Growth, Inflation, and Banking: The Role of Human Capital

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To my parents

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I am, however, solely responsible for any remaining errors.

Preface

This book concentrates on the role of human capital as a growth engine in the process of economic development and its effect on a better explanation of the long-run relationship between growth, inflation and other monetary phenomena. The workhorse of the book is the Lucas growth model with the accumulation of both physical and human capital. The first part of the book provides a general theory of how the development of countries could progress via different stages of growth. Using an extended Lucas model with external benefits of human capital through a diffusion of new knowledge, such a framework allows us to account for different levels of the development of countries over a long span of time and across countries. Specifically, in the first chapter of the book we propose a two-sector model of endogenous growth and show that its dynamics is not only able to capture the two standard features of economic development: stages of sustained growth and underdevelopment traps,[1], but also various transitions between them including non-monotonic transitive phenomena: that of *seemingly sustainable growth*, a *temporary underdevelopment trap* and a *temporary slowdown in productivity growth*. Thus the model of this chapter contributes to the recent search for 'unified growth theories' [see e.g. Galor:2005] that capture the evolution of income per capita, technology and other aspects of human society over the course of its history. Due to the extended set of the transition modes the model is especially useful for modeling the behavior of 'transition countries'. In Chapter 2 an open economy version of the model is developed and used for analyzing the process of the accession of some Central and Eastern European countries to the European Union. Contrary to the neoclassical growth models, which predict 'automatic' convergence, our endogenous growth model supports the existence of multiple equilibria and the divergent behavior of countries. To explain convergence within an endogenous growth framework, the theory has to be amended by the catching-up hypothesis [e.g. (Abramovitz 1986), (Abramovitz 1994)]. This concept assumes that countries must possess a so-called social capability - that includes besides others, human capital, infrastructure capacities and institutional settings - to adopt and use the new technologies successfully. It also implies that investment is only a necessary but not a sufficient condition for convergence. Such an amended endogenous growth model with convergence property still preserves the possibility of divergent behavior. There exists a strong effect of initial

[1] Our model shows that the introduction of 'threshold externalities' can produce a very long endogenous transition from stagnation to economic growth under the assumption of rational expectations - see (Arifovic, Bullard, and Duffy 1997) who used adaptive learning to achieve such transitional behavior.

conditions and parameters on the potential transitional trajectories. Therefore, after the model is calibrated using the available data on a set of three transition countries (the Czech Republic, Hungary, and Poland) we provide not only a quantitative assessment of the speed of convergence to EU standards but also discuss the appearance of phenomena such as an accession boom or accession recession.

The second part of the book proposes an extension of the standard Lucas model by the introduction of money and an explicit banking sector. Such a setup allows to modeling the interactions between real and monetary phenomena. The main contribution of Chapter 3 is that it presents a model in which a reasonable calibration can account for the empirical evidence, across the range of inflation rates, on inflation and growth. The chapter also shows that the inflation-growth explanation is fully consistent with evidence on the existence of the (Tobin 1965)-like effects, including a rise in output per effective labor, even as the balanced-path growth rate declines as a result of an inflation rate increase.

The key mechanism that gives the model proposed in Chapter 3 the added flexibility to explain the evidence is the ability of the representative consumer to choose between competing payment mechanisms, money and credit. The resulting inflation-growth profile is shown to be very nonlinear compared to the model without credit and it qualitatively matches the profile in the evidence, unlike in the previous literature. The use of credit has an implication for the nature of the model's money demand function which can be described as being similar to a general equilibrium version of the (Cagan 1956) function, in that it has an approximately constant semi-interest elasticity (i.e. as the inflation rate rises the interest elasticity rises). In particular, the rising interest elasticity and its correspondence to the nonlinearity of the inflation-growth profile involves a previously unreported systemic link between the strength of the growth and of the (Tobin 1965) effects: when the inflation rate is low and the money demand function is in the relatively inelastic range, the growth and (Tobin 1965) effects are both marginally stronger, that is, of greater magnitude. When the inflation rate is relatively high and the money demand is in a relatively elastic range, these effects are weak and of small magnitude. It is shown there that our banking specification is superior to alternative solutions to the problem of explaining the inflation experience, that rely on popular existing payment mechanisms (e.g. models with (Lucas and Stokey 1987) cash goods and credit goods or models with shopping time technology).

A more careful analysis of several main approaches to modeling the long-run inflation-growth effect and their results along three main dimensions is explored in Chapter 4. The three main dimensions are the following: (i) whether the models exhibit a significant negative inflation effect on long-run growth rate; (ii) what is the nature of the inflation-growth effect across the whole range of the levels of the inflation rate; and (iii) whether the inflation-growth models can at the same time be consistent with evidence on (Tobin 1965)-type effects.

Modeling the monetary aggregates in general equilibrium has been a challenge. The final chapter of the book then expands the simplified growth model[2], the *AK model*, by more developed banking sectors allowing a formulation of the demand for the base money and non-interest bearing demand deposits as well as the demand for the demand for interest bearing demand deposits. In this way our model determines the demands for monetary aggregates M1 and M2.

In this chapter we also show how our model can be used to analyze how subsets of aggregates change according to changes in exogenous factors: in the money supply growth rate (and thus in the nominal rate of interest) and in the banking productivity parameters. Comparative statics of these factors are then applied to explain the actual profiles of the velocity of monetary aggregates, and the profiles of their ratios over time in the US economy.

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January 2007

[2]The simplification of the growth model used in other parts of the book consists of a physical-capital-only version of the original model.

Part 1

Human Capital, Growth, and Development

CHAPTER 1

Stages of Growth in Economic Development

1. Introduction

This chapter analyses a two-sector model of endogenous growth with two features of economic development: stages of sustained growth and underdevelopment traps. By stages, I mean [in the sense of (Rostow 1990)] regimes with persistent differences in sustainable growth rates. In particular, there is a *stage of low growth* before reaching an area of increasing returns. This is followed by a take-off to a *stage of high growth.* The model also demonstrates the transitional issues of a *temporary underdevelopment trap* (underdevelopment trap means here that the economy exhibits a *stage of zero growth*), *seemingly sustainable growth*, and a *temporary slowdown in productivity growth.* A temporary underdevelopment trap occurs when an economy exhibits a regime of extensive growth (i.e. slowly declining growth in physical capital with no growth in human capital), but thereafter starts a transition to sustained growth. Seemingly sustainable growth happens when an economy exhibits a regime of intensive growth (i.e. both capitals grow initially) but the growth of human capital subsequently ceases and the economy eventually finds itself in a zero growth trap. A slowdown in productivity growth occurs when the transition from the low growth stage to the high growth stage is not monotonic[1].

The model follows the Lucas-Uzawa learning-or-doing model (Lucas 1988b) with physical and human capital. We assume that there are positive externalities in the productivity of human capital[2], much like (Azariadis and Drazen 1990)[3], since, as Lucas states [in (Lucas 1988b), p.19], "... human capital accumulation is a social activity, involving groups of people in a way that has no counterpart in the accumulation of physical capital." We further assume that there is a frontier of "theoretical knowledge" that is given exogenously and represents large advances in knowledge, like an industrial revolution. The economy can approach the frontier via the education of people, which facilitates the adoption and implementation of new technologies. In this paper the diffusion of technology leads to threshold or logistic types of externalities in the human capital accumulation process. Because of this, the human accumulation process can exhibit increasing returns to its inputs, depending on the average level of human capital.

[1] I believe that all these phenomena generated by the model and driven by small differences in initial conditions or in model parameters can account for the observed, and well-documented, vast differences in the levels of per capita incomes among countries.

[2] In the original paper by Lucas the external effect was in the goods sector.

[3] In contrast to the authors, who use the overlapping generation framework, we use the infinite lifetimes one.

The model is similar to the (Zilibotti 1995) model except that mine uses two capital stocks as opposed to one and in my model the engine of growth is human capital as opposed to physical capital in 'a Jones-Manuelli-type' production function. My model is also different in that there are no indeterminacies, as there are in both the original Lucas model[4] and Zilibotti's model. And, in contrast to Zilibotti's model, where some kind of structural once-off shock is necessary for a better endowed economy to escape from a zero growth equilibrium, my model does not need any kind of 'first movement' to start the growth engine.

Most of the papers on endogenous growth theory are restricted to steady state analysis. This is caused in part by the assumption that balanced growth regimes can serve as good approximations for the behavior of real economies. There are situations, however, such as wars, natural disasters, and the collapse of communist regimes in Central and Eastern Europe, when the relations between the levels of variables do not correspond to steady state relations. Changes in government policies can also have this effect. In these economies there may appear a transitional period during which they move back to steady states[5]. In such situations, short-run effects cannot be omitted and transitional dynamics have to be studied.

In this paper I go even further by claiming that in my suggested model or "theory of economic development"[6] the transition process can last a very long time, and in many cases the observations on an economy's behavior contain only transitional data. Therefore, if we want to model the behavior of such an economy we are forced to analyze and understand the transitional dynamics of the model. Moreover, the transitional dynamics of the model presented here are not reducible to the development of capital ratios, as they are in the original Lucas model [see (Mulligan and Sala-i Martin 1993)][7], and policy functions are general functions of both state variables.

The rest of the paper is organized as follows. In Section 2 I develop the model and derive the first order conditions for a decentralized economy equilibrium with the use of a step-like approximation of the "learning curve" capturing the process of technology diffusion. This allows later the qualitative analysis of the model's transitional dynamics without the use of numerical methods. Section 3 is devoted to the analysis of steady states and to selected aspects of the model's comparative statics. The stages of low and high growth and of take-off comprise the content of Section 4. Section 5 continues the qualitative analysis of behavior and studies conditions for the existence

[4]Indeterminacies in Lucas' model were discovered by (Benhabib and Perli 1994) and (Xie 1994). I will briefly discuss this problem later and suggest how this kind of indeterminacies could be removed.

[5]When economies are subjected to structural changes, the new steady states may be different from the original ones (see, for instance, the case of the post-socialist CEE countries).

[6]In the sense of Lucas by theory we mean "..an explicit dynamic system, something that can be put on a computer and run" (Lucas 1988b).

[7]For an extensive analysis of transitional dynamics in the Lucas-Uzawa model and its extended variants see (Caballe and Santos 1993), (Chamley 1993) and (Ladron-de Guevara, Ortigueira, and Santos 1997).

of underdevelopment traps. In that section I also use the results of a numerical simulation of the calibrated model to demonstrate the mechanism of productivity slowdown as a result of the transition to a higher balanced growth path.

2. The Model

Following the Lucas-Uzawa framework I assume a two-sector economy with a goods sector and an education sector. There are two ways technology innovation can occur in the model: (i) large discontinuous advances which coincide with important eras like industrial revolutions and (ii) more cumulative and continuous progress during which the society learns how to use this potential. Much like (Zilibotti 1995), I consider the former as being exogenous in the sense that economic activity has no effect on the occurrence of revolutionary advances. The second type of innovation depends on the gap between the present level of technology and the frontier productivity level given by the first type of innovation [see (Nelson and Phelps 1966)]. I assume here, consistent with (Lucas 1988b) and (Azariadis and Drazen 1990), that technical progress is driven by investment in human capital[8] such that:

$$\frac{\dot{B}_t}{B_t} = \psi \frac{B_H - B_t}{B_H} \dot{H}_t \tag{2.1}$$

where B_H means the frontier productivity, $B_H \geq B_t$, $\psi > 0$ is the parameter of the speed of diffusion and H is the average level of human capital in the economy[9]. We can see that the farther from the frontier an economy is, the faster the growth of productivity for a given level of investment is. After solving equation (2.1) we obtain the following logistic solution

$$B(H) = \frac{B_H}{1 + (\frac{B_H}{B_0} - 1)e^{-\psi H}} \tag{2.2}$$

where B_0 is the initial level of productivity related to a zero level of human capital. We can easily see from (2.2) that there is an upper bound of productivity given by B_H (i.e. if H goes to infinity, productivity converges to B_H). Using (2.2) we will generalize the linear Uzawa-Rosen[10] formulation of the production function for human capital assuming that the level of productivity in the education sector, B, depends on the developmental level of a society expressed by the average level of human capital

$$\dot{h} = B(H)(1 - u)h \tag{2.3}$$

where $B(H)$ is given by (2.2). Therefore, the production function in the education sector (see equation (2.3)) exhibits increasing returns to all inputs at the social level

[8]This is different from the standard learning-by-doing models (Arrow 1962) and (Romer 1986) and from the model in (Zilibotti 1995) where technical progress is a by-product of the investment in physical capital.

[9](Benhabib and Spiegel 1994) have provided empirical evidence confirming that per capita income growth depends positively on average levels of human capital.

[10]This specification was used in (Lucas 1988b).

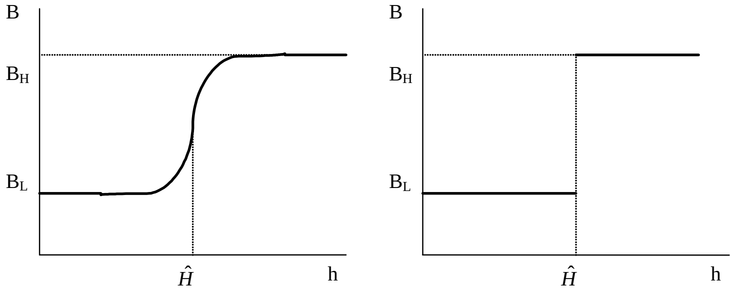

FIGURE 1. (a) Continuous learning curve. (b) Approximated learning curve.

for non-constant levels of productivity (i.e. $B'(H) \neq 0$) and constant returns to private inputs for any level of productivity.

Since the aim of this paper is to describe different stages of development in the model caused by different initial conditions and different values of the parameters, in this section I propose *a "qualitative" approximation*[11] of the model. There are two good reasons for such an approximation. The first one is related to the difficulties of analyzing transitional dynamics, as was mentioned at the end of the last section. Secondly this approximation allows one to describe transitional dynamics qualitatively as a development over different stages, where each stage can be characterized by distinct features.

The main difference between my model and the Lucas-Uzawa model is the "learning" curve (see Figure 1a), which prevents the implementation of the standard procedure of finding a reduced form model with transformed variables exhibiting zero steady state growth. The nonexistence of such a reduced form makes the problem of obtaining a solution difficult, not only from the analytical, but also from the computational point of view. Thus any approximation of the learning curve which significantly simplifies the analysis and at the same time preserves the main features of the model's behavior is beneficial. We claim a step-like approximation of the learning curve can do this job very well.

[11]The necessity of qualitative approximations in obtaining insights into human reasoning, as well as in structuring results in modeling complex systems, are well known in the field of artificial intelligence and physics [see for example (Kuipers 1986) and (Lum and Chua 1991)].

Let's define $B(h)$ as

$$B(h) = \begin{cases} B_L, & 0 < h < \hat{H} \\ B_H, & h \geq \hat{H} \end{cases} \tag{2.4}$$

where $B_H > B_L$. Now assume that there exists a critical value of the average level of human capital, $\hat{H}$, such that below this level the productivity of the education sector is low (B_L), while above this level the productivity of the education sector is high (B_H) (see Figure 1b). The qualitative approximation[12] proceeds in two directions. Firstly, it captures the fact that changes in the level of human capital have a negligible effect on productivity when the level of human capital is sufficiently low or sufficiently high. Secondly, the area in which increasing returns in the education sector are relevant is relatively narrow. This is consistent with the jump in (2.4).

The advantage of such a description of productivity is that it is piece-wise constant. Thus it enables us to split the development of the productivity parameter into three stages: low and high stages of development characterized by B_L and B_H, respectively, and the stage of "take-off" collapsed into the switch from B_L to B_H when the average level of human capital in the economy reaches critical level $\hat{H}$.

Suppose now that the economy contains a constant number, N, of identical, infinitely-lived workers, which we will normalize to 1. All the workers have the same skill level h and devote a fraction u of their (non-leisure) time to current production, and the remaining $1 - u$ to human capital accumulation in the education sector. The effective labor input in production is then $l = uh$. In maximizing their life-time utility, the workers seek an optimal life-time consumption and working pattern, $\{c_t, u_t\}$, which they achieve through the appropriate accumulation of financial and human wealth, a and h respectively:[13]

$$\max_{\{c_t,u_t\}} \int_0^\infty e^{-\rho t} \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) dt \quad \text{s.t.} \tag{2.5}$$

$$\dot{a} = ra + wuh - c \tag{2.6}$$

$$\dot{h} = B(H)(1-u)h \tag{2.7}$$

$$\lim_{t \to \infty} a_t e^{-\int_0^t r_s ds} \geq 0 \tag{2.8}$$

$$0 \leq u \leq 1 \qquad a_0 > 0, \quad h_0 > 0. \tag{2.9}$$

The agents are endowed with perfect foresight with respect to future wages and interest rates, w and r. They know the production for creating new knowledge in the education sector. They cannot, however, influence the average level of accumulated knowledge, H. The price of the consumption good is normalized to one, ρ is the time preference parameter and σ is the intertemporal elasticity of substitution ($\theta = \sigma^{-1}$ is the degree of relative risk aversion). At each date, agents are endowed with a unit

[12]We can look at the approximation of the logistic curve as a piece-wise linearization.

[13]Whenever possible we suppress time indices to avoid cluttered notation. A dot denotes a time derivative.

of time and a stock of financial and human wealth resulting from past accumulation. They choose consumption and allocate time for productive and educational purposes. Equation (2.8) refers to the No-Ponzi-game condition.

The use of the piece-wise constant productivity function violates the standard assumptions regarding the continuity and smoothness of the production functions. However, these shortcomings can be overcome by transforming the problem into a two-stage optimal control problem. During the first stage, period $[0, T)$, the representative agent maximizes his lifetime welfare $V(k_0, h_0)$ subject to the relationships found in the economy having low productivity in the education sector B_L. Lifetime welfare is expressed in (2.10) as the welfare function over period $[0, T)$ together with the scrap value $V_{II}(k_T, h_T)e^{-\rho T}$ given by the welfare function of the second stage discounted to time 0. The first stage finishes at time T when the average level of human capital reaches the critical amount $\hat{H}$ and productivity jumps to B_H. Thus, during the second stage, the agent maximizes his welfare given by $V_{II}(k_T, h_T)$ in (2.15) on the interval (T, ∞) subject to conditions in an economy with high productivity in the education sector:

$$V(k_0, h_0) = \max_{\{c_t, u_t\}} \left\{ \int_0^T e^{-\rho t} \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) dt + V_{II}(k_T, h_T) e^{-\rho T} \right\} \tag{2.10}$$

subject to

$$\dot{k} = rk + wuh - c \tag{2.11}$$

$$\dot{h} = B_L(1-u)h \tag{2.12}$$

$$0 \leq u \leq 1, \qquad k_0 > 0, \quad h_0 > 0 \tag{2.13}$$

$$h_T = \hat{H}, \qquad k_T \text{ is free}, \qquad T \text{ is free}, \tag{2.14}$$

with

$$V_{II}(k_T, h_T) = \max_{\{c_t, u_t\}} \int_T^\infty e^{-\rho(t-T)} \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) dt \tag{2.15}$$

subject to

$$\dot{k} = rk + wuh - c \tag{2.16}$$

$$\dot{h} = B_H(1-u)h \tag{2.17}$$

$$0 \leq u \leq 1, \qquad k_T > 0, \quad h_T = \hat{H} \tag{2.18}$$

$$\lim_{t\to\infty} k_t e^{-\int_T^t r_s ds} \geq 0. \tag{2.19}$$

Using the Pontryagin Maximum Principle of optimal control we get the following necessary conditions for our two-stage optimal control problem

$$\begin{aligned}
(2.20) \quad & \lambda_I &=& \ c^{-\theta}e^{-\rho t} \\
(2.21) \quad & \lambda_I wh &\geq& \ \mu_I B_L h \qquad \text{and } (1-u) \geq 0 \\
(2.22) \quad & \dot{\lambda}_I/\lambda_I &=& \ -r \\
(2.23) \quad & \dot{\mu}_I/\mu_I &=& \ -B_L \\
(2.24) \quad & \lambda_{I,T} &=& \ \frac{\partial V_{II}(k_T,h_T)}{\partial k}e^{-\rho T} \\
(2.25) \quad & \mu_{I,T_}B_L(1-u_{T_})h_T &=& \ \rho V_{II}(k_T,h_T)e^{-\rho T} - \frac{c_T^{1-\theta}-1}{1-\theta}e^{-\rho T}
\end{aligned}$$

for $t \in [0,T)$ and

$$\begin{aligned}
(2.26) \quad & \lambda_{II} &=& \ c^{-\theta}e^{-\rho(t-T)} \\
(2.27) \quad & \lambda_{II} wh &\geq& \ \mu_{II} B_H h \qquad \text{and } (1-u) \geq 0 \\
(2.28) \quad & \dot{\lambda}_{II}/\lambda_{II} &=& \ -r \\
(2.29) \quad & \dot{\mu}_{II}/\mu_{II} &=& \ -B_H \\
(2.30) \quad & \lim_{t\to\infty} & & k_t\lambda_{II,t} = 0 \\
(2.31) \quad & \lim_{t\to\infty} & & h_t\mu_{II,t} = 0
\end{aligned}$$

for $t \in (T,\infty)$. Equations (2.20) and (2.26) associated with low productivity and high productivity, respectively, give the conditions where the maximizing agent is indifferent between consuming another unit of the good or saving it in the form of physical capital because the return from consumption (marginal utility) is the same as the return on investment in physical capital (shadow value λ). The next equations (2.21) and (2.27) state that the marginal return to study must be equal to the marginal return to work if working time u is smaller than 1. The last two equations (2.22) and (2.23), and (2.28) and (2.29) associated with low productivity and high productivity, respectively, describe the development of the shadow prices of capitals. The growth rate of the shadow value of a particular type of capital is given by the net return on that capital. Equation (2.24) is the transversatility condition for a free endpoint with scrap value and (2.25) is the transversatility condition for free time [see e.g. (Kamien and Schwartz 1991)]. λ_I, μ_I and λ_{II}, μ_{II} are the present-value costate variables related to the first and second stage problem respectively.

The initial and TVC conditions given by (2.13) and (2.30)-(2.31), respectively, are straightforward. The TVC conditions for connecting the two stages are (2.24) and (2.25).

On the production side, the economy consists of a large number of identical firms with a constant returns to scale Cobb-Douglas production function $F(k,l) = Ak^{\alpha}l^{1-\alpha}$, $0 < \alpha < 1$, where k is physical capital and l is labor expressed in efficiency units. Each profit maximizing firm makes a static decision on how much labor and physical capital to rent from the agents:

$$\max_{k,l} \pi = F(k,l) - wl - (r+\delta)k \tag{2.32}$$

where δ is the rate of depreciation of physical capital. The maximization of (2.32) gives us the inverse factor demand functions:

$$r = F_k - \delta, \qquad w = F_l \tag{2.33}$$

where $F_k \equiv \partial F/\partial k$ and $F_l \equiv \partial F/\partial l$[14].

All markets clear in equilibrium. The average level of human capital is in equilibrium $H = h$. The competitive equilibrium in the model variables is then given by the following proposition.

PROPOSITION 1. *(Jump[15]) The representative agent problem given by (2.10)-(2.19) with the productivity parameter $B(H)$ characterized in (2.4) leads to the first order necessary conditions (2.20)-(2.25) for $t \in [0,T)$ and (2.26)-(2.31) for $t \in (T,\infty)$. Thus, the competitive equilibrium dynamics of the two-stage problem (2.10)-(2.19) can be expressed by the following two systems of equations*

$$\frac{\dot{k}}{k} = \frac{F}{k} - \delta - \frac{c}{k} \tag{2.34}$$

$$\frac{\dot{h}}{h} = B_I(1-u) \tag{2.35}$$

$$\frac{\dot{c}}{c} = \sigma\left(F_k - \delta - \rho\right) \tag{2.36}$$

$$\frac{\dot{u}}{u} = (B_I+\delta)\frac{1-\alpha}{\alpha} + B_I u - \frac{c}{k} \tag{2.37}$$

where $I \in \{L, H\}$ with further initial, "connecting" and TVC conditions

$$k_0 > 0, \qquad h_0 > 0 \tag{2.38}$$

$$k_{T_-} = k_{T_+}, \qquad h_{T_-} = h_{T_+} = \hat{H}, \qquad c_{T_-} = c_{T_+},$$

$$u_{T_+} < u_{T_-} = \left(\frac{c_T + \delta k_T}{A k_T^{\alpha}}\right)^{\frac{1}{1-\alpha}} \hat{H}^{-1}, \tag{2.39}$$

$$\lim_{t\to\infty} k_t e^{-\int_T^t r_s ds} = 0, \qquad \lim_{t\to\infty} h_t e^{-B_H(t-T)} = 0, \tag{2.40}$$

respectively.

Proof: (see Appendix A.1).

The piece-wise constancy of the productivity parameter enables us to transform the model into two reduced models, related to two productivity levels (B_H and B_L),

[14]Note that w denotes the wage rate per efficiency unit i.e. an agent with human capital h working u fraction of his time endowment earns labor income $wl = wuh$.

[15]The negative jump in the time devoted to work (u) has been derived in a model version with continuous learning function in (Kejak 1998). The analysis confirms that as we let the elasticity of productivity with respect to human capital to go to infinity at $\hat{H}$, the rate of growth of u goes to minus infinity at this point.

expressed in transformed variables[16]: the physical to human capital ratio $x \equiv k/h$, the consumption to physical capital ratio $q \equiv c/k$ and time devoted to work u. These new variables have the convenient property of zero growth rates in the steady state that facilitates further analysis.

By applying the suggested transformations to equations (2.34)-(2.37), the model can be expressed as the following two systems of equations:

$$\frac{\dot{x}}{x} = A(\frac{u}{x})^{1-\alpha} - q - \delta - B_I(1-u) \tag{2.41}$$

$$\frac{\dot{q}}{q} = (\sigma\alpha - 1)A(\frac{u}{x})^{1-\alpha} - \sigma\rho + (1-\sigma)\delta + q \tag{2.42}$$

$$\frac{\dot{u}}{u} = \frac{(B_I + \delta)}{\alpha} - \delta - (1-u)B_I - q \tag{2.43}$$

with $I \in \{L, H\}$ together with the initial, connecting, and TVC conditions given by (2.38)-(2.40).

3. Balanced Growth Path

If we look at equation (2.35) presenting the growth of human capital we can see that the property of locally increasing returns, caused by an upper bound on the externality effect, is critical for the existence of a balanced growth path (BGP). With globally increasing returns, the model would exhibit ever-accelerating growth[17].

Each of the two systems implies, analogously to the original Lucas-Uzawa model[18], a unique BGP with zero growth in the transformed variables. Both capitals and consumption will grow along the BGP at positive growth g_L^* and g_H^* for productivity B_L and B_H, respectively. By applying the condition of BGP to equations (2.34)-(2.37) we obtain the following:

$$g_I^* = \sigma(B_I - \rho) \tag{3.1}$$

$$u_I^* = 1 - \frac{\sigma(B_I - \rho)}{B_I} \tag{3.2}$$

$$q_I^* = \frac{\delta + B_I}{\alpha} - \sigma(B_I - \rho) - \delta \tag{3.3}$$

$$x_I^* = \left(\frac{\alpha A}{\delta + B_I}\right)^{\frac{1}{1-\alpha}} u_I^* \tag{3.4}$$

where $I \in \{L, H\}$.

[16] The same transformation is used in (Mulligan and Sala-i Martin 1993) and (Benhabib and Perli 1994).

[17] The presence of globally increasing returns can also create another kind of problem as in the original (Lucas 1988b) model. In that model human capital externalities in goods production can (for realistic values of the externality factor) cause a "continuum" type of indeterminacy with the implication that the model can exhibit multiple growth rates for given physical and human capital endowments [see (Benhabib and Perli 1994) and (Xie 1994)]. This rather unusual property can be removed by applying upper-bounded increasing returns as is done here.

[18] See (Mulligan and Sala-i Martin 1993).

According to equation (3.1), consumption, physical and human capital grow with a positive balanced growth rate $g_I^* > 0$ only if productivity in the education sector is sufficiently high and/or people are not too impatient. Combined with equation (3.2), we can see that a positive growth rate is possible only if some fraction of the endowed time is spent on education $u_I^* < 1$[19]. Interestingly, for an economy with a high degree of relative risk aversion (i.e. low intertemporal elasticity of substitution σ) where people prefer to smooth the consumption path, the resulting balanced growth rate will be lower while a thriftier and more patient society will enjoy higher growth rates. Equation (3.4) shows that at the steady state net returns from both capitals are identical (i.e. $F_k^* - \delta = B_I$). Moreover, we can see that the parameters which increase returns to physical capital (i.e. productivity A and capital share α) also increase the capitals ratio.

Because of the presence of externalities in the education sector, the sensitivity of the model to changes in the productivity of human capital B_I will be very important. Therefore, we take the derivatives of the steady state values with respect to B_I

$$\frac{\partial g_I^*}{\partial B_I} = \sigma > 0 \tag{3.5}$$

$$\frac{\partial u_I^*}{\partial B_I} = -\frac{\rho\sigma}{B_I^2} < 0 \tag{3.6}$$

$$\frac{\partial q_I^*}{\partial B_I} = \frac{1}{\alpha} - \sigma = \begin{cases} > 0, & \alpha < \sigma^{-1} \\ = 0, & \alpha = \sigma^{-1} \\ < 0, & \alpha > \sigma^{-1} \end{cases} \tag{3.7}$$

$$\frac{\partial x_I^*}{\partial B_I} = -\left(\frac{\alpha A}{\delta + B_I}\right)^{\frac{1}{1-\alpha}} \left[\frac{u_I^*}{(1-\alpha)(B_I + \delta)} + \frac{\sigma\rho}{B_I^2}\right] < 0. \tag{3.8}$$

Equation (3.5) states that the higher the productivity of the education sector is, the higher the balanced growth rate of the economy will be. Growth increases as society's willingness to substitute today's consumption for tomorrow's increases (i.e. σ is higher). Equation (3.6) demonstrates that in an economy with a more effective education sector people study more and work less. This effect is stronger the less patient people are (i.e. where ρ is bigger) and the greater their willingness is to substitute consumption across time. However, the effect diminishes with the productivity of the education sector. Not surprisingly, human capital will be more abundant relative to physical capital on the BGP if the education sector is more productive, as we can see from (3.8). The dependence of the consumption-to-physical-capital ratio on the effectiveness of the education sector given by (3.7) is non-monotonic. In an economy where people strongly prefer to smooth consumption (i.e. the intertemporal elasticity of substitution is smaller than the inverse of the capital share parameter $\sigma < \alpha^{-1}$) the ratio of consumption to physical capital will be higher when the productivity of the education sector is higher.

[19] The situation when people spend all their time in schools and do not work seems to imply maximum growth in human capital. This is not feasible, however, on the BGP because the output F in equation (2.16) is zero and physical capital is, therefore, consumed and steadily declines rather than growing at the rate $g_I^* > 0$. This implies that $u_I^* > 0$.

Notice that a higher B_I means relatively less abundant physical capital and therefore $(c/k)^*$ has to rise to keep consumption at the same level[20]. On the contrary, an economy where people are less willing to smooth consumption ($\sigma > \alpha^{-1}$) will end up with a lower ratio of consumption to physical capital[21].

When $I = H$, the above equations (3.1)-(3.4) characterize the BGP related to high productivity B_H. When $I = L$, the BGP is related to low productivity B_L. However, the global behavior of the model is such that the low BGP will never be reached, even if we allow for infinite time, because a growing economy will always reach the region of higher productivity in finite time. The whole system, therefore, has only one global BGP, the path related to the high productivity level. The BGP related to lower productivity is a fictive BGP or *quasi-balanced growth path* (QBGP) which works only as a temporary attractor in (k, h) space.

4. Transitional Dynamics I: Stages of Growth

Using the above results we can divide the transitional dynamics into three stages: *the stage of low growth*, before the productivity miracle occurs when productivity is low, $B(h) = B_L$, and the economy develops according to (2.41)-(2.43) with $B_I = B_L$; *the take-off stage*, when the miracle happens and the economy switches from low productivity to high productivity in the education sector; and *the stage of high growth*, which occurs in the aftermath of the productivity miracle when productivity is high, $B(h) = B_H$, and the economy develops according to (2.41)-(2.43) with $B_I = B_H$.

In order to study the transitional dynamics of the model it is useful to know more about the local behavior around the BGPs. The standard approach is to derive a log-linear approximation of the model around the BGP. Taking the first-order Taylor expansions of equations (2.41)-(2.43) in logarithmic variables $\tilde{x} \equiv \ln x, \tilde{q} \equiv \ln q$, and $\tilde{u} \equiv \ln u$ at the particular steady states, we obtain the following system of linear differential equations expressed in matrix form (see Appendix 1)

$$(4.1) \qquad \begin{bmatrix} \dot{\hat{x}}_I \\ \dot{\hat{q}}_I \\ \dot{\hat{u}}_I \end{bmatrix} = \begin{bmatrix} -\varepsilon_I & -q_I^* & q_I^* \\ -(1-\sigma\alpha)\varepsilon_I & q_I^* & (1-\sigma\alpha)\varepsilon_I \\ 0 & -q_I^* & B_I u_I^* \end{bmatrix} \begin{bmatrix} \hat{x}_I \\ \hat{q}_I \\ \hat{u}_I \end{bmatrix}$$

where $\varepsilon_I = -\frac{(1-\alpha)}{\alpha}(B_I + \delta) < 0$, $\hat{x}_I \equiv \tilde{x} - \tilde{x}_I^*$, $\dot{\hat{x}}_I \equiv \dot{\tilde{x}}$ and $I \in \{L, H\}$. This system together with the accompanying initial, connecting and final conditions (2.38)-(2.40) serves as a piece-wise log-linear approximation of the model.

As is shown in Appendix A.1, each system has one negative and two positive eigenvalues, $\lambda_{I,1} = \varepsilon_I < 0$, $\lambda_{I,2} = q_I^* > 0$, and $\lambda_{I,3} = B_I u_I^* > 0$ again with $I \in \{L, H\}$. Thus

[20] This situation refers to the "normal case" in (Caballe and Santos 1993) and (Ladron-de Guevara, Ortigueira, and Santos 1997).

[21] This situation refers to the "paradoxical case" in (Caballe and Santos 1993) and (Ladron-de Guevara, Ortigueira, and Santos 1997).

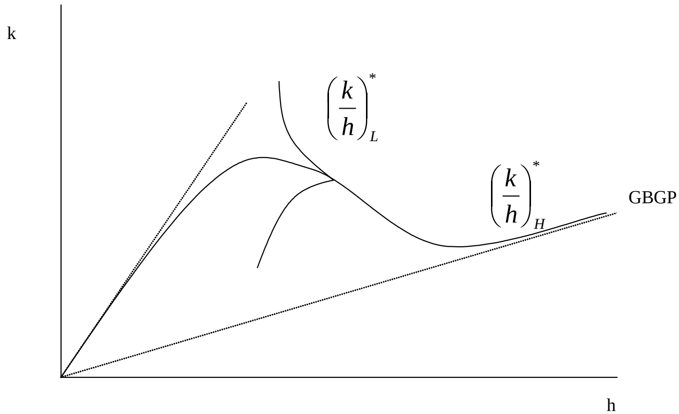

FIGURE 2. Generalised balanced growth path.

the system with two control variables, q, u, and one state variable, x, is saddle-path stable[22].

Stage of High Growth. (See Proposition 10 in Appendix A.1.) If the logistic function has the step-like shape given by (2.4), then the productivity of the education sector of any economy whose average level of human capital is higher than the critical level $\hat{H}$ will be given by B_H. The TVC conditions given in (2.40) imply that the economy will move along the saddle path related to the high BGP. According to the approximation of the behavior along the saddle path given in Proposition 10 in Appendix 1, we can conclude that the policy functions for u and q are both upward-sloping, if the share of physical capital in the goods production function α [or the elasticity of the marginal product of labor with respect to capital as (Ladron-de Guevara, Ortigueira, and Santos 1997) pointed out] is larger than the inverted value of the intertemporal elasticity of substitution σ^{-1} ("paradoxical case"). They are both downward-sloping, if the share of physical capital in the goods production function α is smaller than the inverted value of the intertemporal elasticity of substitution σ^{-1} ("normal case").[23] I examine only the "normal case" in the rest of the paper, when the product of capital share and the intertemporal elasticity of substitution is smaller than one, since I consider it to be more realistic.

Low Growth Stage. (See Proposition 11 in Appendix A.1.) The stage of low growth occurs before the productivity miracle happens at time T. For the analysis of transitional dynamics during the stage of low growth, the connecting conditions given in

[22]Because the eigenvalues of the system do not change their signs for any feasible values of parameters there are no indeterminacies in the model.

[23]The policy functions do not depend on x if $\alpha = \sigma^{-1}$.

(2.39), instead of the TVC conditions, are relevant. As previously stated, the connecting conditions are such that at the moment of the miracle there will be no jump in consumption (i.e. the projection of the path of the economy expressed in (x, q, u) onto the (q, x) plane must lie, at moment T, on the projection of the saddle path related to high productivity). As the connecting condition regarding the time devoted to work shows, the development of working time is composed of two parts: the adjustment to u_{T_-} before the miracle and the jump from u_{T_-} to u_{T_+} on the saddle path related to high productivity at the moment of the miracle (see take-off stage below). According to the Turnpike theorem, if the economy starts with a very low level of human capital, the capitals ratio will move "along" the saddle path close to the steady state ratio. Only when the level of human capital approaches very close to the critical level $\hat{H}$ can an adjustment, approximately along the unstable saddle paths, to u_{T_+} begin. Of course, if the economy has the level of human capital not too far from $\hat{H}$, the transition will be quite different from the saddle path (see below).

Take-off Stage. The take-off stage collapses to the jump in total factor productivity in the education sector from the low level (B_L) to the high level (B_H). Accordingly, the time devoted to work chops further from u_{T_-} to the saddle path, u_{T_+}, which is related to high productivity B_H at the moment of the productivity miracle.

Using the characterizations of the BGP and QBGP in (3.1)-(3.4) we can define *a generalized balanced growth path* (GBGP) as a path which connects the QBGP related to the stage of low growth with the BGP related to the stage of high growth. From the analysis of steady states we know that the slope of the GBGP is determined by the QBGP and the BGP ratios of physical to human capital in the regions associated with the particular level of productivity. The GBGP in (k, h) is depicted in Figure 2 where the adjustment path of capitals between the QBGP and the BGP can be seen. If the economy is initially off the BGP or the QBGP it will first have tendency to move to the GBGP and then along it. Thus, the transitional dynamics of the model can be decomposed into two kinds of transitions: (i) transitions arising due to imbalances in the levels of the stock of capital between the two sectors with respect to the particular steady state (similar to the original Lucas model)[24] and (ii) transitions related to the level of human capital relative to its critical level.

4.1. Development through Stages. The transitional behavior of an economy can now be summarized for different initial levels of human and physical capital. I will show it in the projection of the phase plane $(\tilde{x}, \tilde{q}, \tilde{u})$ into the $(\tilde{x}, \tilde{u})$ plane in Figure 3. There is a "fictive" steady state L related to low growth g_L^* with a stable downward-sloping saddle path $U(\tilde{x}; B_L)$ and a steady state H related to the high growth rate g_H^* with saddle path $U(\tilde{x}; B_H)$. Steady state L is located to the northeast of steady state H, as follows from the results of the effects of a change in productivity given by (3.1)-(3.4) in Section 3, where higher productivity in the education sector leads to a relatively higher level of human capital $\tilde{x}_H^* < \tilde{x}_L^*$ and more time spent in school

[24]For the analysis of transitional dynamics the time elimination method has been used in (Mulligan and Sala-i Martin 1993) and the projection method [introduced for the application in economics by (Judd 1990)] in (Kejak 1993).

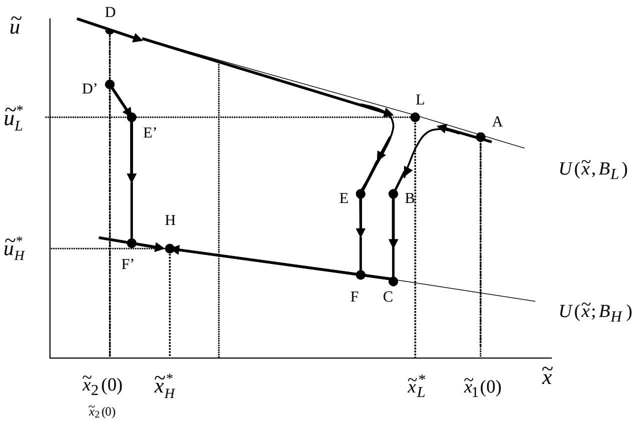

FIGURE 3. Development through stages in plane $(\tilde{u}, \tilde{x})$.

$\tilde{u}_H^* < \tilde{u}_L^*$. According to (1.17) and (1.23) in Appendix A.1, the saddle paths are downward sloping when the intertemporal elasticity of substitution is smaller than the inverse of the capital share[25], $\sigma < \alpha^{-1}$, with the slope decreasing as the productivity increases. As was described in (Mulligan and Sala-i Martin 1992), the downward slope of the policy functions is caused by the fact that people with a strong desire to smooth consumption prefer, when they are poor and have a low level of physical capital, to build their physical capital through increased work effort rather than through increased savings. Thus, as the level of physical capital increases ($\tilde{x}$ grows), they work less ($\tilde{u}$ declines). Let us now examine two typical transitions.

A less developed country with an initial relative lack of human capital. Assume that the economy is initially more abundant in physical then human capital (i.e. $x_H^* < x_L^* < x_0^1$) but is less developed $h_0^1 \ll \hat{H}$[26]. Because $k_0^1/h_0^1 > (k/h)_L^*$, the return to human capital exceeds that to physical capital, motivating people to spend more time on education and thus human capital grows faster than physical capital. The level of human capital is lower than the critical value $\hat{H}$ and therefore the productivity of the education sector is low (B_L). Thus, the economy is initially at point A in Figure 4 and moves "along" saddle path $U(\tilde{x}, B_L)$ toward the "fictive" steady state L, related to the low productivity of education. Since the level of human capital was initially much

[25] As was already mentioned I will confine my analysis only to this case for two reasons. First, it seems that the chosen case is more empirically relevant and second, the analysis of the other case is completely symmetrical to the first case.

[26] We assume here a very low initial level of human capital. Thus, the process of the accumulation of human capital to the level $\hat{H}$ will take a relatively long time and the saddle path, according to the Turnpike theorem mentioned above, will be a good approximation for the development of the economy up to the point where the level of human capital is close to $\hat{H}$.

lower than $\hat{H}$, the economy will move very close to L. This is what we call *the stage of low growth* with the growth rate g_L^*. As the economy continues to develop, the level of human capital grows. Thus, after some finite time, as the economy approaches the critical level of human capital, a fast adjustment process[27] "along" the unstable saddle paths will start (point B). When $\hat{H}$ is reached the sudden increase in the productivity of the education sector increases the return to human capital, motivating people to devote even more time to study. Therefore u jumps to point C on the high productivity saddle path, *the "take-off" stage*. Afterwards, the economy, with a more productive education sector, continues to move along the saddle path $U(\tilde{x}, B_H)$ to the new steady state H with the high growth rate g_H^* characterizing *the stage of high growth*.

A less developed country with initially relatively less abundant physical capital. Now consider an economy that initially has relatively less abundant physical capital $x_0^2 < x_H^* < x_L^*$ and is again less developed $h_0^2 \ll \hat{H}$. Symmetrical to the above case (because $k_0^2/h_0^2 < (k/h)_L^*$), the return on physical capital is higher than that on human capital and people work harder in order to accumulate physical capital faster than human capital. Because the level of human capital is lower than critical value $\hat{H}$, the productivity of the education sector is low (given by B_L). Thus, the economy is initially at point D. The level of human capital is initially much lower than $\hat{H}$ and the economy moves "along" saddle path $U(\tilde{x}, B_L)$ toward the "fictive" steady state[28] L, *the stage of low growth*. As the economy continues to develop, the level of human capital grows. Again an adjustment process will start before the economy manages to reach the critical level of human capital. Once the adjustment process is completed (point E), human capital will reach the critical value and the subsequent sudden increase in the productivity of the education sector will increase the return to human capital, motivating people to increase the time devoted to study. Therefore, u jumps to point F in Figure 4 on the high productivity saddle path, this is *the "take-off" stage*. Thereafter, the economy, having a more productive education sector, proceeds along the saddle path $U(\tilde{x}, B_H)$ to the new steady state H with the high growth rate g_H^* characterizing *the stage of high growth*.

If the level of human capital is initially low, the stage of low growth is prolonged (the transition via F) and the economy with low incentives to study gradually loses its advantage in relatively more abundant human capital. After reaching the critical level of human capital the increased return on education motivates people to study more. This increased investment in education causes a decline in $\tilde{x}$ from F to H. Thus there is an overshooting in the relative level of physical to human capital during the whole transition from D to H. On the other hand, if the economy is initially not very far from the critical level of human capital there will be a large adjustment before the productivity jump and the economy will not come very close to L. In this case there

[27] However, there is already some adjustment in the economy before it reaches the critical value. Because people know all future prices perfectly, they have sufficient time to adjust to the productivity miracle and smooth their consumption stream.

[28] See footnote 26.

will be no (or very little) overshooting during the transition from D' via F' to the new steady state H.

5. Transitional Dynamics II: Underdevelopment Traps and Productivity Slowdown

5.1. Extensive Growth and Underdevelopment Traps. In this section we will study such situations where agents would be willing to spend more time working than they have available. Due to the binding constraint $u \leq 1$, no time is devoted to education and the engine of endogenous growth is stopped with adjustments only in physical capital. We call this development *the stage of extensive growth.* Whether this situation is permanent and the economy gradually approaches a stage of zero growth (i.e. an *underdevelopment trap*) or it is only temporary and the growth of human capital reappears after a certain period of time, will be analyzed in this section.

The necessary condition for the existence of a stage of extensive growth, when there is no accumulation in human capital, is stated in the following Proposition.

PROPOSITION 2. *(Extensive Growth) If the return to study is lower than the return to work ($(1-\alpha)Ax^{\alpha}\lambda_I > \mu_I B_L$), then nobody is willing to study ($u = 1$) and the only capital which adjusts is physical capital.*

PROOF. The proof is straightforward from (2.21). □

When the halt to human capital accumulation during extensive growth is permanent, the transition process finishes at a zero growth steady state and the economy is caught in an underdevelopment trap. The necessary condition for the existence of such an underdevelopment trap is that the education efficiency parameter is smaller than the discount rate as Proposition 3 claims below.

PROPOSITION 3. *(Underdevelopment Trap) An economy can be trapped in the stage of zero growth only if $B_L \leq \rho$, which is characterized by the following conditions*

$$g^*_{UT} = 0 \tag{5.1}$$

$$u^*_{UT} = 1 \tag{5.2}$$

$$q^*_{UT} = \frac{\delta + B_L}{\alpha} - \delta \tag{5.3}$$

$$x^*_{UT} = \left(\frac{\alpha A}{\delta + B_I}\right)^{\frac{1}{1-\alpha}} \tag{5.4}$$

PROOF. The proof is straightforward from (3.2)-(3.4). □

Whether an economy really falls into an underdevelopment trap depends on the initial conditions for physical and human capital, in addition to the above necessary condition. As shown in Propositions 4 and 5 below, we can distinguish two transitions which result in an underdevelopment trap. The first is the transition to an underdevelopment trap via the process of extensive growth. The necessary and sufficient conditions for the transition to an underdevelopment trap via extensive growth are provided by the following proposition.

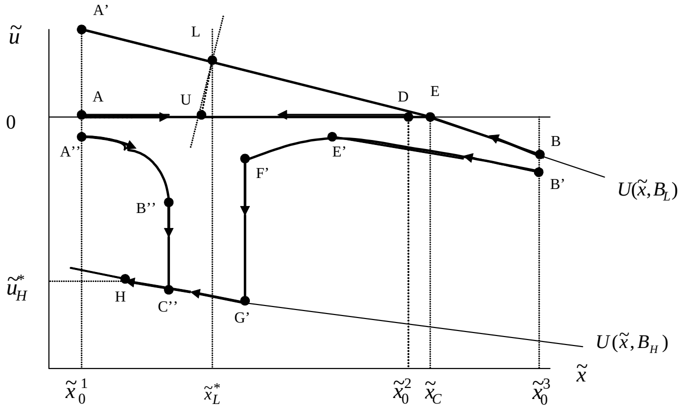

FIGURE 4. Seemingly sustainable growth and underdevelopment traps in plane $(\tilde{u}, \tilde{x})$.

PROPOSITION 4. *(Underdevelopment Trap with Extensive Growth[29]) The economy will reach an underdevelopment trap via adjustments only in physical capital*

- *with a rising level of physical capital: if, and only if,* $B_L < \rho$, $\tilde{x}_0 < \tilde{x}_L^*$, *and* $h_0 < H_C^-(\tilde{x}_0) < \hat{H}$ *where* $H_C^-(\tilde{x}_0) = \inf\{h_0 : \tilde{u}_0(h_0 e^{\tilde{x}_0}, h_0) < 0\}$;
- *with a declining level of physical capital: if, and only if,* $B_L < \rho$, $\tilde{x}_L^* < \tilde{x}_0 < \tilde{x}_C = \tilde{x}_L^* - \frac{\tilde{u}_L^*}{\kappa_L}$, *and* $h_0 < H_C^+(\tilde{x}_0) < \hat{H}$ *where* $H_C^+(\tilde{x}_0) = \inf\{h_0 : \tilde{u}_0(h_0 e^{\tilde{x}_0}, h_0) < 0\}$,

where $\tilde{u}_L^* = \log(1 + \sigma(\frac{\rho}{B_L} - 1))$, $\tilde{x}_L^* = \frac{1}{1-\alpha} \log\left(\frac{\alpha A}{\delta + B_L}\right) + \tilde{u}_L^*$, κ_L *is the slope of the policy function given by equation (1.21) from Proposition 11 in Appendix A.1.* $\tilde{x}_0$, $\tilde{u}_0 = 0$, *and* h_0 *are initial conditions of the log of the capital ratio, log of working time, and human capital, respectively.*

Assume an economy which is initially not very developed, i.e. $h_0^1 < H_C^-(\tilde{x}_0^1)$, and which has relatively more abundant human capital than physical capital ($\tilde{x}_0^1 < \tilde{x}_L^*$). These two conditions imply that the returns to education are much lower than those to work and people have an incentive to work very hard, $u > 1$, (see point A' in Figure 4). Because of the constraint on their time endowment, however, they cannot spend more time at work than $u = 1$ (point A). Spending all their time at work enables them to produce enough to cover their consumption and invest their savings in physical capital.

[29]Due to the lack of space the proof was omitted here and it is available upon the request from the author.

The growing physical capital leads to diminished returns because the effectiveness of labor is fixed. Despite the declining returns people are still motivated to save (till $r > \rho$) and only to work because the returns to the investment in education are even lower ($B_L < \rho$). Thus the economy starts at A and moves to the underdevelopment steady state U with a zero growth rate.

However, as Proposition 4 suggests and Figure 4 demonstrates, there exists a threshold of human capital $H_C^-(\tilde{x}_0^1)$ lower than $\hat{H}$ which enables the economy to escape from the stage of extensive growth with an underdevelopment trap (see trajectory A"B"C"H in Figure 4).

A similar phenomenon occurs when the economy is initially not very developed ($h_0^2 < \hat{H}$) and has relatively more abundant physical capital ($\tilde{x}_0^2 < \tilde{x}_L^*$) but the level of physical capital is still so low $\tilde{x}_0^2 \leq \tilde{x}_C$ that the return to study is smaller than the return to work and, therefore, nobody wants to study. The return on physical capital in this economy with a fixed level of human capital is so small that $r - \rho < 0$ and people have no incentive to save and so consume their physical capital instead. This situation persist until the economy reaches its steady state U with zero growth.

The second kind of transition is the transition to an underdevelopment trap via intensive growth. This transition occurs when the economy initially exhibits intensive growth (i.e. positive growth in both capitals), but is ultimately not successful in escaping from the underdevelopment trap. I call this phenomenon *seemingly sustainable growth.*

PROPOSITION 5. *(Underdevelopment Trap with Seemingly Sustainable Growth*[30]*) The economy will reach an underdevelopment trap even when it has experienced positive growth in both types of capital if, and only if,* $B_L < \rho$, $\tilde{x}_0 > \tilde{x}_{UT}^* = \frac{1}{1-\alpha}\log\left(\frac{\alpha A}{\delta + B_L}\right)$, $h_0 < H_C^{++}(\tilde{x}_0) < \hat{H}$ *with* $H_C^{++}(\tilde{x}_0) = \inf\{h_0 : \sup\{\tilde{u}_t : \tilde{u}_t(h_0 e^{\tilde{x}_0}, h_0) < 0 \text{ and } 0 \leq t < T\}\}$, *and* $\left(\frac{\partial u}{\partial x}\right)_0 = \frac{B_L u_L^*(\tilde{u}_0 - \tilde{u}_L^*) - q_L^*(\tilde{q}_0 - \tilde{q}_L^*)}{\varepsilon_L(\tilde{x}_0 - \tilde{x}_L^*) + q_L^*(\tilde{u}_0 - \tilde{q}_0 + \tilde{q}_L^* - \tilde{u}_L^*)} < 0$ *where* $\tilde{u}_L^* = \log(1 + \sigma(\frac{\rho}{B_L} - 1))$, $\tilde{x}_L^* = \frac{1}{1-\alpha}\log\left(\frac{\alpha A}{\delta + B_L}\right) + \tilde{u}_L^*$, $\tilde{q}_L^* = \log(\frac{B_L + \delta}{\alpha} - \sigma(B_L - \rho) - \delta)$ *and* $\varepsilon_L = \frac{(1-\alpha)}{\alpha}(B_L + \delta)^{1-\alpha}$. $\tilde{x}_0$, $\tilde{u}_0$, $\tilde{q}_0$ *and* h_0 *are initial conditions of the log of the capital ratio, log of working time, log of the consumption-capital ratio, and human capital, respectively.*

Assume an economy that is initially very undeveloped, i.e. $h_0^3 < H_C^{++}(\tilde{x}_0^3)$ and with the relative abundance of physical capital, with this abundance being sufficiently high $\tilde{x}_0^3 > \tilde{x}_C$ (see point B in Figure 4). These two conditions imply that the return to human capital is initially higher than that to physical capital and people study and work in such a way that human capital grows faster than physical capital. During the transition the difference between the two returns diminishes ($\tilde{u}$ is increasing) as the economy draws closer to its steady state. Such an economy will finally become trapped in a pattern of extensive growth followed by a zero steady state growth at U (see trajectory BEU).

However, if the economy is initially endowed with sufficiently more human capital, i.e. $h_0^4 > H_C^{++}(\tilde{x}_0^4)$, the time of the productivity miracle is close enough that people have

[30]See footnote 29.

an incentive to accumulate at least some human capital even in bad times. That is, when the growth of output is close to zero and people produce less than they consume (around the inflexion point E' in Figure 4) eventually they will be rewarded by the productivity miracle at F'. The transition will then continue with high growth from G' to H.

Another kind of behavior is when economies initially exhibit extensive growth but later take off to a stage of high growth. The conditions for such situations are given by the following proposition.

PROPOSITION 6. *(Temporary Underdevelopment Trap[31]) The economy will experience temporary extensive growth via adjustments in physical capital if, and only if, $B_L > \rho$, $\tilde{x}_0 < \tilde{x}_C = \tilde{x}_L^* - \frac{\tilde{u}_L^*}{\kappa_L}$, and $h_0 < H_C(\tilde{x}_0) < \hat{H}$ where $H_C(\tilde{x}_0) = \inf\{h_0 : \tilde{u}_0(h_0 e^{\tilde{x}_0}, h_0) < 0\}$, with $\tilde{u}_L^* = \log(1 + \sigma(\frac{\rho}{B_L} - 1))$, $\tilde{x}_L^* = \frac{1}{1-\alpha} \log\left(\frac{\alpha A}{\delta + B_L}\right) + \tilde{u}_L^*$, κ_L is the slope of the policy function given by equation (1.21) from Proposition 11 in Appendix A.1. $\tilde{x}_0$, $\tilde{u}_0 = 0$, and h_0 are initial conditions of the log of the capital ratio, log of working time, and human capital, respectively.*

If the quasi-steady state related to the lower productivity of the education sector B_L exhibits positive growth, an economy with a low level of human capital $h_0^1 \ll H_C(\tilde{x}_0^1)$ and a ratio of capitals $\tilde{x}_0^1 < \tilde{x}_C$ may experience a return to human capital that is much lower than the return to physical capital and people may have an incentive to work very hard, $u > 1$, (see point A' in Figure 5). Again, because of the constraint on u, they cannot spend more time at work than $u = 1$ and thus the return to study is lower than the return to work, resulting in people working full time and not studying at all. Thus, the economy starts at A and exhibits a stage of extensive growth during which human capital is not accumulated. The increasing level of physical capital leads to a relative decline in the return to physical capital compared to that of human capital. This development also causes a decline in the return to work. It is therefore only a question of time until the return is so low that the economy starts to accumulate human capital again. The growth in the level of human capital means that, given time, the economy will inevitable reach the critical level of human capital and take off. Therefore, extensive growth is only temporary.

A summary of the transitional dynamics with underdevelopment traps is given in Figure 6 where the various transitions are depicted in plane (k, h). The main notations are the same as in Figure 2. In addition line x_C splits the shadowed area related to the less developed economy between 0 and $\hat{H}$ into two parts. The first part, above line x_C, where returns to study are equal to returns to work (area SS in Figure 6), represents the area of seemingly sustainable growth. An economy, which initial conditions lie inside of SS, will initially exhibit growth in both capitals till it crosses line x_C into area UT. The second part of the shadowed area (area UT in Figure 6), where returns to study are lower than returns to work and the time constraint is binding, is the area of underdevelopment traps with extensive growth. As we can see the transition paths

[31]See footnote 29.

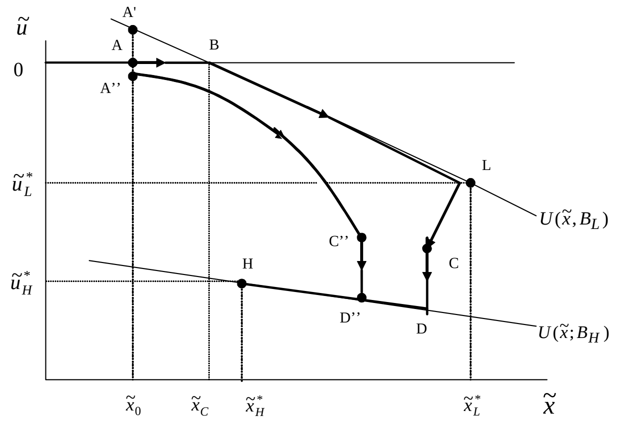

FIGURE 5. Temporary extensive growth in plane $(\tilde{u}, \tilde{x})$.

inside of these boundaries are vertical lines (i.e. there is no accumulation of human capital) converging to line x_L^* representing underdevelopment traps. The underdeveloped economies in the areas to the left of $\hat{H}$, above the curve AB and below the curve DE are those that will not fall into the underdevelopment trap. The behavior of economies with a seemingly underdevelopment trap is illustrated in Figure 7, where the shadowed area SU represents the area of extensive growth.

As demonstrated above, the relationship between the productivity parameter in the education sector and the subjective discount rate is critical for the existence of multiple equilibria or a quasi-steady state. It is represented in the form of a "bifurcation diagram" in Figure 8, which expresses the dependence of the growth rate at steady states on the subjective discount rate ρ[32] for the two given values of productivity in the education sector B_H and B_L.

Propositions 4 and 5 imply that there is one BGP equilibrium with a high growth rate, and one quasi-balanced growth path equilibrium QBGP with a low growth rate for low values of the time preference parameter, $\frac{(1-\sigma)\sigma}{(1-\sigma)\sigma+1}B_H < \rho < B_L$. Likewise, there are two BGP equilibria (one with a high growth rate and one with a zero growth rate) for high values of ρ, $B_L \leq \rho < B_H$. BGP equilibria are pictured as solid lines and QBGP as bold dashed lines in Figure 8. There is, therefore, one bifurcation point,

[32]We allow parameter ρ to take values only from the interval $(\frac{(1-\sigma)\sigma}{(1-\sigma)\sigma+1}B_H, B_H)$ where the lower boundary determines the highest achievable BGP with a bounded lifetime utility and the upper boundary corresponds to the BGP with zero growth related to high productivity B_H.

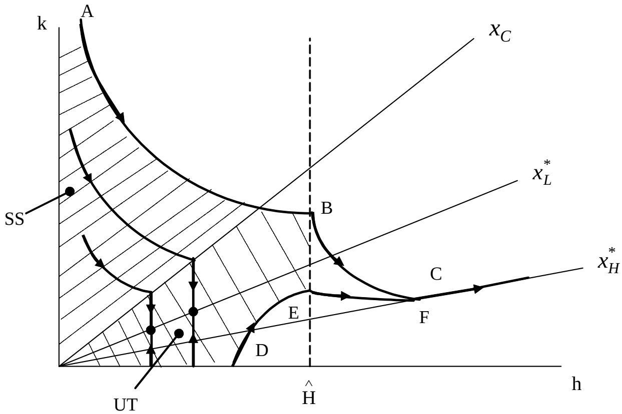

FIGURE 6. Seemingly sustainable growth and underdevelopment traps in plane (k, h).

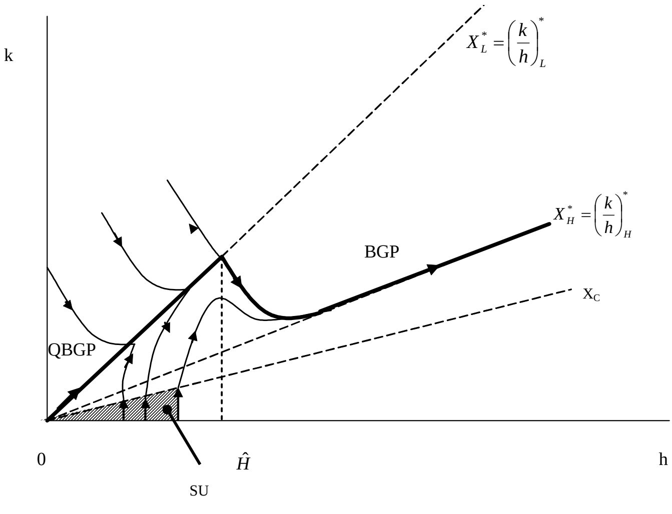

FIGURE 7. Temporary extensive growth in plane (k, h).

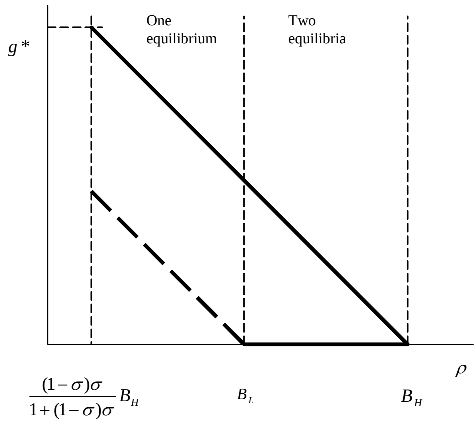

FIGURE 8. Bifurcation diagram.

$\rho = B_L$, at which the economy is structurally unstable and the qualitative behavior of the model changes.

Imagine now that there appears a new productivity miracle (for example a new scientific revolution) such that $B_{HH} > B_H$. What would this imply for the existence of multiple equilibria, underdevelopment traps and quasi-steady states? Clearly, the solid downward sloping line of the balanced growth path in Figure 8 with growth g_H^* would change to a dashed line, implying that the stage of high growth is a quasi-steady state instead of a steady state[33]. Thus, for $\frac{(1-\sigma)\sigma}{(1-\sigma)\sigma+1}B_{HH} < \rho < B_L$, there would be two quasi-steady states related to growth rates g_L^* and g_H^* and one steady state with the very high growth rate g_{HH}^*. Two BGP equilibria, one with a very high growth rate and one with a zero growth rate and one QBGP with an intermediate, high growth rate, will appear for values of ρ, $B_L \leq \rho < B_H$. Ultimately, however, there will be only two BGP equilibria, one with very high growth and one with zero growth for values of ρ, $B_H \leq \rho < B_{HH}$.

5.2. Transition to the High Growth Stage and Productivity Slowdown. The last dynamic phenomenon studied in this paper is related to the transition from a lower growth stage to a higher growth stage, which may be accompanied by *a temporary*

[33]If we assume that the frontier technology, B_{HH}, can be adopted immediately by low developed countries, the solid downward sloping line will disappear.

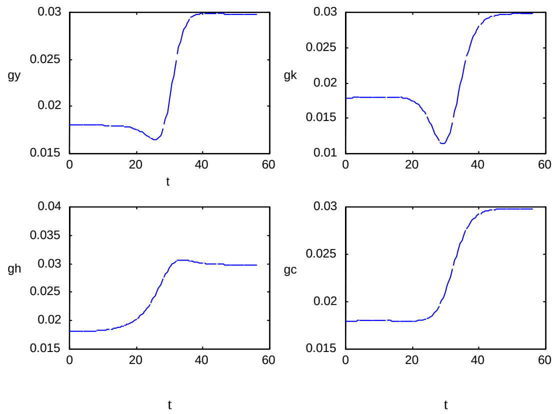

FIGURE 9. Results of model simulation—lower elasticity of intertemporal substitution.

productivity slowdown. To demonstrate this phenomenon, which I am not able to detect from the qualitative analysis of the model, I must numerically simulate the calibrated model.

Let us assume (as is the case at the end of previous section) that our economy is experiencing a new industrial revolution with the appearance of a new frontier for the productivity level B_{HH}[34]. We then calibrate the model using the typical values of parameters found in (Lucas 1988b) and (Mulligan and Sala-i Martin 1993) (see Appendix A.2) with implied values of 2.8% for lower steady state growth and 4% for higher steady state growth.

Looking at the results of the simulation related to a lower value of the intertemporal elasticity of substitution shown in Figure 9, we can observe several distinctive features of the transition. During the early phase of the take-off, at the beginning of the area of increasing returns in the education sector, the path of the economy goes through a narrow region where a temporary decline in the growth rate of physical capital occurs (i.e. growth undershooting in physical capital). The growth rate of human capital is already accelerating at that moment because the agents in the economy, at the moment of take-off, foresee substantially greater future returns to education and, therefore, study more and work less. Lower output together with the agent's preferences for a smooth pattern of consumption (meaning that the intertemporal elasticity of substitution is low) lead to low savings and thus low investment in physical capital.

[34]The similar argument related to the revolution in information technologies has been used in (Greenwood and Yorukoglu 1997).

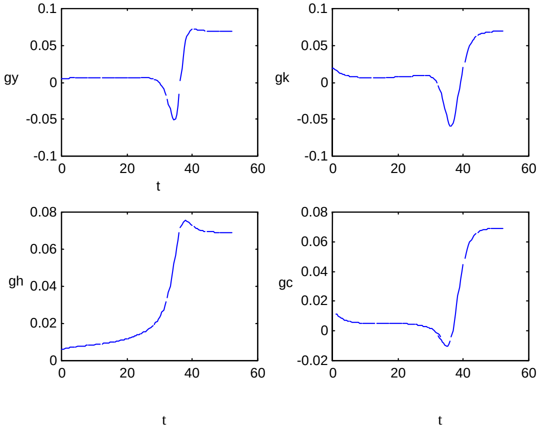

FIGURE 10. Results of model simulation—higher elasticity of intertemporal substitution.

This in turn causes a decline in the growth rate of physical capital. Due to the increased accumulation of human capital and the corresponding higher efficiency of the labor force, people are able to produce more even with a lower fraction of time allocated to work. This results in a general tendency towards higher growth in the economy and the lack of investment in physical capital and the stagnation of consumption growth are only temporary. Once the increasing returns end, there will be a slowdown in the growth rate of human capital (i.e. growth overshooting in human capital).

If the value of the intertemporal elasticity of substitution is high, the agents will be more willing to postpone consumption. Thus there may even appear a temporary decrease in the growth rate of consumption (see Figure 10). With the higher elasticity, the perceived future growth will also be higher and people will have a greater incentive to study. The combination of the low demand for consumption and investment may lead to an apparent decline in output growth (i.e. *a productivity slowdown*) together with an even more profound decline in the growth of physical capital[35]. Again the increased efficiency of the labor force will soon outweigh all the stagnation tendencies in the economy and accelerating growth will start to be a general feature of the economy. After reaching the productivity frontier, increasing returns will be depleted and the economy will converge to the new high steady state growth. The growth overshooting in human

[35]This is consistent with the empirical observations presented in (Bailey and Schultze 1990) which show that the productivity slowdown is accompanied by a decline in the rate of growth of the capital stock.

capital will also be more profound when the intertemporal elasticity of substitution is higher, as can be seen in Figure 10.

6. Conclusions

I have presented a two-sector endogenous growth model with threshold externalities in the process of human capital accumulation. This model can exhibit the two main phenomena of economic development: underdevelopment traps and sustained growth. Without analyzing the transitional dynamics of the model, not much can be determined with regard to its behavior. Even a study of corner solutions has a dynamic dimension in our case when we use an infinite life-time framework. This is the case as well for other dynamic phenomena such as the *temporary underdevelopment trap* and *seemingly sustainable growth*. The *temporary underdevelopment trap* occurs when the economy exhibits slowly declining growth in physical capital and no growth in human capital, followed by a sudden transition to a sustained or quasi-sustained growth path. *Seemingly sustainable growth* occurs when the economy temporarily goes through a transition with positive growth of human capital but is finally trapped in a zero growth stage. Another dynamic phenomenon, a *slowdown in productivity growth*, occurs when the transition from the low growth stage to the high growth stage is not necessarily monotonic and can exhibit a temporary decline in growth rates. Because of increasing returns in the education sector caused by the increasing effect of externalities, people spend relatively more time studying and improving their skills generally. Therefore, the growth of total productivity declines. Given time, the returns from acquiring higher skills will reverse this decline and productivity will grow at a higher rate. Thus our model provides an explanation for the productivity slowdown as a temporary phenomenon during the transition to a stage of higher growth of an economy facing a new "industrial" revolution.

CHAPTER 2

EU Accession, Convergence, and Endogenous Growth

1. Introduction

With the process of transition and integration of many Central and East European (CEE) countries entering its final stages before the accession into the EU, the crucial issue becomes whether they indeed succeed in achieving standards of living pertaining in the European Union, and if so, then in what horizon. The success in this process, commonly referred to as the process of cohesion, is deeply interrelated with the attainment of real convergence, i.e. achieving income per capita levels of the most advanced of the EU countries.

In this paper we analyze qualitatively and quantitatively the potential effect of the accession on the development of several Central and Eastern European (CEE) countries (specifically, the Czech Republic, Hungary, and Poland). In order to achieve the task we design an endogenous growth model of an accession economy which captures the key aspects of development in transition economies (TEs) after the accession to EU and the role of human/knowledge capital in this process. Among the 'accession trajectories' we are mainly interested in the income per capita levels and the quantitative assessment of its speed of convergence to the average EU level. Contrary to the overwhelming use of the results of the 'Barro-type' regression analysis for assessing convergence prospects [see e.g. (Barbone and Zalduendo 1996)], we build a version of a two-sector endogenous growth model of the Uzawa-Lucas style. Since the TEs are typically small open economies (SOEs) and the aspect of their openness is a critical feature of the process of accession, we need to model a small open economy. For this purpose we use here an open version[1] of (Kejak 2003) amended by adjustments costs for investments in physical capital [as in (Turnovsky 1996), and (Osang and Turnovsky 2000)] and with an imperfect credit market in the form of an upward-sloping supply of debt [like in (Osang and Turnovsky 2000)]. The introduction of the second feature into the model is necessary for the existence of balanced growth path (BGP) equilibrium in the open economy with threshold externalities, while the first notion enhances the realism of the calibrated model predictions on the speed of adjustment[2].

[1] In the literature there is only a limited number of endogenous growth models of open economies, they are almost exclusively of the AK-type with physical capital only [e.g. (Frenkel, Razin, and Sadka 1991), (Turnovsky and Bianconi 1992) or (Turnovsky 1996)]. A full analysis of the two-sector endogenous growth open economy model used in this paper can be found in (Kejak 2001).

[2] (Ortigueira and Santos 1997) use adjustment costs in an endogenous growth model of a closed economy to get the speed of convergence consistent with empirical evidence.

Contrary to the neoclassical growth models, which predict convergence, endogenous growth models support the existence of multiple equilibria and divergent behavior of countries[3]. To explain convergence within an endogenous growth framework, the theory has to be amended by the catching-up hypothesis[4] [e.g. (Abramovitz 1986), (Abramovitz 1994)]. This concept assumes that countries must possess a so-called social capability - that includes besides others, human capital, infrastructure capacities and institutional settings - to adopt and use the new technologies successfully. It also implies that investment is only a necessary but not a sufficient condition for convergence. Such an amended endogenous growth model with convergence property still preserves the ability of divergent behavior. However, since we study in this paper the behavior of relatively advanced transition countries the concept of complete divergence (driven by the corner equilibria as in the models of footnote 3) is replaced by a weaker concept of temporary divergence which cannot remove the general tendency of catching up but rather makes the transition non-monotonic and protracted.[5]

The use of the broader definition of human capital as knowledge capital and the explicit modelling of the process of knowledge diffusion can be supported by the perception of the process of accession as a progressive opening of the economy in terms of trade and capital flows, on the one hand, and as a massive technological transfer that enables fast technological catch-up with the technological frontier of the advanced countries, on the other. The knowledge diffusion assumes that there is a frontier of

[3]See for example (Azariadis and Drazen 1990) or (Kejak 2003) and Chapter 1 in this volume who develop an extended Lucas model with these features. Kejak paper differs from (Azariadis and Drazen 1990) (i) by the use of a model with infinitely lived households instead of the overlapping generation model, and (ii) by the explicit derivation of the logistic type of externalities (see Appendix B.1 or Chapter 1).

[4]There is also a recent attempt to include the catching-up hypothesis into the exogenous growth neoclassical framework done by (Parente and Prescott 1991). They show that if there are some barriers to the adoption of private knowledge then countries posses different levels of knowledge and there is no convergence among them even in the standard neoclassical setup.

[5]Obviously, the accession of the candidate countries will have consequences for the employment situation in these economies. The ongoing integration into the European market will make it necessary for companies to achieve higher productivity levels, if they want to become more competitive in the long run. In particular, agricultural and manufacturing industries will be hit. However, new industries will emerge accompanied by the demand for new qualifications and skills. Structural change is a crucial prerequisite for a successful development. However, unemployment might increase due to mismatch and search processes. Furthermore, higher unemployment rates might lead to wage reductions and to a fall in aggregate demand. Companies will then reduce the degree of capacity utilization aggravating the unemployment problem. Moreover, investment activity could decline and thereby lowering the rate of endogenous technical progress. Solving this problem requires beside others an active labour market policy that supports investment in human capital. Such developments will lower the speed of catching up, because they can be considered as a kind of adjustment costs to a new growth path. Since our model already includes adjustment costs for investment and deals with imperfect capital markets, we do not deal with unemployment and lacks of aggregate demand. Hereby, we follow the traditional approach of endogenous growth theory and leave this to further research. However, we are aware that these processes might lead to a longer stagnation in a worst case scenario.

"theoretical knowledge" that is given exogenously and its shifts represent large advances in knowledge, as in the case of an industrial revolution. The economy can approach the frontier via the education of people, which facilitates the adoption and implementation of new technologies. Following (Kejak 2003) and Chapter 1, such a diffusion of technology creates threshold or logistic-type externalities in the knowledge capital accumulation process with possibly increasing returns depending on the average level of knowledge[6].

The model is validated through calibration to stylized facts of the economic development in the EU periphery. In general, the experience of the EU peripheral countries[7] fits the predictions of our model reasonably well. The basic message emerges that the experience of the cohesion countries (and that of the transition countries as well) concurs with the endogenous growth theory in that success in the convergence process is not automatically guaranteed by a simple functioning of economic mechanisms, but rather depends upon a careful policy mix and particular economic circumstances. Hence, the examination of impacts of policy decisions and the evaluation of the particular economic conditions in the cohesion countries at different stages of their development can assist us in empirical validation of the theoretical model which is designed to capture the stylized facts of economic development that the accession Central and Eastern European (CEE) countries will probably experience in the near future. The theoretical model, in turn, improves our understanding of the challenges the transition countries are yet to confront and thus serve as guidance for policy makers in CEE countries in their quest for policy mixes bringing their countries onto a path of balanced long run growth and convergence.

The careful qualitative analysis of the transitional dynamics of the open economy model allows us to describe and study the effect of initial conditions and parameters on the transitional trajectories. Therefore, after the model is calibrated using the available data on a set of three transition countries: the Czech Republic, Hungary, and Poland, we provide not only a quantitative assessment of the speed of convergence to the EU standards but also discuss the appearance of phenomena such as an accession boom or accession recession[8].

The rest of the chapter is organized as follows. In Section 2 we specify the theoretical model of an open economy with imperfect capital mobility, adjustment costs in the accumulation of physical capital and threshold technological externalities in the knowledge sector. The first order conditions and the balanced growth path equilibrium (BGP) are derived in Appendices B.2 and B.3, respectively. The brief analysis of the model using numerical simulations is the content of Section 3. Next, we calibrate the

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[6]There is an alternative way to capture the process of knowledge diffusion in an endogenous growth model done by (Lucas 1993).

[7]See note 16.

[8]It is interesting that such phenomena cannot be observed in a closed economy version of the model. The standard Lucas model [see (Barro and Sala-i Martin 1995)] exhibits monotonic 'convergence property': the growth rate of output out of the steady state always exceeds the steady state output growth rate. Albeit the growth rate of the broadly defined output can be either above or below the steady-state value, the Lucas model still cannot exhibit non-monotonic output level behavior.

model to stylized facts of the economic development in the EU periphery (Section 4) and extend the calibration to the set of chosen transition countries as well (Section 5). In the rest of the section using alternative scenarios with regard to several dimensions we simulate the behavior of the transition countries with different initial conditions. The interplay of the initial conditions and the parameters of the accession generate different transition patterns and also rather different speeds of convergence to the EU average. Section 6 concludes the chapter.

2. Open Economy Model

We consider a two-sector small open economy populated by the continuum of homogenous agents each of whom is endowed with a unit of time, which can be allocated either to the production of good, u, or to the knowledge production, $1-u$. The level of agents' skill/knowledge is h. Thus the effective labor inputs are $l = uh$ and $(1-u)h$ in the goods and knowledge sectors, respectively.

In the goods sector, the economy consists of a large number of identical firms with a constant returns to scale Cobb-Douglas production function $y = F(k,l) = Ak^{\alpha}l^{1-\alpha}, \quad 0 < \alpha < 1$, where k is physical capital.

As in (Kejak 2003) and in Chapter 1 of this volume we use the generalized linear Uzawa-Rosen[9] formulation of the production function for human/knowledge capital assuming that the level of productivity in the education sector, B, capturing the effect of knowledge diffusion, depends on the developmental level of a society expressed by the average level of knowledge[10] H

$$\dot{h} = B(H;\phi)(1-u)h \tag{2.1}$$

with B derived in Appendix B.1 (see also Chapter 1 in this volume) and given by

$$B(H;\phi) = \frac{B_H}{1 + (\frac{B_H}{B_0} - 1)e^{-\phi H}} \tag{2.2}$$

where B_0 is the initial level of productivity related to a zero level of human capital and ϕ is the diffusion parameter that captures institutional barriers to knowledge adoption. Note that there are constant returns to private inputs for any level of productivity in the production function of the knowledge sector and increasing returns to all inputs at the social level for non-constant levels of productivity (i.e. $\partial B(H;\phi)/\partial H > 0$).

The installment of physical capital, which can be imported, is costly with quadratic adjustment costs specified in the following standard way

$$\Psi(\iota, k) = \frac{\psi}{2}\frac{\iota^2}{k} \tag{2.3}$$

with $\psi > 0$.

[9]This specification was used in (Lucas 1988b).

[10]Note that in equilibrium the average level of knowledge capital is equal to the individual knowledge, $H = h$.

The agents in our economy can borrow on the imperfect world capital market, where e.g. creditworthiness of the economy influences its cost of borrowing from abroad at the interest rate which depends on the country's debt-capital ratio, $b = \frac{a}{k}$, i.e.

$$r(b) = r^* + vb \tag{2.4}$$

where a is foreign debt, r^* is the fixed world interest rate and $v > 0$.

In maximizing their life-time utility, the workers seek an optimal life-time consumption, investment and working pattern, $\{c, \iota, u\}$, which they achieve through the appropriate accumulation of financial and human wealth, $k - a$ and h respectively:[11]

$$\max_{\{c_t,\iota_t,u_t\}} \int_0^\infty e^{-\rho t} \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) dt \quad \text{s.t.} \tag{2.5}$$

$$\dot{a} = c + \iota(1 + \frac{\psi}{2}\frac{\iota}{k}) + \delta_k k - y + r\left(\frac{a}{k}\right) a \tag{2.6}$$

$$\dot{k} = \iota \tag{2.7}$$

$$\dot{h} = B(H; \phi)(1-u)h \tag{2.8}$$

$$\lim_{t\to\infty} a_t e^{-\int_0^t r_s ds} \leq 0 \tag{2.9}$$

$$0 \leq u \leq 1 \qquad k_0 > 0, \qquad h_0 > 0, \qquad a_0 \tag{2.10}$$

where the price of the consumption good is normalized to one, ρ is the time preference parameter and θ is the degree of relative risk aversion. Eq. (2.6) defines the current account deficit, $\dot{a}$, where a is foreign debt, c is consumption, $\iota(1 + \frac{\psi}{2}\frac{\iota}{k})$ is the total investment including adjustment costs, $y - \delta_k k$ is net income, and $r(\frac{a}{k})a$ is interest debt payment. The change in capital is equal to the net investment rate ι in Eq. (2.7). According to (2.1) the accumulation of human capital is given by (2.8). Eq. (2.9) refers to the No-Ponzi-game condition. The first order conditions for the agents optimization problem can be found in Appendix B.2.

After introducing transformed variables $s \equiv \frac{c}{k}$, $b \equiv \frac{a}{k}$, and $x \equiv \frac{k}{h}$ with the convenient property of zero growth at steady state we obtain the following equations of the

[11]Whenever possible we suppress time indices to avoid cluttered notation. A dot denotes a time derivative.

dynamic competitive equilibrium[12]:

$$\frac{\dot{x}}{x} = \frac{q-1}{\psi} - B(h;\phi)(1-u) \tag{2.11}$$

$$\frac{\dot{s}}{s} = \frac{1}{\theta}\left(r(b)-\rho\right) - \frac{q-1}{\psi} \tag{2.12}$$

$$\frac{\dot{u}}{u} = \frac{B(h;\phi)-r(b)}{\alpha} + \frac{q-1}{\psi} - \left(B(h;\phi) + \frac{\partial B(h;\phi)}{\partial h}\frac{h}{\alpha}\right)(1-u) \tag{2.13}$$

$$\dot{b} = s + \frac{(q-1)^2}{2\psi} - \frac{F}{k} + r(b)b + \delta_k - \frac{q-1}{\psi}b \tag{2.14}$$

$$\dot{q} = qr(b) - F_k + \delta_k - \frac{q^2-1}{2\psi}. \tag{2.15}$$

Due to the presence of the knowledge externality we need also to include Eqs. (2.1) and (2.2) to capture the development of human capital and the evolution of productivity B, respectively. It implies that transitional dynamics of the model is not reducible to the development of capital ratios, as it is in the original Lucas model [see (Mulligan and Sala-i Martin 1993)][13], and policy functions are general functions of all state variables, as well as sectoral allocation. According to (2.10) in Appendix B.2 the economy asymptotically converges to the BGP when human capital gets sufficiently large.

3. Transitional Behavior of Accession

On a very stylized level, our understanding of the process of accession can be captured by the opening up of the economy to foreign capital markets and by a major improvement in the institutional structure. The latter aspect of the accession is perceived by the accessing country as improved access to the higher knowledge frontier B_H and/or smaller barriers to the knowledge absorption captured by the higher value of diffusion parameter ϕ in Eq. (2.1) and Eq. (2.2).

3.1. No Externalities. To isolate the relevance of the opening of the economy, we will begin with the case of no externalities.[14]

3.1.1. *Case 1.* Let us assume that initially the economy is less abundant in human-knowledge capital than in physical capital, i.e. $(\frac{k}{h})_0 > (\frac{k}{h})^*$ and the debt-capital ratio $b_0 = \frac{a}{k} > 0$ is large. The behavior of the model variables during the transition to BGP is captured in Figure 1. Since physical capital is relatively abundant in the economy, the return to human capital is larger than that to physical capital and investment in human

[12] The log-linearization of the system around the steady state given in Appendix B.3 reveals that the system is saddlepath-stable with three positive and two negative eigenvalues related to the three quasi-control variables, s, u, q, and the two quasi-state variables, x, b, respectively. See (Kejak 2001).

[13] For an extensive analysis of transitional dynamics in the Lucas-Uzawa model and its extended variants see (Caballe and Santos 1993), (Chamley 1993), (Ladron-de Guevara, Ortigueira, and Santos 1997), and (Kejak 2003).

[14] A much more detailed analysis of the transitional dynamics of the model is developed in (Kejak 2001).

capital is larger than in physical equipment. People spend relatively more time in the knowledge sector (u is very small) than in the production process. The large imbalance between the two types of capital and a very high real interest rate (due to the high debt-capital ratio b and according to Eq. (2.4)), create a large incentive to use the channel of capital outflow, while at the same time increased consumption contributes little to the de-cumulation of capital. The ensuing capital outflow is manifested through large negative investment. The whole transition can be decomposed in two phases.[15] During the first one the very fast de-cumulation of capital, i.e. an almost immediate outflow of excessive capital contributes to the current account surplus and a large decline in debt a. Since the decline in capital takes place via capital outflow, the stock of debt declines approximately one-to-one with capital during this phase. Thus, the debt-capital ratio stays roughly constant (see the horizontal trajectory in (x, b)-plane). As the physical capital declines and human capital grows, the marginal product of capital and Tobin's q increase, making people increase their working effort u continuously. The development of the returns to capital is consistent with the decline in the real interest rate r paid on the debt, which is the result of lower debt through capital outflow. During the second phase, when k/h is small and close to its steady state value, a small change in k has a much larger effect on the marginal product of capital and so q increases. Also u responds much faster to these changes and the originally very high growth rate in human capital declines. Since the reduction of k during this phase is larger than the one in a, the debt-capital ratio b falls. Due to the very high growth of productivity, knowledge, and the working time, there is a period of an excessive growth in output along the transition to BGP.

3.1.2. *Case 2.* Let us now consider the initial conditions such that physical capital is relatively scarce and the economy is a net creditor initially, $(\frac{k}{h})_0 < (\frac{k}{h})^*$, $b_0 > 0$. The behavior of the variables during the transition to BGP is captured in Figure 2. Because of the scarcity of physical capital its relative return is higher, the marginal product of capital and q are high and people have an incentive to save more and invest in physical capital. This can be accomplished in one of the three following ways: (i) by increased work effort, (ii) by lowered consumption or (iii) by increased foreign borrowing. Having a relatively low intertemporal elasticity of substitution people prefer to work more, u is high (close to 1), and they spend only a little time on the accumulation of knowledge. As a result, there is almost no growth in human capital. The level of output is small due to the lack of capital and the low productivity. Negative debt makes the interest rate lower than r^* and increases the incentive to finance the large installment of new capital via foreign borrowing. Alternatively, since the domestic return is very high, there is an incentive for a large FDI inflow. Therefore, during rapid transition, the current account is characterized by a large deficit and debt is being accumulated at a high rate. The fast capital accumulation decreases returns to physical capital and simultaneously the domestic interest rate rises (as b increases: a grows faster than k).

[15]This approach is supported by the the fact that the steady state is a saddlepoint with three positive and two negative eigenvalues (see footnote 12). The stable eigenvalue with the large absolute value is responsible for the fast dynamics whereas the other stable eigenvalue represents the slower dynamics.

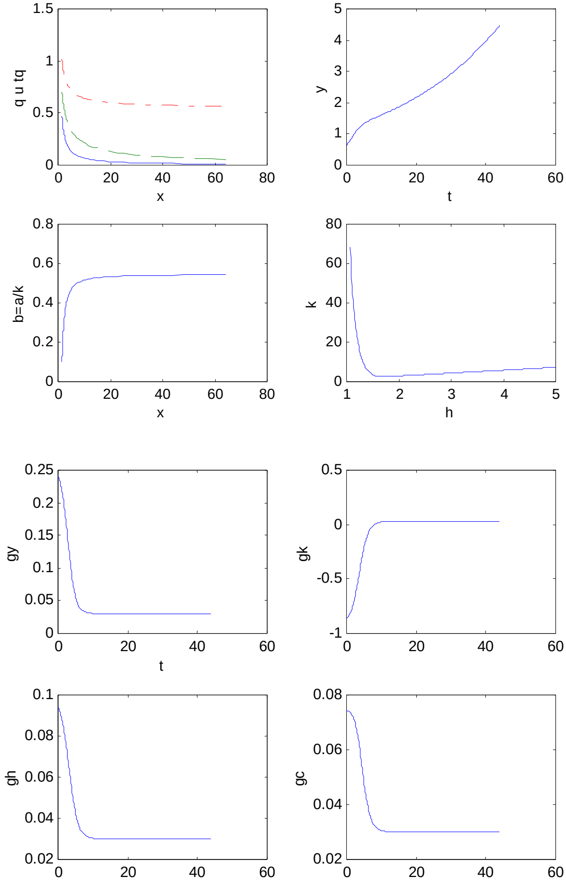

FIGURE 1. Transition dynamics without externality—Case 1.

Because of the initially relatively low level of capital the marginal adjustment costs are large and total investment costs are much larger than net investment.

3.2. Externalities and Catch-Up Factor. In this section we will discuss the behavior of the main model variables when externalities in the knowledge sector are

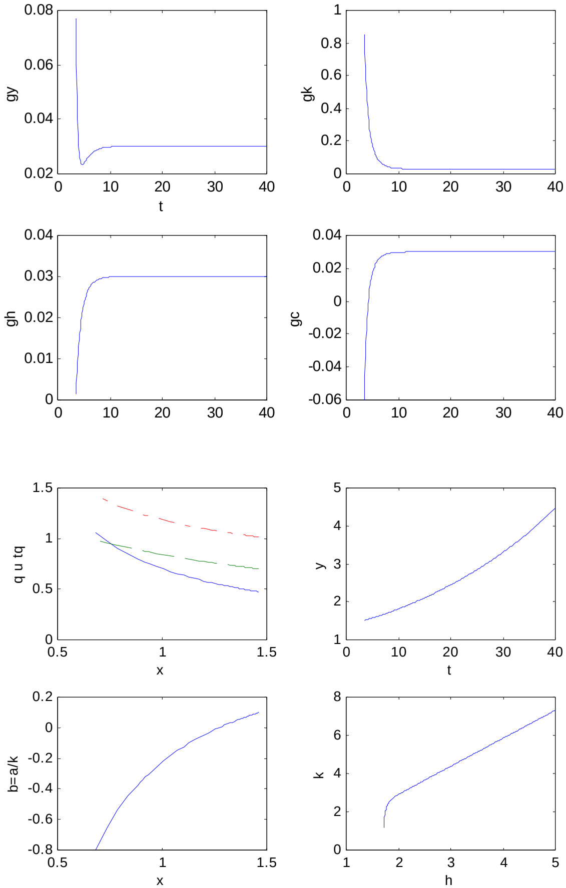

FIGURE 2. Transition dynamics without externalities—Case 2.

present. Let us first start with the economy initially relatively scarce in physical capital, $(k/h)_0 < (k/h)^*$, with initial level of knowledge h_0 such that an externality in the knowledge sector applies, $B(h_0) < B_H$, and the debt is assumed to be initially large, $b > 0$. We can split the transition dynamics of the economy into three phases. During the first phase (similar to case 1 above) the main role is played by relatively high returns to physical capital. Therefore, high investment rates, high working time, high debt, high growth of physical capital and low growth of knowledge are the main features of this phase. However, due to the unlimited access to foreign capital this phase is quite fast (as a matter of fact the phase is not even visible in Figure 3). The driving force of the second phase are the increasing returns in the knowledge sector enabled by the access to the frontier knowledge, which gradually improves productivity of the knowledge sector. As the productivity and returns gradually grow, the composition of physical and human capital changes in favor of human capital. Thus the increase of x slows down and finally reverts in its decline [see the discussion of the low-growth stage and the take-off stage in (Kejak 2003) and in Chapter 1 of this volume]. Tobin's q (capturing the present discounted value of future marginal products of capital) was declining during phase 1 mainly due to the fast decrease in marginal products. However, since the decline in the marginal product of capital slows down and the discount rate r decreases, there is a turning point when q starts to grow again. At this point, investment and capital change the trend and start to increase again. However, returns to knowledge are still increasing (due to the improved access to the frontier) and thus a still higher amount of resources is devoted to knowledge production, u keeps declining and the growth of human capital is faster than that of physical capital. During the first phase of transition there was a decline in output mainly driven by the decrease in working time. Physical output was growing, but the stock of human capital stagnated during this phase. However, the output decline (recession) is only temporary and the economy starts to grow again after it becomes sufficiently productive - during the first phase. The more productive economy also gradually improves its current account and the debt is declining together with the real interest rate.

4. Periphery Countries Accession to EU and Model Calibration

The discussion of the theoretical model in the preceding sections illuminated the factors and decisions that promote either growth and convergence on the one hand, or a relative stagnation and temporary divergence on the other. The exposition also highlighted a plethora of phases and possibilities through which the development of a model economy may take place. The investigation of feasible trajectories of transition countries based on this concept, then, requires a validation by calibrating to stylized facts of their economic development. Although we cannot find in the economic history examples of transitions from centrally planned to market economies, we do have several examples of the integration process into the European Union. One of them, which can provide enough guidance about the likely development paths of the CEE countries and thus help us in calibrating the theoretical model is the recent experience of a subset

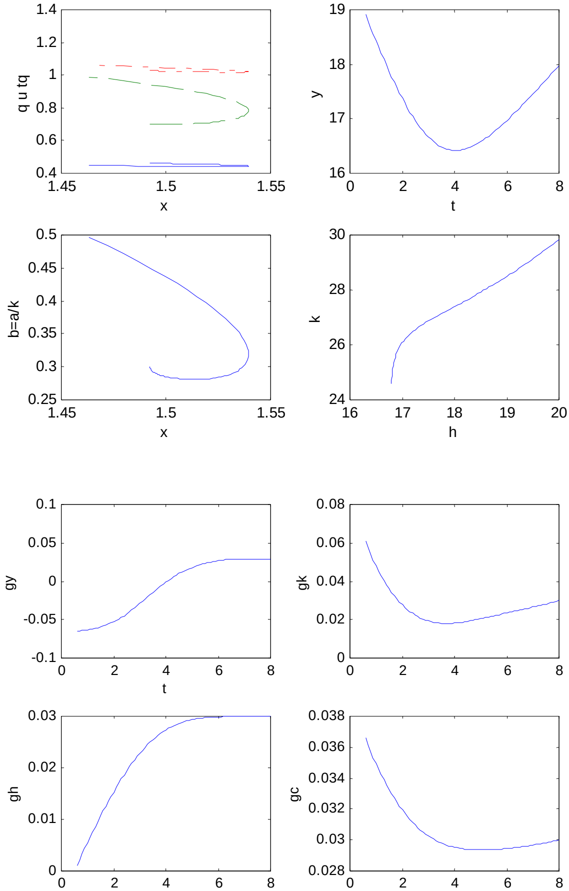

FIGURE 3. Transition dynamics with externalities.

of EU countries, commonly referred to as peripheral[16]. This intuition is supported by several facts.

[16]We use the term 'EU periphery' for countries and regions of the EU, which received 'Objective 1' status within the CSF program in 1989. The term 'periphery' comes from the observation that

First, many of the EU candidate CEE countries have already undergone most of the reforms necessary for accession to the EU. The special features of the transformation process become relatively less important and the economic development is shaped more and more by standard market mechanisms. Second, peripheral countries upon their accession into the EU shared many characteristics (identified in the literature as important for growth) that are similar to the conditions prevailing among the candidate CEE countries. In particular, their income per capita levels were about the same, they were (and still are) relatively small and open economies and their infrastructure was underdeveloped compared to the core of the EU. Third, the economic development of peripheral countries required a massive reallocation of resources on the wake of progressive trade liberalization, rapid technological change, influx of foreign capital (especially in the form of foreign direct investment) and high speed of both physical and human capital accumulation.

These similarities show that the recent economic experience of the EU periphery can indeed become a useful tool for studying the process of convergence of the most advanced among the transition countries. This fact has already been recognized by other studies on transition economies [see for instance, (Barry, Bradley, Kejak, and Vavra 2003), (Kejak and Vavra 1999)]. The calibration of our model is provided in Appendix B.4-6.

4.1. EU Peripheral Countries: Stylized Facts. In a companion paper (Kejak, Seiter, and Vavra 2001a) we examine the post-war development of the cohesion countries, namely Greece, Ireland, Portugal and Spain, hoping to identify the factors responsible for the observed patterns of growth and convergence since the early 1960s. We found that until recently the group as a whole failed to keep up with the predictions of neoclassical growth theory with regard to convergence to the EU average income per capita standards. Most recently only Ireland drifted away from the group and surpassed the EU average.

The pattern of convergence in the EU periphery has not been monotonic, though. Peripheral countries displayed higher than EU average growth rates and partly converged in the period of 1960-1975. Yet, they remained among the poorest economies in Western Europe. This feature prevailed in the decade 1975-1985, when they hardly maintained their relative income per capita standards. It was only in the late 1980s when the process of convergence resumed and the peripheral countries, led by Ireland, began to approach the EU average income levels. It was also then, when the relative homogeneity of the group broke and differences between the forerunners (Ireland and Spain) and a laggard (Greece) became more apparent.

We reviewed most of the basic channels identified by theory as important for long-run growth in order to see whether they are capable of explaining the observed phenomena in the process of peripheral convergence. We found, however, that taken separately, these factors cannot account for most of the observed patterns. In particular, rates of

these regions, originally with 75% or less of the EU average GDP per capita, lie on the western and southern seaboards of the EU. Because of the paucity of data, we nonetheless restrict this meaning to analyzing the actual development of four countries only: Greece, Ireland, Portugal and Spain.

capital accumulation have always been higher and the size of government smaller in the periphery than in the core, yet the countries failed to converge. Enrolment ratios have been rising steadily throughout this period as well as the exposure of the countries to international trade flows. Neither did the combination of the individual growth factors in standard growth regressions prove very helpful. We find that together they predict reasonably well within the group variations in growth rates until the end of the 1980s, but are not capable of explaining the stagnation of 1975-1985. Likewise, they failed to predict differences in individual records over the recent decade, perhaps partly because the measures we have used were no longer adequate.

We vaguely attribute the relative success of these countries since the mid-1980s to the progress of accession and integration within the EU, even though the cohesion countries generally entered the Communities in different periods (notably Ireland being a member since 1973). It is believed that the theoretical framework introduced in this paper and pioneered by (Kejak 1993), (Kejak 2003) (see also Chapter 1 in this volume) enabled us to identify the channels through which the progress of integration could have impacted positively on the recent convergence record in the EU periphery and that it also has the potential of explaining the stagnation of the pre- and early accession periods. If so, the implications for the current transition countries together with relevant policy lessons would immediately follow.

In the quest for the period of technological take-off in the EU periphery we are especially concerned with the periods immediately preceding and following the accession of these countries. We already argued in terms of the model language that the process of accession can manifest itself using two basic channels. On the one hand, as progressive opening of the economy in terms of trade and capital flows, and on the other, as a massive technological transfer that enables fast technological catch-up with the technological frontier of the advanced countries. Hence, it is precisely the developments in the periods around the accession that can be used to validate our theoretical approach.

Taking each channel in turn, we documented in the companion paper (Kejak, Seiter, and Vavra 2001a) that each of the peripheral countries did see their share of exports substantially rise around the time of their entry into the Community. Apart from Greece, all of the countries managed to preserve and increase the shares since then, with Ireland almost doubling its initial value. Higher trade openness went along with improved access to international financial markets. Higher capital mobility envisaged by the process of integration should have enabled better opportunities for intertemporal substitution of consumption as well as for financing the local physical capital build-up. As a consequence of higher local marginal returns to capital, foreign savings would be attracted.

In line with economic theory, current account deficits rose rapidly, financed by various sorts of capital inflows. (Buch 1999) reports that following entry, the structure of capital inflows shifted away from bank loans and investment towards FDI and portfolio investment. In terms of the model language this can be interpreted as a fall in the risk premium on capital investments in the periphery, which is the model variable controlling the degree of openness. With the exception of Greece, all the peripheral countries increased their share of FDI[17] inflows in GDP since accession. Of interest is also the case of Ireland which did not begin to see substantial FDI inflows until the late 1980s, well after its entry into the Community in 1973. This can be partly attributed to protectionist and isolationist economic policies of previous periods, whose legacy survived until some reforms of the 1980s. Hence, we may loosely interpret the mid 1980s as the period when integration came into being, at least in the sense of trade openness and capital liberalization. In the model language, this would also be the moment since when the peripheral countries would have begun to exploit their technological potential.

4.2. EU Peripheral countries: calibration to stylized facts. The purpose of this subsection is to position the economic development of several EU peripheral countries, namely Greece, Ireland, Portugal and Spain, within the highly abstract nature of the theoretical model and to derive model parameters consistent with their individual experiences. In this vein, the experience of the EU periphery will serve as a natural benchmark from which we can quantitatively assess hypothetical trajectories of development for the transition countries of interest: Czech Republic, Hungary, and Poland.

Appendices B.5 and B.6 provide details on the calibration of the main model variables, i.e. mainly human and physical capital stocks. The outcome of the calibration process in terms of the transformed model variables, the consumption-capital and physical-human capital ratios, s and x, respectively, appears in Table 3. Evolution of k and h is also depicted graphically in Figures 4 - 6. In general, the observed patterns conform very well to the theoretical predictions of the model. First, the initial period between 1960 to 1975 was characterized by a high rate of physical capital accumulation alongside with only moderate improvements in knowledge. The trajectory of k versus h is therefore very steep in this period for all peripheral countries. This reflects a relatively low BGP rate of growth through the low productivity in the knowledge

[17] Our accent on FDI, which we share with others [see for instance (Barry and Bradley 1997)], is motivated by a widespread finding that the FDI are carriers of technological transfers and knowledge spillovers [see for instance (Barry and Bradley 1997) or (Coe and Helpman 1995)]. Hence, we take the shares of FDI in GDP as proxies for the progress of technological transfer and catch-up. In sum, both opening to capital market flows and technological transfer began taking place in the EU periphery only in the 1980s, but the process has been very gradual. The speed of the capital account liberalization was hindered by fears of possible financial crises and capital flights that would result from a sudden rise in risk premia imposed on investment in these countries. As a consequence, the process of opening and capital mobility liberalization has been very gradual, often antedating the actual entry into the Community. This accords with the findings of (Buch 1999) who cannot find a statistically significant effect of the EU entry on the degree of capital mobility for a subset of peripheral countries. She too puts the blame on the staggered nature of the process and announcement effects, especially in the case of Spain and Portugal. On the other hand, technological transfer through FDI operated only gradually after the integration process reached a certain irreversible momentum owing to large installation costs. As the cases of Spain and Portugal demonstrate, the inflows of FDI began to rise well before their actual entries into the EU.

TABLE 1. Evolution of knowledge capital per capita in the EU periphery

	Greece	Spain	Ireland	Portugal	Average	Germany
1960	1.7	1.4	3.3	1.8	2.1	3.5
1960-65	1.8	1.5	3.7	2.0	2.2	4.0
1965-70	1.9	1.6	4.2	2.1	2.4	4.5
1970-75	2.0	1.6	4.6	2.3	2.6	5.0
1975-80	2.1	1.7	5.2	2.5	2.9	5.6
1980-85	2.3	1.8	5.9	2.8	3.2	6.3
1985-90	2.5	2.0	7.7	3.4	3.9	6.7
1990-95	2.7	2.3	10.0	4.1	4.8	7.1
1995-2000	3.0	2.6	13.2	5.1	6.0	7.5

Source: own computations.

TABLE 2. Evolution of physical capital per capita in the EU periphery

	Greece	Spain	Ireland	Portugal	Average	Germany
1960	109.0	147.8	156.5	100.3	128.4	302.8
1960-65	124.9	185.9	168.4	125.5	151.2	355.8
1965-70	160.7	262.9	201.9	166.4	198.0	408.0
1970-75	210.8	354.0	249.8	223.7	259.6	461.9
1975-80	253.7	410.0	306.7	263.2	308.4	505.4
1980-85	261.7	431.8	347.7	290.2	332.8	534.6
1985-90	266.9	509.7	368.0	334.7	369.8	576.9
1990-95	274.7	587.1	397.7	396.9	414.1	581.9
1995-2000	308.3	694.1	530.7	493.8	506.7	588.1

Source: own computations.

sector. The fact that the countries did experience fast growth and convergence during this period is due to transitional dynamics along the lines of conventional neoclassical growth models. As a check, we also observe that the German trajectory in this period is much steeper. This corresponds to the intuitive relative positioning of these countries as a technological leader (Germany) and laggards (the periphery). The development across the peripheral countries was also quite uniform, as they maintained their initial relative standing.

TABLE 3. Evolution of consumption to physical capital per capita

	Greece	Spain	Ireland	Portugal	Average	EU15	Germany
1960-65	0.56	0.49	0.56	0.46	0.52	0.41	0.38
1965-70	0.61	0.46	0.55	0.45	0.52	0.40	0.40
1970-75	0.59	0.42	0.49	0.46	0.49	0.39	0.40
1975-80	0.60	0.38	0.46	0.39	0.46	0.39	0.43
1980-85	0.58	0.33	0.38	0.34	0.41	0.38	0.42
1985-90	0.61	0.35	0.40	0.35	0.43	0.40	0.44
1990-95	0.65	0.33	0.43	0.36	0.44	0.37	0.36
1995-2000	0.66	0.31	0.45	0.35	0.45	0.37	0.38

Source: own computations.

Second, we begin to see interesting differences in the 1975-85 period, when the $k-h$ trajectory for the group as a whole began to bend towards that of Germany. This may

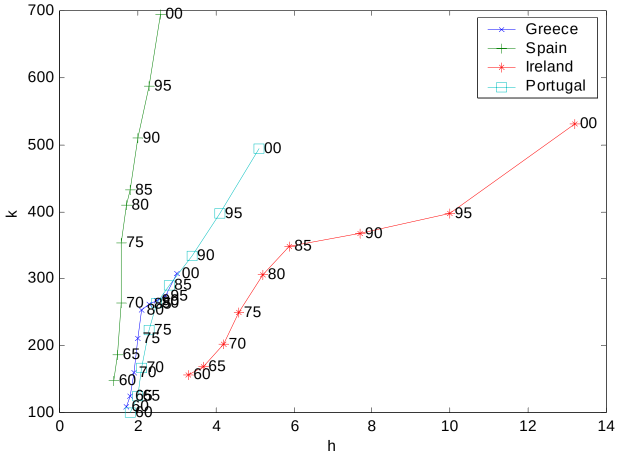

FIGURE 4. Relative evolution of k and h in the EU periphery.

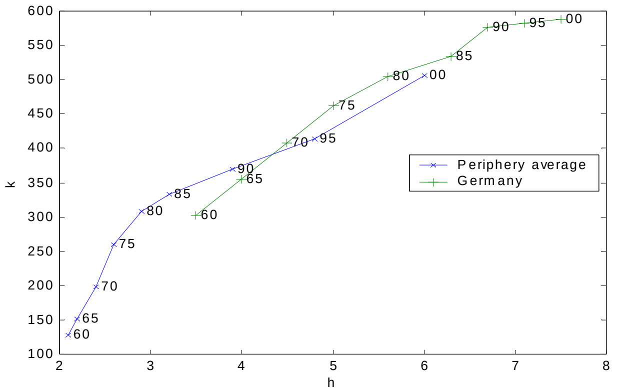

FIGURE 5. Evolution of k and h in the EU periphery and Germany.

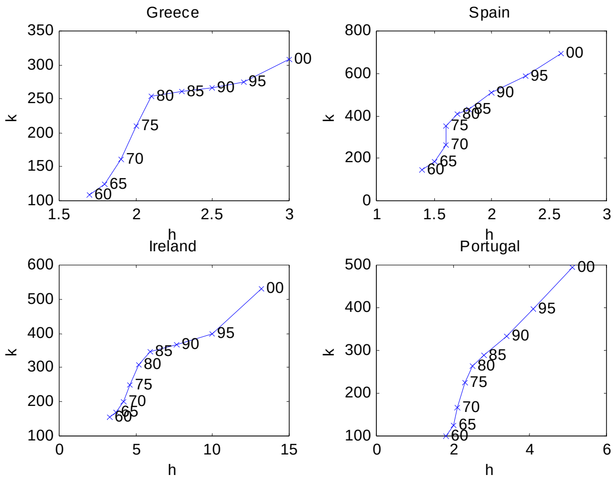

FIGURE 6. Evolution of k and h in the EU periphery.

suggest that the potential of take-off began to be realized already in this phase, at least in some countries. The behavior of the group was influenced by the developments in Portugal and Spain, where the trend from the previous period was broken. We already noted elsewhere that it was in these two countries where the announcement effects of likely accession materialized in high FDI inflows prior to accession. Because we interpret FDI as carriers of technological change, these developments are entirely consistent with the model predictions. Interestingly, trajectories of Greece and Ireland continued to be very steep in this period, pointing to low productivity in the knowledge sector without the catch-up potential.

Third, in the period 1985-2000, bluntly identified as the period of the catch-up, we observe that the slope of trajectories of all countries, inclusive of Ireland and Greece, has approached that of Germany (in the 1985-1990 period). The fact that the EU periphery in this period did begin to converge is therefore not surprising in the light of the model prediction. The failure of Greece (whose trajectory has interestingly the lowest slope since 1980) to converge is pathological and is most likely attributable to social and institutional obstacles that have prevented it so far from capturing the benefits of higher technological potential. Similarly, we have no means of explaining the sudden rise in the slope of the Irish trajectory in the most recent period. We suppose, that it may be linked to accounting and measuring problems. Because this recent experience of Ireland has also shifted the slope of the group average, we do not comment on it, until more country specific research is done.

Taking stock, the experience of the EU peripheral countries in the 1975-1995 fits the model predictions reasonably well. This allows us to use the model in explaining both the seeming puzzle of the relative stagnation in the EU periphery in 1975-1985 and the convergence in the recent period, which could not be accounted for using conventional growth factors and techniques (Kejak, Seiter, and Vavra 2001a). It appears that the stagnation of the 1975-85 period was a consequence of low technological potential as the forces of transitional dynamics through physical capital accumulation of the previous period petered out and technological potential could not be realized because of various (mostly institutional) obstacles. Only increased openness and inflows of FDI, enabled by the EU membership, paved the way for the diffusion of knowledge from the technologically more advanced countries.

5. Accessing Transition Countries

5.1. Transition countries: calibration to stylized facts. Mapping the hitherto experience of transition countries into the "model language" is complicated primarily by the lack of reliable time series. This is especially painful in the case of measuring physical capital, for a bad guess on the initial value would have seriously biased the results. Because our last schooling attainment data were from the mid 1990s, we decided to calibrate the initial conditions for transition economies in 1995.

For the value of the physical capital, we employed the stylized fact for developed economies: $\left(\frac{K}{N}\right)_{1995} = 2.5\left(\frac{Y}{N}\right)_{1995}$, which also corresponds to the situation in the EU periphery (Hall and Jones 1999). Measures of the stock of knowledge were derived with the same procedure as above, using the notional level of knowledge stock in the year 1990 for which we also had the attainment data.

With regard to the model channels of integration, i.e. access to foreign savings and technological transfer, here we find, too, that the process of integration was taking place along two lines: capital inflows (as a proxy for openness or low risk premium) and increase in the share made by FDI (as a proxy for the progress of technological catch-up), which is well documented[18]. The similarity of these developments in several

[18] It is a matter of repetition to note that in terms of market based trade flows most transition countries have completed their journey from an almost autarchy to substantial openness (measured by the ratio of trade flows to GDP) in just a decade. In line with this, the majority of the CEE countries began their transition paths as net creditors, but in line with the theory they soon began importing capital to finance their consumption and capital accumulation. This was reflected in the size of their current account deficits, widely experienced throughout the decade. While in the mid-1990s, the majority of the countries in question, with the exception of Hungary and Poland, still kept positive net investment balances, none of them were able to do so at the end of the decade. These financing needs exercised pressures on the liberalization of their capital accounts, all along with the fears that a sudden capital flight or financial crises may threaten the stability of their currencies. To alleviate these fears most countries took a gradual and cautious approach towards the current account liberalization and throughout the 1990s maintained some sort of capital controls, especially with regard to short term capital. Nevertheless, comparison to peripheral countries in the 1980s undertaken by (Buch 1999) suggests that the extent of capital mobility achieved by CEE countries in the mid 1990s was probably

transition countries with those of the EU peripheral in the 1980s allows us to treat them as in the phase of technological take-off. We believe this is certainly the case for the Czech Republic, Hungary and Poland which we analyze in the next section. For this reason we base our calibration of the knowledge capital stock on the parameters obtained from the EU periphery countries in the period 1985-2000. Our results on this calibration for the three CEE countries are summarized in Table 4.

5.2. Transition countries: Accession trajectories. In this section we parameterize several scenarios in order to analyze the effects of accession on the structure of the CEE economies and simulate their transition process to assess the speed of their convergence to the EU average. As already mentioned above, the accession may have a large impact on the social and institutional infrastructure [see (Kejak, Seiter, and Vavra 2001b)] some of which we capture here:

- by the level of accessible frontier knowledge, B_H, with the direct effect on the long-run growth rate of the accession country;
- by the speed of knowledge diffusion ϕ, a catch-up factor;
- by the parameters of capital accumulation process, ψ and δ.

Using alternative scenarios with respect to these three dimensions we simulate the development of CEE countries under different initial conditions. As we shall see, the interplay of initial conditions and parameters of accession generate different transition patterns and also rather different speeds of convergence to the EU average.

As already explained, we analyze only a subset of transition countries, namely the Czech Republic, Poland and Hungary. We believe that this particular choice displays enough diversity that is characteristic of the group of transition countries as a whole. This diversity translates into the behavior of the model economy through initial values of the relevant variables. Especially, the relative initial position of the stocks of human and physical capital is important with respect to their BGP trajectories. Figure 7 depicts initial conditions for the three countries in the k-h plane and positions them with respect to the BGP trajectories corresponding to a 4% growth rate. We observe that, while both Poland and Hungary are scarce in physical capital (relative to the BGP trajectory), the Czech Republic appears to be capital abundant in this respect. We also note that the initial position of Poland seemed to be very close to a hypothetical BGP trajectory of 4% growth.

We illustrate the mechanics of the transition processes from these initial conditions in a number of alternative simulations, each one designed to capture the effects of a specific isolated channel. The individual factors of interest are those that lay at the forefront of the discussion in the theoretical sections of the paper, namely the steady state-growth rate, the speed of the diffusion process, the rates of depreciation and the installation cost of capital. The simulation results are summarized in terms of the number of years necessary to achieve the current income per capita level of the EU 15 (see Tables 5-8). The tables are accompanied by a set of Figures 8 to 10 depicting the

about the same. FDI inflows accompanied the progress in opening, especially in the case of Hungary and the Czech Republic they were sufficient to finance the current account deficits.

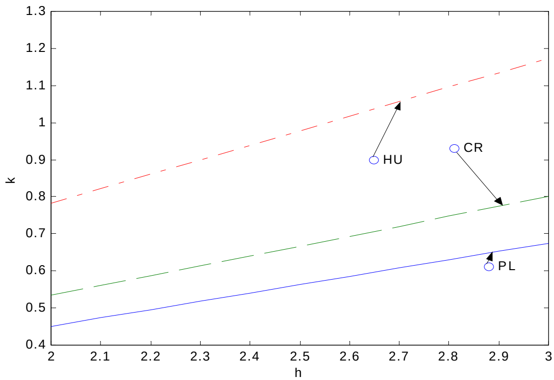

FIGURE 7. Initial conditions in selected transition countries.

trajectory of output of the three countries before the current level of the EU average is reached. In the first set of alternative simulations (Table 5 and Figure 8) we explore the implications of different BGP growth rates, leaving all other factors constant. It is instructive to split the transition process into three phases. The first phase is entirely driven by the effects of the immediate opening of the economy in terms of capital flows. It is therefore very fast: the more accessible foreign capital markets become, the faster it is. Its main feature is the elimination of basic physical capital imbalances with respect to the properties of the final BGP. Hence, this phase is only important for the transition process of economies that are relatively far from their BGPs, such as Hungary and the Czech Republic, and will play only a minor role in a country such as Poland, if we assume 4% BGP growth rate[19].

[19]It should be noted, though, that this characterization depends on the chosen parameters that determine the BGP. For instance, the initial position with respect to the BGP will be different for different BGP growth rates.

TABLE 4. Initial conditions for transition countries in 1995

	Real GDP per capita USD	Relative to EU 15	Physical capital per capita	Knowledge capital per capita	u	A	CA % GDP	FDI % GDP	Cumulative FDI % GDP	Net Investment Position % GDP
Poland	3299.2	24.5	0.61	2.88	0.38	0.28	0.7	2.9	6.2	-21.5
Czech Republic	5025.9	37.3	0.93	2.81	0.49	0.32	-2.6	4.9	14.2	5.9
Hungary	4850.8	36.0	0.90	2.58	0.36	0.39	-3.4	4.6	32.4*	-56.2*

Source: European Economy (1999), IFS, own computations, Note: 1997.

TABLE 5. Long run growth

LR Growth Rate g	3%	4%	5%
Poland	**56.7**	35.4	26.1
Czech Republic	**43.2**	23.0	16.0
Hungary	**41.3**	**29.9**	20.3

TABLE 6. Diffusion process

Catch-Up Factor ϕ	16	8	6
Poland	33.5	35.4	**40.1**
Czech Republic	18.2	23.0	**34.9**
Hungary	24.4	29.9	**36.0**

TABLE 7. Depreciation rate

Depreciation Rate δ	0.06		0.10	
LR Growth Rate g	3%	4%	3%	4%
Poland	**53.3**	**39.9**	57.6	35.4
Czech Republic	**41.7**	24.8	43.2	23.0
Hungary	**38.5**	**33.1**	45.4	29.9

TABLE 8. Capital adjustment costs

Adjustment Costs Parameter ψ	0.25	0.5	1
Poland	35.9	35.4	33.7
Czech Republic	19.4	23	19.6
Hungary	**31.7**	29.9	**28.2**

The initial conditions of these countries also differ in that while the Czech Republic is positioned above all of its hypothetical BGPs, and thus exhibits relative excess of physical capital, the other two economies are found below in the case of the lowest BGP, being relatively scarce in capital. However, the relative initial distance from

BGP depends on the assumed growth rate. In the case of a 3% growth rate, the Czech economy is very close to the respective BGP and so the first phase (characterized by rapid, almost instantaneous, decumulation of the capital stock) will not take place. Instead, the second phase will step in with a much slower decumulation of capital when the existing stock is being consumed or exported. As a consequence, the initial level of debt falls, putting a downward pressure on the prevailing interest rates. The fall in interest rates, in turn, combines with the fall in the capital stock in higher marginal returns to capital and Tobin's q. Increasing marginal returns to capital shifts the consumers' decision away from education to work, since investing and producing becomes relatively more attractive again. Overall, the luxury of having the excess of physical capital at the moment of accession allows people to allocate immediately a lot of time to adopting knowledge with all the benefits of fast growing productivity. Thus, the effects of moderately rising u and the faster fall in the capital stock roughly cancel out each other in their impact on output. In effect, the country is likely to exhibit a stagnation of growth rates during this period. We can well observe this in Figures 5 for the case of 3% BGP growth rate. The depth of stagnation or decline will, however, be dampened by higher returns to knowledge adoption. Poland and Hungary, on the other hand, can exhibit initially very high returns to their low levels of capital stock for the cases of low BGPs. Hence, during the first phase these countries will experience a massive capital inflow, which will fast reduce these returns. More importantly, though, this period will be characterized by a desire to work hard and long, the value of u reaching its upper limit of one. This is a direct consequence of excessive capital returns: it pays off to work and invest at the expense of further education. As a result, the economy is growing extensively in this phase: the growth being entirely driven by capital accumulation at constant u. The second phase is more gradual than the first one, and takes over at the beginning of the technological catch-up process. This process manifests itself by reallocation of resources from production to knowledge accumulation. The reallocation is motivated by the improved productivity of the knowledge sector. This improvement is made possible through the new technological frontier, which becomes more and more apparent as all the three countries move along their BGPs. As a consequence, people work less (u falls), causing a unanimous slowdown in the economic performance. The third and final phase comes into play when the true potential of the new technological frontier is fully reaped and realized. The allocation of resources between work and education will remain steady and so will the rates of physical and human capital accumulation.

In sum, the experience of the three countries along their transition paths will differ in several important respects. First, the countries with lower initial levels of capital stock (Hungary and Poland) may experience an initial period of extensive growth, which will push their level of output higher relative to the capital abundant country (the Czech Republic) suffering from stagnation. These differences are born out at low BGP of 3%. The slowdown of sectoral reallocation then hits all three countries in a comparable way. Second, as the numbers in Table 5 reveal, the actual length of the convergence periods will depend on the initial distance from the respective BGP. In this

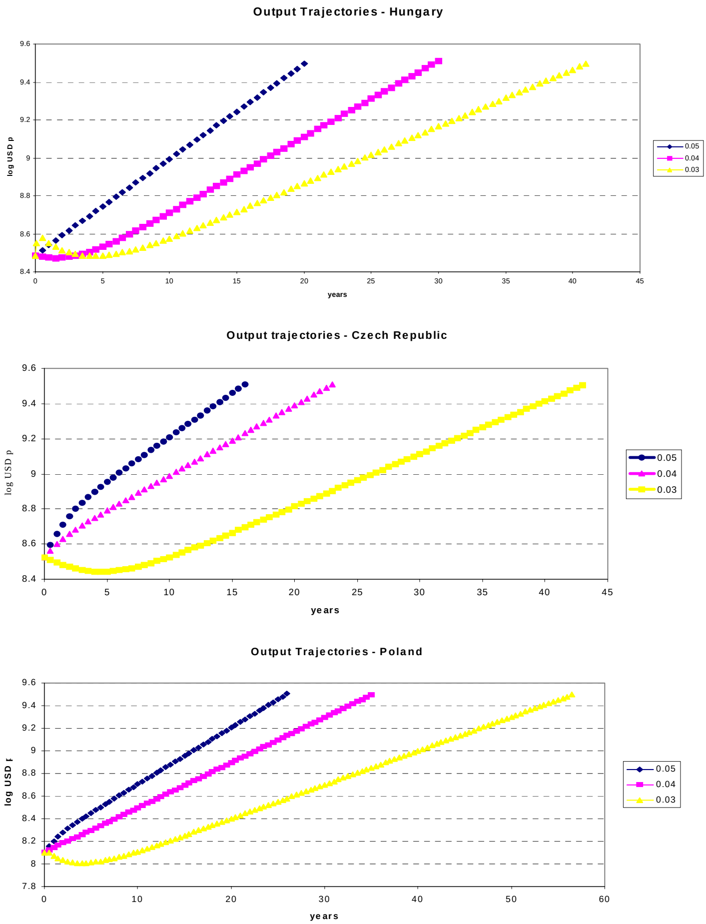

FIGURE 8. Output trajectories for different BGP growth rates.

case, the Czech economy benefits from its relative closeness to its BGP. This effect even dominates the negative effect of relative stagnation noted above. Third, the numbers of Table 5 conceal the fact that the extent and length of the reallocation slowdown

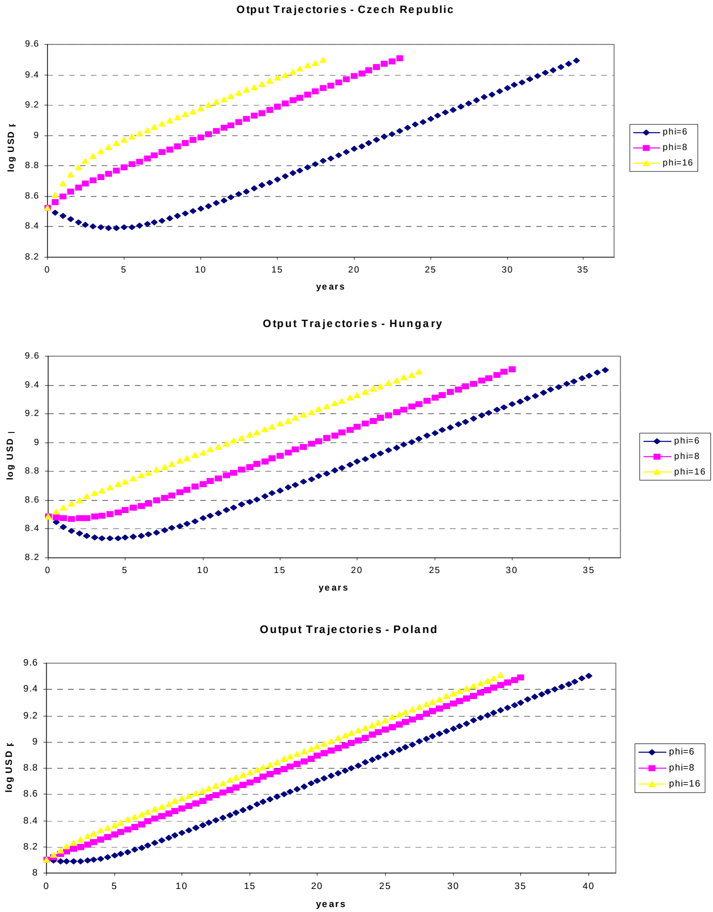

FIGURE 9. Output trajectories for different values of the diffusion parameter.

vary according to the steady state growth rates and may not even be present at all. This is because higher growth rates along the BGP are tantamount to higher levels

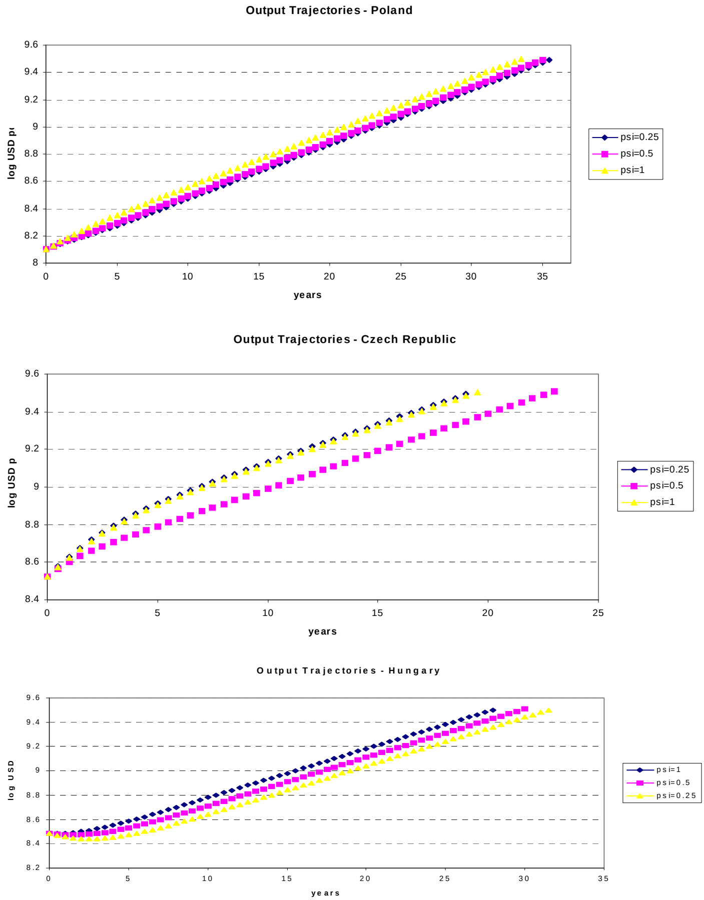

FIGURE 10. Output trajectories for different values of the adjustment parameter.

of the technological frontier. For this reason, the bold format employed in the tables highlights the cases of temporary slowdown.

In Table 5 we can see how long it will take Poland, the Czech Republic and Hungary to reach the current level of income per capita of EU 15 under three different scenarios with different BGPs: 3%, 4% and 5%. The results reflect our discussion above. The lowest BGP of 3% leads to protracted transition hurting mainly the most developed Czech Republic. Hungary and Poland start their accession with a boom followed by a brief reallocation recession before converging to the BGP. Importantly, the initial boom in Hungary makes its catching-up process with EU even faster than the one of the Czech Republic. For the higher growth rates the speeds of convergence are monotonically increasing with the BGP growth rates and the initial values of income. The transitions can be characterized by an accession boom followed by a smooth transition to BGP with only one exception of the accession recession in the case of Hungary with 3% growth rate.

Table 6 captures the effect of improved social infrastructure by removing barriers of knowledge adoption and thus speeding up the catch-up process (the higher ϕ means faster diffusion process). Unambiguously, lower costs to knowledge adoption are linked with a faster catching up process. The transition process is protracted due to accession recession when the adoption costs are high, but it is most painful for the Czech Republic and the least for Hungary.

In Table 7 we have analyzed the role played by a lower depreciation rate of physical capital in the catch-up process. The combination of the lower depreciation rate and a lower growth rate results in all the countries being initially below their BGPs. According to our discussion above there is the three-phase process with an initial accession boom (driven by capital inflow), followed by a recession. The country hit most by this development is the Czech Republic with the smallest boom phase contrary to Hungary's largest one. Interestingly, the Czech Republic is initially very closely situated to its 3% BGP. As such, it enjoys the best parameter setup with a slight initial accession boom followed by a negligible slowdown on its way to the BGP. Contrary to this, Hungary and Poland, initially below their BGPs, will experience the accession recession preceded with none or a negligible accession boom. The results of the last simulation scenarios highlight the role of the costs to physical capital adjustment on the process of accession and are provided in Table 8. In principle, higher adjustment costs harm the transition process by making the adjustment in physical capital more resource-demanding. However, at the same time they bring about relatively higher investment in knowledge. Thus, the total effect is ambiguous. The interplay of these mechanisms and initial conditions make the effect of higher adjustment costs beneficial to the speed of transition for Hungary and Poland. However, in the case of the Czech Republic both higher and lower adjustment costs speed up the catching up process.

6. Conclusions

The contribution of this chapter comes in two parts. First, we built an endogenous growth model of a small open economy with human/knowledge capital of the Lucas style and with S-shaped knowledge externalities. Second, we employ the model to capture the key aspects of development in TEs and to analyze the effect of accession to the EU in terms of their growth prospects. Extensions by adjustment costs to physical capital and an imperfect credit market via an upward-sloping debt supply enabled us to analyze transitional dynamics as well as sectoral adjustments. The model was validated through calibration to stylized facts of economic development in the EU periphery. We find that the experience of the EU peripheral countries fits model predictions reasonably well.

Using alternative scenarios we simulate behavior of model economies for different initial conditions of three transition countries: the Czech Republic, Poland and Hungary. The scenarios are designed to explore the effects of individual factors on the speed of their transition process to the BGPs. These factors include the steady state growth rate, the speed of the diffusion process, rates of depreciation and installation cost of capital. The interplay of initial conditions and parameters generate different accession patterns and also rather different speeds of convergence to the EU average.

According to the first set of scenarios meant to capture the implications of different assumptions about BGP growth rates the individual experiences along the transition path may differ in several important respects. Firstly, the countries with lower initial levels of capital stock (such as Hungary and Poland) may experience an initial period of extensive growth, which would push their level of output higher relative to the capital abundant country (the Czech Republic) suffering from stagnation. The slowdown of sectoral reallocation then hits all the three countries in a comparable way. Second, the actual length of the convergence periods will depend on the initial distance from the respective BGP. We show that countries may profit from the relative closeness to their BGP. This effect can even dominate the negative effect of relative stagnation. Third, the extent and length of the reallocation slowdown vary according to the steady state growth rates and may not even be present at all. This is because higher growth rates along the BGP are tantamount to higher levels of the technological frontier. The second set of scenarios with improved social infrastructure showed unambiguously that lower costs to knowledge adoption are associated with a faster catching up process. The transition process is protracted due to the accession recession when the adoption costs are high, but it is most painful for the Czech Republic and the least for Hungary. The combination of the lower depreciation rate and a lower growth rate results in all the countries being initially below their BGPs. An initial accession boom (driven by capital inflow) will be followed by recession. The country hit most will be the one closest to its BGP. The last scenarios concern the role of the costs of physical capital adjustment in the process of accession. They show that higher adjustment costs hurt the transition process by making the adjustment in physical capital more resource-demanding. However, they also bring about relatively higher investment in knowledge. Thus the total effect is ambiguous. The interplay of these mechanisms and initial conditions make the effect of higher adjustment costs beneficial to the speed of transition for Hungary and Poland. However, in the case of the Czech Republic both higher and lower adjustment costs speed up the catching up process.

Part 2

Human Capital, Growth, and Inflation

CHAPTER 3

The Inflation-Growth Effect and Great Ratios Consistent with Tobin

1. Introduction

The evidence on the effect of inflation on growth has continued to show a strong negative relation. Recent panel studies report strong inflation effects, both for developed and developing country samples. Further in the evidence has emerged a striking nonlinearity of this effect. Here there is a stronger negative effect of inflation at lower rates of inflation, and this becomes weaker as the inflation rate rises. This still makes for a rising cumulative effect of inflation rate increases, but it makes for a significantly weaker, negative, marginal effect on growth as the rate of inflation becomes higher.[1]

The achievement of the theoretical literature in replicating such results has been more mixed. It has been unclear whether a monetary general equilibrium economy with a payments technology can explain the evidence of how inflation affects economic growth and other related activity. One emphasis has been on calibrating the marginal effect on growth of an increase in the inflation rate, from a level typically of 10%, and then matching that to the average estimates in the empirical literature. A variety of endogenous growth models have been offered in this regard, with widely varying results. For example, both (Chari, Jones, and Manuelli 1996), using human capital, and (Dotsey and Sarte 2000), using an AK model with uncertainty, present endogenous growth models with cash-in-advance technologies in which inflation has an insignificant effect on growth. In contrast, for example, both (Gomme 1993), in a human capital model with a cash-in-advance constraint, and (Haslag 1998), in an AK model with money used for bank reserves, find a significant effect of inflation on growth.[2] Thus these models have been ambivalent. And in focusing on just one level of the inflation

[1]A debate has arisen on the effects of inflation below certain "threshold" rates of inflation, with some findings of insignificant inflation effects at inflation rates below the threshold. But this rate has been found to be close to 0 for developed country samples. In developing country samples, the threshold tends to be higher, near 10%, but a strong negative effect is typically re-established at all rates of inflation in all samples when instrumental variables are used, as in (Ghosh and Phillips 1998) and in (Gillman, Harris, and Matyas 2004). These studies also find the marked nonlinearity, as do (Khan and Senhadji 2000) and (Judson and Orphanides 1996). (Bruno and Easterly 1998) provide statistical averages of high inflation episodes whereby high inflation is correlated with lower growth rates than both before and after the episode; (Gylfason and Herbertsson 2001) and (Chari, Jones, and Manuelli 1996) provide reviews of earlier evidence of a negative inflation effect; (Barro 2001) finds a significant negative effect while emphasizing human capital.

[2](Dotsey and Sarte 2000) also present a deterministic AK version of the (Stockman 1981) model in which there is a significant negative effect. And in a more robust reformulation of the (Haslag 1998)

rate, this literature has begged the question of how inflation affects growth over a wide range of inflation rates, and on whether the models can replicate the nonlinear profile of the inflation-growth effect. Also, after a strong appearance in the older exogenous growth literature, the recent growth literature has largely ignored the issue of whether the models generate empirically consistent (Tobin 1965) effects.[3]

The main contribution of the paper here is that it presents a model in which a reasonable calibration can account for the empirical evidence, across the range of inflation rates, on inflation and growth. It does this in a robust fashion, and with an extension of a standard model using human capital and cash-in-advance. The paper also shows that the inflation-growth explanation is fully consistent with evidence on the existence of the (Tobin 1965)-like effects, including a rise in output per effective labor, even as the balanced-path growth rate declines as a result of an inflation rate increase.[4] Further it presents a novel, systemic, link between the strength of the growth effect and the strength of the (Tobin 1965) evidence. This fills another gap in the theoretical literature and opens up a new line of model predictions that have yet to be empirically examined: that the magnitude of the (Tobin 1965) effect is roughly proportional to the magnitude of the growth effect, and that these magnitudes vary monotonically from higher to lower as the inflation rate increases.

The key mechanism that gives our model the added flexibility to explain the evidence is the ability of the representative consumer to choose between competing payment mechanisms, money and credit, so that in equilibrium the marginal cost of each is equal. With such credit available to purchase the good, the nonlinearity is greatly magnified. When inflation rises up, the exchange cost of goods rises, but with credit available it rises by less than otherwise. So the consumer substitutes from goods to leisure, but uses credit to decrease the amount of substitution towards leisure. And this credit is relied upon increasingly more as the inflation rate goes up, and leisure is relied upon increasingly less as a substitution channel. This is because the marginal utility of goods gets increasingly high as less goods are consumed, while the marginal utility of leisure becomes increasingly lower as more leisure is consumed. This inflation-induced distortion in the marginal rate of substitution between goods and leisure is alleviated by the consumer's use of credit, and so accordingly the credit gets used more as the distortion gets bigger. And this results despite the increasing marginal cost of credit use, and in a way that is robust to the nature of the marginal cost specification. Because

model, using instead a cash-in-advance approach, (Gillman and Kejak 2004) also find this strong negative effect. For a comparison of such models, see (Gillman and Kejak 2003).

[3]For example, neither (Dotsey and Ireland 1996), (Aiyagari, Braun, and Eckstein 1998), or (Gomme 1993) indicate Tobin type results, although (Gomme 1993) is clearly consistent with them. The original (Tobin 1965) effect is within an exogenous growth model in which an increase in the inflation rate causes an increase in the capital to labor ratio and in per capita output; see (Walsh 1998) for a review. (Ahmed and Rogers 2000) compare the (Tobin 1965) effect across various exogenous growth models. (Gillman and Kejak 2003) compare Tobin-like effects across endogenous growth models.

[4](Ahmed and Rogers 2000) report long run US evidence showing that inflation has had a negative effect on the real interest rate historically, which would be expected if inflation causes the capital to effective labor ratio to rise as in the (Tobin 1965) effect. (Gillman and Nakov 2003) report long run US and UK evidence of an increase in the capital to effective labor ratio as a result of inflation.

credit gets used increasingly more, and leisure is used increasingly less as a substitution channel, the inflation-growth nonlinearity results. Leisure plays a key role in determining the growth rate: increased leisure use causes a lower return on human capital and a lower growth rate. So the use of increasingly less leisure makes for the decrease in the growth rate to be of increasingly lower magnitude, as the inflation rate rises. The resulting inflation-growth profile is shown to be very nonlinear compared to the model without credit and it qualitatively matches the profile in the evidence, unlike in the previous literature.

The use of credit has a residual implication for the use of money. And the nature of the model's money demand function is an alternative way to explain the basis for the inflation-growth nonlinearity. The money demand can be described as being similar to a general equilibrium version of the (Cagan 1956) function, in that it has an approximately constant semi-interest elasticity. This means that as the inflation rate rises, the interest elasticity rises substantially in magnitude. And this results because of the decreasing use of real money as credit is instead used to ameliorate the rising goods–to-leisure inflation-induced distortion, as the inflation rate rises. As part of this rising magnitude of the interest elasticity, in the model with credit, the use of money is much more interest elastic at all levels of the inflation rate relative to the same model without credit available.[5] And the approximate semi-interest elasticity is a testable model implication that has substantial support, such as in recent international panel evidence by (Mark and Sul 2003). It thereby provides a parallel dimension to the nonlinear inflation-growth evidence.[6]

In particular, the rising interest elasticity and its correspondence to the nonlinearity of the inflation-growth profile involves a previously unreported systemic link between the strength of the growth and of the (Tobin 1965) effects: when the inflation rate is low and the money demand function is in the relatively inelastic range, the growth and (Tobin 1965) effects are both marginally stronger, that is, of greater magnitude. When the inflation rate is relatively high and the money demand is in a relatively elastic range, these effects are weak, of small magnitude. Credit takes most of the substitution burden, instead of leisure, of an increase in the inflation rate when the level of the inflation rate is already high. This results in less growth and capital reallocation effects in re-equilibrating the return on human and physical capital at a lower rate of return.

Alternative solutions to the problem, of explaining the inflation experience, that rely on popular existing payment mechanisms all face inadequacies. The (Lucas 1988b) model with a standard payment mechanism potentially can produce both significant calibrated effects of the inflation-growth effect as well as the (Tobin 1965) effects, but it yields a weakly non-linear inflation-growth profile that is strained to match the evidence. Models with (Lucas and Stokey 1987) cash goods and credit goods, but

[5] As shown in a related model in (Gillman 1993).

[6] Another testable hypothesis here is the models ability to explain velocity; in a closely related model, (Gillman and Kejak 2004) - see also Chapter 5 are able to explain velocity trends for an array of monetary aggregates.

without a payments mechanism specified for credit, can only explain the effects of inflation through the agent's preference for credit goods versus cash goods. The lack of microeconomic evidence for this dichotomy makes the model difficult to calibrate in a non-arbitrary way. And while it has been common to interpret leisure as the credit good, making leisure the credit good in the endogenous growth models simply reduces the model back to the cash-only model with goods and leisure in the utility function.[7] Shopping time economies, a now commonly used alternative approach, in one sense improve on other standard payments mechanisms by allowing time to be used as a substitute to using money. But it is unclear what this shopping time is meant to represent as it has no obvious market analogy. With little to guide the specification, the fashion has been to use a constant interest elasticity to set the shopping time parameters, similar to how the preference-for-money parameters have been set in the money-in-the-utility function approach.[8] Some have interpreted shopping time as banking time, but have not taken the approach of modeling any part of banking. This is precisely what we do with our credit sector. And the result is a (Cagan 1956)-like strongly rising interest elasticity, not a constant one, that is robust to a range of credit production function parameters, and is key to explaining the nonlinear nature of the evidence.

2. The Economy with Goods, Human Capital, and Exchange Production

2.1. The Consumer Problem. The representative consumer's utility at time t depends on goods consumption, c_t, and leisure, x_t, in the constant elasticity form. Lifetime utility is

$$U_0 = \int_0^\infty e^{-\rho t} \frac{c_t^{1-\theta} x_t^{\alpha(1-\theta)}}{1-\theta} dt. \tag{2.1}$$

Output of goods, denoted by y_t, can be turned costlessly into physical capital. Both goods output and human capital are produced with physical capital and human capital -indexed labor in constant-returns-to-scale functions. Let k_t and h_t denote the stocks of physical capital and human capital, with the fixed depreciation rate of the capital stocks denoted by δ_k and δ_h. Let s_{Gt}, and s_{Ht} denote the fraction of capital that the agent uses in the goods production and human capital production, whereby

$$s_{Gt} + s_{Ht} = 1, \tag{2.2}$$

and $s_{Gt}k_t$, and $s_{Ht}k_t$ are the amounts of capital used in each sector. Similarly, let l_{Gt}, l_{Ht}, and l_{Ft} denote the fraction of time the agent uses in the goods, human capital, and credit sectors. This makes the allocation of time constraint

[7](Hodrick, Kocherlakota, and Lucas 1991) found a (Lucas and Stokey 1987)-type economy unable to explain velocity movements.

[8]See (Goodfriend 1997), (Lucas 2000), and (Gavin and Kydland 1999).

$$l_{Gt} + l_{Ht} + l_{Ft} = 1 - x_t, \tag{2.3}$$

and making $l_{Gt}h_t$, $l_{Ht}h_t$, and $l_{Ft}h_t$ the effective labor in each sector.

With $\beta, \varepsilon \in [0, 1]$ and A_G and A_H being positive shift parameters, the goods production function is

$$y_t = A_G \left(s_{Gt}k_t\right)^{1-\beta} \left(l_{Gt}h_t\right)^{\beta}. \tag{2.4}$$

The marginal product of capital $s_{Gt}k_t$, denoted by r_t , and the marginal product of effective labor $l_{Gt}h_t$, denoted by w_t, are

$$r_t = (1-\beta)A_G \left(s_{Gt}k_t\right)^{-\beta} \left(l_{Gt}h_t\right)^{\beta}, \tag{2.5}$$

$$w_t = \beta A_G \left(s_{Gt}k_t\right)^{1-\beta} \left(l_{Gt}h_t\right)^{\beta-1}. \tag{2.6}$$

The human capital equation of motion, given $h_0 > 0$, is

$$\dot{h}_t = A_H[(1 - s_{Gt})\, k_t]^{1-\varepsilon}[(1 - l_{Gt} - l_{Ft} - x_t)\, h_t]^{\varepsilon} - \delta_h h_t. \tag{2.7}$$

Note that this human capital investment equation is the same as in (Lucas 1988b) except that there is also physical capital used as an input along with the effective labor. This follows the (King and Rebelo 1990) extension of the (Lucas 1988b) model which makes it more suitable for calibration purposes. While in the (Lucas 1988b) model the growth rate of human capital is proportional to the labor time devoted to human capital accumulation, or to "learning", here the growth rate is a combination of the fraction of time and the fraction of capital devoted to human capital accumulation. In both the (Lucas 1988b) model and this extension, the balanced-path growth rate equals the human capital stock growth rate, and both are reduced when leisure time increases.

The goods output forms an input into the (Becker 1965b) household production of the consumption good c_t. The goods used as an input for producing the consumption are denoted by y_{ct} . The other input is exchange, denoted by y_{et}, which enters the production function $f_c(\cdot)$:

$$c_t = f_c(y_{ct}, y_{et}). \tag{2.8}$$

The production function for the consumption good is assumed to be Leontieff, with the isoquant ray from the origin having a slope of one:

$$c_t = y_{ct}, \tag{2.9}$$

$$c_t = y_{et}. \tag{2.10}$$

This technology ensures that the amount of consumption goods equals the amount of physical goods, and that the value of the physical goods is equal to the value of the amount that is paid (or exchanged) for the goods. This one-to-one relation is the most intuitively appealing; other specifications are possible but would require some extended justification.

The exchange in turn is produced using two inputs: real money balances, denoted by m_t , and real credit, denoted by d_t. These inputs are perfect substitutes, implying that

$$y_{et} = m_t + d_t. \tag{2.11}$$

Real money balances are defined as the nominal money stock, denoted by M_t , divided by the nominal price of goods output, denoted by P_t ; $m_t \equiv M_t/P_t$. The initial nominal money stock M_0 is given to the consumer. Additional money stock is transferred to the consumer exogenously in a lump sum fashion by an amount V_t . The consumer uses the money to buy some fraction of the output goods with money, and the rest with credit. Let $a_t \in (0, 1]$ denote the fraction of output goods bought with money.[9] Then the agents demand for money is constrained to be this fraction of goods purchased. In real terms,

$$m_t = a_t y_{ct}. \tag{2.12}$$

Substitution from equation (2.9) gives a (Clower 1967) constraint:

$$m_t = a_t c_t; \tag{2.13}$$

$$M_t = P_t a_t c_t. \tag{2.14}$$

Credit demand is the residual fraction of output goods purchases,

$$d_t = (1 - a_t) y_{ct}, \tag{2.15}$$

or substituting in from equation (2.9),

$$d_t = (1 - a_t) c_t. \tag{2.16}$$

With $\gamma \in (0, 1)$, and A_F a shift parameter, the credit production function is specified as

$$d_t = A_F (l_{Ft} h_t)^{\gamma} c_t^{1-\gamma}. \tag{2.17}$$

This function can be interpreted using duality. Because the total cost of production in the credit sector is the wage bill of the effective labor, $w_t l_{Ft} h_t$, equation (2.17) implies the marginal cost (MC_t) function of

$$MC_t = (w_t/\gamma) A_F^{-1/\gamma} (d_t/c_t)^{(1-\gamma)/\gamma}. \tag{2.18}$$

With $\gamma < 0.5$, this gives a marginal cost of credit output, per unit of consumption, that rises at an increasing rate as in a traditional U-shaped cost curve. Figure 1 graphs the three cases of $\gamma = 0.3$ (thicker line), $\gamma = 0.5$ (middle, straight, line) and $\gamma = 0.7$ (and with $w_t = A_F = 0.2$).

[9] An equilibrium with $a = 0$ does not have well-defined nominal prices.

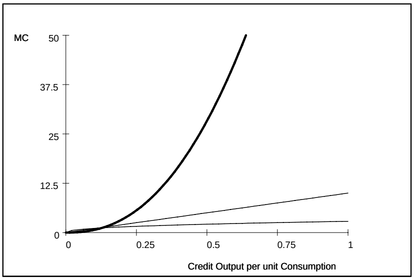

Figure 1. Marginal Cost of Credit

A rising marginal cost function per unit of consumption is the same devise used in (Gillman 1993). The difference is that in that model there was a continuum of goods and of stores each with a different time cost of supplying credit to buy their good. In aggregate the stores present an upward sloping marginal cost curve, so that a unique equilibrium with the nominal interest exists at each nominal interest rate. However here there is only one consumption good and one credit production function, with γ being the diminishing returns parameter that determines the shape of the curve; the unique equilibrium results as long as $\gamma < 1$, although $\gamma > 0.5$ seems unlikely in that they indicate a marginal cost that rises at a decreasing rate in contrast to typical industrial organization evidence.

The upward sloping cost curve, for example, with $\gamma = 0.3$ as in Figure 1, can also be interpreted in terms of the value-added of the credit sector. This requires an explicit price for the credit service through a decentralization of the sector.[10] Given the decentralization, it is found that the price of the credit service is the nominal interest rate. In market clearing equilibrium, this price equals the marginal cost given above. And indeed the equality of the nominal interest rate and the marginal cost of credit is one of the below key equilibrium conditions (equation (2.32)). This "price" can also be used to define the value-added, or total revenues as in national accounts, of the credit sector; this equals the nominal interest rate factored by the quantity of the credit supplied. Given the assumed production function, in equilibrium it can be shown that this value-added is proportional to the cost of production ($(Rc(1-a))\,/\,(wl_Fh) = \gamma$). This gives another way to interpret the assumed production specification. Even more simply the specification implies that the per unit marginal cost is higher than average cost by a fixed proportion for all levels of credit output, resulting in a constant profit rate. Thus the assumption is the same as assuming an upward sloping marginal cost curve, proportional to average cost, with a constant profit rate, which has intuition based firmly in standard price theory.

Note that the output of such a service sector is necessarily proportional to aggregate consumption. Factoring out this proportionality factor to determine what is being

[10]See (Gillman and Kejak 2004).

produced gives the share of the output for which the service is provided. If it is also assumed that the production function has diminishing returns, then the production of the share necessarily includes an "externality" effect from the aggregate consumption. Were constant returns to scale specified for the service, while at the same time there is a substitute price that exhibits a constant marginal cost, which is what the nominal interest rate presents for the marginal cost of real money, then there is no unique equilibrium between the two alternatives. Thus the production function for credit must be specified with diminishing returns in order to have a unique equilibrium, and as a service proportional to aggregate consumption, it must include the externality effect. However consider an illustration of what this really means in the model economy. A credit card company such as American Express, in a decentralized setting, would maximize profit while taking as given how much is spent on goods for consumption. American Express would not try to change this goods expenditure but must consider it in making its optimal credit supply available to the consumer. By making its inputs grow as the consumption of goods grows, it can maintain its share of supplying credit. This simply means that if the aggregate consumption increases, and the credit sector does not increase its effective labor proportionally, then it will lose its share of output for which it provides the service.

Setting credit demand equal to credit supply, in equations (2.16) and (2.17),

$$(1-a_t) = A_F(l_{Ft}h_t/c_t)^{\gamma}. \tag{2.19}$$

Substituting into equation (2.14) for a_t from equation (2.19), the money and credit constraints can be written as

$$M_t = \left(1 - A_F\left(\frac{l_{Ft}h_t}{c_t}\right)^{\gamma}\right) P_t c_t. \tag{2.20}$$

2.2. Government Money Supply. The initial money stock M_0 is given to the representative agent, and the only role of the government is to change the money supply from its initial value. To do this, the government transfers to the consumer each period an exogenous lump sum money supply of V_t at a constant rate of σ;

$$\dot{M}_t = V_t = \sigma M_t. \tag{2.21}$$

The stock V_t is the inflation "proceeds" that result when the government buys output/capital (they are costlessly interchangeable) with freshly printed fiat and then gives this (thereby producing real money) to the consumer as an income transfer. Net government spending equals zero and is omitted for notational simplification. The only effect of such "production" is a relative price distortion if the inflation rate ends up non-optimal.

In real terms, dividing equation (2.21) by P_t implies that the government's investment rate in real money is the supply growth rate minus the inflation-based depreciation of $\pi \equiv \dot{P}_t/P_t$:

$$\dot{m}_t = (\sigma - \pi) m_t. \tag{2.22}$$

2.3. Definition of Equilibrium. The consumer's total nominal financial wealth, denoted by Q_t, is the sum of the money stock M_t and the nominal value of the physical capital stock $P_t k_t$:

$$Q_t = M_t + P_t k_t; \tag{2.23}$$

$$\dot{Q}_t = \dot{M}_t + P_t \dot{k}_t + \dot{P}_t k_t. \tag{2.24}$$

The consumer's change in the financial wealth over time, $\dot{Q}_t$, is equal to the sum of V_t by equation (2.21), plus the nominal value of the change in physical capital $P_t \dot{k}_t$, and plus the nominal price appreciation factor $\dot{P}_t k_t$. The $P_t \dot{k}_t$ term is the output of goods, which can be written in terms of marginal products using equation (2.5) and (2.6), minus the output of goods that are purchased for consumption, which by equation (2.9) equals $P_t c_t$, and minus capital depreciation $P_t \delta_k k_t$. This gives

$$\dot{Q}_t = P_t r_t s_{Gt} k_t + P_t w_t l_{Gt} h_t + V_t - P_t c_t - P_t \delta_k k_t + \dot{P}_t k_t. \tag{2.25}$$

Equations (2.4), (2.5), (2.6), (2.25) and (2.21) imply the social resource constraint

$$y_t = c_t + \dot{k}_t + \delta_k k_t. \tag{2.26}$$

Given M_0, k_0, h_0, and the normalization of $P_0 = 1$, equilibrium consists of the values of the prices $\{r_t, w_t, P_t\}_{t=0}^{\infty}$ and the allocations $\{c_t,\ x_t,\ s_{Gt},\ l_{Gt},\ l_{Ft},\ M_t,\ Q_t,\ k_t\}_{t=0}^{\infty}$ that satisfy i) the representative consumer's maximization of the lifetime utility (2.1) subject to the constraints in equations (2.7), (2.20), (2.23), and (2.25), taking as given the prices and the transfer V_t, ii) the firm's maximization problem taking prices as given, iii) the government supply of money in equation (2.21), and iv) the clearing of all markets in the economy, with equation (2.26) for the goods market.

2.4. Balanced Growth Path. On the balanced-growth path, c_t, k_t, h_t, m_t and y_t grow at the same rate, denoted by g. The variables x_t, l_{Gt}, l_{Ft}, l_{Ht}, s_{Gt}, s_{Ht}, w_t, r_t are stationary.

A balanced growth path reduced set of equilibrium conditions are set out below, with time subscripts dropped and assuming $\delta_k = \delta_h$:

$$\frac{u_c(c,x)}{u_x(c,x)} = \frac{x}{\alpha c} = \frac{1 + aR + w l_F h / c}{wh}, \tag{2.27}$$

$$\frac{w}{r} = \frac{\beta}{1-\beta} \frac{s_G k}{l_G h} = \frac{\varepsilon}{1-\varepsilon} \frac{s_H k}{l_H h}, \tag{2.28}$$

$$g \equiv \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = \frac{\dot{m}}{m} = \frac{r - \delta_k - \rho}{\theta} \tag{2.29}$$

$$= \frac{\varepsilon(1-x)A_H[(s_{Ht}k_t) \,/\, (l_{Ht}h_t)]^{1-\varepsilon} - \delta_h - \rho}{\theta}, \tag{2.30}$$

$$r - \delta_k + \frac{\dot{P}}{P} \equiv R, \tag{2.31}$$

$$R = w / \left(\gamma A_F \left(\frac{l_{Ft}h_t}{c_t} \right)^{\gamma - 1} \right). \tag{2.32}$$

Because of the novel nature of the credit sector, a focus on this last equation (2.32) helps describe the model. In the (Baumol 1952) model, the consumer chooses between two payment mechanisms: the use of money and the use of banking in which interest is earned on the income. The banking of these models is similar to the credit in the model here. Also similar is that the consumer optimally chooses between the two according to the cost of each relative to the other. This choice yields the only equilibrium condition in (Baumol 1952). There is no such margin in the standard cash-only (Lucas 1980) or (Lucas and Stokey 1983) economies. The model here follows (Baumol 1952) and adds this as an additional margin relative to the standard cash-in-advance economy with the following equilibrium condition. The cost of money, R, equals the marginal cost of credit, which is the marginal factor cost of effective labor in the credit sector, w_t, divided by the marginal product of labor in the credit sector. This is a standard microeconomic pricing condition for factor market equilibrium. The existence of this condition, not found in (Baumol 1952), takes the important margin that (Baumol 1952) develops, and places it securely within microeconomic theory, while using the single-good standard neoclassical growth framework.[11] This makes standard monetary theory tractable back to the production structure of credit, unlike in (Baumol 1952).

The marginal rate of substitution of goods relative to leisure is given by equation (2.27), and can be understood as the ratio of the shadow price of the consumption good to leisure. The shadow price of consumption goods is one, the goods cost, plus the exchange cost of $aR{+}wl_F h/c$ per unit. If only money is used in exchange, this is just the nominal interest R. But with credit also used this exchange cost is less than R and can be expressed as a weighted average of money and credit use, or $1+aR+(1-a)\gamma R$. Or with a focus on a, this writes as $1+\gamma R+aR(1-\gamma)$. When the inflation rate goes up the cost of exchange rises. But because of substitution towards credit, the cash share a falls, the shadow exchange price rises by less than proportionately to R, and so it rises by less than in the cash-only model. Thus there is substitution towards leisure as in the cash-only model, but less such substitution.

[11]One comparison in the literature to equation (2.32) can be found in an innovative paper by (Canzoneri and Diba 2005); it follows more of the (Tobin 1965) approach by specifying bonds that back up a non-money exchange service (not dissimilar to credit), and it uses this to solve the price indeterminacy problem.

Other balanced-growth path equilibrium conditions here show that the growth rate equals the return on capital minus the time preference rate, in the log-utility case, and that the returns of human and physical capital are equal; with equal depreciation rates, $r = \varepsilon(1-x)A_H[(s_{Ht}k_t)\,/\,(l_{Ht}h_t)]^{1-\varepsilon}$. This last expression highlights how the increased leisure can act to decrease the growth rate, while the (Tobin 1965) effect towards greater capital intensity in both goods and human capital sectors, as w/r increases because of an inflation increase, can partially offset the decrease in the growth rate.

2.5. Effect of Inflation on Balanced-Growth Path. Technically, the effect of a change in the inflation rate on the balanced-growth path equilibrium can be solved analytically for certain parameter specifications by solving all equations in terms of leisure and then solving for the change in leisure from one implicit equation in terms of only leisure. Then the main results follow and can be summarized in the following two lemmas. For analytic tractability, log-utility is assumed and in addition no physical capital is assumed for the second lemma and its two corollaries. These assumptions are relaxed in the calibration.

Note that the results state what happens when there is an increase in the money supply growth rate. The inflation rate, as in all such models, increases because the exogenous rate of money supply growth is assumed to increase. The inflation rate goes up a bit more than the money supply growth rate increase, because the balanced-path growth rate falls somewhat, while the sum of the inflation rate and the balanced-path growth rate are constrained to equal the money supply growth rate; from equation (2.22), $\pi = \sigma - g$. So while this is generally thought of as the effect of inflation on growth in such models, and this is the usage made in this paper, the inflation-growth relation is more precisely a result of the money supply changes.

LEMMA 1. *An increase in the money supply growth rate σ causes an increase in leisure time, a decrease in the real interest rate, an increase in the capital to effective labor ratio in the goods and human capital production sectors, an increase in the goods capital to output ratio, and a decrease in the balanced-growth path growth rate. It is assumed that $\theta = 1$, $\beta = \varepsilon = \gamma = 0.5$, $A_G = A_H$, and that the change in the money supply growth rate is evaluated at the Friedman optimum of $R = 0$.*

PROOF. Please see Appendix C.1. □

The increase in the exchange cost of goods causes a relative decrease in the opportunity cost of leisure, thereby inducing a shift back in the supply of labor for goods production, while there is a shift of labor into credit production. The real wage rises (by less than does the exchange cost of goods) in order to clear the labor market, inducing firms to realign inputs towards capital and away from labor. The increase in the capital to effective labor ratios, across both goods and human capital production sectors, lowers the marginal product of capital and the real interest rate.[12] Here the rising capital to effective labor effect marks the (Tobin 1965) effect in the human capital model, rather than the rising capital per worker as in the Solow exogenous growth

[12] We thank an anonymous referee for a suggested description here.

model without leisure. Output per effective labor also goes up in a way similar to (Tobin 1965). And a lower real interest rate from an inflation increase can be viewed as part of this (Tobin 1965) effect. But unlike in (Tobin 1965), here the growth rate goes down.

Note that in the (Lucas 1988b) model, only effective labor is used in human capital accumulation and there is no leisure in the utility function; in this case the rate of return on human capital in equilibrium is just proportional to the time spent accumulating human capital, or $A_H l_H$. When the time spent in human capital production goes down, the growth rate goes down. In the monetary extension of the human capital growth model, leisure plays a critical role with respect to inflation. For example, with no physical capital and log-utility (as assumed in the next Lemma), the rate of return on human capital is proportional to the time spent working in all sectors, or $A_H (1 - x)$. And in this case the change in the total time spent working $(1 - x)$ (in all three sectors) is exactly equal to the change in the time spent in human capital accumulation l_H; here the (Lucas 1988b) explanation of the growth rate, as being proportional to the time spent in human capital accumulation, is perfectly interchangeable with the time spent working. With physical capital the growth rate more generally depends on the rate of return to human capital, in which a falling amount of leisure time because of inflation is the primary effect, while an increase in the capital to effective labor ratio is of secondary magnitude, moderating the decrease in the growth rate.

LEMMA 2. *The magnitude of the change in the balanced-path growth rate, from a change in the money supply growth rate, is determined inversely by the magnitude of the interest elasticity of money demand, given that* $\beta = \varepsilon = \theta = 1$, *and given that the interest elasticity is less than one in magnitude. Further with a cash-only restriction* $(a \equiv 1)$, *the inflation-growth profile is exactly linear.*

PROOF. Please see Appendix C.1. □

This is the log-utility and no physical capital case. At the (Friedman 1969) optimum of $R = 0$, the marginal rate of substitution between goods and leisure is undistorted and leisure is a close substitute for goods because there is no tax wedge to force their marginal utilities to diverge. As the inflation rate rises from the optimal rate, leisure tends to be used readily to avoid the inflation tax, while credit use is relegated to a secondary role in avoiding inflation, despite the fact that the marginal cost of credit is relatively low at low inflation rates since there is a rising marginal cost curve. However at higher rates of inflation, the inflation tax wedge makes the use of more leisure increasingly less attractive relative to the use of more credit because leisure's diminishing marginal utility, and goods increasing marginal utility, in effect dominate the rising cost of the credit. Credit is used increasingly more and therefore the interest elasticity of money demand is increasingly high. Because the growth rate effect is dependent directly on how much leisure is used when inflation rises, this effect is strongest when the inflation rate is rising up from the optimum and the wedge in the goods-leisure rate of substitution is at its smallest. The growth rate falls by increasingly less as the inflation rate rises, and the interest elasticity of money demand rises in magnitude.

At a unitary interest elasticity, the growth rate stops falling and actually begins to rise. However the baseline calibration puts this juncture at a hyperinflation rate of inflation, above which the government makes less seigniorage anyway. This suggests that only the range of the inflation rate that induces a less than unitary elasticity is likely to be empirically relevant. Note the relation of this result to (Eckstein and Leiderman 1992). They find that seigniorage in Israel rises at a steadily decreasing rate, which they model with a money demand derived from putting real money balances in the utility function. Our nonlinear inflation-growth profile, and the rising magnitude of interest elasticity, correspond directly to a seigniorage that rises at a diminishing rate. As in the (Cagan 1956) model [but unlike that of (Eckstein and Leiderman 1992)], the total seigniorage would begin to fall once the interest elasticity rose above one in magnitude, but we suggest that this is not an empirically relevant long run range for the elasticity.

COROLLARY 1. *The magnitude of the interest elasticity of the goods-normalized money demand rises with an increase in the inflation rate because the magnitude of the elasticity of substitution between money and credit, and the share of credit in purchases, each rise with an increase in the nominal interest rate.*

PROOF. Please see Appendix C.1. □

A standard factor-price elasticity of substitution between real money and credit, as the two inputs into producing exchange, can be defined as the percentage change in inputs over the percentage change in marginal products. Then the interest elasticity of money demand can be expressed as a price elasticity of the derived input demand, in terms of the elasticity of substitution. In particular, the interest elasticity of money demand (η_m^R) equals the (negative) share of the other input credit $(1-a)$ as factored by the elasticity of substitution between money and credit (ϵ), plus a scale effect (η_c^R); or $\eta_m^R = (1-a)\epsilon + \eta_c^R$.[13] The scale effect is of secondary importance in terms of magnitude, and when normalizing the money demand by consumption, this term drops out (this is the only term in the cash-only economy). As the inflation rate rises, leisure becomes a worse substitute, even while money and credit remain perfect technical substitutes (equation (2.11)). This increases the two-factor elasticity of substitution; the share of credit $1-a$ also rises unambiguously. Note that the isoquant for producing exchange is not linear because of the role of leisure.[14]

The result is insensitive to the specification of the parameters in the credit production function. Given that $\gamma \in (0,1)$ and $A_F > 0$, there is a rising marginal cost of credit, as the credit use per unit of consumption increases. The degree of diminishing returns, γ, affects shape of the marginal cost curve in an unambiguous way, but affects the normalized interest elasticity in an ambiguous fashion that depends on the

[13] See for example (Marshall 1920) or a standard microeconomic text on derived demand elasticiticies.

[14] See (Gillman 2000) for another example of the input price elasticity as applied to real money, in a model using the store continuum as in (Gillman 1993), (Dotsey and Ireland 1996) and (Aiyagari, Braun, and Eckstein 1998). Such a curved isoquant between real money and credit in general equilibrium is graphed in (Gillman 1995).

calibration; the shift parameter A_F does has a clear effect on the magnitude of the normalized interest elasticity (as indicated in the next corollary). But regardless of these specifications, it is the fact of the existence of the credit (with a rising marginal cost), combined with the nature of the goods to leisure marginal rate of substitution, that produces the corollary results, of an increasing interest elasticity with inflation rate increases. This can alternatively be seen by writing the normalized elasticity as $(1-a)\epsilon = -\left[\gamma/\left(1-\gamma\right)\right]\left[\left(1-a\right)/a\right]$. All that is necessary for this elasticity to rise in magnitude is that the normalized money usage (a) falls as the inflation rate rises.

COROLLARY 2. *The magnitude of the interest elasticity of the goods-normalized money demand rises with an increase in productivity in the credit sector, as indicated by an increase in the total factor productivity A_F of the credit production function.*

PROOF. Please see Appendix C.1. □

This corollary brings in one additional factor, the productivity of the credit sector. This can be important for example in analyzing changes in financial regulation. A deregulation is similar to a decrease in the implicit tax on the credit sector that has the effect of shifting up the productivity parameter A_F. Continuing the example, deregulation here has the effect on increasing the demand for credit at each nominal interest rate, making the demand for money in effect more interest elastic. The fall in the price of a substitute to money causes a shift back in the money demand function. Given the same nominal interest rate, this moves the consumer "up" the money demand function to a more interest elastic point.

3. Calibration

The analytic results of the lemmas and corollaries, on how inflation effects the balanced-growth equilibrium, are shown to apply as well in the general model through its calibration. The calibration makes clear that the model produces a significant effect of inflation on growth, within the range of empirical estimates reviewed for example by (Chari, Jones, and Manuelli 1996), while showing the nonlinearity of this effect, the existence of (Tobin 1965) effects, and the link between the magnitude of the growth and (Tobin 1965) effects. Also the calibration shows the robustness of the results to a full range of alternative specifications of the parameters of the credit production function.

3.1. Assumed Parameter Values. Standard parameters values are assumed as in the literature. Table 1 presents the assumed values for the baseline calibration. Leisure is set as in (Jones, Manuelli, and Rossi 1997); risk aversion and Cobb-Douglas parameters for goods and human capital sectors as in (Gomme 1993); depreciation rates as in (King and Rebelo 1990); growth rate as in (Chari, Jones, and Manuelli 1996); the share of cash is similar to (Dotsey and Ireland 1996); leisure preference is set within the range in the literature. For the credit sector technology, the degree of diminishing returns is set to 0.2 as based on the estimated value of this parameter that is found for the US in the money demand estimation of (Gillman and Otto 2002), a companion paper. This parameter is varied below in Table 4 and a fuller set of such variations can be found in (Gillman and Kejak 2002).

TABLE 1. Baseline Parameter and Variable Values

Parameters	ρ	δ_h	δ_k	θ	β	ε	α	γ	A_G	A_H	A_F
	0.04	0.1	0.1	1.5	0.64	0.64	4.692	0.2	1	0.581	0.801
Variables	a	x	g	π	l_G	l_H	l_F				
	0.7	0.7	0.02	0.05	0.1635	0.1355	0.00098				

3.2. The Results. Table 2 shows that the baseline calibration for the negative growth rate effect of a 10% point increase in the inflation rate is a -0.23 percentage point change in the growth rate of output, comparable to the range in (Chari, Jones, and Manuelli 1996). Note that the -0.23 indicates that starting from a baseline of 0.02 percent growth (a 2% growth rate) at an inflation rate of 0.05, the growth rate falls to 0.0177 when the inflation rate rises to 0.15. Figure 2a simulates this in the solid line. The negative growth effect falls in magnitude as the inflation rate rises. This nonlinear relation, of a marginally decreasing magnitude of the negative growth effect, has been found empirically in many studies. And this occurs even while the (Tobin 1965) effect is present through a higher output to effective labor ratio (Figure 2b).

Figure 2a also includes for contrast a dashed line for the cash-only economy that is almost linear, contrary to evidence. Additionally for the economy of Lemma 2, in which there is no physical capital, Figure 2c shows that the inflation growth profile is perfectly linear for the cash-only economy (dashed line) versus the nonlinear Section 2 model with credit (solid line).

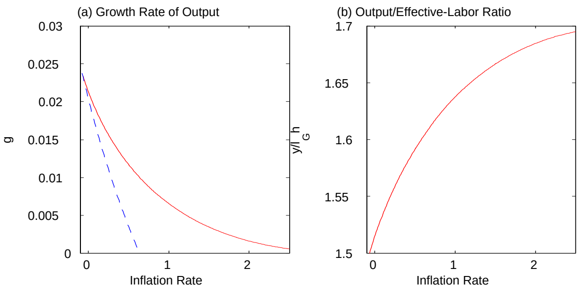

Figure 2a,b. Inflation with Growth and Output per Effective Labor

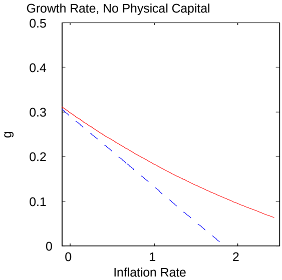

Figure 2c. Credit; Cash-Only (dashed)

Table 2 also shows how leisure rises with inflation (Figure 3a), the real interest rate falls (Figure 3b), the real effective wage rises (Figure 3c), and the capital to effective labor ratio in the goods sector and the investment to output ratio rise (Figures 3d and 3e). The sectorial reallocations are supported empirically in (Gillman and Nakov 2003), while supporting evidence for the positive investment rate effect and negative real interest rate effect are found in (Ahmed and Rogers 2000). Figure 3f simulates the money demand per unit of consumption goods; this is the inverse, endogenous, consumption velocity and it contrasts for example to the assumption in (Alvarez, Lucas, and Weber 2001) that velocity is exogenous. In addition, Table 2 shows the link among the magnitude of the growth and (Tobin 1965) effects and the magnitude of the interest elasticity of money demand.

TABLE 2. Baseline Calibration of the Effect of Increasing the Inflation Rate

Baseline Change in Variable	Inflation Rate Change $5 \to 15\%$	$15 \to 25\%$	$25 \to 35\%$
Growth Rate g	-0.00232	-0.00199	-0.00173
Leisure x	0.00878	0.00824	0.00705
Real Interest Rt r	-0.00320	-0.00304	-0.00263
Real Wage w	0.01054	0.01029	0.00914
Capit/Lab Gds $(s_G k)/(l_G h)$	0.09800	0.09753	0.08810
Capit/Lab Hum $(s_H k)/(l_H h)$	0.09800	0.09753	0.08810
Capit/Output $(s_G k)/y$	0.04086	0.04023	0.03599
Output/Eff.Labor $y/(l_G h)$	0.01647	0.01608	0.01428
Money/Consumption-Goods a	-0.04187	-0.03310	-0.02586
Point Est of Int Elast η_R^m	-0.1276	-0.1757	-0.2220

Table 3 provides a calibration with the goods sector's capital intensity increased above that of the human capital production sector, with $\beta = 0.50$, instead of $\beta = 0.64$ as in the baseline. This shows that with a greater goods sector capital intensity, the inflation-induced substitution from labor to capital is marginally greater, and the

(Tobin 1965) and growth effects stronger, relative to the baseline, while the interest elasticity is of smaller magnitude. This acts to marginally shift up the inflation-growth profile; Figure 3g shows this with the solid line being the baseline and with the dashed line having $\beta = 0.50$ and all other parameters as in the baseline.

TABLE 3. Baseline Calibration Except for an Increase in the Capital Intensity in Goods Production

Baseline Except $\beta = 0.60$			
Change in Variable	Inflation Rate Change		
	$5 \rightarrow 15\%$	$15 \rightarrow 25\%$	$25 \rightarrow 35\%$
Growth Rate g	-0.00232	-0.00200	-0.00174
Leisure x	0.00872	0.00820	0.00703
Real Interest Rt r	-0.00320	-0.00304	-0.00264
Real Wage w	0.01352	0.01327	0.01183
Capit/Lab Gds $(s_G k)/(l_G h)$	0.13379	0.13374	0.12133
Capit/Lab Hum $(s_H k)/(l_H h)$	0.11288	0.11284	0.10237
Capit/Output $(s_G k)/y$	0.04510	0.04452	0.03993
Output/Eff.Labor $y/(l_G h)$	0.02254	0.02211	0.01972
Money/Consumption-Goods a	-0.04063	-0.03212	-0.02507
Point Est of Int Elast η_R^m	-0.1238	-0.1699	-0.2143

Table 4 shows the effect of increasing from its baseline value the parameter that indicates the degree of diminishing returns in the credit sector. It shows that such increases cause a bigger magnitude of the growth effect and of the (Tobin 1965) effects, and a smaller magnitude of the interest elasticity. This calibration is done for a neighborhood of the baseline calibration with respect to changes in γ. Simulation of the inflation-growth effect with a larger γ show that this acts to pivot down the inflation-growth profile. Figure 3h shows this with the solid line being the baseline and with the dashed line having $\gamma = 0.25$ and all other parameters as in the baseline.

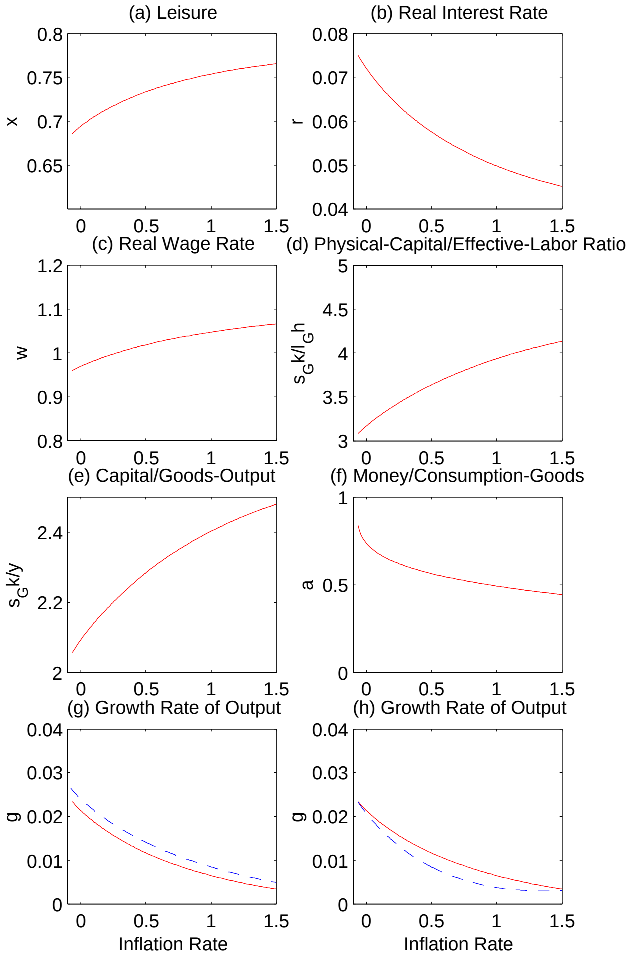

Figure 3. Inflation with Other Balanced-Growth Path Variables

While the role of financial development on the inflation-growth effect has been little studied (although there are sizeable literatures on each the inflation and growth relation, and the financial development and growth relation), (Gillman, Harris, and Matyas 2004) present evidence of differences in the inflation-growth profile for APEC and OECD samples. The profiles compare closely to Figure 3h in that APEC's profile is less steep at every rate of inflation, while the profile starts at about the same point, so that the APEC profile appears pivoted up relative to the OECD profile. The model thus suggests a comparatively greater degree of diminishing returns in credit production, and a more steeply rising marginal cost curve, in the APEC region. This offers one explanation consistent with the different inflation-growth results that cannot be provided with the standard cash-only cash-in-advance exchange technology.

TABLE 4. The Inflation Effects When Increasing the Degree of Diminishing Returns in Credit Production

Baseline: Inflation Rate 5 → 15% Change in Variable	Degree of Diminishing Returns in Credit Production $\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.5$
Growth Rate g	-0.00232	-0.00273	-0.00338
Leisure x	0.00878	0.01148	0.01423
Real Interest Rt r	-0.00320	-0.00421	-0.00524
Real Wage w	0.01054	0.01398	0.01769
Capit/Lab Gds $(s_G k)/(l_G h)$	0.09800	0.13083	0.16724
Capit/Lab Hum $(s_H k)/(l_H h)$	0.09800	0.13083	0.16724
Capit/Output $(s_G k)/y$	0.04091	0.04866	0.06908
Output/Eff.Labor $y/(l_G h)$	0.01647	0.02184	0.02764
Money/Consumption-Goods a	-0.04187	-0.05434	-0.03080
Point Est of Int Elast η_R^m	-0.12757	-0.11737	-0.08745

4. Comparison to Other Payment Mechanisms

One type of comparison that can be further detailed is to use the same human capital model but with different payment mechanisms.

4.1. Cash-only Economy. The most standard is the cash-only economy of (Lucas 1980). Here the consumer can use only money to buy goods. This case results from the Section 2 model when $a \equiv 1$ is imposed. Or this can be derived by having credit be prohibitively expensive (A_F close to zero). Figure 2a (dashed line) shows the resulting inflation-growth profile with the baseline calibration. The almost linear profile indicates that the growth rate becomes negative quickly as the inflation rate rises, contrary to evidence. The cash-only model overstates the inflation effect on growth at every level of the inflation rate for $R > 0$, in comparison to the Section 2 model. The reason is that when inflation increases, with cash-only the consumer can only substitute towards leisure, and so uses more leisure for each marginal increase in the inflation rate than if credit was available. So instead of having much smaller leisure increases as the inflation rate goes higher, which is what happens when credit is available, the increases in leisure only decrease in magnitude slightly.

4.2. The Shopping Time Economy. The (Lucas 2000) shopping time model focuses on the use of resources in exchange activity. Calling this activity "shopping time" after (McCallum and Goodfriend 1987), and showing the sense in which it exactly equals the welfare cost of inflation in the economy (with no leisure), he specifies the shopping time exchange constraint so as to induce a constant interest elasticity. This strategy of specifying the exchange technology so as to have a constant interest elasticity is also used in (Goodfriend 1997), who cites an earlier version of the (Lucas 2000) paper, and in (Gavin and Kydland 1999).

By assuming a constant interest elasticity, the free parameters of the shopping time function can be constrained in a non-arbitrary way. However the problem with the constant interest elasticity assumption is that it is in conflict with evidence. (Lucas 2000) describes how a constant-like interest elasticity model seems to breakdown for US data during the 1980s, after which he concludes that a constant semi-interest elasticity model seems to be the preferred model. (Mark and Sul 2003) find substantial cointegration panel data evidence in support of the constant semi-interest elasticity model.

If in fact a constant semi-interest elasticity is the appropriate model, then the key fact here is that the interest elasticity rises as the interest rate rises, rather than remaining constant as in the shopping time models. In this case the shopping time models are forcing an undue lack of non-linearity upon the inflation effects with respect to growth and (Tobin 1965) variables. This means that the constant interest elasticity will make the effects too weak for low values of the inflation rate and too strong for higher values of the inflation rate, depending on the particulars of which constant interest elasticity is chosen.

The model of section 2 can in fact be viewed as a special case of the shopping time economy. The special case is that the shopping time of the (McCallum and Goodfriend 1987) exchange constraint becomes instead the banking time of an explicit credit production technology.[15] The credit technology parameters determine only how quickly the interest elasticity of money demand rises with the inflation rate. Corollary 1 explains why a rising interest elasticity with inflation does not depend on the exact specification of these parameters, a result confirmed with calibration. Rather through their effect on the interest elasticity they determine the degree of nonlinearity of the inflation-growth profile. Extreme values can reproduce the cash-only economy ($A_F = 0$ or $\gamma = 0$).

5. Conclusion

The chapter shows that, contrary to what has become generally accepted, growth models with (Lucas 1988b) human capital, and well-defined payments mechanisms, can successfully explain major facets of how inflation affects long run economic activity. First it makes clear that point estimates, of significant magnitude, of the negative effect of inflation on the balanced-path growth rate can be found with a standard calibration

[15]In a related paper, (Gillman and Yerokhin 2003) detail this connection. One implication is that shopping time function in an endogenous growth setting should include human capital in its specification, unlike in (Love and Wen 1999).

that is robust to varying the parameters of the credit production function. Second the credit allows the consumer to use less leisure as inflation increases, so that the economy exhibits a significantly non-linear inflation-growth relation as has been found repeatedly in empirical studies. Third the model shows that related (Tobin 1965) effects are at work in the economy, with a decrease in the real interest rate to the real wage ratio, an increase in the capital to effective labor ratios across sectors, and a rise in the output per effective labor input. This inflation-tax-induced increase in the output per effective labor hour is a result of the household trying to moderate the growth rate decrease by realigning inputs towards capital as labor becomes scarce and leisure in greater use.

The model has household production of consumption using goods and exchange. The exchange is produced interchangeably with money or a credit sector. This offers a direction alternative to general transaction cost models such as the shopping time models. The approach is related to the cash-credit framework of (Aiyagari, Braun, and Eckstein 1998), who assume a constant semi-interest elasticity of money demand. Here such a money demand is generated endogenously as the consumer equalizes the marginal cost of alternative payment mechanisms. As a result, links between the money demand function and the inflation effects are pervasive and, unlike previous work, are made explicit. The money demand's interest elasticity inversely determines the strength of the growth and (Tobin 1965) effects in a way that fills out intuition of these events. This presents also an alternative research strategy towards further developing and calibrating such models: to use structural parameters of the credit production technology in addition to so-called behavioral parameters of the partial equilibrium money demand functions. This may further advance understanding of how inflation affects international growth and other aspects of the structure of the economy.

CHAPTER 4

Contrasting Models of the Effect of Inflation on Growth

1. Introduction

There are three main controversies in the literature on the long run effect of inflation on growth. First is whether models can exhibit a significant negative effect of stationary inflation on the balanced-path growth rate. Second is the nature of the inflation-growth effect across the whole range of the levels of the inflation rate. Third is whether the inflation-growth models and evidence can at the same time be consistent with evidence on (Tobin 1965)-type effects.

The contribution of the paper is to first bring together for comparison several main approaches to modelling the inflation-growth effect by nesting them within a general model. This shows what factors determine the magnitude of the inflation-growth effect across these different approaches. And it yields the following notable result: a robustness for the ability to generate a strong magnitude of the inflation-growth effect. In addition the paper explains the source of the effect by showing that the key distinguishing feature of competing approaches is whether inflation acts mainly as a tax on physical capital or on human capital. The outcome of this determines whether the inflation-growth effect accompanies an inverse or positive (Tobin 1965) effect. Finally evidence on the growth and (Tobin 1965) effects is brought to bear on these competing models, as a way to lend support to favoring one approach over another.

The paper's approach to explaining this literature is to allow for a mix of physical and human capital in the production of goods, and for a mix of exchange means, money and credit, in buying the goods. The general model starts with an exchange technology extended from the standard cash-in-advance genre using microfoundations in such a way that it also encompasses a special case of the shopping time model. The extension specifies the production of credit, which is used as an alternative to money. This helps distinguish the overall inflation-growth effect in terms of its theoretical characteristics over the range of (non-hyperinflation) inflation rates, as well as some additional "money and banking" facts.

Starting with (Ireland 1994) "Money and Growth: An Alternative Approach" that compares to transitional inflation-growth effects found in (Sidrauski 1967) "Money and Growth", the paper sets out a model that puts (Ireland 1994) approach within an aggregate consumption good setting. From this capital-only economy that includes credit, the paper next covers a case of (Stockman 1981) capital-only economy with investment as a "cash good"; this model with uncertainty added is used in (Dotsey and

Sarte 2000).[1] The paper then turns to human capital only models, that compares to (Gillman, Kejak, and Valentinyi 1999) and (Stokey and Lucas 1989) (section 5.8). Then the paper sets out models with both types of capital that compare to (Gomme 1993), and to the capital accumulation process of (Chari, Jones, and Manuelli 1996). Finally, an extension to (Gomme 1993) is put forth that includes credit, as in (Gillman and Kejak 2002).

Most evidence finds a negative inflation-growth effect. For example by way of large changes in the inflation rate, (Gylfason and Herbertsson 2001) list some 17 studies for which all but one find a significant decrease in the growth rate from increasing the inflation rate from 5 to 50%. More by the way of a marginal increase in the inflation rate, (Chari, Jones, and Manuelli 1996) review the empirical results from increasing the inflation rate from 10 to 20 percent; they report a significant fall in the growth rate within a range of 0.2 to 0.7 percentage points; for example the growth rate falls from an initial level of 3% at a 10% inflation rate to between 2.8 and 2.3% at a 20% inflation rate. Recent findings for example of (Barro 2001) compound the evidence of a strongly significant negative effect of inflation on growth.

In addition evidence suggests that the negative effect is marginally stronger at low inflation rates and marginally weaker as the inflation rate rises. This negative and highly nonlinear effect is strongly supported in (Judson and Orphanides 1996), (Ghosh and Phillips 1998), (Khan and Senhadji 2001) and (Gillman, Harris, and Matyas 2004). Some evidence is qualified by findings of a "threshold" rate of inflation, above which the effect is strongly significant and negative, but below which the effect is insignificant and positive. For industrialized country samples, this threshold level has been tested for and found to be very low, at a 1% inflation rate (Khan and Senhadji 2001), although others have assumed (without such testing) higher thresholds of 2.5% (Ghosh and Phillips 1998) or 10% (Judson and Orphanides 1996). The "threshold" for developing country samples has been found through testing to be at 11% (Khan and Senhadji 2001), below which again the inflation-growth effect is insignificant and positive. However when using instrumental variables in order to adjust for possible inflation-growth endogeneity bias, the negative nonlinear inflation-growth effect has been reinstated at all positive inflation rate levels for both developed and developing country samples [(Ghosh and Phillips 1998), (Gillman, Harris, and Matyas 2004)]. This suggests no inconsistency in a modelling approach that focuses only on a negative effect of inflation on growth.[2]

(Tobin 1965) evidence includes an inflation-induced decrease in the real interest rate, an increase in the average investment level and decline in the consumption level, normalized by output, and a rise in the aggregate capital to effective labor ratio. (Ahmed and Rogers 2000) find a variety of (Tobin 1965) long run evidence for the US including a decrease in the real interest rate because of permanent inflation increases. Similarly (Rapach 2003) finds that permanent inflation increases lower the long run

[1] See also (Haslag 1998) for Ak economies in which money is required as bank reserves; these models have negative inflation-growth effects in a way similar to (Stockman 1981) model.

[2] As an exception, (Paal and Smith 2000) present an overlapping generations model in which a threshold inflation rate exists.

real interest rate in 14 out of 14 countries studied. There is also the related evidence in (Gillman and Nakov 2003) of inflation Granger-causing increases in the capital to effective labor ratios in the US and UK postwar data, which is consistent with a (Tobin 1965) effect of increased capital intensity.

The literature on how to model such evidence extends the traditional (Tobin 1965) modification of the Solow exogenous growth model, whereby money is introduced as an alternative to capital. Similar to the original IS-LM model, in the (Tobin 1965) model an increase in the money supply growth rate, or in the inflation rate, causes investment, capital, and output to rise. But the growth rate is exogenous and so is unaffected by the inflation rate. The extensions from the (Tobin 1965) framework are classes of the Cass-Koopman neoclassical model that endogenizes the savings rate of the exogenous Solow growth model through utility maximization; this gives the Euler equation results whereby the growth rate equals the marginal product of capital, net of time preference, and (for CES utility) normalized by a utility parameter. Further, the extensions are of the endogenous growth genre as extended to a monetary setting, whereby the rate of return on real money, being based on the inflation rate, can affect the marginal product of capital and the growth rate. In the endogenous growth models, inflation typically causes the growth rate to fall, while the output as a balanced-growth path ratio relative to different variables can rise or fall, resulting in either (Tobin 1965) effects or inverse (Tobin 1965) effects. The idea of an inverse (Tobin 1965) effect follows from (Stockman 1981), whereby the inflation rate increase causes a capital stock decrease.

Therefore, in the endogenous growth models, the inflation rate affects the growth rate because it affects the marginal product of capital, either that of physical capital as in Ak models, that of human capital as in Ah models, or that of both physical and human capital in combined capital models. Some models have produced insignificant long run inflation-growth effects, for example the Ak models of (Ireland 1994) and (Dotsey and Sarte 2000) and the physical and human capital model of (Chari, Jones, and Manuelli 1996). While at least equally diverse models have produced significant and negative inflation-growth effects, including the Ak models of (Haslag 1998) and (Gillman and Kejak 2004) (see also Chapter 5 in this volume), the Ah model of (Gillman, Kejak, and Valentinyi 1999), and (Gylfason and Herbertsson 2001) model with money in the goods production function, (Gomme 1993) physical and human capital model, and (Gillman and Kejak 2002) extension of (Gomme 1993) and (Gillman and Kejak 2005) (see also Chapter 3 in this volume). Using a nesting as based on Ak, Ah, or a combination of physical and human capital can illustrate the results from most of the models, and so this is the approach taken here.

Note that these are balanced-path, stationary-state, results. And it is actually the rate of money supply growth in these models that is exogenous, changes in which "cause" changes in the stationary inflation rate, and simultaneously cause changes in the output growth rate. However since, for example, long run evidence finds that money Granger- causes inflation (Crowder 1998), and since some evidence also finds that inflation Granger- causes the output growth rate [see (Nakov and Gillman 2003), (Gillman and Wallace 2003), (Cziraky and Gillman 2006)] this literature on inflation and growth tends to discuss how inflation affects the output growth rate. This convention is also used here. Further, with log-utility as we assume throughout the paper, the nominal interest rate depends on only the money supply growth rate and the rate of time preference so that we can calibrate how increases in the nominal interest rate affect the economy in a way equivalent to a money supply acceleration.

The calibration strategy is to examine the change in the nominal interest rate on the balanced-path growth rate, the real interest rate and the capital to labor ratio in the goods sector across the different models. For all models the growth rate and the real interest rate are fixed at the same values at the optimum, of three percent for the growth rate and six percent for the real interest rate (nine for the gross real interest rate). Given the greater number of degrees of freedom in the more complicated models relative to the simpler models, calibrating them so that they have a common point at the optimum allows for a normalized comparison of how inflation affects the economies as it rises up from its optimal level.

2. The General Monetary Endogenous Growth Economy

The nesting model has three sectors that each use both physical capital and human capital-indexed labor: goods production, human capital investment, and credit production. The notation, parameter assumptions and production specifications are presented in the Table 2.1.

TABLE 1. Notation and Assumptions

Variables		**Parameters**
Real	l_{Ht} : HC Share, HC	$\beta \in [0,1]$
y_t : Goods Output	l_{Ft} : Credit Share, HC	$\varepsilon \in [0,1]$
c_t : Consumption Goods	r_t : Interest Rate	$\gamma_1, \gamma_2 \in [0,1)$
x_t : Leisure Time	w_t : Effective Wage Rate	$A_G > 0$
k_t : Physical Capital (PC)	*Nominal*	$A_H > 0$
h_t : Human Capital (HC)	M_t : Money Stock	$A_F > 0$
i_t : PC Investment	P_t : Goods Price	$\delta_K \geq 0$
i_{Ht} : HC Investment	V_t : Money Transfer	$\delta_H \geq 0$
s_{Gt} : Goods Share, PC	*Definitions*	$\alpha > 0$
s_{Ht} : HC Share, PC	$m_t \equiv M_t/P_t$	$\sigma \geq -\rho$
s_{Ft} : Credit Share, PC	$a_t \equiv m_t/c_t$	$\rho \in (0,1)$
l_{Gt} : Goods Share, HC	$d_t \equiv (1-a_t)c_t$	$a_2 \in [0,1]$
	Production Functions	
Goods	Human Capital Investment	Credit
$y_t = A_G (s_{Gt}k_t)^{\beta} (l_{Gt}h_t)^{1-\beta}$	$i_{Ht} = A_H (s_{Ht}k_t)^{\varepsilon} (l_{Ht}h_t)^{1-\varepsilon}$	$\frac{d_t}{c_t} = A_F \left(\frac{s_{Ft}k_t}{c_t}\right)^{\gamma_1} \left(\frac{l_{Ft}h_t}{c_t}\right)^{\gamma_2}$

Current period utility is of the constant elasticity of substitution form, whereby

$$u(c_t, x_t) = \ln c_t + \alpha \ln x_t. \tag{2.1}$$

The consumer allocates time to labor supplied to the goods producer, to self-production of human capital, and to self-production of credit. With an endowment of one unit of time, the time constraint is similar to an adding-up-of-shares constraint:

$$1 = x_t + l_{Ft} + l_{Gt} + l_{Ht}. \tag{2.2}$$

Similarly a share of the capital stock is used potentially in each of the three production functions, and the shares must add to one:

$$1 = s_{Gt} + s_{Ht} + s_{Ft}. \tag{2.3}$$

The physical capital investment equation is standard in its assumption of no costs of adding to the capital stock except the actual capital:

$$k_{t+1} = k_t (1 - \delta_K) + i_t. \tag{2.4}$$

The human capital investment technology function follows (Becker 1975). The equation for motion of the accumulation is $h_{t+1} = h_t (1 - \delta_H) + i_{Ht}$. The investment in human capital i_{Ht} requires effective labor and capital whereby

$$h_{t+1} = h_t (1 - \delta_H) + A_H (s_{Ht} k_t)^{\varepsilon} (l_{Ht} h_t)^{1-\varepsilon}. \tag{2.5}$$

The consumer receives income from the human capital augmented labor for goods production and from the rental of capital to the goods producer; there is also the lump sum transfer of money V_t from the government that the consumer receives. Given the consumer's endowment of initial money stock M_0, and dividing the income between goods purchases and investment, the equation of motion for the consumer's nominal income, or the income constraint, can be put in terms of the change in the nominal money stock:

$$M_{t+1} - M_t = P_t w_t l_{Gt} h_t + P_t r_t s_{Gt} k_t - P_t c_t - P_t k_{t+1} + P_t k_t (1 - \delta_K) + V_t. \tag{2.6}$$

The consumer can buy the consumption good at a price of P_t either using the money (carried over from the end of the last period) or using the credit. The fraction of goods bought with money can vary between zero and one, with $a_t \in (0, 1]$, and with $1 - a_t$ being the residual fraction of goods that is bought with credit. A fixed fraction $a_2 \in [0, 1]$ of physical capital investment is also bought with money. The rest of the investment is a "costless" credit good requiring neither money nor credit as is standard in this literature (think of retained earnings).This makes the so-called (Clower 1967) constraint

$$M_t = a_t P_t c_t + a_2 P_t i_t, \tag{2.7}$$

which is that of (Stockman 1981) if $a_t = a_2 = 1$.

The consumer's choice of a_t is determined by how much labor the agent decides to spend supplying the alternative to money, this being the credit. Here the total real credit d_t equals the residual real amount of consumption goods not bought with money, or $d_t \equiv c_t(1 - a_t)$, and is given as

$$d_t = c_t A_F (s_{Ft} k_t / c_t)^{\gamma_1} (l_{Ft} h_t / c_t)^{\gamma_2}. \tag{2.8}$$

This exhibits constant returns to scale in its three factors, c_t, $s_{Ft} k_t / c_t$, and $l_{Ft} h_t / c_t$, resulting in an upward sloping marginal cost of credit supply per unit of consumption as long as $\gamma_1 + \gamma_2 < 1$. With $\gamma_1 + \gamma_2 < 1$ the consumption velocity of money, the inverse of a_t, is stationary along the balanced growth path as in the evidence. The Cobb-Douglas case of $\gamma_1 + \gamma_2 = 1$ is problematic because it creates an equilibrium that is not-well defined since then both money and credit have a constant marginal cost; and also velocity would not be stationary. Dividing the above equation by c_t and using the definition $d_t \equiv c_t(1 - a_t)$, the share of credit in purchases $(1 - a_t)$ can be written as

$$(1 - a_t) = A_F \left(s_{Ft}k_t/c_t\right)^{\gamma_1} \left(l_{Ft}h_t/c_t\right)^{\gamma_2}. \tag{2.9}$$

In this specification the effective labor and capital inputs are proportional to total consumption, so that the share of credit use remains constant when consumption is growing only if the effective labor and capital inputs grow at the same rate.

A combined exchange constraint for money and credit, which are perfect substitutes, results by solving for a_t from equation (2.9) and substituting this into equation (2.7):

$$M_t = P_t \left[c_t - A_F \left(s_{Ft}k_t/c_t\right)^{\gamma_1} \left(l_{Ft}h_t/c_t\right)^{\gamma_2} c_t + a_2 P_t i_t\right]. \tag{2.10}$$

This results in the standard (Lucas 1980) "cash-only" (Clower 1967) constraint when $a_2 = 0$ and $A_F = 0$, so that credit is prohibitively costly to produce.

The goods producer maximizes profit subject to the CRS production technology, with the following first-order conditions, and zero profit in equilibrium:

$$w_t = (1 - \beta) A_G \left(s_{Gt}k_t\right)^{\beta} \left(l_{Gt}h_t\right)^{-\beta}; \tag{2.11}$$

$$r_t = \beta A_G \left(s_{Gt}k_t\right)^{\beta-1} \left(l_{Gt}h_t\right)^{1-\beta}. \tag{2.12}$$

The government supplies nominal money through the lump sum transfer V_t at a steady rate σ, whereby

$$M_{t+1} = M_t + V_t \equiv M_t (1 + \sigma). \tag{2.13}$$

This money supply process is used without alteration in all models of the paper.

With social resources being that output is divided between consumption and investment, the social resource constraint can be found to be

$$y_t = c_t + i_t. \tag{2.14}$$

This resource constraint holds for all of the calibrated models below.

The consumer maximizes the preference discounted stream of utility in equation (2.1) subject to the constraints (2.5), (2.6), and (2.10), with respect to c_t, x_t, M_{t+1}, k_{t+1}, h_{t+1}, s_{Gt}, l_{Gt}, s_{Ft}, and l_{Ft}. The first-order conditions are presented in Appendix D.1. The stationary variables on the balanced-growth path are the shares l_G, l_H, l_F, x, s_G, s_H, s_F, while the variables that grow at the rate g are y_t, c_t, $m_t \equiv M_t/P_t$, k_t, h_t, i_t, i_{Ht}. Equilibrium can be characterized by the marginal rate of substitution between goods and leisure, the balanced-path growth rate, the equivalence between the returns on physical and human capital, and the marginal condition between credit and money use, made known by (Baumol 1952).

The goods-leisure marginal rate of substitution is

$$\frac{\alpha x}{c_t} = \frac{1 + aR + w(l_F h/c) + r(s_F k/c)}{w h_t} = \frac{1 + aR + (1 - a)(\gamma_1 + \gamma_2) R}{w h_t}. \tag{2.15}$$

This rate equals the ratio of the shadow price of goods to that of leisure. The goods shadow cost is one plus the shadow cost of exchange, $aR + w(l_F h/c) + r(s_F k/c)$ with $w(l_F h/c) + r(s_F k/c)$ being a real resource cost of inflation avoidance through credit activity. Or the shadow cost can be written equivalently as the weighted average of cash and credit $aR + (1-a)(\gamma_1 + \gamma_2) R$.

The balanced-path growth rate can be expressed by

$$1 + g = \frac{1 + \frac{r}{1+a_2 R} - \delta_K}{1+\rho} = \frac{1 + (1-\varepsilon) A_H (l_H h / s_H k)^{-\varepsilon} (1-x) - \delta_H}{1+\rho}. \tag{2.16}$$

The growth rate is decreased because of the a_2 factor if $R > 0$, where the need to use money to buy investment goods acts as a tax, as in (Stockman 1981). And here, given that $\delta_K = \delta_H$, the return on physical capital $r/(1 + a_2 R)$ is equal to the return on human capital $\varepsilon A_H (l_H h / s_H k)^{\varepsilon - 1} (1-x)$. An increase in leisure works directly to bring down the human capital return.

The linkage between R, σ and π along the BGP in all of the models is first through the Fisher equation

$$1 + R \equiv (1+\pi)(1 + [r/(1+a_2 R)] - \delta_K), \tag{2.17}$$

that can be derived by introducing government bonds.[3] Second using the Fisher equation plus the (Clower 1967) constraint (2.7) and the growth rate equation (2.16), the nominal interest rate and money growth rate are related by

$$1 + R = (1+\sigma)(1+\rho). \tag{2.18}$$

Changes in the nominal interest rate are directly caused by changes in the money supply growth rate. And in response to an increase in σ, and in R, it is important to realize that the gross real interest rate r falls, even as π rises. The change in r is emphasized throughout the paper as part of the (Tobin 1965) effect, and is simultaneous with an increasing capital to effective labor ratio across sectors as a result of a higher σ.

The factor input ratios in the goods and human capital sectors are given by:

$$r/w = [\beta/(1-\beta)] [l_G h_t / (s_G k_t)] = [\varepsilon/(1-\varepsilon)] [l_H h_t / (s_H k_t)]. \tag{2.19}$$

The credit sector input equilibrium is determined by the (Baumol 1952)-type conditions:

$$R = w / \left[\gamma_2 A_F (l_F h_t / c_t)^{(\gamma_2 - 1)} (s_F k_t / c_t)^{\gamma_1}\right]; \tag{2.20}$$

$$R = r / \left[\gamma_1 A_F (l_F h_t / c_t)^{\gamma_2} (s_F k_t / c_t)^{(\gamma_1 - 1)}\right]. \tag{2.21}$$

Each of the above two exchange conditions set the marginal cost of money, R, equal to the marginal factor cost divided by the marginal factor product in producing credit. With this general equilibrium setting for the (Baumol 1952) condition, combined with the existence of an explicit credit sector, these conditions are nothing more than a standard microeconomic sectoral condition whereby the marginal cost of output equals the

[3] Denote nominal discount bonds that are purchased at time t by B_{t+1}, and their price by q_{t+1}. Then the receipts B_t and costs $-\ q_{t+1} B_{t+1}$ are added to the income constraint in equation (2.6), and the derivative with respect to B_{t+1} gives that $(1 + R_{t+1}) \equiv (1/q_{t+1}) = (1 + g_{t+1})(1 + \pi_{t+1})(1+\rho)$. This combined with equation (2.16) gives the Fisher equation (2.17).

factor price divided by its marginal product. This implies, in equalizing the marginal costs of different exchange means as in the original (Baumol 1952) model, that the marginal cost of credit is the nominal interest rate; it is verified with a decentralized formalization for an explicit credit sector, and an explicit price of the credit service, that the price of credit is the nominal interest rate (Gillman and Kejak 2005) (see also Chapter 3) and (Gillman 2000).

The (Baumol 1952) conditions determine the equilibrium demand for money and its interest elasticity. For example when only money is used, such as in the standard (Lucas 1980) cash-in-advance model, then $m = c$, and the interest elasticity of money demand is simply the interest elasticity of consumption. (Gillman 1993) shows in a related economy that this type of model gives a very low magnitude of the interest elasticity of money, while when credit is produced to avoid the inflation, the interest elasticity rises in magnitude by several-fold. (Gillman and Kejak 2002) show that the higher is the interest elasticity in magnitude the lower is the inflation-growth effect in magnitude, along with the (Tobin 1965)-type effects. And a substantially rising interest elasticity as inflation increases produces a highly nonlinear inflation-growth profile as is similar to evidence.

3. Physical Capital Only Models

3.1. Ireland (1994). (Ireland 1994) uses only physical capital in an aggregate production function with a constant marginal product of capital; the Ak model. In addition the consumer avoids inflation through a sector that provides credit for buying goods instead of using money. The credit is produced using only goods and is used across a continuum of stores, selling a continuum of goods, with a different monotonically changing cost of credit at each store. As the inflation rate goes up, credit is used at more stores, with each marginally added store having a somewhat higher cost of producing the credit. This has the effect in aggregate of establishing a rising marginal cost of credit as more credit is used to avoid inflation.

Such a continuum of stores, with a continuum of goods and each with a different cost of credit that can be used to buy the goods, is also found in (Gillman 1993), (Gillman 2000), (Aiyagari, Braun, and Eckstein 1998) and (Erosa and Ventura 2000). The credit supply in these models is therefore very similar except that they use time, rather than goods or capital as in (Ireland 1994), to provide the credit. To illustrate (Ireland 1994) model in a way compatible with the standard neoclassical growth and business cycle paradigm, consider using a single aggregate consumption good as in the Section 2 model. Here the production function for credit explicitly has an increasing marginal cost, rather than this resulting in aggregate from a continuum of stores. And since goods are costlessly convertible into capital in the (Ireland 1994) economy, here it can be assumed that capital is used (rather than goods) in the production of the credit.

Assume the following special case of the Section 2 economy. Let there be a zero, instead of one, time endowment. Set the utility value of leisure to zero; $\alpha = 0$. Assume there is no human capital investment, including that $h_0 = 0$, and $A_H = 0$. Also with only physical capital being used, $\beta = 1$, goods output is CRS, and this gives the Ak function. Also here the money is used only for consumption goods, so that $a_2 = 0$. For the credit production, let $\gamma_2 = 0$ so that there is only capital used with diminishing returns, and an increasing marginal cost.

TABLE 2. Assumptions for Special Case: 3.1 Economy

Parameters	Production Functions
$\alpha = h_0 = A_H = \gamma_2 = a_2 = 0;$	$y_t = A_G s_{Gt} k_t;$
$\beta = 1.$	$d_t = A_F \left(\frac{s_{Ft}k_t}{c_t}\right)^{\gamma_1} c_t.$

The credit production function then is given by $(1 - a_t)c_t = A_F \left(\frac{s_{Ft}k_t}{c_t}\right)^{\gamma_1} c_t$, and the shares of capital add to one, $s_{Gt} + s_{Ft} = 1$. The (Clower 1967) constraint (2.7) with $a_2 = 0$ is $M_t = a_t P_t c_t$. The (Clower 1967) constraint can be combined with the credit production function to make the combined exchange constraint (2.10) now as given by $M_t = P_t c_t - P_t A_F (s_{Ft}k_t)^{\gamma_1} c_t^{1-\gamma_1}$. In the above equation, when $s_{Ft}k_t = 0$ so that no credit is produced, the standard "cash-only" (Clower 1967) constraint results in which $a_t = 1$. Note also that the amount of resources that the consumer willingly uses in credit production to avoid inflation is the capital $s_{Ft}k_t$. The rental value of this capital is the amount that corresponds precisely to (Lucas 2000) measure of the welfare cost of inflation. However where that cost was the value of the shopping time spent, here the welfare cost is the rental value of the capital used in credit production. Solving for credit capital $s_{Ft}k_t$ from the last equation gives $s_{Ft}k_{Ft} = \left[\left(1 - \frac{m_t}{c_t}\right) \diagup A_F\right]^{1/\gamma_1} c_t$. Analogous to the shopping time model, the credit capital falls with increases in m_t and with decreases in c_t.

With $\alpha = h_0 = A_H = \gamma_2 = a_2 = 0$, and $\beta = 1$, the consumer maximizes the preference discounted stream of utility in equation (2.1) subject to the constraints (2.6), and (2.10), with respect to $c_t, M_{t+1}, k_{t+1}, s_{Gt}$. The real interest rate from the firm problem is $r = A_G$. The balanced-growth rate is constant, as given by $1 + g = (1 + A_G - \delta_K) / (1 + \rho)$.

The shadow price of goods is $1 + R - R(1 - \gamma_1) A_F (\frac{s_F k_t}{c_t})^{\gamma_1}$, showing that credit use decreases the shadow exchange cost of goods below R as it would be with only money. The single (Baumol 1952) condition, comparable to equation (2.21), is $R = A_G / \left[\gamma_1 A_F \left(\frac{s_{Ft}k_t}{c_t}\right)^{\gamma_1 - 1}\right]$. This condition implies that capital in credit production, relative to consumption, rises as the nominal interest rate rises. This gives the diversion of capital from goods production when the inflation rate rises, one of (Ireland 1994) main results.[4]

[4]See (Otto and Crosby 2000) for some related empirical work.

The "great ratios" can be found to equal as $\frac{c_t}{y_t} = \frac{(\rho/A_G)(1+A_G-\delta_K)}{(1+\rho)+[A_G-\rho(1-\delta_K)]\left(\frac{R\gamma_1 A_F}{A_G}\right)^{\frac{1}{1-\gamma_1}}}$, $\frac{i_t}{y_t} = 1-\frac{c_t}{y_t}$, $\frac{c_t}{k_t} = \frac{[\rho/(1+\rho)](1+A_G-\delta_K)}{1+A_G\left(\frac{R\gamma_1 A_F}{A_G}\right)^{\frac{1}{1-\gamma_1}}}$, and $\frac{y_t}{k_t} = \frac{c_t}{k_t} + \frac{A_G-\rho(1-\delta_K)}{1+\rho}$. When the nominal interest rate R rises, c_t/y_t, c_t/k_t, and y_t/k_t fall, while i_t/y_t rises, similar to (Tobin 1965). However the real interest rate r is constant while the (Gillman and Nakov 2003) evidence indicates that r falls with increases in inflation in the long run.

The solution for the share of money usage is $a = 1 - \left[\left(\frac{R\gamma_1}{A_G}\right)^{\gamma_1/(1-\gamma_1)} A_F^{1/(1-\gamma_1)}\right]$. Since both a and c_t/k_t fall with an increase in R, it can be seen that the money to capital ratio also falls with an increase in the nominal interest rate, in that $\frac{m_t}{k_t} = \frac{ac_t}{k_t}$. This indicates substitution from real money to capital when inflation rises, again as in (Tobin 1965).

(Ireland 1994) demonstrates how the increase in capital coming from the diversion of capital into banking decreases the growth rate along the transition, but not in the stationary state at the limiting end of the transition. In the model above also there is no long run growth effect of inflation, contrary to evidence. However in this model and in (Ireland 1994) are some of the empirically supported (Tobin 1965) effects.

3.2. Stockman/Dotsey and Sarte. A special case of the (Stockman 1981) economy, also used in (Dotsey and Sarte 2000), results by assuming only a goods sector, with no human capital and no credit production, and by assuming a constant marginal product of capital, whereby $y = A_G k_G$. This is the same Ak production function as in the last section except that now $s_G = 1$. With only money used in exchange, $a_t = 1$. Also as in (Stockman 1981) let investment be purchased with money as well as goods, so that $a_t = a_2 = 1$. By using the Ak function, this puts the (Stockman 1981) model in an endogenous growth setting.

The assumptions are summarized in Table 3.

TABLE 3. Assumptions for Special Case: 3.2 Economy

Assumptions	Production functions
$\alpha = h_0 = A_H = A_F = 0$; $a_t = a_2 = s_G = \beta = 1$	$y = A_G k_G$
Calibration	
Parameters	Variables
$\rho = \delta_K = 0.03$, $A_G = 0.0909$	$R = 0, g = 0.03, r = 0.0909$

△ Nom Int Rt	△ Growth Rt	△ Real Int Rt
$0.00 \rightarrow 0.10$	-0.0080	0
$0.10 \rightarrow 0.20$	-0.0067	0

With $\alpha = h_0 = A_H = A_F = 0$ and $a_t = a_2 = s_G = \beta = 1$, the consumer maximizes the preference discounted stream of utility in equation (2.1) subject to the constraints (2.6), and (2.10), with respect to c_t, M_{t+1}, k_{t+1}.The balanced–path solution

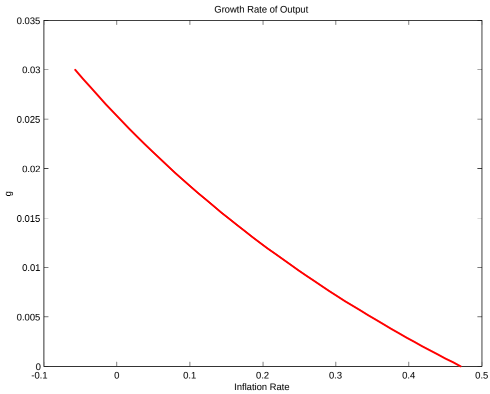

FIGURE 1. Inflation and Growth Calibration - Model 3.2

has a growth rate of $1+g = \frac{1+\frac{A_G}{1+R}-\delta_K}{1+\rho}$, so that an increase in the nominal interest rate lowers the growth rate. The after-inflation-tax marginal product of capital $A_G/(1+R)$ falls since investment must be purchased with money. The shadow price of goods is $1+R$, and the rest of the solution is $\frac{c_t}{y_t} = 1 - \frac{(g+\delta_K)}{A_G}$, $\frac{i_t}{y_t} = \frac{g+\delta_K}{A_G}$, and $\frac{m_t}{k_t} = A_G$.

An increase in the nominal interest rate causes the growth rate to fall and c/y to rise and i/y to fall, which as (Stockman 1981) noted is similar to an "inverse" (Tobin 1965) effect; r is constant. Further, with m/k constant there is no substitution between money and capital as in (Ireland 1994) and (Tobin 1965) model. In fact, there is no possibility to avoid the inflation tax as there is only one good produced, one means of exchange, and one input to utility.

However the calibration along the balanced-growth path, given in Table 3.2 and graphed in Figure 1, shows that a significant negative growth effect can result robustly in this model. Note that Figure 1, as well as the subsequent Figure 2, graphs the nominal interest rate against the growth rate, rather than the inflation rate, because the analytic solution for R versus g is simple while that solution for π and g is quite complex.

The inflation-growth effect, of -0.67% in Table 3.2, falls within the (Chari, Jones, and Manuelli 1996) range. But over the whole range of inflation rates there is only a marginal nonlinearity resulting in a counter-empirical negative growth rate as inflation gets above 50%.

4. Human Capital Only Models

Two models using only human capital are reviewed here. Both use a linear production of goods using only human capital -indexed labor. The difference is in the nature of the human capital investment function. This function can be "costless" in the sense that a certain amount of output can be costlessly transformed into human capital; this is the analogue to the standard physical capital investment accumulation equation (2.4). Or the human capital investment can be "costly", as in (Becker 1975) and (Lucas 1988b), whereby labor time and possibly physical capital inputs with diminishing returns to each input are transformed into human capital; (King and Rebelo 1990) describe this as the analogue of costly physical capital investment, such as the "adjustment cost" in (Lucas 1967).

TABLE 4. Assumptions for Special Case: 4.1 Economy

Assumptions	Production functions
$\alpha = k_0 = A_F = a_2 = \beta = \varepsilon = 0$;	$y_t = A_G l_{Gt} h_t$;
$a_{\ t} = 1$.	$i_{Ht} = A_H(1 - x_t - l_{Gt})h_t$.
I. Baseline Calibration	
Parameters	Variables
$\rho = \delta_H = 0.03,\ \alpha = 3, A_G = 0.1836, A_H = 0.1836$	$R = 0, g = 0.03, \widetilde{r} = 0.0909$
II. Calibration	
Parameters	Variables
$\alpha = 2, A_G = 0.1836, A_H = 0.1527$	$R = 0, g = 0.03, \widetilde{r} = 0.0909$

	I.	II.
△ Nom Int Rt	△ Growth Rt	△ Growth Rt
$0.00 \rightarrow 0.10$	-0.0082	-0.0056
$0.10 \rightarrow 0.20$	-0.0081	-0.0056

Table 4 summarizes the costly human capital model specification. And here define the gross marginal product of capital as $\widetilde{r} \equiv A_H(1-x)$. With $\alpha = k_0 = A_F = a_2 = \beta = \varepsilon = 0$, the consumer maximizes the preference discounted stream of utility in equation (2.1) subject to (2.5), (2.6), and (2.10), and with respect to c_t, x_t, M_{t+1}, h_{t+1}, and l_{Gt}. The real wage rate from the firm problem is given by $w = A_G$, while the growth rate is expressed as $1+g = \frac{1+A_H(1-x)-\delta_H}{1+\rho}$. The negative effect of inflation comes through its induced decrease in leisure time, and the resulting change in the marginal product of human capital. The marginal rate of substitution between goods and leisure is $x/(\alpha c_t) = (1+R)/(A_G h_t)$, and the closed-form solution of the economy is $\frac{c_t}{h_t} = \frac{\rho A_G(1+A_H-\delta_H)}{A_H(1+\rho[1+\alpha(1+R)])}$, $x = \frac{\rho\alpha(1+R)(1+A_H-\delta_H)}{A_H(1+\rho[1+\alpha(1+R)])}$, and $1+g = (1+A_H-\delta_H)/(1+\rho[1+\alpha(1+R)])$.

Table 4.1 shows a significant negative inflation-growth effect, of 0.81%. Instead of the baseline value of α =3 and $A_H = 0.1836$, this growth effect becomes smaller in magnitude, 0.56% when α is set equal to 2, and A_H recalibrated to 0.1527. This shows sensitivity but still robustness in generating a large magnitude. But as Figure 2 shows the inflation-growth profile is almost linear, and the growth rate becomes negative at a relatively low inflation rate, while in the long run evidence the growth rate stays positive for all inflation rates.

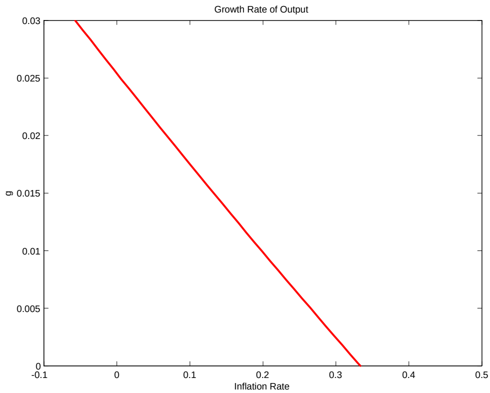

FIGURE 2. Inflation and Growth Calibration - Model 4.1

Note how this compares to a similar but non-nested model. Continue to assume that $\beta = \varepsilon = 0$ and that $A_F = 0$, so that only human capital is used in production and there is no credit available. But now assume also that $A_H = 0$, so that human capital investment does not take place through a production process. Instead, in a slight deviation from the Section 2 model, assume that goods output can be costlessly turned into human capital, as compared to assuming that goods output can be turned into physical capital in the Section 2 model. With $\widetilde{i}_{Ht}$ denoting these goods that become human capital, the social resource constraint becomes $y_t = c_t + \widetilde{i}_{Ht}$. Table 5 summarizes the specification.

TABLE 5. Assumptions for Special Case: 4.2 Economy

Assumptions	Production functions
$A_F = A_H = a_2 = \beta = 0$; $a_t = 1$	$y_t = A_G l_G h_t$
Calibration	
Parameters	Variables
$\rho = \delta_H = 0.03$, $A_G = 0.1836$	$R = 0, g = 0.03, r = 0.0909$

The human capital accumulation equation is $h_{t+1} = h_t(1 - \delta_H) + \widetilde{i}_{Ht}$. This accumulation equation is the approach taken in (Chari, Jones, and Manuelli 1996), although there physical capital also is used. With no time in human capital accumulation, the time constraint simplifies even further from that of the last subsection to $1 = x_t + l_{Gt}$.

With $A_F = A_H = a_2 = \beta = 0$, the consumer maximizes the preference discounted stream of utility in equation (2.1) subject to (2.5), (2.6), and (2.10), and with respect to c_t, x_t, M_{t+1}, $h_{t+1,}$ and $\widetilde{i}_{Ht}$. The first-order conditions imply that the solution for the growth rate in terms of leisure is $1+g=\frac{1+A_G(1-x)-\delta_H}{1+\rho}$, where $x=\frac{\alpha\rho(1+R)(1+A_G-\delta_H)}{A_G(1+\rho[1+\alpha(1+R)])}$, and $\frac{c_t}{h_t}=\frac{\rho(1+A_G-\delta_H)}{1+\rho[1+\alpha(1+R)]}$. This makes the growth rate equal $1+g=\frac{1+A_G-\delta_H}{1+\rho[1+\alpha(1+R)]}$. The growth rate is identical to the previous model of Section 4.1 if $A_G=A_H$, with the same calibration.

The singularity of the two models can also be viewed as implying that both models have two sectors, although the latter model has a simple technology for the human capital sector that is usually viewed as being a one-sector model only in goods production. And it means that the non-nested model here can be made equivalent to the nested model of Section 4.1.

5. Models with Physical and Human Capital

In a model with both physical and human capital, a standard (Clower 1967) constraint, and with human capital as the source of endogenous growth, the inflation effect on growth depends on the nature of the human capital investment function. The differences are shown by examining a model with a simple accumulation equation (Chari, Jones, and Manuelli 1996) versus one with a (Becker 1965a) - (King and Rebelo 1990) human capital investment function.

5.1. Simple Human Capital Accumulation . The simple human capital accumulation equation, in which say $\widetilde{i}_{ht}=h_{t+1}-h_t(1-\delta_H)$, sidesteps the traditional literature on human capital in which time is involved in human capital accumulation [(Schultz 1964), (Becker 1975)]. An approach sympathetic with this simple accumulation equation, but still fully nested within the (Becker 1975) human capital investment function, is to assume that the production function for the human capital investment uses only capital and no labor. Here, instead of assuming that $\widetilde{i}_{ht}$ are transformed goods output, assume instead that the (Becker 1975) human capital function has the form of equation (2.5) but assumes that $\varepsilon=0$ and that A_H equals 1, so that only physical capital is used to produce the human capital. Since goods output can be costlessly transformed into physical capital in these models, the use of physical capital instead of goods output allows for a nesting of this modified simple accumulation equation.[5] Table 6 provides the specification details.

Comparing the two definitions, of $\widetilde{i}_{Ht}$ from the last Section 4, and i_{Ht} in this section, it could be stated that $\widetilde{i}_{Ht}=s_{Ht}k_t$; just as physical capital instead of goods are used in the Section 3.1 model that compares to (Ireland 1994). The human capital accumulation process now becomes $h_{t+1}=h_t(1-\delta_H)+s_{Ht}k_t$ and the accounting of the shares of human and physical capital are now $1=s_{Gt}+s_{Ht}$ and $1=x_t+l_{Gt}$. The resource constraint is the same as in Section 2, in equations (2.4) and (2.14).

With $A_F=a_2=0$, and $a_{\ t}=A_H=\varepsilon=1$, the consumer maximizes the preference discounted stream of utility in equation (2.1) subject to (2.5), (2.6), and (2.10), and with respect to $c_t, x_t, M_{t+1}, k_{t+1}, h_{t+1,}s_{Gt}$. The growth rate is given by

[5]For an alternative view, see (Barro and Sala-i Martin 1995), footnote 13, page 181.

TABLE 6. Calibration for Section 5.1 Economy

Assumptions		Production functions	
$A_F = a_2 = 0$;		$y_t = A_G(s_{Gt}k_t)^\beta(l_{Gt}h_t)^{1-\beta}$	
$a_t = A_H = \varepsilon = 1$		$i_{Ht} = s_{Ht}k_t$	
Baseline Calibration			
Parameters		Variables	
$\rho = \delta_H = \delta_K = 0.03$, $\beta = 0.4$		$R = 0, g = 0.03$, $r = 0.0909$	
$A_G = 0.0877$		$s_G k/l_G h = 0.0184$	
△ Nom Int Rt	△ Growth Rt	△ Real Int Rt	△ Cap-Lab-Ratio
0.00 → 0.10	−0.00281	−0.00298	0.01167
0.10 → 0.20	−0.00258	−0.00273	0.01167

$$1 + g = \frac{1 + r - \delta_K}{1 + \rho} = \frac{1 + \frac{w}{r}(1 - x) - \delta_H}{1 + \rho}, \tag{5.1}$$

whereby balanced-growth implies an equivalence of the marginal products of physical capital and human capital.

Solving the economy numerically, the baseline calibration of the change in the growth rate is given in Table 6. The growth rate decreases are significant and within the range of empirical estimates. A problem however is that with human capital so productive relative to goods production, in that $A_H = 1$ and $A_G = 0.0877$, most of the capital is directed to human capital. The capital to effective labor ratio at the optimum of $R = 0$ is only $(s_G k/l_G h) = 0.0184$. This is not a very plausible ratio and represents an indication of the problem with the general specification. However qualitatively, the calibration shows the positive (Tobin 1965) effect of a rising capital to effective labor ratio as the nominal interest rate rises, while the growth rate falls.

Figure 3 graphs the inflation-growth profile over a range of inflation rates. The line representing the model is the dot-dash one. it shows a marginal degree of nonlinearity that tends to be much less than found empirically. Although there is no exact empirical measure for the degree of nonlinearity, evidence indicates that the growth rate never becomes negative, while in the model here it does become negative.

The model with the alternate assumption of using goods in the simple human capital accumulation equation was also calibrated, but is not shown here since it is not nested. The results for the growth rate are quite similar, and so in this respect the models compare closely. In both the model of this section and the alternate the effect of inflation on growth is about half the magnitude of that effect when the human capital function also includes time, as in the next section.[6]

[6](Chari, Jones, and Manuelli 1996) use a (Lucas and Stokey 1983) cash-good, credit-good, preference function, which cannot be nested in the Section 2 model of this paper, and report an insignificant inflation-growth effect. It is possible that if the (Lucas and Stokey 1983) preference parameters are specified so that cash and credit goods are near perfect substitutes, while at the same time there is

5.2. Becker-Lucas Model. Using a more general (Becker 1975) function as in the Section 2 model relaxes the constraint on A_H and ε, as in (Gillman and Kejak 2002). The human capital investment function uses both effective labor and capital. Table 7 summarizes the specification.

TABLE 7. Calibration for Section 5.2 Economy

Assumptions			Production functions	
$A_F = a_2 = 0$			$y_t = A_G (s_{Gt}k_t)^{\beta} (l_{Gt}h_t)^{1-\beta}$	
$a_t = 1$			$i_{Ht} = A_H (s_{Ht}k_t)^{\varepsilon} (l_{Ht}h_t)^{1-\varepsilon}$	
I. Baseline Calibration				
Parameters			Variables	
$\rho = \delta_H = \delta_K = 0.03, \beta = 0.4, \varepsilon = 0.3, \alpha = 3,$ $A_G = 0.3110,\ A_H = 0.2609$			$R = 0, g = 0.03, r = 0.0909$	
II. Calibration				
Parameters			Variables	
$\varepsilon = 0.4,\ A_G = 0.3110,\ A_H = 0.3318$			$R = 0, g = 0.03, r = 0.0909$	
III. Calibration				
Parameters			Variables	
$\alpha = 2,\ A_G = 0.3110,\ A_H = 0.2609$			$R = 0, g = 0.03, r = 0.0909$	
I.			II. $\varepsilon = 0.4$	III. $\alpha = 2$
△ Nom Int Rt	△ Growth Rt	△ Real Int Rt	△ Growth Rt	△ Growth Rt
$0.00 \to 0.10$	-0.00572	-0.00607	-0.00492	-0.00439
$0.10 \to 0.20$	-0.00538	-0.00570	-0.00459	-0.00420

With $A_F = a_2 = 0$, and $a_t = 1$, the consumer maximizes the preference discounted stream of utility in equation (2.1) subject to (2.5), (2.6), and (2.10), and with respect to $c_t, x_t, M_{t+1}, k_{t+1}, h_{t+1}, s_{Gt}, l_{Gt}$. The balanced path growth rate can be expressed as $1 + g = \frac{1+\beta A_G\left(\frac{l_G h}{s_G k}\right)^{1-\beta} - \delta_K}{1+\rho} = \frac{1+(1-x)(1-\varepsilon)A_H\left(\frac{s_H k}{l_H h}\right)^{\varepsilon} - \delta_H}{1+\rho}$.

Solving for this system numerically, the baseline calibration is given in Table 5.2. The general nature of the human capital investment function makes the magnitude about double of the previous model in Section 5.1. The only significant problem here in matching the evidence is the marginal degree of nonlinearity in the inflation-growth effect. The inflation-growth profile is graphed in Figure 3 as the dashed line. It is nearly linear and indicates a negative growth rate as inflation increases contrary to evidence.

5.3. Becker-Lucas Model with a Credit Sector. Finally, consider adding a credit sector to the model of Section 5.2. This adds one more margin to the consumer

no real resource cost to using the credit, which is true in the (Lucas and Stokey 1983) model, then inflation can cause near perfect substitution to the credit good, with close to zero increase in leisure, resulting in an insignificant growth effect.

and yields considerable flexibility to avoid the inflation tax. This gives the model the ability to explain not only a significant inflation-growth effect, and (Tobin 1965) effects, but also the nonlinearity of the inflation-growth effect. In addition, while not detailed here (see (Gillman, Harris, and Matyas 2004)), it can show differences in the inflation-growth effect across regions as based on financial development. Here a credit sector is added using only effective labor, in contrast to the capital-only model of Section 3.1. The assumptions are given in Table 5.3.

TABLE 8. Calibration for Section 5.3 Economy

Assumptions	Production functions		
$a_2 = \gamma_1 = 0;$	$y_t = A_G\left(s_{Gt}k_t\right)^{\beta}\left(l_{Gt}h_t\right)^{1-\beta}$		
	$i_{Ht} = A_H\left(s_{Ht}k_t\right)^{\varepsilon}\left(l_{Ht}h_t\right)^{1-\varepsilon}$		
	$d_t = (1-a_t) = A_F\left(l_{Ft}h_t/c_t\right)^{\gamma_2} c_t$		
Baseline Calibration			
Parameters	Variables		
$\rho = \delta_H = \delta_K = 0.03, \beta = 0.4$	$R = 0, g = 0.03, r = 0.0909$		
$\varepsilon = 0.3, \alpha = 3, \gamma_2 = 0.3,\ A_F = 0.5184$	$s_G k/l_G h = 1.6871$		
$A_G = 0.3110, A_H = 0.3279,$			
$\triangle$ Nom Int Rt	$\triangle$ Growth Rt	$\triangle$ Real Int Rt	$\triangle$ Cap-Lab-Ratio
$0.00 \to 0.10$	-0.00472	-0.00500	0.16696
$0.10 \to 0.20$	-0.00381	-0.00401	0.15398

With $a_2 = \gamma_1 = 0$, the consumer maximizes the preference discounted stream of utility in equation (2.1) subject to (2.5), (2.6), and (2.10), and with respect to c_t, x_t, M_{t+1}, k_{t+1}, h_{t+1}, s_{Gt}, l_{Gt}, l_{Ft}. The added first-order condition is the (Baumol 1952) equation $R = w/\left[\gamma_2 A_F(l_F h_t/c_t)^{(\gamma_2-1)}\right]$.

The calibration results are reported in Table 5.3 and details of similar calibrations can be found in (Gillman and Kejak 2002). Figure 3 graphs the inflation-growth profile in the solid line. This conforms roughly to evidence on the shape of the nonlinearity [see (Gillman, Harris, and Matyas 2004)] and in which the growth rate does not become negative as the inflation rate increases.

6. Comparison of Models

Table 9 summarizes the findings across the different models. The table shows, in its second column, that all models but the first have inflation-growth effects close to the (Chari, Jones, and Manuelli 1996) range of -0.2% to -0.7% for a change in the inflation rate from 10 to 20% (although here we report the results for similar changes in R). While some of the models have calibrations that are a bit high, none are too low. This establishes clearly a robust significant negative inflation-growth effect across a range of models. Distinguishing further among the models requires use of the third and fourth columns, on nonlinearity and (Tobin 1965)-type effects. With growing evidence of a strong nonlinearity, whereby the inflation-growth effect is marginally weaker as

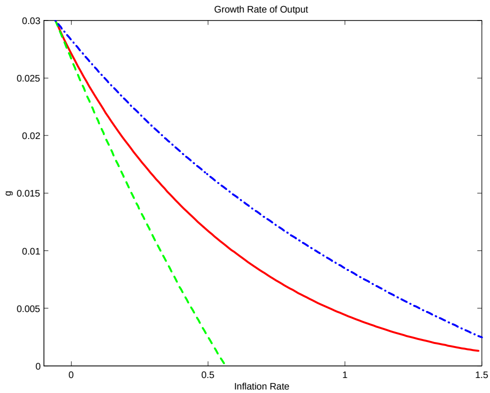

FIGURE 3. Calibration of Inflation and Growth - Section 5 Models

higher levels of the inflation rate, and on positive (Tobin 1965)-type effects, only two models, 5.1 and 5.3, meet all criterion. The model of Section 5.1 however does not provide a sense of plausibility, in that the parameter assumption of $A_H = 1$ leads to a nearly insignificant capital to effective labor ratio. Also its nonlinearity is only marginal and generates negative levels of the growth rate. The model of Section 5.3 has no such plausibility problems and has a strong nonlinearity without negative levels of the growth rate.

TABLE 9. Summary of Growth and Tobin Effects

Models of Section	BGP Inflation-Growth Decrease R: 0→ 0.10	R: 0.10→ 0.20	Nonlinearity	Tobin Effect
3.1	0	0	n.a.	positive
3.2	−0.0080	−0.0067	Marginal	inverse
4.1	−0.0082	−0.0081	Near Linear	none
5.1	−0.0028	−0.0026	Marginal	positive
5.2	−0.0057	−0.0054	Near Linear	positive
5.3	−0.0047	−0.0038	Highly	positive

Only the last model also is jointly consistent with the (Aiyagari, Braun, and Eckstein 1998) money and banking findings, that the banking sector expands in size in conjunction with the level of the inflation rate. Further (Gillman and Kejak 2002) show that this Section 5.3 model yields a money demand closely comparable to a (Cagan 1956)-type constant semi-interest elasticity model for which (Mark and Sul 2003) find recent broad-based cointegration support.

7. Conclusions

The paper presents a general monetary endogenous growth model with both human and physical capital, and then categorizes a set of models as being nested within this model. The first subset of models considered are Ak models in which inflation acts as a tax on physical capital with a negative long-run (Tobin 1965)-type effect. Next presented are Ah models in which inflation acts as a tax on human capital and there is a positive (Tobin 1965) effect. Then come the more general models with human and physical capital, in which inflation acts more as a tax on human capital and there is a positive (Tobin 1965) effect.

While there is no unemployment per se in any of these three classes of models, the employment rate moves in the opposite direction of the inflation rate in the models with human capital. This direction and causality of the employment effect is not inconsistent with evidence in (Shadman-Mehta 2001). They find cointegration of inflation and unemployment for historical UK data, including Phillips original sample period, and that inflation Granger-causes unemployment in the long run.

The reviewed models show a strong linkage between the magnitude of the inflation-growth and the (Tobin 1965)-type effects, and between the nonlinearity of both of these effects. The growth rate decrease, when the inflation rate rises from 0 to 10% and then to 20%, is proportional in its strength of magnitude and its degree of nonlinearity directly to the real interest rate decrease and the capital to effective labor rate increase. This linkage is a general characteristic across models that acts as a key distinguishing feature. If the nonlinearity is in fact significant, as evidence suggests, then models without this overstate significantly the inflation effects for rates of inflation above the baseline level. The explanation of this nonlinearity comes back to the money demand elasticity that underlies the model. A rising interest elasticity, with inflation rising, leads to easier substitution away from inflation and causes the nonlinearity. A near constant interest elasticity money demand, as in the standard cash-in-advance model, leads to a near linear response.

Debate on the monetary growth models and on the existence within these models of (Tobin 1965)-type effects on the real interest rate and the Great Ratios, c/y and i/y, goes back to when monetary growth models used the Solow model as modified by (Tobin 1965) to include money as the basis of debates (see for example (Johnson 1969) and (Niehans 1969)). The advent of endogenous growth theory as ushered in by (Lucas 1988b) marked a substantial leap in progress that reframed this debate once money was included within these models using the (Lucas 1980) approach (Gomme 1993). The resulting endogeneity of the growth rate relative to changes in the inflation rate, as working through the labor-leisure channel, allowed for calibration of the inflation-growth effect within the estimated empirical range. However the Ak models also have been able to accomplish the same feat, making unclear what approach is more advantageous. Updating the traditional focus on the Great Ratios has allowed for a re-focusing on how these models can be differentiated. The (Gomme 1993)-type models capture general equilibrium decreases in the real interest rate and the consumption to output ratio, and increases in the investment to output ratios, all as a result of inflation, and as consistent with evidence. This makes the simpler Ak models more dated, in that they cannot so easily, if at all, achieve similar results. A further distinguishing factor, going beyond the magnitude of the inflation-growth effect, and beyond the direction of the (Tobin 1965)-type effects, is how these effects behave over the range of inflation rate levels. Evidence shows a strong nonlinearity in the inflation-growth effect. And the (Lucas 1988b)-(Gomme 1993) model that is extended to include credit production as a substitute to cash, in a (Baumol 1952)-type fashion, can account for this nonlinearity by producing an implied interest elasticity of money demand that rises in magnitude with the inflation rate, as in the successful (Cagan 1956) model. There are currently few such models that link the evidence in favor of near constant semi-interest elasticities of money demand, with the nonlinearity of the inflation-growth effect, along with a significant negative magnitude of this effect, while also capturing the (Tobin 1965)-type effects.

CHAPTER 5

The Demand for Bank Reserves and Other Monetary Aggregates

1. Introduction

Modeling the monetary aggregates in general equilibrium has been a challenge. There are some examples such as (Chari, Jones, and Manuelli 1996), and (Gordon, Leeper, and Zha 1998), who present models that are compared to Base money. (Ireland 1995) presents one that he relates to M1-A velocity. And these models have been employed as ways to explain the actual monetary aggregate time series evidence. However (McGrattan 1998), for example, argues that the simple linear econometric model in which velocity depends negatively on the nominal interest rate, may do just as well or better in explaining the evidence.

The chapter here takes up the topic by modeling a nesting of the aggregates that uses a set of factors that expands from the nominal interest rate by including the production of banking services. Through this approach the productivity factor of banking enters, as well as a cost to using money, sometimes thought of as a convenience cost. With this general equilibrium model, and its comparative statics, an explanation of velocity is provided that depends in part on the nominal interest rate, similar in spirit to (McGrattan 1998). Also using technology factors, the paper explains US evidence on monetary base velocity, M1 velocity, and M2 velocity, as well as for the ratios of various aggregates. This more extended explanation than previous work highlights the limits to a nominal interest rate story, while revealing a plausible role of technological factors in determining the aggregate mix.

The original literature on the welfare cost of inflation, well-represented by (Bailey 1992), assumes no cost to banks in increasing their exchange services as consumers flee from currency during increasing inflations.[1] Similarly, (Johnson 1969) and (Marty 1969) assume no real costs for banks in producing "inside money".[2] The approach here builds upon the more recent literature of (Gillman 1993), (Aiyagari, Braun, and

[1] "The presence of the banking system has no real effect whatever but merely alters the nominal rate of inflation necessary to achieve a given real size of the government budget" (P.234, 1992); in the model here the latter statement is true, but not the former since capital is used up in banking activities, and since reserve requirements affect the real interest rate when there is a non-Friedman optimum rate of interest.

[2] (Johnson 1969), for example, writes that "a banking organization could issue non-interest-bearing deposites, assumed to be costless to administer"; p. 32. (Marty 1969) discusses demand deposits and assumes that "the cost of setting up and running a bank is zero"; p. 106. Here both authors are focusing on the wealth effects of inside and outside money.

Eckstein 1998), and (Lucas 2000) that assumes resource costs to avoiding the inflation tax by using alternative exchange means. In particular, the paper specifies production functions for banking instruments, both demand deposits (inside money) and credit, that require real resource use. This gives rise to the role of banking productivity factors in explaining the movement of aggregates.[3]

The next section reviews Haslag's (1998) model and shows how it is sensitive to the distribution of the lump sum inflation proceeds. This sensitivity makes tentative the growth effect of inflation with the model. The demand for reserves can be made insensitive to the distribution of the inflation tax transfer by framing it within a model in which the bank must hold money in advance as in the timing of transactions that is pioneered in (Lucas 1980). This is done in Section 3 using Haslag's (1998) notation, Ak production technology, and full savings intermediation. The resulting real interest rate depends negatively on the nominal interest rate, and so inflation negatively affects the growth rate, similar in fashion to the central result of (Haslag 1998). A parallel consumer cash-in-advance demand for goods is also added, as in (Chari, Jones, and Manuelli 1996), to give a model of reserves plus currency.

The chapter then expands the model to give a formulation of the demand for the base plus non-interest bearing demand deposits, or an aggregate similar to M1.[4] Following a credit production approach used in a series of related papers,[5] the paper then adds credit, or interest bearing demand deposits, to give a formulation for an aggregate similar to M2.

2. Sensitivity to Lump Transfers

In (Haslag 1998), all savings funds are costlessly intermediated into investment by the bank. The bank must hold reserves in the form of money. This gives rise to a bank demand for money in order to meet reserve requirements on the savings deposits. The consumer-agent does not use money although the lump sum inflation tax is transferred to the agent. Instead the agent simply holds savings deposits at the bank and earns interest as the bank intermediates all investment. The bank's return is lowered by the

[3](Hicks 1935) seeks a theory of money based on marginal utility, with cash held in advance of purchases, as (Lucas 1980) follows. Hicks shunts aside both Keynes's alternative to Fisher's quantity theory as found in his Treatise (see (Gillman 2002) on flaws in this theory), and considers "Velocities of Circulation" as in Fisher's quantity theory an "evasion". He reasons that money use suggests the existence of a friction and that "we have to look the friction in the face". The "most obvious sort of friction" is "the cost of transfering assets from one form to another". And Hicks says that we should consider "every individual in the community as being, on a small scale, a bank. Monetary theory becomes a sort of generalisation of banking theory." In alignment with Hicks, the agent in this paper acts as a bank in part, and the bank has costs from creating new instruments such as demand deposits and credit. But in contrast, here velocity is endogenously determined as a fundamental part of the resulting equilibrium. And Hicks's and Lucas's approaches converge with Fisher's.

[4]This abstracts from the interest that is earned on some demand deposit accounts included in the US M1 aggregate, since this interest tends to be of nominal amounts compared to the savings accounts included in M2.

[5]See (Gillman and Kejak 2002), (Gillman, Harris, and Matyas 2004), and (Gillman and Nakov 2003).

need to use money for reserves. Further, the timing of the model is such that inflation decreases the real return to depositors, and therefore also the growth rate, through the requirement that reserves be held as money.

The following model gives the reported result in (Haslag 1998).[6] With the gross return on invested capital being $1 + A - \delta$, as in an AK model, with the time t capital stock denoted by k_t, the savings deposits denoted by d_t, the nominal money stock by M_t, the price level by P_t, and the net return paid on deposits denoted by r_t, the nominal profits are given as

$$\Pi_t = P_t(1 + A - \delta)k_t + M_{t-1} - P_t(1 + r_t)d_t. \tag{2.1}$$

This is stated as a maximization problem with respect to k_t, M_{t-1}, d_t and subject to two constraints. The constraints (with equality imposed) are that the sum of capital and last periods real balances equals deposits:

$$k_t + M_{t-1}/P_{t-1} = d_{t,} \tag{2.2}$$

and that a fraction γ_{t-1}, given in the last period, of time t deposits is held as real money balances in time $t - 1$:

$$M_{t-1}/P_{t-1} = \gamma_{t-1} d_t. \tag{2.3}$$

Assuming zero profit this yields through simple substitution the return reported by (Haslag 1998):

$$1 + r_t = (1 + A - \delta)(1 - \gamma_{t-1}) + \gamma_{t-1}(P_{t-1}/P_t). \tag{2.4}$$

The result is sensitive to who gets the lump sum cash transfer from the government. If the transfer instead goes to the bank, the only user of money in the model, then there is no growth effect of inflation. This can be seen in the following way: Let the money supply process be given as in (Haslag 1998) as $M_t = M_{t-1} + H_{t-1}$, where H_{t-1} is the lump sum transfer by the government. With the transfer given to the bank, the profit of equation (2.1) becomes

$$\Pi_t = P_t(1 + A - \delta)k_t + M_{t-1} + H_{t-1} - P_t(1 + r_t)d_t. \tag{2.5}$$

Let the balanced growth rate of the economy be denoted by g_t, and the consumer's time preference by ρ, whereby the consumer's problem in (Haslag 1998) with log utility

[6] However to get this result, three changes were made to the model actually published in (Haslag 1998), indicating incidental errors in the published paper: the money stock in the profit equation (2.1) is in time $t - 1$, instead of t as published; and the money stock and the price level in equation (2.2) are in time $t - 1$ instead of time t as published. The actual return in the paper as published is that $r_t = (A - \delta)\left[1 - \gamma_t (1 + g_t)\right]$, where g_t denotes the balanced-path growth rate; it is independent of the inflation rate.

gives that $1 + g_t = (1 + r_t)/(1 + \rho)$. With this growth rate in mind, the zero profit equilibrium now gives a rate of return to depositors of

$$1 + r_t = (1 + A - \delta)(1 - \gamma_{t-1}) + \gamma_t(1 + g_t), \tag{2.6}$$

and there is no inflation tax on the return or on the growth rate.

Alternatively let the profit function be given as equation (2.5). Then assume that the stock and reserve constraints, equations (2.2) and (2.3), are all in terms of current period variables, as in a standard cash-in-advance economy where here the reserve constraint now would look like a (Clower 1967)-type constraint. Substituting in M_t for $M_{t-1} + H_{t-1}$, then the model is exactly as in (Chari, Jones, and Manuelli 1996). This gives the result, also found in (Einarsson and Marquis 2001), that

$$1 + r_t = (1 + A - \delta)(1 - \gamma_t) + \gamma_t. \tag{2.7}$$

The return is lowered because reserves are idle but there is no inflation tax.

3. Models of Monetary Aggregates

3.1. Monetary Base. The financial intermediary has a demand for nominal money, denoted by M_t^r, as created by the need for reserves, with the reserve ratio denoted by $\gamma \in [0, 1]$. But here, as in (Chari, Jones, and Manuelli 1996), the reserve constraint is considered as the bank's (Clower 1967) constraint, and structured accordingly in a fashion parallel to the consumer's, being that

$$M_t^r = \gamma P_t d_t. \tag{3.1}$$

In addition the asset constraint adds together the current period real money stock with the current period capital stock to get the current period real deposits. In real terms this is written as

$$k_t + M_t^r / P_t = d_t. \tag{3.2}$$

And, unlike (Chari, Jones, and Manuelli 1996), the bank has to set aside cash in advance of the next period's accounting of the reserve requirement in order to meet any increase in its reserve requirements. The bank has revenue from its return on investment, and costs from payment of interest to depositors, and from any increase in money holdings for reserves.

The technology for the output of goods, as in (Haslag 1998), is an Ak production function, making the current period profit function:

$$\Pi_t^r = P_t(1 + A - \delta)k_t + M_t^r - M_{t+1}^r - P_t d_t(1 + r_t). \tag{3.3}$$

The profit maximization problem is dynamic because of the way in which money enters the bank's profit function in two different periods, the same dynamic feature of the consumer problem. The competitive bank discounts the nominal profit stream by the nominal rate of interest, and maximizes the time 0 discounted stream, denoted by $\widehat{\Pi}_0^r$, with respect to the real capital stock, k_t, the real deposits, d_t, and the money stock used for reserves, denoted by M_{t+1}^r, and subject to the (Clower 1967)-type reserve and asset stock constraints of equations (3.1) and (3.2):

$$
\begin{aligned}
(3.4) \qquad \underset{d_t, M_{t+1}^r, k_t}{Max} \widehat{\Pi}_0^r = & \sum_{t=0}^{\infty} \prod_{i=0}^{t} \left(\frac{1}{1+R_i} \right)^t \{ [P_t(1+A-\delta)k_t \\
& + M_t^r - M_{t+1}^r - P_t(1+r_t)d_t] \\
& + \lambda_t \left[P_t d_t - M_t^r - P_t k_t \right] \\
& + \mu_t \left[M_t^r - \gamma P_t d_t \right] \} .
\end{aligned}
$$

Assuming a constant money supply growth rate, so that the nominal interest rate is constant over time, the first-order conditions imply that the rate of return is

$$
(3.5) \qquad 1 + r = (1 + A - \delta)(1 - \gamma) - \gamma R.
$$

Using the Fisher equation of nominal interest rates (presented below), with the above equation, shows that there is a negative effect of inflation on the return. Combined with the consumer's problem and the derivation of the balanced-growth rate as depending on the real interest rate, inflation therefore causes a negative effect on the balanced-path growth rate.

The bank does not receive any lump sum transfer from the government; the consumer-agent receives it all. However the distribution only affects how much profit the intermediary makes. Since the profit is transferred to the consumer, just as is the lump sum transfer of inflation proceeds, the distribution of the inflation proceeds between the bank and the consumer can be changed without affecting the allocation of resources in the economy. For example, if the intermediary gets part of the inflation proceeds transfer, by an amount at time t equal to $M_{t+1}^r - M_t^r$, then in equilibrium the money terms cancel from the profit function, and $\Pi_t^r / (P_t k_t) = R [\gamma / (1-\gamma)]$. At the Friedman optimum, this profit is zero.[7]

Consider a consumer problem as in (Haslag 1998) except that now the consumer uses cash as in (Lucas 1980). The problem then includes the setting aside of the consumer's cash in advance of trading in the next period, denoted by M_{t+1}^c, and the receipt of the lump sum government transfer of inflation proceeds, denoted by H_t.

The consumer's (Clower 1967) constraint is

$$
(3.6) \qquad M_t^c = P_t c_t.
$$

[7]See (Bailey 1992) for an early discussion of intermediary earnings during inflation. If current period non-negative profit is required for the bank intermediary to exist, then a transfer to the bank as in the above-described transfer scheme, with $\Pi_t^r / (P_t k_t) = R_t [\gamma / (1-\gamma)]$, would satisfy this at all inflation rates.

The consumer also makes real (time) deposits, denoted by d_t, with the real return, denoted by r_t, as the form of all savings and wholly intermediated through banks, as in (Haslag 1998). This involves choosing the next period deposits d_{t+1} and receiving as real income $(1 + r_t)\, d_t$. The nominal current period profit of the intermediation bank, Π_t^r, is received by the consumer each period as a lump sum income source. This makes the consumer current period budget constraint of income minus expenditures as in the following:

$$P_t(1 + r_t)d_t + H_t + \Pi_t^r + M_t^c - M_{t+1}^c - P_t c_t - P_t d_{t+1} = 0. \tag{3.7}$$

The problem is to maximize the time preference discounted stream of current period utility, where $\beta \equiv 1/(1 + \rho)$ denotes the discount factor, subject to the income and (Clower 1967) constraints:

$$\begin{gathered} \underset{c_t, d_{t+1}, M_{t+1}^c}{Max} \; L = \textstyle\sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_t\right) \right. \\ +\lambda_t \left[P_t(1 + r_t)d_t + H_t + \Pi_t^r + M_t^c - M_{t+1}^c - P_t c_t - P_t d_{t+1} \right] \\ \left. +\mu_t \left[M_t^c - P_t c_t \right] \right\}. \end{gathered} \tag{3.8}$$

The first-order conditions are

$$u_{c_t} = \lambda_t P_t (1 + \mu_t/\lambda_t), \tag{3.9}$$

$$\lambda_t/(\lambda_{t+1}\beta) = (1 + r_{t+1})(1 + \pi_{t+1}) \equiv (1 + R_{t+1}),^{8} \tag{3.10}$$

$$\lambda_t/(\lambda_{t+1}\beta) = 1 + \mu_{t+1}/\lambda_{t+1}. \tag{3.11}$$

These imply that

$$u_{c_t} = \lambda_t P_t (1 + R_t), \tag{3.12}$$

so that the nominal interest rate is the shadow exchange cost of buying a unit's worth of consumption. Using this later equation to form an Euler equation, then along the balanced-growth equilibrium with log utility it follows that the growth rate of consumption, where $1 + g_{t+1} = c_{t+1}/c_t$, is constant and given by

$$1 + g = \frac{1 + r}{1 + \rho}. \tag{3.13}$$

The demand for money is given by the (Clower 1967) constraint, $M_t^c = P_t c_t$. This standard (Lucas 1980) demand function can be thought of as a demand for "currency", in this, the simplest version of the model.

[8] Including the market for nominal bonds as in (Lucas and Stokey 1987) would give R_t as the price of the bonds and would explicitly derive the Fisher equation.

The total demand for money is the sum of the bank's and the consumer's, and this is set equal to the total money supply as a condition of market clearing in equilibrium:

$$M_t^r + M_t^c \equiv M_t^b. \tag{3.14}$$

The total money supply equation is that this period's money base, denoted by M_t^b plus the lump sum transfer equal next period's base supply of money:

$$M_t^b + H_t = M_{t+1}^b. \tag{3.15}$$

Assume that the money supply growth rate is constant at σ, where $\sigma \equiv H_t/M_t^b$.

3.2. M1. Now consider an extension in which the consumer suffers a nominal cost of using money that is proportional to the amount of cash used to make purchases. This can be thought of as the "convenience" cost of using money. This can be related to the average amount stolen in robberies by pickpockets, lost by carelessness, and spent on protection against crime and carelessness. And, it can be (Karni 1974) time costs or (Baumol 1952) shoe-leather costs. These costs can be affected by the availability of bank locations, and now ATM locations.[9] Let this amount be given by ϕM_t^c, with $\phi \in [0, 1)$. Second assume that a second bank exists, a bank that supplies only non-interest bearing deposits, denoted by $M_t^{dd,s}$, that can be used in exchange. This money can be thought of demand deposits as in the US or as a debit card as is more common in Europe.[10] The bank charges a nominal fee of P_t^{dd} per unit of real deposits, so that it receives from the consumer total such receipts equal to $P_t^{dd}(M_t^{dd,s}/P_t)$; and the bank produces these non-interest bearing deposits through a production process. The consumer receives from the deposit bank its nominal profit, denoted by Π_t^{dd}, and the profit from the intermediation bank, and the lump sum inflation tax transfer from the government. The consumer's demand for the real non-interest bearing deposits is denoted by M_t^{dd}/P_t. Also the consumer invests in capital that is rented out by the demand deposit bank at the rate of r_t, with this capital denoted by k_t^{dd}. This makes the bank similar to a "mutual" customer-owned bank, and its capital does not get intermediated through the savings deposit bank. The depreciation rate on this capital

[9]We are indebted to Bob Lucas for originally suggesting the concept of the cost from crime, and to Rowena Pecchenino for comments on this. Note that these costs are on the consumer side of the problem, while costs of alternative instruments for exchange are on the banking firm side of the problem. The so-called shopping time costs (Lucas 2000) actually compare better to the bank firm costs in this problem, as is shown below in footnote 13. Karni's and Baumol's costs are a story more about the costs on the consumer side. The diffusion of ATMs plausibly affects both banking productivity and the consumer's cost of using money.

[10]In Russia, after losing confidence in the bank sector during its collapse in 1997, people are again starting to use banks to hold cash. '"I'm used to carrying all my cash with me, but with a [debit] card its easier," said Denis Tafintsev, a 25-year-old warehouse manager. "If you lose your card you don't lose your money."' (Wall Street Journal Europe, May 28, 2003, "Retail Banking Grows in Russia", p.M1; brackets in article).

is assumed to be zero, so that the consumer invests in this capital each period by the amount of $k_{t+1}^{dd} - k_t^{dd}$.

The consumer chooses what fraction of purchases to be made with cash, denoted by $a_t^c \in [0,1]$, and what fraction to be made with non-interest demand deposits, $a_t^{dd} \in [0,1]$; where

$$a_t^c + a_t^{dd} = 1. \tag{3.16}$$

The (Clower 1967) constraints become

$$M_t^c = a_t^c P_t c_t; \tag{3.17}$$

$$M_t^{dd} = (1 - a_t^c) P_t c_t. \tag{3.18}$$

And the consumer problem now is:

$$\begin{aligned} \underset{c_t, d_{t+1}, k_{t+1}^{dd}, M_{t+1}^c, M_{t+1}^{dd}, a_t^c}{Max} \quad & L = \textstyle\sum_{t=0}^{\infty} \beta^t \{u(c_t) \\ & +\lambda_t \left[P_t(1+r_t)d_t + H_t + \Pi_t^r + \Pi_t^{dd} + M_t^c + M_t^{dd} - M_{t+1}^c - \phi M_t^c \right. \\ & \left. -M_{t+1}^{dd} - (P_t^{dd}/P_t)M_t^{dd} - P_t c_t - P_t d_{t+1} - P_t k_{t+1}^{dd} + P_t k_t^{dd}(1+r_t)\right] \\ & +\mu_t^c [M_t^c - a_t^c P_t c_t] \\ & +\mu_t^{dd} \left[M_t^{dd} - (1-a_t^c) P_t c_t\right]\} . \end{aligned} \tag{3.19}$$

The first-order condition with respect to a_t^c gives that $\mu_t^{dd} = \mu_t^c$. In combination with the first-order conditions with respect to the two money stocks, M_{t+1}^c and M_{t+1}^{dd}, this implies that the interior solution satisfies

$$P_t^{dd}/P_t = \phi. \tag{3.20}$$

And note that the shadow cost of buying goods with cash is given by the marginal condition:

$$u_{c_t} = \lambda_t P_t (1 + R_t + \phi), \tag{3.21}$$

so that the shadow exchange cost now is equal to $R_t + \phi$ instead of only R_t as in the previous subsection.

The demands for the cash and for the demand deposits are given by the (Clower 1967) constraints in equilibrium, where the a_t^c variable is determined by finding the equilibrium bank supply of demand deposits and setting this equal to the demand for demand deposits.

The original bank, the capital intermediation bank, has the same problem as stated previously. Now consider the specification for the production function of the new bank. This bank uses real resources in the process of producing demand deposits and so is costly unlike the intermediation bank. With an $\widehat{A}K$ type production function for the non-interest bearing demand-deposit bank, it can be shown that the equilibrium would not be well defined. If the $\widehat{A}$ parameter equals ϕ, then there is no unique equilibrium; and if $\widehat{A}$ equals any other value then there is an equilibrium either with no demand for cash or with no demand for credit. A unique equilibrium is satisfied by specifying a diminishing returns technology whereby there is a margin at which the fixed ϕ is equal to the variable marginal cost of producing the demand deposits. Initially assume that the new demand deposit bank faces the following production function that is diminishing in its capital input. Denoting the shift parameter by $\widehat{A}_{dd}$ and the capital input by k_t^{dd}, and with $\alpha \in (0,1)$, let the function be specified as

$$M_t^{dd,s}/P_t = \widehat{A}_{dd}(k_t^{dd})^{\alpha}. \tag{3.22}$$

The demand deposit bank gets revenue from "printing" new demand deposits, $M_{t+1}^{dd} - M_t^{dd}$, and from the fee the consumer pays for the services, and on the cost side rents capital from the consumer at the market real interest rate of r_t. The current period profit, Π_t^{dd}, are given as the revenue minus the costs,

$$\underset{k_t^{dd}, M_{t+1}^{dd}}{Max}\ \Pi_t^{dd} = \left(P_t^{dd}/P_t\right) M_t^{dd} - P_t r_t k_t^{dd} + M_{t+1}^{dd} - M_t^{dd}. \tag{3.23}$$

With a constant money supply growth rate, the nominal interest rate is constant at R and the deposit bank faces the following dynamic profit maximization problem:

$$\begin{aligned} \underset{d_t, M_{t+1}^r, k_t}{Max}\ \widehat{\Pi}_0^r &= \sum_{t=0}^{\infty}\left(\frac{1}{1+R}\right)^t \{[\left(P_t^{dd}/P_t\right) M_t^{dd} - P_t r_t k_t^{dd} + M_{t+1}^{dd} - M_t^{dd}] \\ &\quad + \lambda_t \left[P_t \widehat{A}_{dd}(k_t^{dd})^{\alpha} - M_t^{dd}\right]\right\}. \end{aligned}$$

The first-order conditions imply that

$$R + P_t^{dd}/P_t = r_t \Big/ \left[\widehat{A}_{dd}\alpha(k_t^{dd})^{\alpha-1}\right], \tag{3.24}$$

which when combined with the consumer's equilibrium condition (3.20) gives that

$$R + \phi = r_t \Big/ \left[\widehat{A}_{dd}\alpha(k_t^{dd})^{\alpha-1}\right]. \tag{3.25}$$

This equation sets the marginal cost of credit to the marginal cost of capital divided by the marginal product of capital in producing demand deposits, a standard microeconomic pricing condition.[11]

Solving for the equilibrium capital stock,

[11] This result extends the traditional literature, such as (Marty 1969) who postulates that "If bank money were the only money, competitively produced bank money not subject to outside constraints will result in equality of the price of money with its cost of production. Since these costs are zero, the price of money would in equilibrium be zero"; p. 105. Here with positive production costs, the marginal cost in equilibrium is equal to the cost of the substitute, cash, which is $R + \phi$. This can be zero only at the Friedman optimum of $R = 0$ combined with the case that $\phi = 0$, in which case there will be no demand for demand deposits.

$$k_t^{dd} = \left[\widehat{A}_{dd}\alpha(R+\phi)/r_t\right]^{1/(1-\alpha)}, \tag{3.26}$$

and substituting this into equation (3.22) gives the supply of demand deposits as

$$M_t^{dd,s}/P_t = \widehat{A}_{dd}^{1/(1-\alpha)}\left[\alpha(R+\phi)/r_t\right]^{\alpha/(1-\alpha)}. \tag{3.27}$$

As the cost of using money $R+\phi$ falls due to a nominal interest falling towards $R=0$, there is still production due to cost ϕ. If in addition ϕ goes to zero, the capital used in produced non-interest bearing deposits, and the output, also goes to zero, and then the consumer uses only cash.

Here $M_t^{dd,s}/P_t = M_t^{dd}/P_t$ and the M1 aggregate can be represented as follows:

$$M_t^c + M_t^{dd} \equiv M1_t. \tag{3.28}$$

The problem with this specification is that, in the equilibrium, with a positive growth rate g_t, the ratio of M_t^c/M_t^{dd} is increasing towards infinity. While there may be some trend in this ratio empirically, this trend should be explainable by changes in other exogenous factors that determine the ratio; with constant exogenous factors, theoretically the trend should be stable on the balanced growth path. To see that the ratio is not stable, equations (3.17) and (3.18) imply that $M_t^c/M_t^{dd} = a_t^c/(1-a_t^c)$. The solution for a_t^c is found by setting equal the supply and demand from equations (3.18) and (3.27), giving that $a_t^c = 1 - \left(\widehat{A}_{dd}^{1/(1-\alpha)}\left[\alpha(R+\phi)/r_t\right]^{\alpha/(1-\alpha)}\right)/c_t$, with $r_t = (1+A-\delta)(1-\gamma) - \gamma R_t - 1$ by equation (3.5). This implies that $a_t^c/(1-a_t^c) =$ $\left\{1 - \left[\left(\widehat{A}_{dd}^{1/(1-\alpha)}\left[\alpha(R+\phi)/r_t\right]^{\alpha/(1-\alpha)}\right)/c_t\right]\right\} \Big/ \left\{\widehat{A}_{dd}^{1/(1-\alpha)}\left[\alpha(R+\phi)/r_t\right]^{\alpha/(1-\alpha)}/c_t\right\}$, or $a_t^c/(1-a_t^c) = \left\{c_t \Big/ \left(\widehat{A}_{dd}^{1/(1-\alpha)}\left[\alpha(R+\phi)/r_t\right]^{\alpha/(1-\alpha)}\right)\right\} - 1$. By inspection it is clear that with c_t rising when there is positive growth on the equilibrium path, and with the real interest rate being stable given that there is a stationary inflation rate, the ratio $a_t^c/(1-a_t^c)$ also rises towards infinity towards a cash-only solution with no demand deposits.

An alternative production function that gives a stationary ratio of M_t^c/M_t^{dd} is one that includes an externality that affects the shift parameter $\widehat{A}_{dd}$. In particular let $\widehat{A}_{dd} = A_{dd}c_t^{1-\alpha}$, so that the production function is CRS in terms of capital and goods consumption:

$$M_t^{dd,s}/P_t = A_{dd}{c_t}^{1-\alpha}(k_t^{dd})^{\alpha}. \tag{3.29}$$

This function is a type of positive externality in which the goods output is complementary to the bank's output; see also (Romer 1986). It has the property that the share of goods bought with demand deposits, a_t^{dd}, is a function of the capital to goods ratio; by equations (3.16), (3.18), and (3.29),

$$a_t^{dd} = A_{dd}(k_t^{dd}/c_t)^{\alpha}. \tag{3.30}$$

This means that the bank takes the aggregate consumption as given and demands capital and produces demand deposits in proportion to the aggregate consumption. Substituting the alternative production function into the profit maximization problem of equation (3.23), with $\widehat{A}_{dd} = A_{dd}c_t^{1-\alpha}$, the solution is

$$k_t^{dd}/c_t = \left[A_{dd}\alpha(R+\phi)/r_t\right]^{1/(1-\alpha)}. \tag{3.31}$$

And from equations (3.30) and (3.31), the solution for the equilibrium share of demand deposits is

$$a_t^{dd} = A_{dd}^{1/(1-\alpha)}\left[\alpha(R+\phi)/r_t\right]^{\alpha/(1-\alpha)}.[12] \tag{3.32}$$

Figure 1 graphs the equilibrium for the demand deposit bank. To graph this the current period profit function was solved along the balanced growth path. The dynamic nature of the bank problem brings into the consumer's rate of time preference into the equilibrium profit function, when the growth rate is substituted in for using equation (3.13). The resulting profit solution can be written as

$$\frac{\Pi_t^{dd}}{P_t c_t(\sigma+\phi)} = \frac{M_t^{dd}}{P_t c_t} - \frac{r\left(k_t^{dd}/c_t\right)}{(\sigma+\phi)}, \tag{3.33}$$

which is graphed as the straight line in Figure 1.

With the production function of equation (3.29), the balanced-growth path exists and the ratio M_t^c/M_t^{dd} is stationary along it. Stationarity of M_t^c/M_t^{dd} follows directly from above where it is shown that $M_t^c/M_t^{dd} = a_t^c/(1-a_t^c)$.By equation (3.16) this can be written as $M_t^c/M_t^{dd} = (1-a_t^{dd})/a_t^{dd}$ and by inspection of equation (3.32) can be seen to be stationary.

3.3. M2. The model can be expanded to its full form by allowing the agent the choice of using costly credit to make purchases, or "exchange credit", along with cash or non-interest bearing demand deposits. Here the credit is like a credit card, such as the American Express card, rather than a debit card. The agent must pay a fee for this service that is proportional to the amount of the exchange credit; this is like the percentage fee paid by stores using the American Express card (without a roll-over debt feature). Denoting the time t nominal amount of exchange credit demanded by the consumer by M_t^{cd}, and the nominal fee by P_t^{cd}, the consumer's expenditure on such fees is given by $(P_t^{cd}/P_t)M_t^{cd}$.The consumer again owns the exchange credit bank, receives

[12] Note that if $a_t^{dd} = 1$, and so $a_t^c = 0$, there would be no consumer demand for cash. The monetary equilibrium would still have well-defined nominal prices as long as $\gamma > 0$, so that there was a reserves demand for cash by the intermediation bank. This could then be characterized solely as a "legal restrictions" demand for money. At $a_t^c = 0$, and $\gamma = 0$, and with a positive supply of money, prices may not be well-defined.

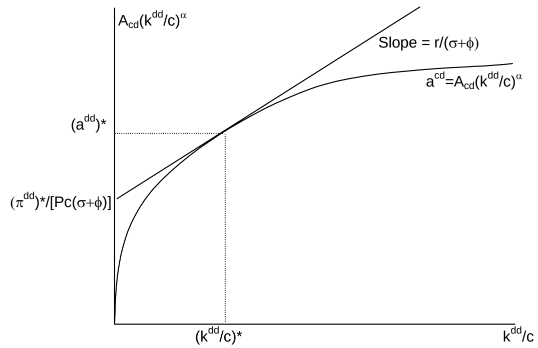

FIGURE 1. Equilibrium in the Demand Deposit Bank Sector

the nominal profit, denoted by Π_t^{cd}, and rents non-depreciating capital k_t^{cd} and invests in the capital each period by an amount $k_{t+1}^{cd} - k_t^{cd}$. And the consumer must pay off the debt incurred using the exchange credit at the end of the period. But this credit saves the agent from having to set aside money in advance of trading, and so allows avoidance of the inflation tax. And now with three types of exchange, let the share of consumption good purchases made by cash and by non-interest bearing demand deposits remain notated by a_t^c and a_t^{dd}, and the share of consumption good purchases made by exchange credit by a_t^{cd}, where the shares sum to one:

$$a_t^c + a_t^{dd} + a_t^{cd} = 1. \tag{3.34}$$

This adds a third (Clower 1967) constraint to the consumer's problem, allowing the three constraints to be written as

$$M_t^c = P_t c_t a_t^c, \tag{3.35}$$
$$M_t^{dd} = P_t c_t a_t^{dd}, \tag{3.36}$$
$$M_t^{cd} = P_t c_t (1 - a_t^c - a_t^{dd}). \tag{3.37}$$

The consumer problem now buys goods with cash or demand deposits as before, but also has a debit of $a_t^{cd} P_t c_t$ for credit purchases, and has a debit of $(P_t^{dd}/P_t)M_t^{dd}$ due to the credit fee.

This makes the consumer problem

$$
\begin{aligned}
&\underset{c_t,d_{t+1},k_{t+1}^{dd},k_{t+1}^{cd},M_{t+1}^{c},M_{t+1}^{dd},M_t^{cd},a_t^{c},a_t^{dd}}{Max} \quad L=\textstyle\sum_{t=0}^{\infty}\beta^t\left\{u\left(c_t\right)\right. \\
&+\lambda_t\left[P_t(1+r_t)d_t+H_t+\Pi_t^r+\Pi_t^{dd}+\Pi_t^{cd}+M_t^c+M_t^{dd}-M_{t+1}^c\right. \\
&-P_t k_{t+1}^{dd}+P_t k_t^{dd}\left(1+r_t\right)-P_t k_{t+1}^{cd}+P_t k_t^{cd}\left(1+r_t\right) \\
&\left.-\phi M_t^c-M_{t+1}^{dd}-(P_t^{dd}/P_t)M_t^{dd}-(P_t^{cd}/P_t)M_t^{cd}-P_t c_t-P_t d_{t+1}\right] \\
&+\mu_t^c[M_t^c-a_t^c P_t c_t] \\
&+\mu_t^{dd}[M_t^{dd}-a_t^{dd}P_t c_t] \\
&\left.+\mu_t^{cd}\left[M_t^{cd}-(1-a_t^c-a_t^{dd})P_t c_t\right]\right\}.
\end{aligned} \tag{3.38}
$$

The first-order conditions imply that the interior solution satisfies

$$P_t^{dd}/P_t = \phi \tag{3.39}$$

$$P_t^{cd}/P_t = R+\phi, \tag{3.40}$$

and the shadow cost goods is again, as in the last section, given by

$$u_{c_t} = \lambda_t P_t(1+R_t+\phi). \tag{3.41}$$

Denote the name for the exchange credit banking firm as Amex. Amex is assumed to supply the exchange credit, denoted by $M_t^{cd,s}$, using only capital, denoted by k_t^{cd}, in a diminishing returns fashion similar to the technology for the demand deposit bank. While this technology could be given as $(M_t^{cd,s}/P_t)=\widehat{A}_{cd}(k_t^{cd})^{\theta}$,where $\widehat{A}_{cd}>0$ and $\theta\in(0,1)$, for a general diminishing returns case, the problem would arise that the equilibrium share of the Amex credit would trend down towards zero if there was a positive growth rate g_t, making infeasible the existence of a balanced-growth path. Therefore consider a technology similar to equation (3.29), which gives a stable share of exchange credit in purchases. In particular, let the function be specified with a complementary goods externality that affects the shift parameter $\widehat{A}_{cd}$, whereby $\widehat{A}_{cd}=A_{cd}c^{1-\theta}$, so that

$$M_t^{cd,s}/P_t = A_{cd}c_t^{1-\theta}(k_t^{cd})^{\theta}. \tag{3.42}$$

The profit maximization problem is static and given by

$$\underset{k_t^{cd}}{Max}\Pi_t^{cd} = P_t^{cd}A_{cd}c_t^{1-\theta}(k_t^{cd})^{\theta}-P_t r_t k_t^{cd}. \tag{3.43}$$

The equilibrium conditions of the consumer and Amex bank imply that

$$R_t+\phi = P_t^{cd}/P_t = r_t/[A_{cd}\theta(k_t^{cd}/c_t)^{\theta-1}]; \tag{3.44}$$

$$k_t^{cd}/c_t = [A_{cd}\theta(R_t+\phi)/r_t]^{1/(1-\theta)}. \tag{3.45}$$

This means that as the nominal interest rate rises the Amex bank expands credit supply and k_t^{cd}/c_t rises in equilibrium. *And it means that the marginal costs of exchange are equated to* $R+\phi$ *across all of the different forms of exchange*, being cash, demand deposits, or credit. This equalization of the marginal costs of the various means of

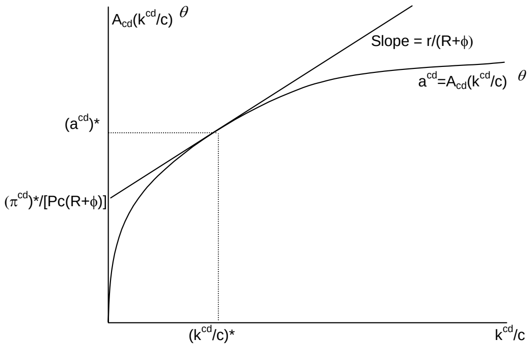

FIGURE 2. Equilibrium in the Credit Bank Sector

exchange, the basis of Baumol's (1952) equilibrium, is one of the most important features of the general equilibrium.

Equating the supply and demand for the Amex credit, from equations (3.37) and (3.42), and using the above equation (3.44), the share of exchange credit can be found to be

$$a_t^{cd} = A_{cd}^{1/(1-\theta)}[\theta(R_t + \phi)/r_t]^{\theta/(1-\theta)}, \tag{3.46}$$

also rising as the nominal interest rate goes up. And note that by substituting equation (3.46) into equation (3.34), so that $1 - a_t^{cd} = a_t^c + a_t^{dd}$, and then substituting in equation (3.32), the solution for a_t^c is found.[13]

Figure 2 illustrates the equilibrium for the credit bank. At the Friedman optimum of $R = 0$, some credit would still be provided as long as $\phi > 0$. This use of credit at $R = 0$ contrasts to zero such use of credit in (Gillman 1993), (Ireland 1994), and (Gillman and Kejak 2002).

[13]Alternatively the exchange credit sector can be kept implicit by having the consumer engage in "self-production" of the exchange credit. This can be done by constraining the consumer's problem by the technology constraint (3.42), combining this constraint with equation (3.37), solving for a_t^{cd}, and using this to substitute in for a_t^{cd} in the consumer problem (3.38), with the consumer now choosing k_t^{cd} instead of a_t^{cd}. This approach would make the revised Clower contraint (3.37) equal to $M_t^{cd} = P_t A_{cd} c^{1-\theta}(k_t^{cd})^{\theta}$. Setting $\gamma = 0$ and $\phi = 0$, then $M_t^c/(P_t c_t) = 1 - a_t^{cd} = a_t^c$, and only this one Clower constraint would be necessary. Now solve this constraint for k_t^{cd}, and it would take a form exactly analogous to a special case of the (McCallum and Goodfriend 1987) shopping time constraint, but in capital instead of time, that depends on real money balances and goods in the same direction: $k_t^{cd} = c_t[1 - (M_t^c/P_t)/c_t]^{1/\theta}(1/A_{cd})^{1/\theta}$; with $\partial k_t^{cd}/\partial(M_t^c/P_t) < 0$, and $\partial k_t^{cd}/\partial c_t > 0$ [See (Walsh 1998), on shopping time models.]

The money market clearing condition here is that the demand for the exchange credit equals the supply of the exchange credit. This can also be further aggregated to

$$M_t^c + M_t^{dd} + M_t^{cd} \equiv M2_t, \tag{3.47}$$

and can be considered an aggregate like M2. It includes the monetary base, demand deposits, plus the exchange credit that allows funds to interest during the period, as do certificates of deposit, and is then paid off with"money market mutual funds" invested in short-term government securities. So it is a mixed set of non-interest bearing aggregates that suffer the inflation tax, and are traditionally thought of as money-like in nature, and of the Amex credit and money market accounts that avoid the inflation tax, unlike "money".

4. Changes in Aggregates Over Time

The model of M2 can be used to analyze how subsets of aggregates change according to changes in exogenous factors. In particular the focus is on changes in the money supply growth rate, σ, or more simply in the nominal rate of interest since this is given by $R = \sigma + \rho + \rho\sigma$. Also the focus is on changes in the banking productivity parameters A_{dd} and A_{cd}, and the banking cost parameter ϕ. Comparative statics of these factors are then applied to explain the actual profiles of the velocity of monetary aggregates, and the profiles of their ratios.

The explanation of the aggregates relies on changes in productivity that result from changes in US bank law. This approach can be formalized by adding a proportional tax to the credit firm's proceeds from selling the credit, denoted by τ , whereby the price received by the firm is $P_{cd}(1-\tau)$. Now assume that the tax proceeds are destroyed, as regulations sometimes are modeled. Then the equilibrium is such that in equation (3.44) the productivity factor is factored by $(1-\tau)$. An increase in regulations makes τ bigger, and effective (net) productivity smaller, while deregulation causes τ to decrease and effective (net) productivity to increase. The same regulations can likewise affect A_{dd}.Now consider the following brief review of major US deregulatory laws in banking in order to indicate how and when the effective productivity factor might shift.

4.1. Financial Deregulation and the Increase in Bank Productivity. Significant US financial deregulation manifested with the Depository Institutions Deregulation and Monetary Control Act of 1980, The Garn-St.Germain Financial Modernization Act of 1882, the Riegle–Neal Interstate Banking and Branching Efficiency Act of 1994, and the Gramm-Leach-Bliley Act of 1999. The 1980 law phased out interest ceilings and allowed banks to pay more interest on deposits. The 1982 law allowed banks to offer money market accounts in order to compete with mutual funds. The 1994 act allowed national bank branching and consolidation:

"Congress passed significant reform legislation in the 1990s. In 1994, the Riegle–Neal Interstate Banking and Branching Efficiency Act repealed the McFadden Act of 1927 and Douglas Amendments of 1970, which had curtailed interstate banking. In particular, the McFadden Act, seeking to level the playing field between national and state banks with respect to branching, had effectively prohibited interstate branch banking. Starting in 1997, banks were allowed to own and operate branches in different states. This immediately triggered a dramatic increase in mergers and acquisitions. The banking system began to consolidate and for the first time form true national banking institutions, such as Bank of America, formed via the merger of BankAmerica and NationsBank." (Guzman 2003).

The 1999 law permitted mergers between banks, brokerage houses, and insurance companies, "allowing banking organizations to merge with other types of financial institutions under a financial holding company structure" (Hoenig 2000).

4.2. Comparative Statics and Comparison to the Evidence. The income velocity of money is defined as income divided by a particular monetary aggregate. The income in the economy comes from the goods production function; this makes it equal to $(A-\delta)\,k_t$,which equals $(A-\delta)\,(1-\gamma)\,d_t$. The velocity of the monetary aggregates can then be defined as $(A-\delta)\,(1-\gamma)\,d_t/M_t^b$.

PROPOSITION 7. *Given* $g=0$, *and along the balanced growth path, the base money velocity rises with the nominal interest rate, or*
$\partial[(A-\delta)\,(1-\gamma)\,d_t/M_t^b]/\partial R>0$.

PROOF. The solution for the Base velocity is
$(A-\delta)\,(1-\gamma)\,d_t/M_t^b=\left[(A-\delta)\,(1-\gamma)\,(d_t/c_t)\right]/\left[1-a^{dd}-a^{cd}+\gamma\,(d_t/c_t)\right]$, where $a^{dd}=A_{dd}^{1/(1-\alpha)}\left[\alpha(R+\phi)/r_t\right]^{\alpha/(1-\alpha)}$, $a^{cd}=A_{cd}^{1/(1-\theta)}[\theta(R+\phi)/r]^{\theta/(1-\theta)}$, $r=(A-\delta)\,(1-\gamma)-\gamma(1+R)$, $(1+g)=(1+r)\,/\,(1+\rho)$ and
$d_t/c_t=\frac{1+\phi\left(1-a^{dd}-a^{cd}\right)+g\left(\frac{k_t^{dd}+k_t^{cd}}{c_t}\right)}{(A-\delta)(1-\gamma)-g-\gamma}$. At $g=0$, $d_t/c_t=\frac{1+\phi\left(1-a^{dd}-a^{cd}\right)}{(A-\delta)(1-\gamma)-\gamma}$ and
$(A-\delta)\,(1-\gamma)\,d_t/M_t^b=\frac{(A-\delta)(1-\gamma)}{\frac{(A-\delta)(1-\gamma)+\gamma}{\frac{1}{1-a^{dd}-a^{cd}}+\phi}+\gamma}$. Substituting in for a^{dd}, a^{cd}, it can be seen that $\partial[(A-\delta)\,(1-\gamma)\,d_t/M_t^b]/\partial R>0$. Note that the solution of d_t/c_t requires substituting into the budget constraint of the problem in equation (3.38), using equations (3.3), (3.23), (3.34), (3.35), (3.43). □

Figure 3 shows the post 1959 US base money velocity and the 10 year bond, US Treasury, interest rate. (McGrattan 1998) presents such a graph and argues, in her comment on (Gordon, Leeper, and Zha 1998), that the nominal interest rate goes a long way to explaining base money velocity.[14] And this is the implication of the result of Proposition 1. The difference from (McGrattan 1998) is that she uses a simple linear econometric equation, as found in (Meltzer 1963) and (Lucas 1988a), to argue that the nominal interest rate has a direct effect on velocity. Here the velocity is derived analytically to make the point from the general equilibrium perspective.[15]

[14](McGrattan 1998) argues that the long term rate is better to use than the short term rate that (Gordon, Leeper, and Zha 1998) use. "Low frequency movements in velocity are well-explained by low frequency movements in observed interest rates."

[15]Note that (Gillman and Otto 2002) take the time series approach of (Meltzer 1963) and (Lucas 1988a) while including a data series on the productivity in banking in order to capture changes in productivity. They find cointegration of money demand with the productivity series, but without it

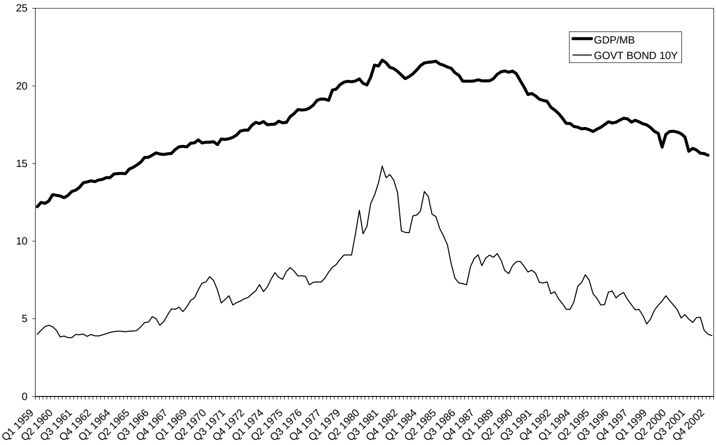

FIGURE 3. US Base Velocity and Nominal Interest Rates: 1959-2003

Comparative statics for the other factors, A_{cd}, A_{dd}, and ϕ, are ambiguous in general because of the d_t/c_t factor, but holding d_t/c_t constant then all three factor have a positive effect on base velocity. This positive direction of the effect of these factors is also readily apparent in calibrations. While these other factors do not provide any obvious help in interpreting base velocity empirical evidence, they do provide an explanation as based on the model of the evidence on the ratio of reserves to currency.

Figure 4 shows the post 1959 US reserves/currency ratio against the long term interest rate. There is a marked trend down, with a flattening out period during the 1980s, and a rather more pronounced downward direction after 1994. In the model, M_t^r/M_t^c is the notation for the reserves to currency ratio, and is given $M_t^r/M_t^c = \frac{\gamma d_t/c_t}{1-a^{dd}-a^{cd}}$. With d_t/c_t held constant, the reserves to currency ratio rises with increases in each R, ϕ, A_{cd}, and A_{cd}. Since the US reserves/currency trend is downward, while the effect of the nominal interest is upward in the 1959-1981 period, it appears that the nominal interest plays no role in explaining this ratio. In contrast, the hypothesis of a downward trend in the cost of using money, ϕ, serves well to explain the evidence.

M1 velocity is defined by $(A-\delta)(1-\gamma)\, d/M1_t = (A-\delta)(1-\gamma)(d_t/c_t) / \left(1-a_t^{cd}\right)$. With d_t/c_t held constant, along the balanced growth path, M1 velocity rises with the nominal interest rate because a_t^{cd} rises. Similarly, an increase in A_{cd} and ϕ cause M1 velocity to go up.

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the money demand appears to be "instable". Or as (Parry 2000) asserts, "Once deposit interest rates began to vary with market rates, the demands for M1 and M2 - the primary guides to monetary policy - became unstable."

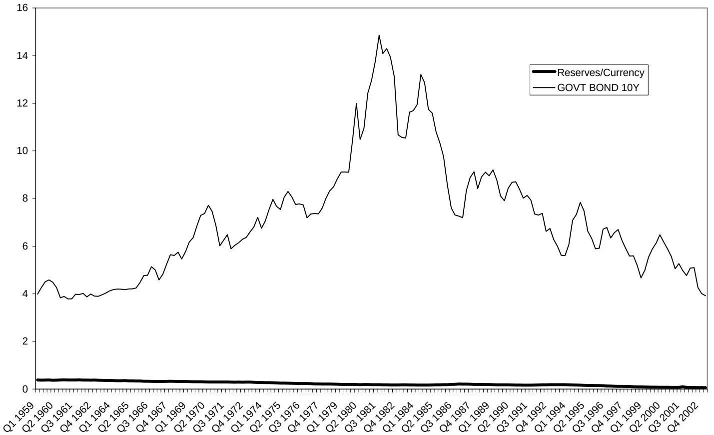

FIGURE 4. US Reserves to Currency Ratio and Interest Rates: 1959-2003

Figure 5 shows the US M1 velocity and the 10-year US Treasury interest rate from 1959 to 2003. The rise in velocity from 1959 to 1981 is consistent with the rise in the nominal interest rate. While still following changes in the nominal interest rate in the 1980s, M1 velocity appears to level off rather than fall during this period by as much as would be expected from the decrease in the nominal interest rate. Deregulation of the 1980s, and an associated increase in A_{cd} presents an explanation of the leveling off of velocity in the 1980s. The striking trend upwards in velocity after 1994, as with the reserves to currency ratio is consistent with an accelerated increase in A_{cd} that can be from the deregulation of interstate branching that led to national branching and the diffusion of ATMs, as well as the banking consolidation because of the 1999 act. Thus the two factors of the nominal interest rates and the banking productivity each play a distinct role in this explanation.[16]

A way to see further into the M1 velocity profile is to look at the ratio of its components, currency and demand deposits. Analytically the demand deposit to currency ratio in the model is M^{dd}/M^{c}.

PROPOSITION 8. *The demand deposit to currency ratio, M_t^{dd}/M_t^c, rises with increases in each R, ϕ, A_{cd}, and A_{cd}.*

[16](Ireland 1995) compares US M1-A velocity with 6-month Treasury bill interest rates. He explains velocity as following a continuous upward trend due to financial innovation.

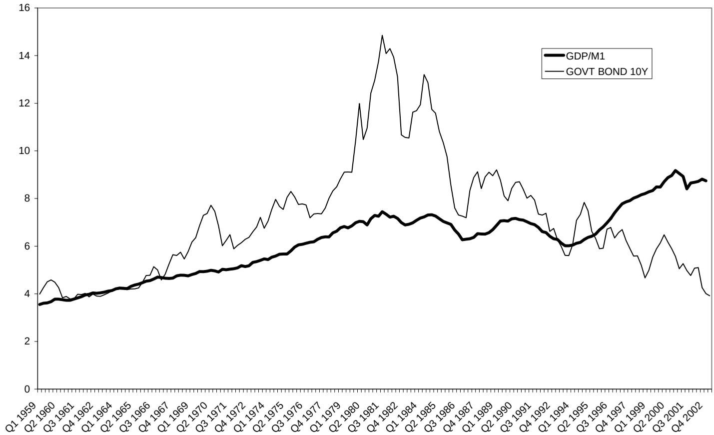

FIGURE 5. US M1 Velocity and Nominal Interest Rates: 1959-2003

PROOF. From equations (3.32), (3.34), (3.35), (3.36), (3.37), and (3.46), $\frac{M^{dd}}{M^c} = \frac{A_{dd}^{1/(1-\alpha)}[\alpha(R_t+\phi)/r_t]^{\alpha/(1-\alpha)}}{1-A_{dd}^{1/(1-\alpha)}[\alpha(R_t+\phi)/r_t]^{\alpha/(1-\alpha)}-A_{cd}^{1/(1-\theta)}[\theta(R_t+\phi)/r_t]^{\theta/(1-\theta)}}$, and $\partial\left(M^{dd}/M^c\right)/\partial R > 0$, $\partial\left(M^{dd}/M^c\right)/\partial A_{cd} > 0$, $\partial\left(M^{dd}/M^c\right)/\partial A_{dd} > 0$, and $\partial\left(M^{dd}/M^c\right)/\partial \phi > 0$. □

Figure 6 shows the US demand deposit to currency ratio, and the 10-year US Treasury interest rate for the same 1959-2003 period. In a first look, the ratio simply trends down. But looking more closely shows a simple trend down, from 1959 to 1981, that levels off in the 1980s, as with M1 velocity, and then moves down steadily post 1994 at an accelerated rate compared to the earlier period.

A downward trend in ϕ well explains the downward trend in the demand deposit to currency ratio in a way the nominal interest rate's pre-1981 upward trend and a possible upward trend in A_{cd} and A_{dd} cannot. However the role of A_{cd} and A_{dd} again emerges as the only way to explain the leveling off of the trend in demand deposits to currency in the 1980s, when there was financial deregulation and a surge in A_{cd} and A_{dd}. Further the accelerated downward trend in the ratio after 1994 is consistent with an accelerated decrease in ϕ because of the ATM diffusion.[17]

Now consider the velocity of the broader aggregate M2. In the model, M2 velocity is defined by $(A-\delta)(1-\gamma)d/M2_t$. This is given by $(A-\delta)(1-\gamma)d/M2_t =$

[17]Note that stable deposit to currency ratios were reported by (Cagan 1956) for the hyperinflations he studied (an exception was post WWII Hungary that Cagan suggests is due to data problems). This indicates a small role of the nominal interest rate in causing changes in this ratio, and is consistent with the small role given here to the nominal interest rate in explaining the US ratio's postwar movement.

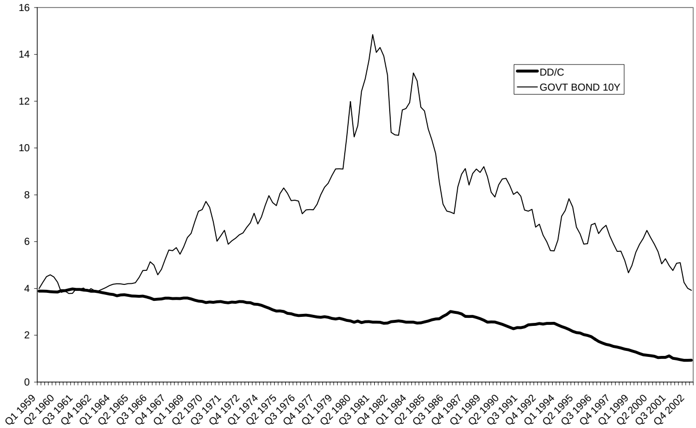

FIGURE 6. US Demand Deposits to Currency Ratio and Interest Rates: 1959-2003

$(A-\delta)(1-\gamma)\, d / \left[c_t\left(a_t^c+a_t^{dd}+a_t^{cd}\right)\right]=(A-\delta)(1-\gamma)\left(d_t / c_t\right).$ The comparative statics of the M2 velocity are therefore as the comparative statics of the ratio of savings to consumption. The effects of R, A_{cd}, A_{dd}, and ϕ are ambiguous in general, although with $g=0$, it is true as shown above that $\partial\left(d_t / c_t\right) / \partial R>0$. But the $\left(d_t / c_t\right)$ factor does not appear to play any significant role in the explanation of base or M1 velocity. Figure 7 indeed shows that US M2 velocity has been remarkably constant relative to the 10-year US Treasury bond rate. Thus the explanation from the model is that the magnitude of changes in $\left(d_t / c_t\right)$, because of the factors considered here, is small. It is easy to confirm this with calibrations, although this exercise is not reported. However one aspect of this is worth noting. With a relatively unchanging d_t / c_t as the explanation for a stable M2 velocity, it is internally consistent with the previous analysis that the comparative statics of A_{cd} , A_{dd}, and ϕ, with d_t / c_t held constant, can be used to explain Base and M1 velocity.

Breaking down the components of M2 is more revealing. Consider the ratio of M2 to M1. In the model this is given by

$$M2_t / M1_t=1 / \left[1-A_{cd}^{1/(1-\theta)}\left[\theta\left(R_t+\phi\right) / r_t\right]^{\theta/(1-\theta)}\right].$$

PROPOSITION 9. *Along the balanced growth path, the ratio $M2_t / M1_t$ rises with an increase in the nominal interest rate, or $\partial\left(M2_t / M1_t\right) / \partial R>0$.*

PROOF. $\partial\left(1-A_{cd}^{1/(1-\theta)}\left[\theta\left(R_t+\phi\right) / r_t\right]^{\theta/(1-\theta)}\right) / \partial R>0.$ □

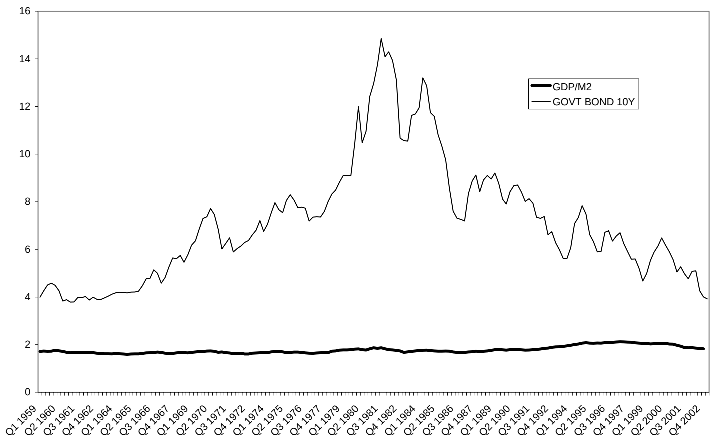

FIGURE 7. US M2 Velocity and Nominal Interest Rates: 1959-2003

The other comparative statics with respect to A_{cd} and ϕ are ambiguous because of the d_t/c_t factor; holding d_t/c_t constant, the ratio $M2_t/M1_t$ rises with each of these. Now consider Figure 8, which shows the US ratio of M2 to M1 from 1959 to 2003, along with the 10-year US Treasury bond rate. Proposition 3 provides a way to explain the upward trend in M2/M1 from 1959 to 1981, and perhaps the fall in M2/M1 from 1990 to 1994. The leveling off of M2/M1 in the 1980s can be explained by financial deregulation and increases in A_{cd}; note that the downward change in R during this period, and a downward trend in ϕ during this period cannot explain the leveling off of M2/M1, as these factors work to make the ratio go down. The trend upwards after 1994 again can be explained by upward increases in A_{cd} because of national branching being allowed, ATM diffusion, and consolidation.

5. Discussion

The demand for bank reserves that (Haslag 1998) put forth helps pave the way for modeling the demand for a range of monetary aggregates. The model as revised here acts as a missing link that ties together conventional money demand functions from the cash-in-advance approach with an analogue to the monetary aggregates widely studied, by adding a bank's demand for cash reserve. An inflation tax on the deposit rate of return results because, as in the cash-in-advance economies, the intermediation bank must in effect put aside cash-in-advance in order to meet the demands of the reserve requirement. This is similar to (Stockman 1981) in which the (Clower 1967) constraint

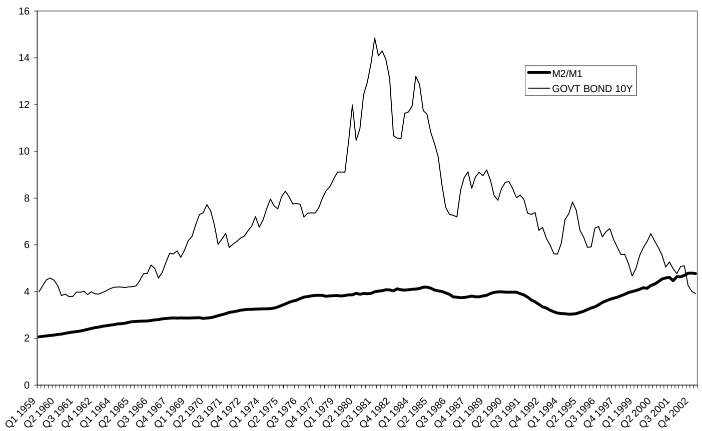

FIGURE 8. US Ratio of M2 to M1 and Interest Rates: 1959-2003

is applied to all investment; here however, the intermediation bank's (Clower 1967) constraint applies only to the reserve fraction of the investment.

On the basis of the intermediation bank's demand for reserves plus the imposition of a standard (Clower 1967) constraint on the consumer's purchase of goods, the demand for an aggregate similar to the monetary base, reserves plus currency (cash), is constructed whereby the inflation rate can affect the real return to intermediated investment under an AK technology because of the need to hold cash reserves. This model is extended to include non-interest bearing deposits, unlike previous work, and in a way that gives an aggregate analogous to $M1$. The model further is extended to include exchange credit, to give an aggregate analogous to $M2$. In this fully extended model comparative statics are presented for Base, M1 and M2 velocity, and the ratios of demand deposits to reserves, demand deposits to currency, and M2/M1. With these analytics the empirical evidence on both the velocities and various ratios of the aggregates are explained in an internally consistent way. This requires more than only the nominal interest rate. In addition the productivity of the credit bank sector plays a critical role in explaining aggregate movement during the financial deregulation era. And the convenience cost of using money has a unique role in explaining the trends in the reserve to currency and in the demand deposit to currency ratios.

The models here enable the consumer to choose the least expensive source of exchange means. As a result, the (Clower 1967) constraints are not "exogenously" imposed upon the consumer but rather left as a consumer choice to bind certain fractions of purchases to particular exchange means only to the extent that the particular exchange means is efficient for the consumer to use. This consumer choice amongst alternative means of exchange might be seen as ameliorating the strength of the criticism of the "deep" models of money that the (Clower 1967) constraint is exogenously imposed, or even as offering an alternative approach to the search for deep models.[18]

Note that the model of the exchange credit sets the quantity of credit that is produced equal to the value of the output of the consumption good that is being bought on credit. (Aiyagari, Braun, and Eckstein 1998) instead model credit as a service that is produced, and then enters as an input into a production function for credit goods. The credit goods production is Leontieff in its inputs of the credit service and of the value of the consumption goods being bought with the credit. This Leontieff technology in equilibrium implies as a special case the condition that the credit services output equals the value of the of consumption goods being bought with the credit.[19] In the paper here, as in the continuum-of-stores approach in (Gillman 1993), (Ireland 1994), and (Erosa and Ventura 2000), there are no credit or cash goods per se, but only the consumption good that can be bought with cash or credit. This in a sense can be thought of as collapsing the (Aiyagari, Braun, and Eckstein 1998)-type credit goods and credit services into a single technology called credit, whereby the equilibrium condition that is implied by the special case of the Leontieff technology of (Aiyagari, Braun, and Eckstein 1998) is implicitly applied. And the advantage of the model here over the continuum-of-stores approach is that here the velocity can be solved more simply.

The model's implications for growth are that inflation lowers growth because it lowers the real interest rate, a result supported in (Ahmed and Rogers 2000). However, this feature combined with an Ak goods production technology cannot account for the substitution from effective labor to capital, as induced by inflation, that (Chari, Jones, and Manuelli 1996) describe and that (Gillman and Nakov 2003) further elaborate; (Gillman and Nakov 2003) find evidence in support of this substitution for the postwar US and UK data. Thus while the AK model provides easier analytic tractability, a goods production function with both labor and capital as in (Gomme 1993) and (Gillman and Kejak 2002) also can account for a negative effect of inflation on growth.[20] And since this approach also involves the inflation-induced labor to capital substitution, it may be useful to nest the models of monetary aggregates within the (Gomme 1993) framework.[21]

(Gillman and Kejak 2002) go partly in this direction by extending (Gomme 1993) so as to include credit, as in Section 3.3 of this paper. One advantage of having monetary aggregates more fully embedded in the (King and Rebelo 1990)-type of endogenous

[18]See (Bullard and Smith 2001), and (Azariadis, Bullard, and Smith 2000), for example, for an alternative approach to modeling "inside" money, based on a three-period model. They apply this to anaylse the optimality of restricting inside money; (Gillman 2000) analyses the optimality of such restrictions in a model similar to the paper here.

[19]The case is that $q = 1$ in (Aiyagari, Braun, and Eckstein 1998) model, using their notation.

[20]See also (Jones and Manuelli 1995).

[21]Changes in the real interest rate in the Ak model presented here occur only through changes in the inflation rate, and are discussed in this fashion. In a model with labor and capital, the real interest rate could move endogenously with velocity. At business cycle frequencies this simultaneity may be interesting to investigate.

growth model is that this provides the leisure channel by which to substitute away from inflation and so make the inflation tax less burdensome to the individual consumer. As (Gillman and Otto 2002) show, the (Gillman and Kejak 2002) model with leisure and the credit substitute, in addition, creates an interest elasticity of money demand that rises significantly in magnitude with inflation. This feature also exists in this model of this paper, and this is the central feature of the (Cagan 1956) model. Or as Martin Bailey put it "Cagan's principal conclusion, indeed, is that the demand for real cash balances ... has a higher and higher elasticity at higher and higher rates of inflation" (Bailey 1992). And (Mark and Sul 2003) report recent international panel evidence in support of the (Cagan 1956) money demand function. Only with such an elasticity, within the general equilibrium money demand function, are (Gillman, Harris, and Matyas 2001) able to explain international evidence on inflation and growth.[22]

The current AK model of the paper implies that an increase in the nominal interest rate causes the same degree of a growth rate decrease, no matter what the level of the nominal interest rate. But rather than this linear relation, the evidence shows a high degree of non-linearity, with stronger negative inflation effects at low inflation rate levels. An inflation-induced rising interest elasticity makes substitution towards leisure less, and towards credit more, making the decrease in the growth rate less. Therefore, as inflation rises, the additional leisure and credit channels help explain both effective-labor-to-capital substitution and a rising interest elasticity that leads to a falling magnitude of the marginal decrease in the growth rate.

[22](Paal and Smith 2000) offer an overlapping generations model in which low inflation can cause a positive effect on growth, while higher inflation causes a negative level. This is supported in the panel evidence of (Ghosh and Phillips 1998), (Khan and Senhadji 2001), (Judson and Orphanides 1996), and (Gillman, Harris, and Matyas 2004) in which a threshold level of inflation is found after which the inflation-growth effect is negative. However the positive effect at low inflation rates is found to be insignificant in these papers. And (Gillman, Harris, and Matyas 2004) show that using instrumental variables, the effect of inflation on growth is negative for all positive levels of inflation, across both OECD and APEC regions, as well as in the full sample; (Ghosh and Phillips 1998) also find this for a full sample.

APPENDIX A

Stages of Growth in Economic Development

1. Log-linearization of the Reduced Model

The system of differential equations for the reduced model is derived from Eqs. (2.41)-(2.43) in Chapter 1 by

$$\frac{\dot{x}}{x} = A(\frac{u}{x})^{1-\alpha} - q - \delta - B(1-u) \tag{1.1}$$

$$\frac{\dot{q}}{q} = (\sigma\alpha - 1)A(\frac{u}{x})^{1-\alpha} - \sigma\rho + (1-\sigma)\delta + q \tag{1.2}$$

$$\frac{\dot{u}}{u} = \frac{(B+\delta)}{\alpha} - \delta - (1-u)B - q. \tag{1.3}$$

We will log-linearize the model at the steady state (x^*, q^*, u^*) given from Eqs. (3.1)-(3.4) in Chapter 1 by

$$g^* = \sigma(B-\rho) \tag{1.4}$$

$$u^* = 1 - \frac{\sigma(B-\rho)}{B} \tag{1.5}$$

$$q^* = \frac{\delta + B}{\alpha} - \sigma(B-\rho) - \delta \tag{1.6}$$

$$x^* = \left(\frac{\alpha A}{\delta + B}\right)^{\frac{1}{1-\alpha}} u^*. \tag{1.7}$$

Taking a first-order Taylor expansion of Eqs. (1.1)-(1.3) in logarithmic variables $\ln x, \ln q$, and $\ln u$ we obtain

$$\frac{d\ln x}{dt} \approx -A(1-\alpha)(\frac{u^*}{x^*})^{1-\alpha} d\ln x + q^* d\ln q \tag{1.8}$$

$$+(A(1-\alpha)(\frac{u^*}{x^*})^{1-\alpha} + Bu^*)d\ln u \tag{1.9}$$

$$\frac{d\ln q}{dt} \approx -(1-\alpha)(\sigma\alpha - 1)A(\frac{u^*}{x^*})^{1-\alpha} d\ln x + q^* d\ln q +$$

$$+(1-\alpha)(\sigma\alpha - 1)A(\frac{u^*}{x^*})^{1-\alpha} d\ln u \tag{1.10}$$

$$\frac{d\ln u}{dt} \approx -q^* d\ln q + Bu^* d\ln u. \tag{1.11}$$

If we introduce the notation $\tilde{x} \equiv \ln x$, $\dot{\tilde{x}} \equiv \frac{d}{dt}(\ln x)$, $\hat{x} \equiv \tilde{x} - \tilde{x}^*$ and $\dot{\hat{x}} \equiv \dot{\tilde{x}} - \dot{\tilde{x}}^*$ then these equations can be rewritten in matrix form as:

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{q}} \\ \dot{\hat{u}} \end{bmatrix} = \begin{bmatrix} -\varepsilon & -q^* & q^* \\ -(1-\sigma\alpha)\varepsilon & q^* & (1-\sigma\alpha)\varepsilon \\ 0 & -q^* & Bu^* \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{q} \\ \hat{u} \end{bmatrix} \tag{1.12}$$

where $\varepsilon = -\frac{(1-\alpha)}{\alpha}(B+\delta)^{1-\alpha} < 0$.

Let us denote the matrix as $\mathbf{D}$. Then we can compute the eigenvalues of the system from the characteristic matrix equation $\det(\lambda\mathbf{E} - \mathbf{D}) = 0$. Because $\det(\lambda\mathbf{E} - \mathbf{D}) = (\lambda - \varepsilon)(\lambda - q^*)(\lambda - Bu^*)$, the system has one negative and two positive eigenvalues, $\lambda_1 = \varepsilon < 0$, $\lambda_2 = q^* > 0$, and $\lambda_3 = Bu^* > 0$. Thus the system with two control variables and one state variable is saddle-path stable.

Solving the characteristic equations $(\lambda_i\mathbf{E} - \mathbf{D})\boldsymbol{\xi}_i$ for $i = 1, 2, 3$ we obtain the eigenvectors $\boldsymbol{\xi}_i$ related to eigenvalues λ_i. Thus we get the following result

$$\boldsymbol{\Xi} = [\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \boldsymbol{\xi}_3] = \begin{bmatrix} q^* - \sigma\alpha\varepsilon & q^* & 1 \\ (1-\sigma\alpha)\varepsilon & -\frac{1-\alpha}{\alpha}(\delta + B) & 0 \\ (1-\sigma\alpha)\varepsilon & q^* & 1 \end{bmatrix}. \tag{1.13}$$

If the vector of the model variables is denoted as $\mathbf{Z} \equiv [x, q, u]^T$, then the system of log-linear differential equations can be written in the form $\dot{\hat{\mathbf{Z}}} = \mathbf{D}\hat{\mathbf{Z}}$ and the solution is given by

$$\hat{\mathbf{Z}}_t = e^{\mathbf{D}(t-t_0)}\hat{\mathbf{Z}}_{t_0} = \boldsymbol{\Xi} e^{\boldsymbol{\Lambda}(t-t_0)}\boldsymbol{\Xi}^{-1}\hat{\mathbf{Z}}_{t_0} \tag{1.14}$$

where $\boldsymbol{\Lambda} = diag[\lambda_1, \lambda_2, \lambda_3]$ and $\mathbf{D} = \boldsymbol{\Xi\Lambda\Xi^{-1}}$. Thus we can express the general solution in the form $\hat{\mathbf{Z}}_t = \sum_{i=1}^{3} \boldsymbol{\xi}_i \boldsymbol{\Theta}_i e^{\lambda_i(t-t_0)}$ where $\boldsymbol{\Theta} = \boldsymbol{\Xi}^{-1}\hat{\mathbf{Z}}_{t_0}$.

PROPOSITION 10. *(Stage of High Growth) The behavior of the economy given by (2.41)-(2.43) of Chapter 1 at the stage of high growth with $I = H$ can be approximated by the policy functions*

$$\tilde{q}_t - \tilde{q}_H^* = \kappa_H(\tilde{x}_t - \tilde{x}_H^*) \tag{1.15}$$

$$\tilde{u}_t - \tilde{u}_H^* = \kappa_H(\tilde{x}_t - \tilde{x}_H^*). \tag{1.16}$$

where the slope of policy functions is given by

$$\kappa_H = \frac{\xi_{H,12}}{\xi_{H,11}} = \frac{\xi_{H,13}}{\xi_{H,11}} = \frac{(1-\alpha\sigma)\varepsilon_H}{q_H^* - \alpha\sigma\varepsilon_H} = \begin{cases} > 0, & \alpha\sigma > 1 \\ = 0, & \alpha\sigma = 1 \\ < 0, & \alpha\sigma < 1. \end{cases} \tag{1.17}$$

PROPOSITION 11. *(Stage of Low Growth) The behavior of the economy given by (2.41)-(2.43) of Chapter 1 at the stage of low growth with $I = L$ can be approximated* *by*

$$\tilde{x}_t = \sum_{i=1}^{3} \xi_{L,i1} \mathbf{\Theta}_{L,i} e^{\lambda_{L,i}t} + \tilde{x}_L^* \tag{1.18}$$

$$\tilde{q}_t = \sum_{i=1}^{3} \xi_{L,i2} \mathbf{\Theta}_{L,i} e^{\lambda_{L,i}t} + \tilde{q}_L^* \tag{1.19}$$

$$\tilde{u}_t = \sum_{i=1}^{3} \xi_{L,i3} \mathbf{\Theta}_{L,i} e^{\lambda_{L,i}t} + \tilde{u}_L^* \tag{1.20}$$

with $\mathbf{\Xi}_L$ *given by (1.13) and* $\mathbf{\Theta}_L = \mathbf{\Xi}_L^{-1}\hat{\mathbf{Z}}_{L,0}$. *However, for a level of human capital much lower than* $\hat{H}$, *the saddle path related to the BGP with low growth* g_L^* *is a good approximation:*

$$\tilde{q}_t - \tilde{q}_L^* = \kappa_L(\tilde{x}_t - \tilde{x}_L^*) \tag{1.21}$$

$$\tilde{u}_t - \tilde{u}_L^* = \kappa_L(\tilde{x}_t - \tilde{x}_L^*). \tag{1.22}$$

The slopes of the policy functions are given by

$$\kappa_L = \frac{\xi_{L,12}}{\xi_{L,11}} = \frac{\xi_{L,13}}{\xi_{L,11}} = \frac{(1-\alpha\sigma)\varepsilon_L}{q_L^* - \alpha\sigma\varepsilon_L} = \begin{cases} > 0, & \alpha\sigma > 1 \\ = 0, & \alpha\sigma = 1 \\ < 0, & \alpha\sigma < 1. \end{cases} \tag{1.23}$$

2. The Calibrated Model

In Section 5 of Chapter 1 we use numerical simulations to obtain precise interior solutions. Firstly, we specify the smooth function of the "learning curve", which captures the spillover effects of human capital accumulation, as a logistic (or S-) curve

$$B(h) = \frac{B_H - B_L}{1 + e^{-\epsilon B_H (h - \hat{H})}} + B_L \tag{2.1}$$

where ϵ and $\hat{H}$ are parameters controlling the steepness and the position of the inflexion point.

To calibrate the model I use parameter values similar to those in (Lucas 1988b) and (Mulligan and Sala-i Martin 1993): the intertemporal elasticity of substitution $\sigma = 0.5$, the capital income share $\alpha = 0.3$, the depreciation rate $\delta = .08$, and the rate of time preference $\rho = 0.08$. I further calibrate the model in such a way that the low stage growth g_L is equal to 2.8% the average US growth rate during the 1950s and 1970s. I also extend the analysis in the sense that I assume that the model economy is facing a new era of increasing returns related to the increased social effects of human capital accumulation. I hypothesize further that in the 1980s the USA was on the verge of a higher growth stage g_H^*, let's say, 4%. Thus the values of the efficiency of education are $B_L = 0.136$ and $B_H = 0.24$. The chosen parameters of the learning curve are $\epsilon = 8$ and $\hat{H} = 8$. By means of equations (3.2)-(3.4) we can calculate steady state values $q_L^* = (c/k)_L^* = 0.612$, $x_L^* = (k/h)_L^* = 1.2697$, and $u_L^* = 0.7941$ for the lower growth stage and values $q_H^* = (c/k)_H^* = 0.680$, $x_H^* = (k/h)_H^* = 1.0316$, and $u_H^* = 0.75$ for the higher one. The results of simulation are shown in Figure 9 and Figure 10.

APPENDIX B

EU Accession, Convergence, and Endogenous Growth

1. Threshold Externalities

There are two ways technological innovation can occur in the model: (i) large discontinuous advances which coincide with important eras like industrial revolutions and (ii) more cumulative and continuous progress during which the society learns how to use this potential. Much like (Zilibotti 1995), we consider the former as being exogenous in the sense that economic activity has no effect on the occurrence of revolutionary advances. The second type of innovation depends on the gap between the present level of technology and the frontier productivity level given by the first type of innovation [see (Nelson and Phelps 1966)].

Accordingly, technical progress is driven by investment in human capital[1] such that:

$$\frac{\dot{B}_t}{B_t} = \phi \frac{B_H - B_t}{B_H} \dot{H}_t \tag{1.1}$$

where B_H means the frontier productivity, $B_H \geq B_t$, $\phi > 0$ is the parameter of the speed of diffusion and H is the average level of human capital in the economy[2]. Consistently with Parente and Prescott (2000), the diffusion parameter is a measure of the barriers to knowledge adoption. We can see that the farther from the frontier an economy is and higher the diffusion parameter, the faster the growth of productivity for a given level of investment is. After solving Eq. (1.1) we obtain the following logistic solution

$$B(H;\phi) = \frac{B_H}{1 + (\frac{B_H}{B_0} - 1)e^{-\phi H}} \tag{1.2}$$

where B_0 is the initial level of productivity related to a zero level of human capital. We can easily see from (2.2) that the productivity monotonically increases with the level of human capital, $\partial B/\partial H \geq 0$, and there is an upper bound of productivity given by B_H (i.e. if H goes to infinity, productivity converges to B_H).

---

[1] The existence of human capital makes our model different from the standard learning-by-doing models [(Arrow 1962), (Romer 1986)] and from the model in (Zilibotti 1995) where technical progress is a by-product of the investment in physical capital.

[2] (Benhabib and Spiegel 1994) have provided empirical evidence confirming the existence of human capital externalities and the positive dependency of per capita income growth depends positively on average levels of human capital.

2. First Order Conditions

The agents know the technology for creating new knowledge in the education sector, they, however, take the average level of accumulated knowledge, H, as given. We find that the following necessary conditions for the dynamic optimization problem given by Equations (2.5)-(2.9) in Chapter 2 are

$$
\begin{aligned}
&(2.1) \quad & \lambda &= c^{-\theta} e^{-\rho t} \\
&(2.2) \quad & \lambda A(1-\alpha)\left(\frac{x}{u}\right)^{\alpha} &= \mu B(H;\phi) \\
&(2.3) \quad & q &= 1 + \psi \frac{\iota}{k} \\
&(2.4) \quad & \dot{q} &= qr(b) - \frac{y}{\alpha k} + \delta_k - \frac{q^2-1}{2\psi} \\
&(2.5) \quad & \dot{\lambda}/\lambda &= -r\left(\frac{a}{k}\right) \\
&(2.6) \quad & \dot{\mu}/\mu &= -B(H) \\
&(2.7) \quad & \lim_{t\to\infty} & \; k_t q_t \lambda_t = 0 \\
&(2.8) \quad & \lim_{t\to\infty} & \; b_t \lambda_t = 0 \\
&(2.9) \quad & \lim_{t\to\infty} & \; h_t \mu_t = 0 \\
&(2.10) \quad & \lim_{H\to\infty} & \; B(H) = B_H
\end{aligned}
$$

where $H = h$ in equilibrium. Eq. (2.1) gives the condition that the maximizing agent is indifferent between consuming another unit of the good or saving it in the form of physical capital because the return from consumption (marginal utility) is the same as the return on investment in physical capital (shadow value λ). Eq. (2.2) states that the marginal return to study must be equal to the marginal return to work if working time u is smaller than 1. Eq. (2.3) specifies the static relation between Tobin q and the investment rate. Eq. (2.4) claims that the net return on domestic capital equates to the marginal cost borrowing. The last two Eqs., (2.5) and (2.6), describe the development of the shadow prices of capitals. The growth rate of the shadow value of knowledge capital is given by $-B(H)$. The TVC conditions given by (2.7)-(2.9) are straightforward.

3. Balanced Growth Path

Using the first order conditions given by Eqs. (2.1)-(2.10) and accumulation equations (2.6)-(2.8) in Chapter 2 we can obtain the specification of the balanced growth path:

$$g^* = \frac{(B_H - \rho)}{\theta} \tag{3.1}$$

$$b^* = \frac{B_H - r^*}{v} \tag{3.2}$$

$$q^* = 1 + \psi g^* \tag{3.3}$$

$$u^* = 1 - \frac{1}{\theta}\left(1 - \frac{\rho}{B_H}\right) \tag{3.4}$$

$$q^* B_H = F_k^* - \delta_k + \frac{(q^* - 1)^2}{2\psi} \tag{3.5}$$

$$x^* = \left(\frac{\alpha A}{F_k^*}\right)^{\frac{1}{1-\alpha}} u^* \tag{3.6}$$

$$s^* = (g^* - B_H)b^* - \frac{q^{*2} - 1}{2\psi} + \frac{F_k^*}{\alpha} - \delta_k. \tag{3.7}$$

According to Eq. (3.1), consumption, physical and human capital and debt grow with a positive balanced growth rate $g^* > 0$ only if productivity in the education sector is sufficiently high and/or people are not too impatient. Combined with Eq. (3.4), we can see that a positive growth rate is possible only if some fraction of the endowed time is spent on education $u^* < 1$[3]. For an economy with a high degree of relative risk aversion (i.e. low intertemporal elasticity of substitution σ) where people prefer to smooth the consumption path, the resulting balanced growth rate will be lower while a thriftier and more patient society will enjoy higher growth rates. Eq. (3.5) shows the relation between the marginal product of capital F_k^*, Tobin q^*, given by (3.3) and the return to human capital B_H. It extends the standard result that in the presence of no capital adjustment costs net returns from both capitals are identical at the steady state (i.e. $F_k^* - \delta_k = B_H$). The relation between the capitals ratio x^* and the marginal product of capital is in (3.6). The last Eq. (3.7) captures the expression for the consumption-capital ratio s^*, it depends positively on debt b^* when preferences are logarithmic.

4. Model Calibration

The aggregate production function introduced earlier has the following general form:[4]:

$$Y = AK^\alpha (huN)^{1-\alpha}. \tag{4.1}$$

[3]The situation when people spend all their time in schools and do not work seems to imply maximum growth in human capital. This is not feasible, however, on the BGP because the output F is zero and physical capital is, therefore, consumed and steadily declines rather than growing at the rate $g^* > 0$. This implies that $u^* > 0$.

[4]We use this formulation, because we feel, that once the average knowledge capital is correctly accounted for, there should be only little or no place for additional factors of labour input augmentation.

The effective labour input, huN, derives from the decision of all representative households to allocate the total time of their members to work, uN, augmented by their average level of knowledge capital, h. In this interpretation, N would stand for the total hours available in the economy. Because we lack reliable data on working hours for all the countries in question, we rather interpret N to be the total size of population, assuming away variations in average working hours. Instead of assuming that each person splits its time between work and knowledge accumulation, we simply suppose that each representative household decides on how many of its members spend their time working (fraction of u), while others are occupied with knowledge accumulation or leisure. This interpretation then implies that we measure labor input using the number of workers, L, such that $L = uN$. The normalized versions of this production function in terms of the total population and total workers can therefore be written as:

$$\frac{Y}{L} \equiv \frac{Y}{uN} = A\left(\frac{K}{uN}\right)^{\alpha} h^{1-\alpha}. \tag{4.2}$$

Because the theoretical model has been cast in the former representation, we also adopt GDP per capita in 1995 prices as our measure of economic performance and formulate all intensive variables with respect to total population. We choose the year 1960 as our starting period and normalize the starting level of GDP per capita so that EU15 equals 100. Collecting the stylized facts for the peripheral countries that could be matched across the model properties meant especially the computation of variables featuring in the normalized system variables of $s = c/k$ and $x = k/h$. While the evolution of consumption per capita, c, was trivial to obtain, it was more difficult to construct the physical and knowledge capital intensities, k and h. Thanks to our interpretation of the labor input, a measure of u can simply be computed from the available time series as a fraction of working population, $\frac{L}{N}$.

5. Calibration of Knowledge Capital

As for the knowledge, accumulated by the non-working population, we assume that it is costlessly transmitted within households, so h refers to the average level of knowledge capital per worker. Like many other researchers, we lack direct measures of knowledge capital that would be relevant for our model framework. Many of them employ (variously amended) concepts of educational attainment [e.g. (Hall and Jones 1999), (Temple 2001)] or enrolment ratios as the closest available measures. We too intend to use the easily available information on educational attainment in the construction of the stock of knowledge capital. Unlike the standard practice though, we don't think the educational attainment is directly applicable as a proxy for knowledge capital (as we use it), essentially because the two are related but distinct concepts.

As already highlighted in the introductory parts, our concept of knowledge is directly related to the concept of human capital of (Lucas 1988b) and (Uzawa 1965). These concepts can jointly be understood as pools of ideas and inventions increasing productivity. On the other hand, education improves the ability to adopt and implement new technologies and ideas, both of domestic and foreign origins. (Temple 2001) in his extensive survey of literature on growth effects of education speaks about 'skills acquired through education.' The distinctive feature of these two concepts lies in the fact that while a simple allocation of time to production of ideas and inventions improves productivity (by expanding the available pool of knowledge), better education has no such direct consequences. This distinction is crucial for any empirical work on level or growth accounting, because unless a direct measure of knowledge (or human) capital is available, one cannot employ measures of education attainment directly in place of production factors.

This is also the reason why we think most research studies failed to find significant impacts of human capital (or knowledge) accumulation on growth performance of countries when they use educational attainment as a factor of production [see for instance, (Temple 2001), (Hall and Jones 1999)]. In their influential empirical contribution, (Benhabib and Spiegel 1994), inspired by the earlier work of (Nelson and Phelps 1966), recognize this by arguing that "the role of human capital is indeed one of facilitating adoption of technology from abroad and creation of appropriate domestic technologies rather than entering on its own as factors of production."

In their empirical treatment, though, 'human capital' is simply a measure of school attainment, so the concepts of education and human capital become equivalent. We consider it a gross simplification that leads to the (in our view false) conclusion that it is the stock of human capital which affects the growth of per capita income. This is certainly not the case of productivity enhancing stock of knowledge (or disembodied human capital) that we have in mind here. Nevertheless the conclusions of Benhabib and Spiegel become directly relevant for ours, once we strictly interpret their concept of 'human capital' as educational attainment, which it effectively is.

To sum up, we interpret education in line with Benhabib and Spiegel as the capacity to absorb new ideas and creatively build upon the existing (productivity improving) knowledge stock in generating new ones. As a consequence, it is the level of education (measured, for instance, by average attainment) that matters for flows of productivity improving inventions, and hence for the growth of knowledge stock. This also means that we cannot use measures of school attainment instead of knowledge capital as a factor of production. The construction of a measure of knowledge capital from the available data on school attainment is described in more detail in (Kejak and Vavra 2004). It suffices for our present purposes to register the accumulating equation which is consistent with the effect of education on growth of knowledge capital:

$$\log h_t = \log h_s + \gamma \int_s^t \log E_\tau d\tau. \tag{5.1}$$

This functional specification forms the basis of our measure of knowledge capital. We proxy the level of skills acquired through education, E, by data on the average school attainment in the population aged 25 and more which come from (Barro and Lee 1993) and go back to 1960. Because of the five year paucity of the attainment data we approximate the integral in (5.1) for any year T for which the data are available as:

$$h_t = h_{t-\Delta t} e^{\gamma \frac{\log E_{t-\Delta t} + \log E_t}{2} \Delta t}, \tag{5.2}$$

where $\Delta t = 5$. Note that this specification is entirely consistent with the 'ad hoc' specification used by Benhabib and Spiegel in testing whether the level of education (which they refer to as 'human capital') impacts on growth. They use the average of the log of attainment to proxy the percentage change in the human capital as the factor of production.

The last piece of our exercise requires calibration of the scale parameter γ. It follows from the discussion in the main text that γ should be high during the productivity take-off, and lower before (or after). We adopt the growth rate of income per capita as a proxy discriminating between these cases. Because we are mostly interested in explaining the productivity performance of the EU peripheral countries in two different stages of their development, we conveniently allow only for two values of γ: before (and inclusive of) 1985 and after. We substitute the productivity growth rate for the rate of knowledge accumulation in (3.1) and compute γ for every year of data on educational attainment. We then average the gammas in the period of 1986-2000 (representing the take-off) and before[5].

6. Calibration of Capital Stock

As regards the data for the value of capital per worker, $\frac{K}{L}$, we used the perpetual inventory formula starting with a notional value in 1960. To derive this notional value, $\left(\frac{K}{L}\right)_{1960}$, we took the average growth rate of investment per worker over the 1960s, g, and assumed the rate of depreciation, δ, of 7%[6]. Then, we determined $\left(\frac{K}{L}\right)_{1960}$ as $\frac{(I/L)_{1960}}{(g+\delta)}$. Using this approach, the potential bias in the notional value would rapidly fall and arguably will have disappeared by the onset of the 1980s, which is our period of interest. To check for this bias, we also experimented with the notional value set according to stylized fact for developed economies: $\left(\frac{K}{L}\right)_{1960} = 2.5\left(\frac{Y}{L}\right)_{1960}$.

[5]The construction of knowledge capital stock using (5.2) is additionally complicated by the possibility of transitional dynamics out of BGP that make h behave differently from (3.1). Because we assume the effect of this distortion is greater the further we go back in history (the possibility that the country is far from its BGP increases), we account for it by averaging γ only in the 1975-1985 period, instead of 1960-1985. We then use this γ_{75-85} in constructing the human capital index for the whole 1960-1985 period and γ_{86-00} for the 1986-2000 period with the help of (5.2). Our estimates of the evolution of knowledge stock in the four peripheral countries since 1960 are summarised in Table 1.

[6]We, however, experimented with other rates of depreciation, ranging from 2-20%, without significant alteration to our results.

APPENDIX C

The Inflation-Growth Effect and Great Ratios Consistent with Tobin

1. Proofs

PROOF. (**Lemma 1**) The equilibrium conditions, including the marginal product definitions in equations (3.5) and (2.6), imply that the balanced-growth solution of all of the variables of the economy can be written in terms of $1-x$; in addition is an implicit equation in $1-x$. The implicit equation, derived from $1-x = l_F + l_G + l_H$, is $1-x = \frac{r^{(1-\varepsilon)/\beta}}{[A_G(1-\beta)]^{(1-\varepsilon)/\beta}} \frac{\left(A_G\left(\frac{l_G h}{s_G k}\right)^{\beta}(r-\rho)+\left(\frac{c}{h}\right)\left(\frac{l_H h}{s_H k}\right)^{\varepsilon}\right)}{\beta A_G\left(\frac{l_G h}{s_G k}\right)^{\beta}+\rho} + \frac{w^{\gamma/(\gamma-1)}x}{\alpha\gamma A_F R^{1/(\gamma-1)}\left[1+aR+w\left(\frac{l_F h}{c}\right)\right]}$. With $\varepsilon = \beta = \gamma = 0.5$, and $A_G = A_H = 1$ this gives the following polynomial in $z \equiv (1-x)^{0.5}$, where $\Omega \equiv \left[A_F\left(\sigma+\rho\right)\right]^2$.

$$0 = -0.5\Omega z^3 + 2[2\alpha\rho\Omega - (1+\rho\Omega)]z^2 - [4\alpha\rho\left(1+\sigma+\rho\right) - 0.5\Omega]z + 1 + \rho\Omega. \tag{1.1}$$

Differentiating with respect to σ and z, and solving for $\partial z/\partial\sigma$, we have $\frac{\partial z}{\partial \sigma} = \frac{(\partial\Omega/\partial\sigma)\{-0.5z^3+\rho(2\alpha-1)z^2+0.5z+\rho\}-4\alpha\rho}{\Omega\{1.5z^2-2\rho(2\alpha-1)z+0.5\}+2z+4\alpha\rho(1+\sigma+\rho)}$, where $\partial\Omega/\partial\sigma = 2A_F^2\left(\sigma+\rho\right)$. Evaluating $\frac{\partial z}{\partial \sigma}$ at the optimum of $\sigma+\rho = 0$, implies that $\frac{\partial z}{\partial \sigma} = -\frac{2\alpha\rho}{z+2\alpha\rho}$. Since $\alpha, \rho > 0$ and $z = 1-x \in (0,1)$, $\frac{\partial z}{\partial \sigma} = \frac{\partial(1-x)}{\partial \sigma} < 0$. Then the equilibrium values of all variables can be examined in terms of their change with respect to $1-x$ and σ. With the above parameter restrictions these are given by $r = 0.5(1-x)^{0.5}$, with $\partial r/\partial(1-x) > 0$, and $\partial r/\partial\sigma < 0$; $w = 0.5(1-x)^{-0.5}$, $\partial w/\partial(1-x) < 0$; $\partial w/\partial\sigma > 0$; $\frac{s_G k}{l_G h} = \frac{s_H k}{l_H h} = (1-x)^{-1}$; $\partial\left(s_G k/l_G h\right)/\partial\sigma < 0$; $(s_G k)/y = 1/[r(1-\beta)]$, $\partial[y/(s_G k)]/\partial\sigma > 0$; $g = r - \delta_k - \rho$, $\partial g/\partial\sigma < 0$.

Finally we derive the unique solution for x at the optimum. Evaluating equation (1.1) at the optimum of $\sigma+\rho = 0$, implies that $z^2 + 4\alpha\rho z + 1 = 0$. The quadratic equation has two solutions: $z_{1,2} = 2\alpha\rho(-1 \pm \sqrt{1+1/(4\alpha^2\rho^2)})$. One solution gives a negative x, outside of its feasible range. And it can be shown that the unique solution for leisure, $x \in [0,1]$, is $1 - 4\alpha^2\rho^2(-1+\sqrt{1+1/(4\alpha^2\rho^2)})^2$. □

PROOF. (**Lemma 2**) Under the assumptions of $\beta = \varepsilon = \theta = 1$ the economy uses no physical capital and has log-utility. Here the growth rate is determined by the marginal product of human capital and is given by $g = A_H(1-x) - \delta_h$, and $\partial g/\partial\sigma = -A_H\partial x/\partial\sigma$. The economy has a closed form solution and $x = (\rho\alpha/A_H)[(1 + aR + A_G l_F h/c)/(1 + A_G l_F h/c)$. Since $R = \sigma + \rho$, it follows that $\partial g/\partial\sigma = \partial g/\partial R$. Using this fact and the expression for x, $\partial g/\partial\sigma$ can be written as $\partial g/\partial R = -\alpha\rho[a/(1+$

$A_G l_F h/c)][1 + \eta_R^a - \eta_R^{l_F h/c}(A_G l_F h/c)/(1 + A_G l_F h/c)]$, where η_R^a is the elasticity of a with respect to R and is given by $\eta_R^a = -[\gamma/(1-\gamma)][(1-a)/a]$, and $\eta_R^{l_F h/c}$ is a similar elasticity given by $\eta_R^{l_F h/c} = 1/(1-\gamma)$. Further, $-\eta_R^{l_F h/c}(A_G l_F h/c)/(1+A_G l_F h/c) = \eta_R^c$, and so $1 + \eta_R^a - \eta_R^{l_F h/c}(A_G l_F h/c)/(1 + A_G l_F h/c) = 1 + \eta_R^a + \eta_R^c = 1 + \eta_R^m$, where $\eta_R^m \leq 0$ is the interest elasticity of money demand in equation (3.6). Therefore $\partial g/\partial R = -\alpha\rho[a/(1 + A_G l_F h/c)][1 + \eta_R^m]$. At $R = 0$, $\eta_R^m = 0$. As R rises the elasticity becomes increasingly negative, and $1 + \eta_R^m$ gets smaller. Because it can be shown that the other term also falls unambiguously as R rises, that is $\partial[a/(1+A_G l_F h/c)]/\partial R < 0$, the growth rate decrease that occurs for $\eta_R^m \geq -1$ becomes increasingly smaller as R increases; and its decrease is made directly less by the rising interest elasticity of money demand and the falling magnitude of the $1 + \eta_R^m$. Now if $a \equiv 1$, then from above it is clear that $\partial g/\partial R = -\alpha\rho$, which implies a linear inflation-growth relation. □

PROOF. (**Corollary 1**) Define the elasticity of substitution between cash and credit as $\epsilon \equiv \left[\partial\left(\frac{ac}{(1-a)c}\right)/\partial\left(\frac{R}{A_G/\gamma A_F^{1/\gamma}}\right)\right]\left[\left(\frac{R}{A_G/\gamma A_F^{1/\gamma}}\right)/\left(\frac{ac}{(1-a)c}\right)\right]$, which is solved as $\epsilon = -[\gamma/(1-\gamma)]/a$. In turn the interest elasticity of money is $\eta_R^m = \eta_R^a + \eta_R^c$, and this writes as $\eta_R^m = (1-a)\ \epsilon + \eta_R^c$. Normalizing the money demand m by dividing by the goods consumed, c, this gives $m/c = a$. And $\eta_R^a = (1-a)\ \epsilon$. Since $1-a = A_F^{1/(1-\gamma)}(R\gamma/A_G)^{\gamma/(1-\gamma)}$, by equations (2.19) and (2.32), then $\partial\,(1-a)\,/\partial R \geq 0$, $\partial\epsilon/\partial R \geq 0$, and so $\partial\eta_R^a/\partial R \leq 0$; for $R > 0$, $\partial\eta_R^a/\partial R < 0$. □

PROOF. (**Corollary 2**) By Lemma 2 and Corollary 1, $\eta_R^a = -[\gamma/(1-\gamma)][(1-a)/a] = -[\gamma/(1-\gamma)][A_F^{1/(1-\gamma)}(R\gamma/A_G)^{\gamma/(1-\gamma)}]/[1-A_F^{1/(1-\gamma)}(R\gamma/A_G)^{\gamma/(1-\gamma)}]$, and $\partial\eta_R^a/A_F \leq 0$ so that the magnitude of η_R^a rises as A_F rises. □

APPENDIX D

Contrasting Models of the Effect of Inflation on Growth

1. First-Order Conditions

Define η_t, λ_t, and μ_t as the Lagrangian multipliers for the human capital, income, and money constraints respectively, of equations. The first order conditions of the model in Section 2 of Chapter 4 are:

$$c_t : \frac{1}{c_t} = \lambda_t P_t \left\{ 1 + \left(\frac{\mu_t}{\lambda_t}\right) \left[\gamma_1 + \gamma_2 + (1-\gamma_1-\gamma_2)\left(1 - A_F \left(\frac{s_{Ft}k_t}{c_t}\right)^{\gamma_1} \left(\frac{l_{Ft}h_t}{c_t}\right)^{\gamma_2}\right)\right]\right\};$$

$$x_t : \frac{\alpha}{x_t} = \eta_t A_H (1-\varepsilon) h_t (s_{Ht}k_t)^{\varepsilon} (l_{Ht}h_t)^{-\varepsilon};$$

$$M_{t+1} : -\lambda_t + \left(\frac{1}{1+\rho}\right)\left(\lambda_{t+1} + \mu_{t+1}\right) = 0;$$

$$k_{t+1} : -\lambda_t P_t - \mu_t P_t a_2 + \left(\frac{1}{1+\rho}\right) P_{t+1} r_{t+1} s_{Gt+1} + \left(\frac{1}{1+\rho}\right) P_{t+1}(1-\delta_K)$$
$$+ \left(\frac{1}{1+\rho}\right) \mu_{t+1} a_2 (1-\delta_K) P_{t+1} + \left(\frac{1}{1+\rho}\right) \mu_{t+1} \gamma_1 \frac{s_{Ft+1}}{c_{t+1}} P_{t+1} A_F \left(\frac{s_{Ft+1}k_{t+1}}{c_{t+1}}\right)^{\gamma_1 - 1} \left(\frac{l_{Ft+1}h_{t+1}}{c_{t+1}}\right)^{\gamma_2} c_{t+1}$$
$$+ \left(\frac{1}{1+\rho}\right) \eta_{t+1} \varepsilon (s_{Ht+1}) A_H (s_{Ht+1}k_{t+1})^{\varepsilon - 1} (l_{Ht+1}h_{t+1})^{1-\varepsilon} = 0;$$

$$h_{t+1:} - \eta_t + \left(\frac{1}{1+\rho}\right) \lambda_{t+1} P_{t+1} w_{t+1} l_{Gt+1} + \left(\frac{1}{1+\rho}\right) \eta_{t+1}(1-\delta_H)$$
$$+ \left(\frac{1}{1+\rho}\right) \mu_{t+1} P_{t+1} \gamma_2 l_{Ft+1} A_F \left(\frac{s_{Ft+1}k_{t+1}}{c_{t+1}}\right)^{\gamma_1} \left(\frac{l_{Ft+1}h_{t+1}}{c_{t+1}}\right)^{\gamma_2 - 1}$$
$$+ \left(\frac{1}{1+\rho}\right) \eta_{t+1}(1-\varepsilon) l_{Ht+1} A_H (s_{Ht+1}k_{t+1})^{\varepsilon} (l_{Ht+1}h_{t+1})^{-\varepsilon} = 0;$$

$$s_{Gt} : \lambda_t P_t r_t - \eta_t \varepsilon A_H \left(\frac{l_{Ht}h_t}{s_{Ht}k_t}\right)^{1-e} = 0;$$

$$l_{Gt} : \lambda_t P_t w_t - \eta_t (1-\varepsilon) A_H \left(\frac{l_{Ht}h_t}{s_{Ht}k_t}\right)^{-e} = 0;$$

$$s_{Ft} : \mu_t \gamma_1 P_t A_F \left(\frac{s_{Ft+1}k_{t+1}}{c_{t+1}}\right)^{\gamma_1 - 1} \left(\frac{l_{Ft+1}h_{t+1}}{c_{t+1}}\right)^{\gamma_2} - \eta_t \varepsilon A_H \left(\frac{l_{Ht}h_t}{s_{Ht}k_t}\right)^{1-e} = 0;$$

$$l_{Ft} : \mu_t \gamma_2 P_t A_F \left(\frac{s_{Ft+1}k_{t+1}}{c_{t+1}}\right)^{\gamma_1} \left(\frac{l_{Ft+1}h_{t+1}}{c_{t+1}}\right)^{\gamma_2 - 1} - \eta_t (1-\varepsilon) A_H \left(\frac{l_{Ht}h_t}{s_{Ht}k_t}\right)^{-e} = 0.$$

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