given by

$$\begin{array}{l} y_t = \underset{(0.103)}{0.467} \cdot x_{t-1} + \underset{(0.371)}{1.213} \cdot \left(\dfrac{1}{1+e^{-\underset{(0.900)}{1.389}(x_{t-2}-\underset{(0.523)}{0.053})}} \right) \\[2ex] x_t = \underset{(0.891)}{-1.561} + \underset{(0.059)}{0.475} \cdot x_{t-1} - \underset{(0.119)}{1.680} \cdot y_{t-1} + \\[2ex] \quad + \underset{(0.580)}{1.489} \cdot y_{t-1} \cdot \left(\dfrac{1}{1+e^{-\underset{(3.288)}{2.592}(y_{t-1}+\underset{(0.994)}{0.614})}} \right) \end{array} \tag{19}$$

It can be deducted from Table 7 that models (18) and (19) are comparable in terms of fit, if model (19) is not even slightly better. Therefore it would be wise to consider also (19) when searching for the model with the best forecasting properties.

Table 7: Comparing the Fit of Models (18) and (19)

Value	Model	First equation		Second equation	
	logL	R^2	*S.E.*	R^2	*S.E.*
Model (18)	-327.14	0.352	0.754	0.253	0.718
Model (19)	-322.20	0.369	0.744	0.287	0.701

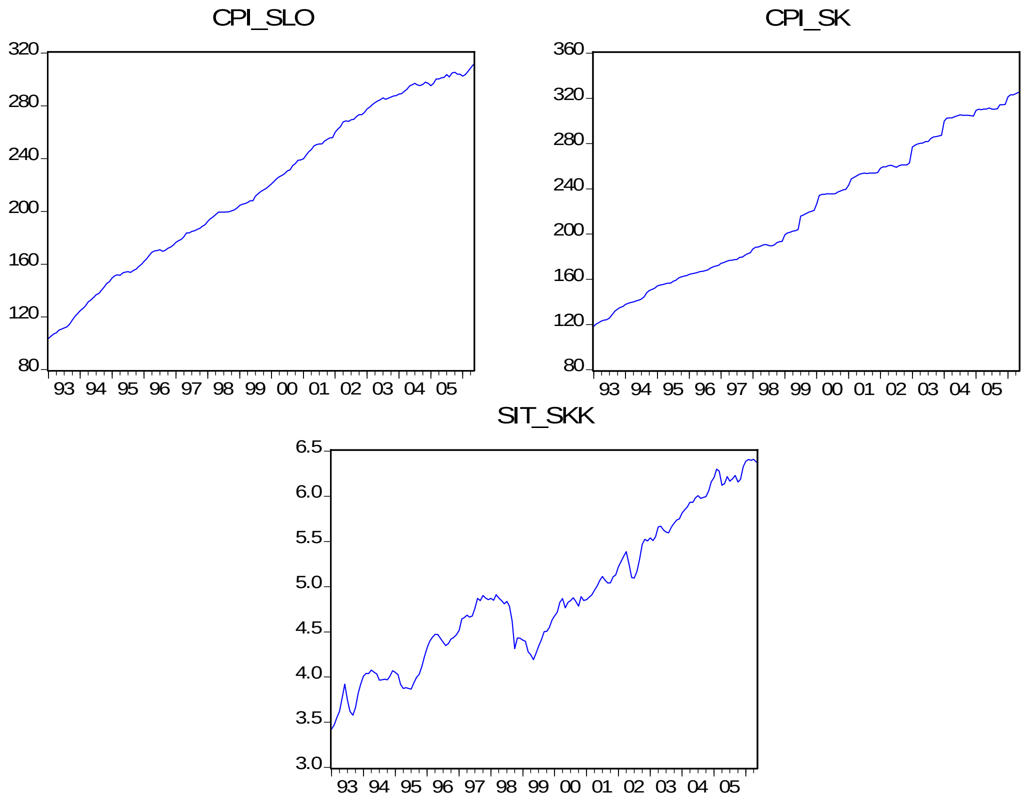
The modeling procedure proposed by Camacho has several drawbacks. Firstly, the system linearity test based on auxiliary regression (11) is rejected if at least one of the equations includes nonlinear terms. Specifying every equation as nonlinear based only on the rejection of the system linearity test thus neglects the possibility of a system involving linear and nonlinear equations and can yield inefficient estimates. Secondly, the limitations in the specified functional form can be justified neither by the relationships postulated within the economic theory nor by taking into account the very few existing applied studies. Thus, the specification with the same transition variable and the same type of the transition function in every equation of a smooth transition vector autoregressive model is too restrictive and should not be imposed a priory.

4. Empirical analysis

In this section, a three-variable smooth transition vector autoregressive model of the consumer price index for Slovenia, consumer price index of Slovakia and the nominal exchange rate between the currencies of both countries is discussed. The investigation applies the proposed augmented specification procedure to a small model of the real exchange rate, decomposed into its three components, domestic prices (P_t), foreign prices (P_t^*) and the nominal exchange rate (S_t).

Monthly data for the period from January 1992 till May 2006 were obtained from the Vienna Institute for International Economics Studies (WIIW), the administrator of the Monthly Database on Central and Eastern Europe. Due to the fact that Slovenia declared independence in June 1991 and introduced its own currency (tolar) in October of the same year, only the data for the period from January 1993, when Tolar was already an established currency, were used in the study.

Figure 1: Consumer Price Indices in Slovenia and Slovakia and Tolar/Koruna Nominal Exchange Rate



Notes: CPI_SLO stands for consumer price index in Slovenia, CPI_SK for consumer price index in Slovakia and SIT_SKK for nominal exchange rate of Slovenian tolar in Slovak koruna.

Source of data: WIIW Monthly Database on Central and Eastern Europe.

Figure 1 presents the graphs of consumer price indices in Slovenia and Slovakia and tolar nominal exchange rate, defined as a price of Slovak koruna in Slovenian tolar. It can be seen that the developments of consumer prices in the two observed economies are rather similar, although there are a few particularities. In the first years of the observed period inflation in Slovenia was higher than in Slovakia. Slovenian consumer prices show similar steady increases throughout the observed period with an exception of last two years when the increases were lower. This is due to the fact that Slovenia entered ERMII in June 2004 and has kept inflation low since then in order to fulfil the Maastricht criteria. In Slovakia steady increases in consumer price index fade away in 1998. Since then the consumer prices evidenced much bigger fluctuation. The third part of Figure 1 illustrates the movements in tolar/koruna nominal exchange rate. It can be clearly seen that tolar experienced nominal depreciation against koruna. Falling from 3.43 to 6.38 tolars for koruna in the whole observed period results in 86% of nominal depreciation. (Taking into account the inflation in the period 1993-2006 in both economies the real depreciation amounts to 104%.) The steady depreciation of tolar against koruna was interrupted in 1998, when koruna depreciated. But the tolar appreciation against koruna was short-lived since Bank of Slovenia continued with the policy of steady depreciation throughout the existence of tolar while Slovak National Bank reacted to depreciation of koruna in 1998 with a change in exchange rate regime by introducing managed float (Amerini 2003) and kept the koruna rather stable thereafter.

The econometric model employs variables expressed in growth rates with the help of the logarithmic transformation. Therefore small letters are used to denote the transformed variables, where s_t stands for the logarithm of nominal exchange rate of tolar, p_t for logarithm of consumer price index in Slovenia and p_t* represents logarithm of consumer price index in Slovakia.

4.1. Does the purchasing power parity hold?

We start our empirical investigation by testing for the purchasing power parity between Slovenia and Slovakia. The results of the augmented Dickey – Fuller (ADF) unit root tests for each of the three variables and their first differences are given in Table 8. The null hypothesis of unit root is rejected (at the 5 % level) only for the differenced variables, therefore s_t, p_t and p_t^* are all integrated of order 1.

Table 8: ADF Unit Root Test Results for Slovenia and Slovakia

Variable	s_t	p_t	p_t^*
Test statistic (p-value)	-3.3527 (0.0617)	-2.6867 (0.2437)	-1.9948 (0.5991)
Variable	Δs_t	Δp_t	Δp_t^*
Test statistic (p-value)	-8.2812 (0.0000)	-9.3496 (0.0000)	-10.4961 (0.0000)

Both the trace statistic and the max-eigenvalue statistic of the Johansen cointegration test indicate one cointegrating equation at the 5 % level. The normalized cointegration coefficients yield the following cointegrating equation:

$$s_t = 4.5637 \ + 4.3801 \cdot \underset{(1.4164)}{p_t} \text{-} 3.809361 \cdot \underset{(1.4693)}{p_t^*} + \xi_t . \tag{20}$$

The standard errors of the coefficient estimates are given in brackets. Note that the parameters α_1 and α_2 (as denoted in equation (1)) are significantly different from zero and the signs are in accordance with purchasing power parity. The main advantage of the Johansen cointegration test over the Engle-Granger test when checking the purchasing power parity lies in the possibility to test linear restrictions imposed on the parameters of the cointegrating vectors. We shall test both the symmetry and the proportionality condition with the help of the likelihood ratio test. The proportionality condition $\alpha_1 = \alpha_2 = 1$ (p = 0.4294) and the symmetry condition $\alpha_1 = \alpha_2$ cannot be rejected (p = 0.4294), thus PPP holds in its strictest form.

4.2 Real exchange rate model

In a preliminary specification, the linear vector error-correction model was specified. This simplifies the search for an appropriate nonlinear specification. As neglected autocorrelation structure may lead to false rejections of the linearity hypothesis (Teräsvirta, 1994), the order of autoregression was chosen on the basis of the serial correlation tests. Thus, a VECM model with 1 lag and 1 cointegrating equation (given in Equation (20)) was indicated as the best choice. The Schwarz information criterion also selected VECM(1), whereas according to Akaike information criterion 2 lags should be specified.

The system linearity tests and the single equation linearity tests are performed in the next step. The variables Δs_{t-1}, Δp_{t-1}, Δp_{t-1}^* and ce_t are regarded as candidates for the transition variable. Since in the augmented specification procedure different transition variables are allowed in different equations of the system, there are $4^3 = 64$ possible transition variable triplets. The results of the system linearity tests as well as single equation linearity tests are shown in Table 9. Note that for the third equation, the null hypothesis of linearity is never rejected. Therefore we shall specify a linear relationship when Δp_t^* is the dependent variable. The system linearity test and the first equation linearity test are most strongly rejected (i.e. p-val.<0.0001), if Δs_{t-1} is selected for the transition variable in the first equation. In this case there are 2 candidates for the transition variable in the second equation, namely Δs_{t-1} and ce_{t-1}. We have chosen ce_{t-1} to examine different regimes in the Δp_t equation associated with different levels of the deviation from PPP.

Table 9: Linearity Test Results (p-values)

LINEARITY TESTS (p-values)						
Transition variables			Test results			
1st eq.	2nd eq.	3rd eq.	System	1st eq.	2nd eq.	3rd eq.
Δs_{t-1}	Δs_{t-1}	Δs_{t-1}	0.0000	0.0000	0.0307	0.4145
Δs_{t-1}	Δs_{t-1}	Δp_{t-1}	0.0000	0.0000	0.0249	0.2364
Δs_{t-1}	Δs_{t-1}	Δp^*_{t-1}	0.0000	0.0000	0.0236	0.8297
Δs_{t-1}	Δs_{t-1}	ce	0.0000	0.0000	0.0414	0.3505
Δs_{t-1}	Δp_{t-1}	Δs_{t-1}	0.0000	0.0000	0.7254	0.4538
Δs_{t-1}	Δp_{t-1}	Δp_{t-1}	0.0000	0.0000	0.8304	0.4104
Δs_{t-1}	Δp_{t-1}	Δp^*_{t-1}	0.0000	0.0000	0.7014	0.8922
Δs_{t-1}	Δp_{t-1}	ce	0.0000	0.0000	0.7151	0.3081
Δs_{t-1}	Δp^*_{t-1}	Δs_{t-1}	0.0000	0.0000	0.1218	0.4197
Δs_{t-1}	Δp^*_{t-1}	Δp_{t-1}	0.0000	0.0000	0.1691	0.3756
Δs_{t-1}	Δp^*_{t-1}	Δp^*_{t-1}	0.0000	0.0000	0.0831	0.7825
Δs_{t-1}	Δp^*_{t-1}	ce	0.0000	0.0000	0.1496	0.3376
Δs_{t-1}	ce	Δs_{t-1}	0.0000	0.0000	0.0362	0.3837
Δs_{t-1}	ce	Δp_{t-1}	0.0000	0.0000	0.0399	0.2769
Δs_{t-1}	ce	Δp^*_{t-1}	0.0000	0.0000	0.0406	0.8818
Δs_{t-1}	ce	ce	0.0000	0.0000	0.0672	0.3969
Δp_{t-1}	Δs_{t-1}	Δs_{t-1}	0.0075	0.0097	0.0486	0.3946
Δp_{t-1}	Δs_{t-1}	Δp_{t-1}	0.0043	0.0094	0.0452	0.2495
Δp_{t-1}	Δs_{t-1}	Δp^*_{t-1}	0.0239	0.0093	0.0423	0.8198
Δp_{t-1}	Δs_{t-1}	ce	0.0066	0.0093	0.0692	0.3538
Δp_{t-1}	Δp_{t-1}	Δs_{t-1}	0.0782	0.0091	0.6502	0.4665
Δp_{t-1}	Δp_{t-1}	Δp_{t-1}	0.0657	0.0084	0.7558	0.4183
Δp_{t-1}	Δp_{t-1}	Δp^*_{t-1}	0.1771	0.0084	0.6086	0.8650
Δp_{t-1}	Δp_{t-1}	ce	0.0491	0.0082	0.6196	0.3032
Δp_{t-1}	Δp^*_{t-1}	Δs_{t-1}	0.0137	0.0066	0.1031	0.4246
Δp_{t-1}	Δp^*_{t-1}	Δp_{t-1}	0.0123	0.0064	0.1424	0.3835
Δp_{t-1}	Δp^*_{t-1}	Δp^*_{t-1}	0.0317	0.0064	0.0694	0.7576
Δp_{t-1}	Δp^*_{t-1}	ce	0.0106	0.0064	0.1223	0.3360
Δp_{t-1}	ce	Δs_{t-1}	0.0044	0.0078	0.0279	0.3682
Δp_{t-1}	ce	Δp_{t-1}	0.0028	0.0076	0.0327	0.2783
Δp_{t-1}	ce	Δp^*_{t-1}	0.0177	0.0074	0.0334	0.8576
Δp_{t-1}	ce	ce	0.0046	0.0072	0.0569	0.3983
Δp^*_{t-1}	Δs_{t-1}	Δs_{t-1}	0.2288	0.7595	0.0378	0.3742
Δp^*_{t-1}	Δs_{t-1}	Δp_{t-1}	0.1568	0.7341	0.0360	0.2397

Δp^*_{t-1}	Δs_{t-1}	Δp^*_{t-1}	0.4381	0.7591	0.0336	0.8316
Δp^*_{t-1}	Δs_{t-1}	ce	0.2132	0.7529	0.0553	0.3504
Δp^*_{t-1}	Δp_{t-1}	Δs_{t-1}	0.7326	0.7442	0.5593	0.4459
Δp^*_{t-1}	Δp_{t-1}	Δp_{t-1}	0.6786	0.7096	0.6817	0.4198
Δp^*_{t-1}	Δp_{t-1}	Δp^*_{t-1}	0.9091	0.7427	0.5310	0.8874
Δp^*_{t-1}	Δp_{t-1}	ce	0.6234	0.7360	0.5459	0.3151
Δp^*_{t-1}	Δp^*_{t-1}	Δs_{t-1}	0.3612	0.7374	0.0938	0.4152
Δp^*_{t-1}	Δp^*_{t-1}	Δp_{t-1}	0.3309	0.7094	0.1338	0.3830
Δp^*_{t-1}	Δp^*_{t-1}	Δp^*_{t-1}	0.5330	0.7194	0.0599	0.7643
Δp^*_{t-1}	Δp^*_{t-1}	ce	0.3117	0.7211	0.1107	0.3354
Δp^*_{t-1}	ce	Δs_{t-1}	0.2167	0.8049	0.0319	0.3509
Δp^*_{t-1}	ce	Δp_{t-1}	0.1604	0.7905	0.0389	0.2732
Δp^*_{t-1}	ce	Δp^*_{t-1}	0.4625	0.8023	0.0385	0.8671
Δp^*_{t-1}	ce	ce	0.2248	0.7924	0.0666	0.4052
ce	Δs_{t-1}	Δs_{t-1}	0.0057	0.0064	0.0389	0.4087
ce	Δs_{t-1}	Δp_{t-1}	0.0030	0.0057	0.0354	0.2439
ce	Δs_{t-1}	Δp^*_{t-1}	0.0183	0.0060	0.0332	0.8280
ce	Δs_{t-1}	ce	0.0049	0.0061	0.0555	0.3595
ce	Δp_{t-1}	Δs_{t-1}	0.0466	0.0037	0.4840	0.4654
ce	Δp_{t-1}	Δp_{t-1}	0.0381	0.0036	0.6090	0.4212
ce	Δp_{t-1}	Δp^*_{t-1}	0.1160	0.0037	0.4463	0.8727
ce	Δp_{t-1}	ce	0.0283	0.0036	0.4591	0.3074
ce	Δp^*_{t-1}	Δs_{t-1}	0.0095	0.0036	0.0815	0.4409
ce	Δp^*_{t-1}	Δp_{t-1}	0.0080	0.0034	0.1132	0.3822
ce	Δp^*_{t-1}	Δp^*_{t-1}	0.0220	0.0035	0.0517	0.7645
ce	Δp^*_{t-1}	ce	0.0070	0.0034	0.0933	0.3354
ce	ce	Δs_{t-1}	0.0052	0.0098	0.0416	0.3841
ce	ce	Δp_{t-1}	0.0032	0.0091	0.0478	0.2699
ce	ce	Δp^*_{t-1}	0.0207	0.0090	0.0468	0.8550
ce	ce	ce	0.0054	0.0090	0.0808	0.4078

Notes: ce stands for the cointegrating vector given by Equation (20) and d denotes the first difference operator.

Next, the question of the type of the transition function in each of the equations has to be investigated. The sequence of nested hypotheses (17) is performed with the results given in Table 10. The F3 test yields the lowest p-value for both equations thus indicating the LSTR2 transition function in both cases. The F3 test for the first equation is strongly significant, whereas for the second equation the hypothesis H_{03} is rejected only at the 10% significance level.

Table 10: Tests for Choosing the Type of Transition Function (p-values)

NESTED TESTS (p-values)							
Transition variables		First equation			Second equation		
1st eq.	2nd eq.	F4	F3	F2	F4	F3	F2
Δs_{t-1}	ce	0.0492	0.0000	0.1548	0.3057	0.0671	0.2121

The estimated coefficients of the specified smooth transition vector error-correction model and the results of the diagnostic tests are given in Table 11 below. The transition function in the i-th equation is denoted by $G_i(s_t)$ with

$$G_1(\gamma, c_1, c_2; s_t) = 1 \Big/ \Big(1 + \text{Exp}(-\underset{(0.7613)}{1.3714}(\Delta s_{t-1} + \underset{(0.0046)}{0.0660})(\Delta s_{t-1} - \underset{(0.0020)}{0.0256}) / \sigma^2(\Delta s_{t-1}))\Big) \tag{21}$$

$$G_2(\gamma, c_1, c_2; s_t) = 1 \Big/ \Big(1 + \text{Exp}(-\underset{(19.8342)}{14.9249}(ce_t + \underset{(0.0359)}{0.9370})(ce_t - \underset{(0.0137)}{0.4152}) / \sigma^2(ce_t))\Big)$$

$$G_3 = 0$$

Table 11: Smooth Transition Vector Error-Correction Model

	Equations		
Regressors	Δs_t	Δp_t	Δp_t^*
$const \cdot (1 - G_i(s_t))$	0.0045 (0.0017)	0.0056 (0.0007)	0.0068 (0.0012)
$\Delta s_{t-1} \cdot (1 - G_i(s_t))$	0.6100 (0.0866)	0.0721 (0.0299)	
$\Delta p_{t-1} \cdot (1 - G_i(s_t))$	-0.3946 (0.2093)	0.1727 (0.0824)	-0.2269 (0.1372)
$\Delta p_{t-1}^* \cdot (1 - G_i(s_t))$			0.1614 (0.0816)
$ce_t \cdot (1 - G_i(s_t))$	-0.0084 (0.0031)	-0.0068 (0.0015)	-0.0051 (0.0020)
$const \cdot G_i(s_t)$		-0.0011 (0.0019)	
$\Delta s_{t-1} \cdot G_i(s_t)$	-1.0523 (0.3537)	0.0949 (0.0536)	
$\Delta p_{t-1} \cdot G_i(s_t)$	6.6055 (1.4409)		
$\Delta p_{t-1}^* \cdot G_i(s_t)$	-4.7047 (2.9421)	1.1366 (0.1617)	
$ce_t \cdot G_i(s_t)$	0.0253 (0.0121)		
R_{nl}^2	0.3984	0.4822	
$S.E._{nl}$	0.0121	0.0048	
R_{lin}^2	0.1453	0.3937	0.0717
$S.E._{lin}$	0.0141	0.0048	0.0085
$\hat{\sigma}_{nl} / \hat{\sigma}_{lin}$	0.8557	0.9394	
Diagnostic tests (p-values)			
AR(12)	0.6415	0.1719	0.0458
ARCH(12)	0.2915	0.8918	0.1908

White	0.2693	0.2675	0.5643

Notes: standard errors are given in brackets. AR(12) denotes the serial correlation LM test with no autocorrelation up to 12 lags under the null hypothesis. The ARCH(12) notation is analogous.

According to linearity tests the best transition variable for the first equation in Table 11 (with Δs_t as dependent variable) is the lagged change in nominal exchange rates and LSTR2 type of transition function is appropriate. The estimates of c_1 and c_2 are statistically significant (Equation 21). Thus, one regime is identified by small changes in nominal exchange rates (G=0) and is represented in the first part of Table 11, while the other regime reflects large changes in nominal exchange rates (G=1, the second part of Table 11). According to small adjustment parameter (γ=1.3714), the transition among regimes (from G=0 to G=1) is slow. When changes in nominal exchange rates are small, the coefficient of -0.0084 for deviations from PPP (ce_t) indicates that exchange rates adjust to equilibrium, while in the second regime (G=1) the exchange rates move away from the equilibrium, which is indicated by coefficient of 0.0253 for ce_t. It is interesting to note that in the second regime, when nominal changes in exchange rates are high, the signs of coefficients for both price indices indicate the validity of PPP. When nominal exchange rate is far away from the equilibrium, the price movements work in favor of converging to PPP.

Deviation from PPP (ce_t) turned out to be an appropriate transition variable for the second equation (with Δp_t as dependent variable), while the best type of transition function is again LSTR2. Threshold parameters c_1 and c_2 are again statistically significant (Equation 21) indicating the existence of two regimes. One of them (G=0) is presented in the first five rows of Table 11 and is determined by small deviations from PPP, while the other regime in the second equation is characterized by large deviations from PPP and is presented in the second five rows of Table 11 (G=1). A relatively high value of γ in the second transition function (γ=14.9249) indicates a rapid transition from G=0 to G=1. STVECM estimate of the coefficient for lagged domestic prices in the regime, when the deviations from PPP are small, confirms the trend of Slovenian price index in Figure 1.

The third equation (with Δp_t^* as dependent variable) in Table 11 is estimated as linear relationship among variable, as the null hypothesis of linearity was not rejected due to linearity tests and $G_3 = 0$, which indicates only one regime. The coefficient of the lagged change in p* is in line with the increasing trend of consumer price index in Slovakia in Figure 1.

Diagnostic tests are presented in the third part of Table 11. The error variance ratios of nonlinear relative to linear models for the first and the second equation show that the nonlinear models have a better fit ($\hat{\sigma}_{nl}/\hat{\sigma}_{lin} < 1$).

5 Conclusion

After reviewing the literature and presenting an example of testing of PPP in linear framework, the article proceeds with an overview of smooth transition regression approach in different frameworks (single equation regression, system of equations with one transition variable and system of equations with different transition variables).

While traditional unit root test rejected the null of non-stationarity in level real exchange rate for Slovenian tolar against Slovak koruna in the period of January 1993 to December 2006, Johansen cointegration test confirmed the validity of PPP among the two economies.

Within the methodological overview of nonlinear framework we argue that system of equations with one transition variable and one transition function is not realistic. Indeed, in our empirical case the linearity tests show that different transition variables are appropriate to use, while the type of transition function is the same (LSTR2) in two equations. In the case of the third equation, linearity tests cannot be rejected and the equation was estimated as linear relationship among variables. The model reflects some evidence in favor of PPP theory. Since both of the analyzed countries are transition economies, such result was expected.

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