THE INTERACTION BETWEEN PRIVATE AND PUBLIC IPR PROTECTION IN A SOFTWARE MARKET: A POSITIVE AND NORMATIVE ANALYSIS

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The Interaction between Private and Public IPR Protection in a Software Market: a Positive and Normative Analysis

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Abstract

Two software developers, each offering a product variety of different (exogenously given) quality, compete in prices for heterogeneous users who choose from purchasing a legal version, using an illegal copy, and not using a product at all. Using an illegal version violates intellectual property rights (IPR) and is thus punishable when disclosed. If a developer considers the level of piracy as high, he can introduce protection for his product in the form of restricting support and other services to illegal users. We study the positive and normative implications of the interaction between a regulator's IPR protection and the IPR protection that producers themselves may undertake to protect their IPR against the end users' software piracy. In particular, we aim to establish when the two forms of IPR protections (public and private) act as complements and when as substitutes to each other. Finally, we explore the situations in which there is (or is not) a conflict of interest between the regulator and the developers in this respect.

Keywords: Vertically differentiated duopoly, Software Piracy, Bertrand competition, Private and public intellectual property rights protection

JEL Classification: D43, L11, L21, O25, O34

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Abstrakt

Dva vývojáři softwaru, z nichž každý nabízí varietu produktu rozdílné (exogenně dané) kvality, cenově konkurují o heterogenní uživatele, kteří se rozhodují mezi koupí legálně verze, používáním nelegální kopie nebo nepoužíváním produktu vůbec. Užívání nelegální verze porušuje práva duševního vlastnictví (IPR) a může tak být v případě odhalení potrestáno. Jestliže vývojář pokládá úroveň pirátství za vysokou, může zavést ochranu svého produktu ve formě omezení podpory a jiných služeb nelegálním uživatelům. Studujeme pozitivní a normativní důsledky interakce mezi ochranou IPR ze strany regulátora a ochranou IPR, kterou proti softwarovému pirátství koncového uživatele výrobci mohou zavést sami. Zejména se snažíme ukázat, kdy tyto dvě formy ochrany IPR (veřejná a soukromá) fungují jako komplementy a kdy jsou navzájem subsituty. Závěrem analyzujeme situace, ve kterých (ne)existuje v tomto ohledu konflikt zájmů mezi regulátorem a vývojáři.

1 Introduction

The key factors contributing to the creation of intellectual property rights' (IPR) violation or illegal imitations are low costs and the low technical requirements of such imitation. In this light, the natural leaders for IPR violation are "information" products, (also known as digital content products), i.e. software, movies, music, or e-books[1]. These products have two idiosyncratic attributes: imitations are often 100% identical to the original and the direct costs of copying are negligible. According to the report of the Business Software Alliance, the proportion of pirated software within the total software installed in 2008 climbed to 41% resulting in the global loss in excess $50 billion. Even in the US, where the rate of illegal usage is the lowest, 20% of software is illegal, while in Western Europe the proportion is closer to one third. At the top of the list are Georgia, Pakistan, Indonesia, and China where 80% and more of installed software is illegal[2].

The expansion of DVD burners, alongside the penetration of broadband internet, does not only increase the opportunity for illegal copying[3] but also eliminates mass illegal producers from the market. Illegal copies are, nowadays typically made (installed) by the end users themselves who do it wittingly and only for themselves[4]. This attribute changes the essentials of the fight against IPR violation. While, say, in pharmaceuticals, luxury goods, or electronics markets, end users might be often perceived as victims, in "information" markets, end-users of illegal copies are predominantly the ones that actually carry out IPR violation[5]. Thus, the fight in digital content markets is now aimed mainly against end users (meaning both retail and corporate users)[6].

[1]For information about mp3, movie, or e-book protection and their illegal copying see www.ifpi.org, www.riaa.com, or www.pro-music.org.

[2]see also The Economist, May 16th, 2009.

[3]Most of the illegal copies of digital content are easily accessible using P2P networks (direct connect, torrent trackers) or data sharing (Rapidshare). Note that easy downloading could be accompanied by relatively complicated installation/usage of illegal versions.

[4]In this paper, we omit the problem of the black market with DVDs/CDs or software in the suburbs. These kinds of piracy experienced a boom a decade ago and are now declining strongly, especially in developed countries.

[5]However, companies try to distinguish between intentional piracy and the unconscious usage of an illegal version, e.g., Microsoft replaces fake versions with legal ones to users who bought a fake version of its software in good faith.

[6]A well-known example aimed at end-users is suing students at US/EU universities for sharing software on university servers. Note that these actions are often accompanied with legal actions against the means

We focus on such digital content markets (like the software market) where only the end users violate IPR and study both the positive and normative aspects of this phenomenon. More specifically, we analyze strategic interactions among software developers who compete in prices but may also undertake private IPR product protections against end users' piracy. On the other hand, public IPR protection (say in the form of copyrights) also exists. The core of our analysis then, is the interaction between the public (or government) on one hand and the private IPR protection on the other. In particular, we put forward a dynamic three-stage duopoly model, where, in the last stage, two developers compete in prices for users with different price sensitivity on the same market. That is, we rely on a quality competition model (see, for instance, Shaked and Sutton, 1984, and Tirole, 1988[7]). In the second stage of the game, each developer has an option to choose a level of its private IPR protection. Finally, in the first stage the government commits to a penalty for those users who are caught in their illegal use of software. Like most of the literature, we assume that the government's punishment (or public protection) is broad-based in a sense that it raises the piracy costs to all consumers[8].

We concentrate on a particular and very simple practice of private IPR protection that the developers' use to protect their products. This practice appears in the form of decreasing product value (or degradation of product quality, in the language of Bae and Choi, 2006) to illegal users by denying various services related to the efficient use of software (say, by eliminating updates in antivirus or tune-up utilities[9], restricting access to users' manuals, etc.). We refer to this type of private protection as restricting services to illegal users.

On the other hand, most of the other analyses of private IPR protection consider some kind of physical protection, or more generally, a so called DRM (Digital Rights Management)

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of sharing e.g. closing Napster as the first famous case or the current hot suit against torrent tracker, The Pirate Bay, with the intention to close it.

[7] Shy (1999) addresses the same problem using a Hotelling-type spatial competition model.

[8] However, there is also an alternative approach in which public protection mostly targets institutional and corporate users rather than individual users, see Harbaugh and Khemka, 2010 and the relevant literature cited there on such an approach.

[9] Illegal versions of some antivirus software, e.g., Symantec Antivirus, might not update their installed databases of viruses and thus the PC is more vulnerable in the case of the latest virus attack. Similarly, tune-up utilities may not update their internal list of supported problems, so some new errors cannot be corrected. For other examples of decreasing product value, see Bae and Choi, 2006

system [10]. The main drawback of the DRM system, however, is that it also imposes costs for legal users so that the quality of the original may be even lower than the quality of a copy. Moreover it often restricts the usage far beyond the copyright laws (e.g., not only against illegal copying, but even legal usage, such as restricting the use of a legally bought e-book to only one device). DRM is thus considered a controversial and somewhat problematic approach to protecting IPR. For instance, Belleflamme and Peitz, 2012 note that ”recent development in digital distribution suggest that DRM is on a losing path since not only Apple but also Amazon and Walmart have started selling DRM-free digital music”.

The remarkable difference between DRM and private protection in the form of service restrictions is that the latter typically does not impose any costs for the developers on one hand, while also not adversely affecting the utility of the legal users on the other.

To capture the regulator’s role in a simple manner, we assume that imposing a penalty is the sole instrument for reducing or eliminating the illegal usage of the product that is under copyright protection[11]. Thus, the government commits to a penalty in order to maximize social welfare. We assume that both developers are, from the point of view of the government, domestic ones, so the regulator sets the level of copyright protection that maximizes the sum of profits and consumer surplus[12]. Since the legal environment as well as the regulator’s activity is publicly observable, a user can estimate the probability of being caught and then convicted for copyright violation and so he can correctly calculate the expected size of the penalty. Thus, if a user decides to use an illegal version, he can evaluate the expected penalty which can be considered as the cost of illegal usage.

The software market may distinguish itself from other digital content markets due to po-

[10] DRM is an umbrella term for various technologies that limit the usage of digital content in an unintended way by the developer. Most major content providers such as Microsoft, Sony, Amazon, or Apple used to exploit DRM. Nowadays most content providers experiment with DRM-free alternatives, mainly in music (see more on DRM in Belleflamme and Peitz, 2010 and also Scotchmer, 2004).

[11] The government’s reliance on taxes and subsidies as an instrument of IPR protection is not considered very realistic in the given context and is thus assumed away in the further analysis.

[12] In setting the level of IPR protection, however, a government may favor one of the developers in the case of a domestic developer competing with a foreign developer. The government may, for instance, adapt IPR enforcement to favor the domestic developer or vice versa. Such special cases can easily be handled within our framework. For illustration, we could use a comparison among countries that have strong developers (e.g., the US) and quite a severe level of protection of IPR with countries where no strong local developers exist, e.g., Finland, Sweden, or Norway, where protection of IPR is moderate and more "open ".

tentially high Network Effects (NEs) coming from software usage. NEs mean that increasing the base of users by, say, allowing the copying of a product to some other users, raises the utility of all users and thus adds extra value to the product. We, on the other hand, consider NE unimportant and assume them away (see, however, Střelický and Žigić, 2011 for how NE could easily be incorporated in our set-up).

It is important to stress at the outset that our approach is somewhat different from the current literature on software piracy. According to Belleflamme and Peitz's comprehensive survey (2012), our approach can be classified as i) end-user piracy models that ii) includes the competitive effects meaning that there are two producers of substitutable and piratable digital products that directly compete with each other (see Belleflamme and Peitz). As can be clearly seen from Belleflamme and Peitz (2012), there are indeed only a few articles which deal with the positive and normative issues of digital piracy while explicitly modeling direct firms' competition. Moreover, all of these papers, in general, rely on the notion of horizontal product differentiation (see the next section for a brief survey of the related literature).

The papers that are closest to the spirit of our analyses are the works of Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010). Much like ourselves, these authors also study the interaction between public and private IPR protection in the circumstances in which strategic interactions prevail. There are, however, several important differences in the two approaches. Firstly, we focus on direct strategic interaction between the developers where the two firms compete in prices in a vertically differentiated market, whereas the strategic interactions in Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010) are the indirect ones stemming from the different copying technologies. Secondly, in addition to the different focus (direct versus indirect competition), the other key difference between our set-up and that of Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010) is that in their settings the original products have the same quality, while in our set-up, the original products are vertically differentiated and thus have distinct qualities to begin with. Thirdly, since we focus on the software market, we do not allow for a different copying technology as it is typically the case with multiple, initially independent digital products. Thus, the cost of consuming illegal copies is constant in our setting, while it may be decreasing with the number of different originals copied in the settings of Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010). Last but not least, private IPR protection appears in the form of service restrictions in our approach while Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010) assume rather standard DMR protection.

As already mentioned above, the focus of our analysis is to study both the positive and the normative aspects of the interaction of private and public IPR protection. In examining the positive aspect we explore how the expected penalty affects market structure, market coverage, and the developers' IPR protection, especially in relation to whether private and public IPR protections are substitutes or complements to each other. As regards the normative aspect, we look for the optimal public IPR protection and study how the absence/presence of the private IPR protection affects the optimal public protection. In order to focus on the differing impact of the absence or presence of private IPR protection on optimal public IPR protection, we assume that the product qualities are given, so the need for the public IPR protection does not come from the incentives to innovate.

Regarding the positive aspect, we show that the government's and the developers' IPR are always substitutes in a sense that for the given developers' optimal protection, the public IPR protection could be substantially lower (compared to the situation with no private IPR protection) in order to fully eliminate illegal usage. Moreover, the government can, by its choice of IPR protection (that is, via the size of the expected penalty), affect the market configuration and market coverage since the height of the expected penalty has an effect on equilibrium prices and profits and thus on the toughness of price competition. For instance, for the size of the expected penalty that falls between two prices, there might occur a market configuration with two unconnected segments of legal users. In this case, the high quality developer serves the upper part of the market and earns (constrained) monopoly profit, while the lower quality developer serves the lower end of the market. In the middle of these two segments, there is a "buffer" composed of illegal users. If on the other hand, the government sets the penalty rather low so that both prices are bigger than the expected penalty, then direct duopoly competition might be restored.

Our analysis of the normative aspect indicates that the very presence of private protection can change the government's IPR incentive in a fundamental way. More specifically, in the absence of private IPR protection it is always optimal for the regulator to set a zero (or by, say, an international agreement, minimally required) IPR protection. In the presence of private IPR protection, however, it is in general optimal for the government to set strictly positive IPR protection. Thus, the optimal public and private IPR protections are complements in this sense. Finally, we show that, depending on the setup, there may or may not be a conflict of interests between the regulator and the developers.

The paper is structured as follows: following the literature survey, we present the set-up of our model in Section 3 and the brief analysis of monopoly market structure. We then move on to our key analysis of the standard duopoly market interactions[13] in Section 4. We analyze the last stage of the game that deals with price competition among the developers and then briefly study how the change in private and public IPR protection affect equilibrium prices and profits. We continue with the second stage of the game which deals with the choice of the optimal private IPR protection. In Section 5, we focus on the key normative analysis of the optimal choice of public IPR protection and its interaction with private IPR protection. Finally, Section 6 is reserved for concluding remarks.

2 Survey of the literature

To put our analysis into context, we follow the very recent comprehensive and influential survey of digital piracy by Belleflamme and Peitz, 2012 (see also Peitz and Waelbroeck, 2006). The pioneering article on this subject is that of Shy and Thisse (1999), who analyze piracy in the Hotelling-type duopoly competition where users have exogenous preferences for a particular developer[14]. They show that a developer's decision to introduce protection against illegal copying depends mainly on the NEs, and that under strong NEs, each developer decides not to implement protection in order to make his software more attractive and to

[13] Given our framework in which the high quality firm competes with the low quality firm, it seems that Stackelberg competition might be also an appropriate way to model it. Střelický, J and Žigić, K, (2011), however, show that Stackelberg setup does not bring any qualitatively new insight so we do not consider it here..

[14] There is, however,a mistake in the article; see Peitz, (2004) for the correction of the mistake.

raise the users base. Jain (2008) builds upon the model of Shy and Thisse (1999) and assumes that firms can choose a level of IPR protection so that only a proportion of consumers with low product valuations (who are, by assumption, the only consumers interested in copying) can copy its product. In the absence of NE, Jain shows that, in such a set-up, piracy can change the structure of the market and, thereby, reduce price competition between firms. The reason is that copying by low, more price-sensitive types enables firms to credibly charge higher prices to the segment of consumers that do not copy. Furthermore, this positive effect of piracy on firms' profits can sometimes outweigh the negative impact due to lost sales. So, even in the absence of network effects, firms may prefer weak copyright protection in equilibrium.

Finally, there is a recent paper by Minnitti and Vergari (2010), who also rely on the Hotelling differentiated-product duopoly framework. They, however, deal with a rather specific form of piracy similar to a private file sharing community and study how its presence affects the pricing behavior and profitability of producers of digital products.

From the perspective of our approach, it is very important to note that digital developers' competition can also occur in a multi-product framework, where piracy can generate a kind of indirect competition between horizontally differentiated digital products as demonstrated by Belleflamme and Picard (2007). They show how the copying technology displaying increasing returns to scale can create an interdependence between the demands for digital products that would be unrelated otherwise. Moreover, the underlying demand is, much like in our approach, obtained in a vertical differentiation manner. However, the vertical differentiation does not, as in our set-up, arise from the different quality levels of the developers but from the existence of original and copied digital products in a market where the originals are assumed always to be of higher quality than the copies, thus ensuring that all consumers unambiguously prefer the original product over the copy. In this set-up Belleflamme and Picard (2007) study how piracy affects prices and profits and, interestingly enough, they show that depending on the parameters of the model, prices can be either strategic substitutes or strategic complements. If the fixed cost of copying is low enough, there is no equilibrium in pure strategies. Firms may then randomize between several prices, leading to price dispersion.

Following the approach of Belleflamme and Picard (2007), Choi, Bae, and Jun (2010) likewise use a Hotelling horizontal differentiation model and analyze the situation in which also the interdependence between the firms stems from their strategies against piracy rather than from direct competition on prices.

Finally, there are by now numerous scholarly articles that deal with the issue of digital piracy and optimal public IPR protection in the monopoly set-up (see, for instance, Yoon, 2002, Banerjee, 2003; King and Lampe, 2003; Kúnin, 2004; Bae and Choi, 2006, Banerjee, et al., 2008. Takeyama, 2009, Ahn, and Shin, 2010). Thus, for instance, Banerjee (2003) demonstrates that the socially optimal level of IPR protection differs from a monopoly developer's optimum and stresses the role of NEs. King and Lampe (2003) show that the monopoly allows illegal users in the case when the network effect is present, while Takeyama (2009) shows that under asymmetric information about product quality, the copyright has to be imperfect in order to avoid adverse selection. Kúnin (2004) provides an explanation for why a software manufacturer may tolerate widespread copyright infringement in developing countries and often even offer local versions of their software. He showed that if NEs are present and there is an expected improvement in copyright, then software manufacturers enter the market even if they incur losses in the beginning when copyright enforcement is weak. For a deeper and systematic review of the literature on the piracy of digital products, the interested reader is advised to look at the two excellent and comprehensive surveys in Peitz and Waelbroeck (2006) and Belleflamme and Peitz (2012).

3 The Model

3.1 Industry set-up

Consider two developers, A and B who compete in prices on a particular market and offer product varieties of different quality[15]. Developer A releases a product of quality q_A, while the quality of the second developer B is q_B and we assume, without loss of generality, in the

[15] We will use the term "value" instead of "quality" when quality contains multiple dimensions.

rest of the article that developer A offers higher quality ($q_A > q_B$). Product qualities q_A, q_B, in the whole article are assumed to be exogenous and cannot be changed by developers[16]. We assume that both developers already existed before meeting and competing on the market under consideration and that both of them had already determined their respective qualities. Consequently, both developers are assumed to have already incurred fixed set-up costs and fixed costs associated with software development (R&D costs). As these sunk costs do not enter in the economic optimization under consideration we have left them out of the profit function. With the qualities exogenously given, we can conceive our analysis as a short (or medium) run analysis as opposed to a long-run analysis in which firms choose the qualities. Finally, the unit variable costs are assumed to be constant and normalized to zero.

One of the implications of the above set-up is that the developer(s) may not operate in the market under consideration if the developers' own IPR protection, and the lack of public IPR protection, do not enable them to earn non-negative profit. We, however, assume that even if a developer is not present in the market under consideration, it may be active in another market (country) so the users may still be able to obtain an illegal version via copying.

We further assume that developers cannot directly restrict illegal usage of the product itself but could restrict part of the services related to the product. This restriction lowers the product value for illegal users. That is, the value of the legal version differs from the illegal version since a developer provides parts of the valuable services only to legal users (such as online help and technical support, live updates, a discount for upgrades or even free training, access to user manuals, etc.). Probably the most famous example of restricting services to illegal users, familiar to everybody, is the one with Microsoft Windows. Microsoft's Windows Genuine Program allows a user to run an illegal version of the product only up to a certain point. In order to install selected patches/updates, the user has to validate the originality of the program online. If a particular copy is identified as illegal, some functions

[16] In the more elaborated versions of these kind of models that deal with long-run, there is also a choice of qualities preceding the pricing decision (see, for instance, Shaked and Sutton, 1982 and 1983; Kúnin and Žigić, 2006). In principle, it should be possible to extend our approach to such a long-run analysis and allow the developers to choose and compete in qualities too, but given that our focus is on the short/medium run we leave this issue for future research.

are disabled, and the illegal user is irritated with constant messages about buying the legal version. If a user decides not to validate the program online, he cannot update his Windows further for selected components (e.g. Windows Media Player or Internet Explorer)[17]. The implementation of such a restriction is technically easy since the developer could use the standard tools that restrain access to those services that require user authorization based on personal information verification. In the case of automatic access to those services, a developer can use very reliable tools as authorization is based on the IP address or hard-locks.

Denote the value (perceived quality) of the legal version as q_i. Then, the value of the product for the illegal users is decreased to αq_i so these users face "degradation costs" that are proportional to consumers' valuation of the original product (see Bae and Choi, 2006). The variable $\alpha \in (\underline{\alpha}, 1)$, and $\underline{\alpha} > 0$ stands for technically the highest possible level of restriction (that is, the lowest possible α) beyond which it is impossible to further restrain services[18]. The exclusive part of the product value that only the legal users can enjoy is $1 - \alpha$.

As for the developers' costs of restricting services, it seems reasonable to assume that these costs are negligible given that the developers already exist and have chosen their quality levels and the accompanying level of consumer services previously. So, we assume these costs to be zero, but we do discuss the implications of nonzero costs for the optimal choice of α in the Appendix II.

3.2 The regulator's role

The regulator's role is limited to monitoring software usage and to the penalization of those users, who use products illegally and are disclosed. The regulator chooses the level of pun-

[17]Another examples is antivirus programs (e.g., Symantec Antivirus), when often after updates, the program recognizes that a particular installation is illegal and does not allow further updates of its virus database for new viruses. Finally, for many computer games, online playing is allowed only for the legal users. For other examples of decreasing product value, see Bae and Choi, 2006.

[18]Technically, the maximal possible private protection ($\alpha = \underline{\alpha}$) is the one at which quality q_A to legal users is still not adversely affected. Decreasing α below $\underline{\alpha}$ would require such strong protection/verification tools (e.g. manual online authorization), which would become annoying even for legal users resulting in quality drop below q_A.

ishment and the monitoring efforts in order to maximize social welfare (that is, the sum of profits, consumer surplus and revenue collected from penalty net of the monitoring costs - see section 5 for the precise definition). More specifically, the regulator has under its control the expected penalty, X, that is defined as the product of the size of penalty (say Y) and the perceived probability by the developers and consumers (say, P) that the infringer would be caught, where P is assumed to be the same for all consumers and developers, thus, $X = PY$. The regulator sets Y by law and also put costly efforts (say, E) in monitoring and catching infringers. The effort, E, is observable by the developers and consumers and it maps into the perceived probability P. So, for given Y, $X = P(E)Y$ is fully determined by E. It is reasonable to assume that there are "decreasing returns" in E, so that $P'(E) > 0$ but $P''(E) < 0$ with $Lim_{E \to \infty} P(E) = 1$ and $P(0) = 0$. This, in turn, implies that the regulator's costs of monitoring, E, is (for a given Y) an inverse function of X that we label as $E \equiv C(X)$ such that $C(0) = C'(0) = 0, C'(X) > 0$ and $C''(X) > 0$.

In a sense, X can be interpreted as the piracy costs that include the risks and inconveniences that consumers bear when making the copies (see Ahn, I., Shin, I. 2010).

3.3 Users set-up

We assume that every user has access to all the versions: to both legal versions A, B and to the illegal versions of A, B and decides based on the product prices and values. Utility for a user θ is then:

$$U_P(\theta) = \begin{cases} \theta q_i - p_i & \text{... if he buys software.} \\ \theta \alpha q_i - X & \text{... if he uses software illegally.} \\ 0 & \text{... if he does not use software at all.} \end{cases} \tag{1}$$

There are always some users (top-end users with high θ) that prefer to buy the legal version rather than the restricted illegal one even if both versions (legal and illegal) are available and even when X goes to zero. From utility function (1), we can identify 6 types of users indifferent between some two actions. Those users appear on the market under different levels of X, q_A, q_B, and α. Only some of the indifferent users exist on a particular market but never all of them. Here are the 6 types of indifferent users:

1. θ_{PA} ... The user indifferent between using legal product A and its illegal version.
2. θ_{0P} ... The user indifferent between using illegal version A and using nothing at all.
3. θ_{0A} ... The user indifferent between using legal product A and using nothing at all.
4. θ_{0B} ... The user indifferent between using legal product B and using nothing at all.
5. θ_{BP}, θ_{PB} ... The user indifferent between using legal product B and using illegal version A.
6. θ_{BA} ... The user indifferent between using legal product A and using legal product B.

3.4 The public and private IPR protection and the market structure

Before proceeding to the very analysis of the underlying three-stage game, it is instructive to see briefly how the two forms of IPR protection may interact and affect the market structure and market coverage and to preliminarily introduce the market structures that we focus on. These market structures are those in which all three versions of the product (one illegal and two legal) are present in the market equilibrium. We start with stressing the role of public IPR protection in isolation. If there is only public protection available, it is clear that the expected penalty X, when high enough, would completely protect the developers from piracy and result in the standard Bertrand competition in prices making the private IPR protection redundant. In the case of the medium expected penalty (where X is lower than the optimal Bertrand price of developer A), however, the developer A may either not be active in the market or would have to decrease the price. In such a market situation, X fosters competition and forces both developers to decrease prices, but at the same time, too low an X could squeeze one of the developers out of the market since he may no longer be able to recover his set-up costs (connected with the presence in the market under considerations). From the government's point of view, X serves as an artificial price that must either be accepted by developer A, or he has to be inactive in that market. In the case of a very low X (where X is now lower than the optimal Bertrand price of developer B), none of the developers would operate in the market (see Střelický, J and Žigić, K, 2011 for the detailed analysis of IPR protection in the absence of private IPR protection). Note that in the absence of private IPR protection, the users do not perceive a quality (value) difference between the original product and its illegal version, and thus, users always choose the version with a lower "cost" per quality unit ($\frac{p_i}{q_i}$ in the case of a legal version and $\frac{X}{qi}$ in the case of an illegal version).

When, on the other hand, private IPR protection is present as well, the legal and illegal versions are not perfect substitutes. That is, the value of the legal version for a user θ is θq_i and it is bigger than the corresponding value of the illegal version for the same user that is valued only $\alpha\theta q_i$ due to the services restriction for the illegal usage. Now there will always be legal users and at least one developer present in the market under consideration even if public IPR is absent ($X = 0$). The reason is, as we already know, that there are always users with high θ that prefer to buy the legal version rather than the illegal one. More specifically, if the illegal version of the high quality product has a still higher value than the legal version of the low quality product (that is, when $q_A > \alpha q_A > q_B$), then the absence of public IPR protection would result in monopoly of developer A, since he would sell the legal version of software to the top users while the end tail of the market will be comprised of pirate users where a user θ of the product A obtains utility of $\alpha\theta q_A$. Product B will not be in the market. If, however, there is even a very small level of public IPR protection, then, all three versions will again be present in the market, since developer B could set the price $p_B < X$ (implying $\frac{p_B}{q_B} \leq \frac{X}{\alpha q_A}$) given our assumption of zero marginal production costs and absence of entry sunk costs. This is the first of the two market structures that we are interested in and that we label as Case 1 or piracy duopoly *(p1)*.

If, on the other hand, the value of both legal versions are higher than the value of the illegal version of the high quality product (that is $q_A > q_B > \alpha q_A$), then both developers would introduce IPR protection and be present in the market even if public protection is very small or absent. So again all three versions of the product (that is, q_A, q_B and αq_A) will be in the market (despite that fact that the "cost" per quality unit would be such that $\frac{X}{\alpha q_A} \leq \frac{p_B}{q_B} \leq \frac{p_A}{q_A}$). We label this market situation as Case 2 or piracy duopoly (*p2*).

In what follows, we first briefly review the monopoly market structure and then focus on our key market structure under consideration - duopoly. Thus, in the last stage in the duopoly game the developers choose the prices and we analyze the impact of different α and X on equilibrium prices and on the resulting market structure and coverage. We then proceed with the second stage of the game in which the firm(s) choose(s) private protection, that is, the degree of service restrictions. We conclude with the government's choice of public protection and the analysis of the interaction between these two types of IPR protections.

3.5 Monopoly

For a better illustration of the model behavior, we start with the monopoly case (of developer A). In this case, developer A can compete only with an illegal version of his own product. If the expected penalty X is high enough that nobody is willing to use software illegally, we obtain the pure monopoly. This situation occurs when $\frac{1}{2}\bar{\theta}\alpha q_A \leq X$.

In the case where $X \leq \frac{1}{2}\bar{\theta}\alpha q_A$, there are users who prefer to use the illegal version and so setting α as low as possible is the right thing to do in order to increase the demand for the legal version. In order to work out the monopolist's demand, we find user θ_{PA}, who is indifferent between the legal and illegal product, and so this user is described by $\theta_{PA} = \frac{p_M - X}{q_A - \alpha q_A}$. The demand for product A is then $D_A = (\bar{\theta} - \theta_{PA})$, and the monopolist profit is $\pi_M = (\bar{\theta} - \theta_{PA})p_M$, while the demand for the illegal version is $D_P = (\theta_{PA} - \theta_{0P})$. Equilibrium price and profit are:

$$p_M^* = \frac{X + \bar{\theta} q_A (1 - \alpha)}{2}, \ \pi_M^* = \frac{1}{4} \frac{\left(X + \bar{\theta} q_A (1 - \alpha)\right)^2}{q_A (1 - \alpha)}. \tag{2}$$

This results in the distribution of users on the market as captured in Figure 1 .

Note that the monopolist which faces illegal usage, but has an option to increase the

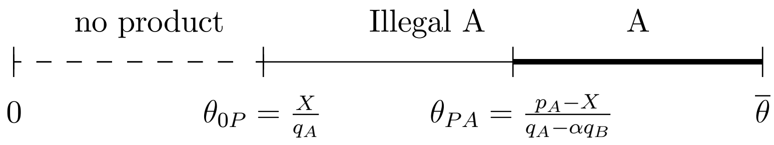


Figure 1: The decreased quality to illegal users on the monopoly market

number of legal users by restricting additional services, generates uniformly higher profit than the monopolist that could only set $p_M^* = X$.

Clearly, now the profit of the monopolist increases not only in the level of expected penalty but also in the degree of restrictiveness to the additional services (that is, the lower the α, the higher the monopolist's profit). Thus, the maximal restriction of services to the illegal users is optimal, that is $\alpha_M^* = \underline{\alpha}$.

4 Duopoly

4.1 The last stage - competition in prices

A user's decision to use an illegal version depends on the user's sensitivity to product quality θ as well as on the expected penalty X and the service restriction α.We first focus on the situation in which only a developer of a higher quality product uses the restriction in services. If X is high enough such that in equilibrium $p_B < p_A < X$, then[19] illegal usage is fully suppressed, the private IPR protection is redundant and the market is divided between both developers.

Assuming that illegal usage is not eliminated, and legal versions are on the market, then there are two interesting cases in which both developers operate on the market. The first one is when $q_B < \alpha q_A$ (implying that in equilibrium $\frac{p_B}{q_B} \leq \frac{X}{\alpha q_A} \leq \frac{p_A}{q_A}$), and the second one when $\alpha q_A < q_B$ (implying that in equilibrium $\frac{X}{\alpha q_A} \leq \frac{p_B}{q_B} \leq \frac{p_A}{q_A}$). In all other cases, either the legal version of product B is eliminated so there is a monopoly for developer A, or the illegal usage of product A is eliminated and there is pure or constrained (non-piracy) duopoly (see Graph

[19] In this part, whenever we write $p_B < p_A$ we mean $\frac{p_B}{q_B} < \frac{p_A}{q_A}$, which is a necessary condition for product B to be in the market.

1). *Supplementary Material* containing the complete analysis of the last stage of the game and all price equilibria can be found on Michael Kúnin's home page[20] or can be obtained from the authors upon request.

In the first case when $q_B < \alpha q_A$, developer A competes with an illegal version of his own product to capture users with relatively high θ, while developer B competes with an illegal version of product A to capture users with relatively low θ (see Figure 2)[21]. In the second case, when $\alpha q_A < q_B$, developer A competes with developer B for users with high θ, while developer B competes with the illegal version of A for users with low θ (see Figure 3). The second case leads to a tougher competition between developers A and B, where q_A and q_B are relatively close, while the first case describes a market where developer B produces a significantly less valuable product than developer A, and thus, he can hardly compete with his legal version[22].

Finally, note that developer B has incentives to restrain services to illegal users only in the second case. Since the expected penalty is the same for whatever product is used illegally, the users would always prefer to use the illegal version of product A to illegal version of product B . So the only case when developer B would introduce the restriction of his services is, as we will see, when X is "low," and A introduces strict restrictions of his services (low α). In that case, the lack of developer B implementing protection would result in the illegal usage of product B (in this case, an illegal version of B has quality $q_{B,}$ while an illegal version of A has αq_A, which is lower than q_B). In other words, no user would use product B legally unless developer B also implements protection (see Case 2 below).

4.1.1 Case 1: Piracy duopoly (*p1*) when $q_B < \alpha q_A$

This situation corresponds to a product distribution over the market in which there are three types of indifferent users:

[20] http://home.cerge-ei.cz/kunin/

[21] This is in line with a real life situation where the developer of a product with a lower quality competes strongly with an illegal version of a better product developer. Both developers know that introducing sophisticated protection could only discourage legal users from their services, while illegal users would always prefer to use a better product.

[22] E.g., competition between the Microsoft Office 2010 package against small alternative developers such as 602 and its package known as "OpenOffice.org Software 602 Edition."

1. A user indifferent between buying product A and its illegal usage: $\theta_{PA} = \frac{p_A - X}{q_A - \alpha q_A}$,

2. A user indifferent between the illegal usage of product A and buying product B: $\theta_{PB} = \frac{X - p_B}{\alpha q_A - q_B}$, and

3. A user indifferent between buying product B and not using any product at all: $\theta_{0B} = \frac{p_B}{q_B}$.

All users with $\theta \in (\theta_{BP}, \theta_{PA})$ use an illegal version of product A. The users of the illegal version split the market into two sub-markets and to put it roughly, the illegal users recruit themselves from the middle part of the market. The profit function for each developer is then $\pi_A = \left(\bar{\theta} - \theta_{PA}\right) p_A = \left(\bar{\theta} - \frac{p_A - X}{q_A - \alpha q_A}\right) p_A$, and $\pi_B = \left(\theta_{BP} - \theta_{0B}\right) p_B = \left(\frac{X - p_B}{\alpha q_A - q_B} - \frac{p_B}{q_B}\right) p_B$. Equilibrium prices and profits are the following:

$$p_A^* = \frac{\bar{\theta} q_A (1-\alpha)}{2} + \frac{X}{2},\ p_B^* = \frac{q_B}{2\alpha q_A} X \tag{3}$$

$$\pi_A^* = \frac{1}{4} \frac{\left(\bar{\theta} q_A (1-\alpha) + X\right)^2}{q_A (1-\alpha)}, \pi_B^* = \frac{1}{4} \frac{q_B}{\alpha q_A} \frac{1}{\alpha q_A - q_B} X^2. \tag{4}$$

and resulting market coverage is the following:

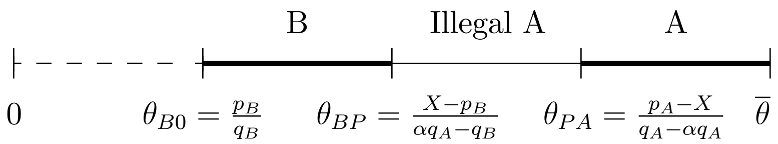


Figure 2: BC with illegal users in the middle of the market

Note that in the equilibrium, we have $\frac{p_B}{q_B} \leq \frac{X}{\alpha q_A} \leq \frac{p_A}{q_A}$ implying that $p_B^* < X < p_A^*$.

Lemma 1 *The necessary and sufficient condition with respect to X for piracy equilibrium (p1) to exist is:*

$$0 < X < X_{\alpha 1} = \frac{\bar{\theta} \alpha q_A (\alpha q_A - q_B)(1-\alpha)}{(2-\alpha)\alpha q_A - q_B}.$$

Proof. See *Supplementary Material.* ■

In this special case, only developer A has the incentive to choose service restriction in the second stage. Moreover, note that the developers do not directly compete against each other because users who are using product A illegally create a "buffer" between the legal users of products A and B. Thus, the profit of each developer is independent of the competitor's price and the driving factors of the profit are the level of the expected penalty X, and the level of restricted services α. Moreover, note that the market coverage, equilibrium price, and, consequently, profit of developer A are the same as if he were a monopolist constrained by $X \leq p_A^*$ (implying that $X \leq \frac{1}{2}\bar{\theta}\alpha q_A$, see sub-section 3.5).

Remark 1 *Developer A's decision to implement α and then set the price to $p_A = X$ is never optimal in the given set-up.*

Lemma 2 *In the case of duopoly competition when $\frac{p_B}{q_B} \leq \frac{X}{\alpha q_A} \leq \frac{p_A}{q_A}$ and $q_B < \alpha q_A$, the equilibrium profit and price of developer A as well as developer B are decreasing in α as long as $q_B \leq \alpha q_A$ holds.*

Proof. The behavior of $p_A^*(\alpha)$, $p_B^*(\alpha)$, $\pi_B^*(\alpha)$ can be seen immediately from equilibrium (3) and (4), proof that $d\pi_A^*(\alpha)/d\alpha < 0$ could be also easily derived. ■

Intuitively as α decreases, the illegal usage becomes more costly and consequently shrinks. Since both developers compete directly only with an illegal version of product A, this improves their competitive advantage by making legally accessible quality more attractive compared to the illegal one allowing in turn both prices to increase in equilibrium. Finally it is straightforward to show that the developers' profit increase in X at the relevant range $0 < X < X_{\alpha 1}$.

4.1.2 Case 2: Piracy Duopoly (*p2*) when $q_B > \alpha q_A$

Note that in this set-up, developer B would also be forced to introduce the IPR protection of, say α_B, in order to stay in the market. Otherwise the users who do not buy a legal version of product A, would prefer to use the illegal version of product B, whose quality would be $q_B > \alpha q_A$. As a consequence of IPR implementation by both developers, there would be a direct competition between the two developers, but their payoffs depend on the level of X and the developer's A IPR protection[23], α. A user indifferent between A and B is $\theta_{BA} = \frac{p_A - p_B}{q_A - q_B}$, and a user indifferent between illegal usage of A and buying B is $\theta_{PB} = \frac{p_B - X}{q_B - \alpha q_A}$. Users with $\theta \in \left(\frac{X}{\alpha q_A}, \frac{p_B - X}{q_B - \alpha q_A} \right)$ use an illegal version of product A. The profits for developers are: $\pi_A = \left(\bar{\theta} - \theta_{BA}\right) p_A$ and $\pi_B = \left(\theta_{BA} - \theta_{PB}\right) p_B$. This situation leads to the following distribution on the market:

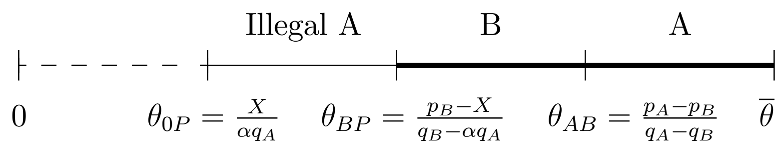


Figure 3: BC with illegal users at the low end of the market

This results in the following equilibrium prices and profits:

$$p_A^* = (q_A - q_B) \frac{\bar{\theta} 2 q_A (1 - \alpha) + X}{4q_A - 3\alpha q_A - q_B}, \; p_B^* = (q_A - q_B) \frac{\bar{\theta} (q_B - \alpha q_A) + 2X}{4q_A - 3\alpha q_A - q_B}. \tag{5}$$

$$\begin{aligned} \pi_A^* &= (q_A - q_B) \left(\frac{2\bar{\theta} q_A (1 - \alpha) + X}{4q_A - 3\alpha q_A - q_B} \right)^2, \\ \pi_B^* &= (q_A - q_B)(1 - \alpha) q_A \frac{\left(\bar{\theta}(q_B - \alpha q_A) + 2X\right)^2}{(q_B - \alpha q_A)(4q_A - 3\alpha q_A - q_B)^2}. \end{aligned} \tag{6}$$

Note that in the equilibrium, we have $\frac{X}{\alpha q_A} \leq \frac{p_B}{q_B} \leq \frac{p_A}{q_A}$ implying that $X < p_B^* < p_A^*$.

Lemma 3 *A necessary and sufficient condition for the existence of piracy equilibrium (p2) is satisfied only for X and α such that:*

$$0 \leq X < X_{\alpha 2} = \frac{\bar{\theta}(q_A - q_B)(q_B - \alpha q_A)\alpha q_A}{4q_A q_B - q_B^2 - 2\alpha q_A^2 - \alpha q_A q_B}.$$

Proof. See *Supplementary Material.* ■

Lemma 4 *The equilibrium profit and price of developer A as well as developer B are decreasing in α when $q_B > \alpha q_A$ holds.*

[23] We assume that $\alpha_B q_B < \alpha q_A$. It is straightforward to figure out the outcome when the opposite is true but it brings no new insights.

Recall that the competition in Case 2 is tougher than in Case 1 since developers now compete directly with each other, and the increase in market share of one developer automatically implies a decline in the share of the other developer. Again it is straightforward to show that the developers' profit increases in X at the relevant range $0 < X < X_{\alpha 2}$.

4.1.3 Non-piracy Duopoly Equilibria

If X exceeds the applicable threshold $X_{\alpha 1}$ or $X_{\alpha 2}$, then it turns out that the illegal product has zero market share in equilibrium, and two cases are possible.

- **Constrained Non-piracy Duopoly (*d1* and *d2*)**: $D_P = 0$ but the developers' prices depend on X.
- **Unconstrained Non-piracy Duopoly(*D*)**: $D_P = 0$, and the developers' prices do not depend on X and are the same as under the classical Bertrand competition (see Graph 1):

$$p_A = \frac{2\bar{\theta} q_A (q_A - q_B)}{4q_A - q_B}, \quad p_B = \frac{\bar{\theta} q_B (q_A - q_B)}{4q_A - q_B}.$$

It can be shown that unconstrained non-piracy equilibria occur in case 1 for

$$X > X_{\alpha 1+} = \frac{\bar{\theta}(2\alpha q_A - (1+\alpha) q_B) q_A}{4q_A - q_B},$$

and in case 2 for

$$X > X_{\alpha 2+} = \frac{\bar{\theta}(q_A - q_B)\alpha q_A}{4q_A - q_B},$$

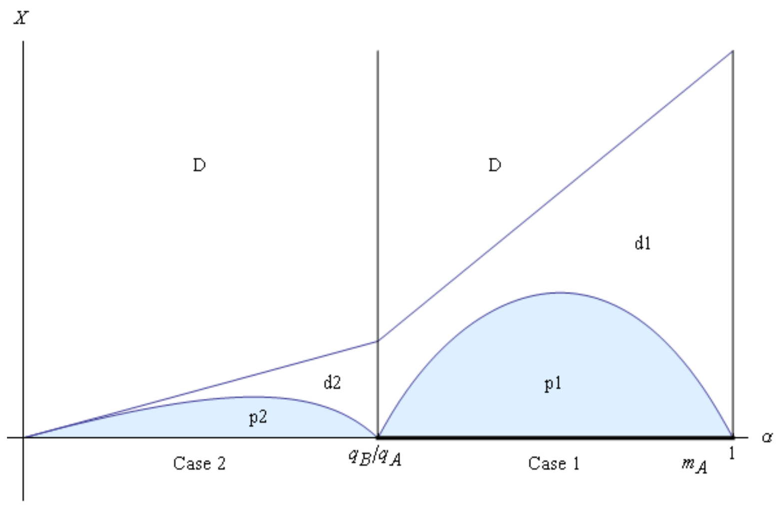
and that the intermediate values of X lead to constrained non-piracy equilibria. In case 2 (see the area d2 in Graph 1), such equilibria are characterized by

$$p_A = \frac{1}{2}\left(X \frac{q_B}{\alpha q_A} + \bar{\theta}(q_A - q_B)\right), \quad p_B = X \frac{q_B}{\alpha q_A}.$$

However, in Case 1 constrained non-piracy equilibria are non-unique as there is a convex and compact set of price pairs (p_A, p_B) such that $\theta_{BP} = \theta_{PA}$ and both developers' profits are maximized ($\partial\Pi/\partial p \geq 0$ at $p-0$, $\partial\Pi/\partial p \leq 0$ at $p+0$) given the other developer's price (see the area d1 in Graph 1). See Appendix I and *Supplementary Material* for details.

From the above analysis, it follows that $X_{\alpha 1} = 0$ when $\alpha = 1$ or $q_B = \alpha q_A$, and $X_{\alpha 1} > 0$ between these limiting values. Similarly, $X_{\alpha 2} = 0$ when $\alpha = 0$ or $q_B = \alpha q_A$, and $X_{\alpha 2} > 0$ between these limiting values. However, both $X_{\alpha 1+}$ and $X_{\alpha 2+}$ strictly increase in α and that $X_{\alpha 1+} = X_{\alpha 2+}$ when $q_B = \alpha q_A$. Thus, while the task of simply driving the pirate product out of the market is easiest when the pirate product is close in quality to one of the legal products or to zero, the task of completely removing the influence of the pirate product becomes increasingly difficult as its quality increases.

Having discussed all the market structures that may appear in the last-stage game equilibrium, we picture them in the Graph 1 below.



Graph 1

Note that the upper borders of the piracy regions, $X_{\alpha 1}$ and $X_{\alpha 2}$ are not monotonic in α. The reason for that is the nature of price competition where the distance between the qualities matter. To see that, let us first start from the point where $X = 0$ and $\alpha = 1$ and describe the competitive forces around this point. For developer A to be in the market it is necessary that he introduces a little bit of private protection (i.e., sets α slightly below unit) in order to capture some consumers with the highest valuation for quality at the top-end of the market. For developer B, (whose quality is lower than the pirated version of product A) to be active in the market, it is necessary for public protection to be strictly positive.

For α slightly below unit, the high quality product A competes with its illegal version, which is a very close substitute to the legal version at this point, so the price competition is intense and therefore a rather small X, ("the price" of the illegal version) is enough to eliminate the pirated product from the market. As α decreases, the substitutability becomes poorer and, consequently, the competition softer and so a larger X is needed to eliminate the piracy. This explains the falling part of $X_{\alpha 1}$ in Graph 1. To explain the increasing part of $X_{\alpha 1}$, note that beyond a certain point (to the left of maximum of $X_{\alpha 1}$), the illegal version starts to become a closer substitute to the product B as α goes towards $\frac{q_B}{q_A}$ and so lower X is needed to eliminate the illegal version. At the value $\alpha = \frac{q_B}{q_A}$, product B and the illegal version are perfect substitutes, so for X even slightly above zero, piracy is eliminated. The analogous logic applies to the shape of $X_{\alpha 2}$. Finally, when $X = 0$ and there is Case 1, then the ensuing market structure is the constrained monopoly of the developer A that we label as m_A.

4.2 The second stage - the optimal firms' service restrictions

The optimal service restriction is rather simple in our set-up given the assumption of no costs for restraining services. (Recall that profit functions in both Cases 1 and 2 decrease in α.) Thus, the optimal service restriction is always such that $\alpha^* = \underline{\alpha}$ irrespective of the level of X (provided that the size of X is such that it requires the imposition of a service restriction by at least one developer, that is, $X < p_A$). What is more interesting here is to see how X and the optimal α affect the emerging market structure and market coverage in the second stage equilibrium. We start with the buffer case: Case 1.

4.2.1 $\alpha^* = \underline{\alpha}$, and $p_B < X < X_{\alpha 1}$

This case appears in equilibrium when $\alpha^* q_A > q_B$, and this is typically the case when the quality of the first developer is "substantially" larger than the quality of the second developer. The interesting (comparative static) question to ask here is what would happen if the regulator increases X to be at $X_{\alpha 1}$ or larger. If X exceeds $X_{\alpha 1}$, then piracy becomes too costly and, consequently, the buffer of illegal users is completely eliminated (that is,

$\underline{\alpha} q_A \bar{\theta} - X < 0$ for all θ). Thus an expected penalty high enough (such that $X > X_{\alpha 1}$) coupled with the private protection $\alpha^* = \underline{\alpha}$ is able to restore the competition without illegal users. The intuition is that the usage of an illegal version becomes non-attractive when α falls so low that a legal version of product B has the same (or only slightly lower) value for consumers but is offered at a lower price, $p_B < X$. In other words, when $\underline{\alpha} q_A$ approaches to q_B the required public protection $X_{\alpha 1}$ to eliminate the piracy decreases and collapses to zero in the limiting case when $\underline{\alpha} q_A = q_B$ (see Graph 1). Moreover, a pair of private and government protections $\{\underline{\alpha}, X_{\alpha 1+}\}$restores pure Bertrand competition. Without private protection, the regulator would have to set a much higher expected penalty to achieve the same outcome (see Graph 1).

4.2.2 $\alpha^* = \underline{\alpha}$, and $X < p_B < X_{\alpha 2}$

Clearly, this situation appears when the difference between the qualities is not large, so that $\underline{\alpha} q_A < q_B$, and there is direct duopoly competition (Case 2 above) in which illegal usage occurs only at the lowest tail of the market. Note that it would now be optimal for both developers to introduce service restrictions. Moreover, both developers choose the technically minimal possible α, i.e., $\alpha^* = \alpha_B^* = \underline{\alpha}$. Let us assume again that the regulator sets the expected penalty that exceeds the critical value for Case 2 to occur, (that is, $X > X_{\alpha 2}$). In that case, $\frac{X}{\alpha q_A} > \frac{p_B}{q_B}$ (or $\theta_{0P} > \theta_{0B}$) implying that no one would use an illegal version. Thus, in this set-up too, a pair of $(\underline{\alpha}, X_{\alpha 2+})$ restores a pure Bertrand competition and like in the case of 4.2.1 above, $X_{\alpha 2}$ is substantially lower than the expected penalty that would alone achieve complete elimination of illegal usage. Much like in the 4.2.1 , when $\underline{\alpha} q_A$ approaches (now from below) to q_B, the required public protection $X_{\alpha 2}$ to eliminate the piracy falls and reaches zero when $\underline{\alpha} q_A = q_B$.

Note that in the 4.2.2 set-up, developer A may in some situations have an option to set service restriction at, say, $\alpha_1 > \underline{\alpha}$ such that $\alpha_1 q_A > q_B$ (while $\underline{\alpha} q_A < q_B$). Thus, the developer may opt to be in 4.2.1 and has to compare the profits from the two cases. It turns out, however, that for all feasible α and X, the optimal profit in 4.2.2 is bigger than its counterpart in 4.2.1 that is, $\pi_{A2}^*(\underline{\alpha}) \geq \pi_{A1}^*(\alpha_1)$ for all $\alpha \in [\underline{\alpha}, 1)$ and X such that $X \leq X_{\alpha 1} \vee$

$X \leq X_{\alpha 2}$ (where π^*_{Ai} stands for the profits in the two respected cases, $i = 1, 2)$[24].

Proposition 1: *The private and public IPR protections are substitutes in the sense that the presence of private protection enables the regulator to eliminate the illegal software usage with a lower (and less costly) expected penalty compared with the case when there is no private IPR protection.*

Proof: Both $X_{\alpha 1+}$ and $X_{\alpha 2+}$ evaluated at $\alpha = \underline{\alpha}$ are strictly increasing in $\underline{\alpha}$ indicating that a rise in the private protection (fall in $\underline{\alpha}$.) leads to fall in $X_{\alpha i+}$.

Finally, as a robustness check, we show in the Appendix II that the substitutability between the private and public IPR protection readily extends to the case of costly private protection.

5 The optimal public IPR protection and its interaction with private IPR protection

5.1 The first stage - the optimal IPR protection

Fully eliminating piracy might not necessarily be optimal from the social point of view. In this sense, Proposition 2 above is not a normative but rather positive statement that answers the question of what the effect would be of private IPR protection on its public counterpart if the aim is to fully eliminate piracy. In this section we characterize the socially optimal public IPR protection and tackle a different question: how the presence/absence of private IPR protection affects socially optimal IPR protection, X^*. Thus, we start with the issue of the optimal IPR protection in the absence of private IPR protection. Note that it is optimal to completely give up on public IPR protection when there is no private protection, that is,

[24]Namely, the equilibrium profit π^*_A decreases in α for piracy equilibria in Case 1, and for piracy and constrained non-piracy equilibria in Case 2. For the constrained non-piracy equilibria in Case 1, where there is equilibrium multiplicity, it can be shown that both maximum and minimum π^*_A across the set of multiple equilibria at given X decrease in α. In addition, it is shown in *Supplementary Material* that the equilibrium in the model (viewed as a correspondence) is continuous in X and α, i.e., there is no discontinuity either between different equilibrium structures in Case 1 and Case 2 or between Case 1 and Case 2 at $\alpha = q_B/q_A$. This completes the proof that developer A will always prefer α to be as low as possible.

$X^* = 0$ in this case. The reason is well known; there are zero marginal production costs for producing a unit of a (low or high) quality good. In addition, there are no restrictions on accompanied services and, consequently, no decrease in the perceived values of the product given the absence of private IPR protection.

The situation, however, changes dramatically, as we will see, when the developers introduce private protection. Thus, we first focus on solving the first stage of the game and on characterizing the optimal X^* in the presence of private IPR protection. We also continue to assume that private protection is not costly. So while private protection is endogenously chosen, it becomes de facto exogenous as the outcome of the developers' choice is $\alpha = \underline{\alpha}$ regardless of X.

The government's objective function is social welfare $W(X)$, which is comprised of the following components.

- Consumer surplus for each of the products present in the market, that is, $CS_A(X)$, $CS_B(X)$ and $CS_P(X)$. It follows from the above that both legal products (or, under monopoly, the only legal product) are present in equilibrium, whereas the illegal product may or may not be present.

- Expected penalty revenue $X \cdot D_P$, if the illegal product is present.

- Developers' profits, $\Pi_A(X)$ and $\Pi_B(X)$. Here it matters whether, and which of, the developers are domestic or foreign—only the domestic developers' profits, if any, are included.

- Protection costs. In general, implementing the expected penalty level X would cost $C(X)$. Recall that $C(0) = C'(0) = 0$ and $C'' \geq 0$.

  Thus for the benchmark case when both developers are domestic we have:

  $W(X) = CS_A(X) + CS_B(X) + CS_P(X) + \Pi_A(X) + \Pi_B(X) + X \cdot D_P - C(X)$ or $W(X) = G(X) - C(X)$

  where $G(X)$ stands for the gross social welfare, that is, $G(X) = CS_A(X) + CS_B(X) + CS_P(X) + \Pi_A(X) + \Pi_B(X) + X \cdot D_P$

In the first stage, the regulator chooses the public protection in the form of the expected penalty level X that maximizes the social welfare function, $W(X)$. The regulator faces the following short-run trade-off. On the one hand, lower values of X (such that the outcome is not unconstrained duopoly) increase market coverage and lower the social costs of IPR protection, so they have a positive impact on social welfare. On the other hand, low values of X lead to higher shares of low-quality goods, which decreases consumer surplus and firms' profits and thus social welfare.

It can be shown that in all possible situations the social welfare function is continuous[25] in X , and, since the outcome for X high enough, i.e, $X \geq X_{\alpha i+}$, is the classical Bertrand outcome (or the classical monopoly outcome), which does not depend on X, the welfare is bounded in X. Thus, social welfare attains a global maximum at $X = X^*$ such that $0 \leq X^* \leq X_{\alpha i+}$ for any cost function $C(\cdot)$ satisfying the restrictions above. It is clear that the optimal X when the public protection costs are $C(\cdot) > 0$ cannot exceed the optimal X when $C(X) \equiv 0$. For this reason, it seems insightful to characterize the optimal X when there are no protection costs, that is, when $C(X) \equiv 0$. Note that this would define the upper bound of the optimal public protection. We then make an inference of what would be the optimal solution in the general case when $C(X) > 0$. Thus, before moving to our benchmark case of two domestic developers (or domestic monopoly), we first briefly discuss the general properties of the optimal public IPR irrespective of the concrete market structure.

There are three situations to consider when $C(X) \equiv 0$:

- $X^* = 0$ If the optimal solution for $C(X) \equiv 0$ is $X^* = 0$, it stays like that under non-zero public protection costs, that is for $C(X) > 0$.

- $0 < X^* < X_{\alpha i}$ (piracy equilibrium) If the optimal solution for $C(X) \equiv 0$ is interior in the area where the illegal product is present, it stays like that since in this case $\partial W/\partial X = \partial G/\partial X > 0$ at $X = 0$ while $C'(0) = 0$ (unless W is maximized at $X = 0$).

- $X_{\alpha i} \leq X^* \leq X_{\alpha i+}$ (non-piracy equilibrium) If the optimal solution for $C(X) \equiv 0$ is such that the illegal product is not present ($X^* \geq X_{\alpha i}$), the equilibrium outcome

[25] As a correspondence, see *Supplementary Material.*

may or may not change under non-zero protection costs. In particular, if the optimal solution for $C(X) \equiv 0$ is a constrained non-piracy equilibrium, then it may remain a constrained non-piracy equilibrium or switch to a piracy equilibrium, and if the outcome at $C(X) \equiv 0$ is an unconstrained non-piracy equilibrium, then it remain an unconstrained non-piracy equilibrium or switch to either a constrained non-piracy equilibrium or to a piracy equilibrium (however, the outcome cannot switch to zero public protection in either case since $\partial W/\partial X = \partial G/\partial X > 0$ at $X = 0$).

Note that if $C(X) \equiv 0$ and the welfare is maximized at $X = X_{\alpha i+}$, then any $X \geq X_{\alpha i+}$ yields the same welfare value. However, $X > X_{\alpha i+}$ can be never optimal when $C(X)$ is non-zero. For this reason, we assume that if $C(X) \equiv 0$ and $X \geq X_{\alpha i+}$ is optimal, then $X = X_{\alpha i+}$ is chosen by the regulator.

The actual equilibrium outcome depends on the nature of the public protection costs, more specifically, on the steepness of the $C(X)$ function. For instance, if $C(X)$ increases very steeply in X then the optimal X may well be in piracy range even though the optimal X for for $C(X) \equiv 0$ would yield a non-piracy equilibrium.

5.1.1 Monopoly

Social welfare has an interior maximum in the range of X where the illegal product has a positive market share: the regulator can be said to use X to dilute the monopoly power to some extent. The welfare increases in α in all applicable cases: the better the “alternative” product, αq_A, the higher the social welfare.

5.1.2 Duopoly

In Case 1 ($q_B < \alpha q_A$), it turns out that the welfare has an interior local maximum (see Appendix III) in the piracy range, $0 < X^* < X_{\alpha 1}$, which is global if q_B is not too close to αq_A, i.e., if there is a sufficient quality difference between product B and illegal product A[26]. Thus, the socially optimal market structure in this case is a piracy duopoly (*p1*) - see

[26] When q_B is "very close" to αq_A , then the global optimum appears in the constrained non-piracy range $X_{\alpha 1} < X < X_{\alpha +}$ (see region *d1* in Graph 2) where there is multiplicity of price equilibria. We disucuss this case below.

Graph 2 below. The intuition beyond this outcome is that while the illegal product is not of the highest quality on the market, neither is it the worst one, so it is optimal to allow it in the market. Introducing the public protection at the margin enables the second developer to earn positive profit and, more importantly, increases the consumer surplus of legal users at the expense of the illegal users and brings (expected) revenue for the treasury resulting in $W'(0) = G'(0) > 0$.

Social welfare increases in α in the piracy range in all applicable cases for the same reason as under a monopoly; that is, the higher the quality of the illegal version due to an increase in α, the gain in consumer surplus exceeds the loss in profits of developer A, and so social welfare increases. Thus there is a conflict of interest between the regulator and the developers; while the regulator would prefer to have no private protection, the developer A prefers maximal private protection and the developer B also benefits from it.

In Case 2 ($q_B > \alpha q_A$), welfare is maximized at $X = X_{\alpha 2+}$ when $C(X) \equiv 0$, (see Appendix III) or when $C(X) > 0$ but the protection costs are not "very sensitive" to increase in X. The socially optimal outcome is then a classical Bertrand duopoly (D) - see Graph 2. The reason is that now the illegal product is the worst one, and a possible gain in market coverage due to its presence cannot offset the welfare lost due to other consumers switching to lower qualities. Of course, if increase in X causes protection cost to rise rather steeply, then $X_{\alpha 2} \leq X^* < X_{\alpha 2+}$ or there is an interior (piracy) optimum X^*such that $0 < X^* < X_{\alpha 2}$.

Contrary to the results in Case 1 and also in the related monopoly literature (see, for instance, Bae and Choi, 2006), there is no conflict of interest between the regulator and the developers since now social welfare decreases in α for $X < X_{\alpha 2+}$. The lower α, the easier it is for the legal products to compete with the inferior illegal product; in other words, the higher α, the more difficult it is to drive the illegal product out of the market and, since this is the product of the lowest quality, its presence is harmful for social welfare.

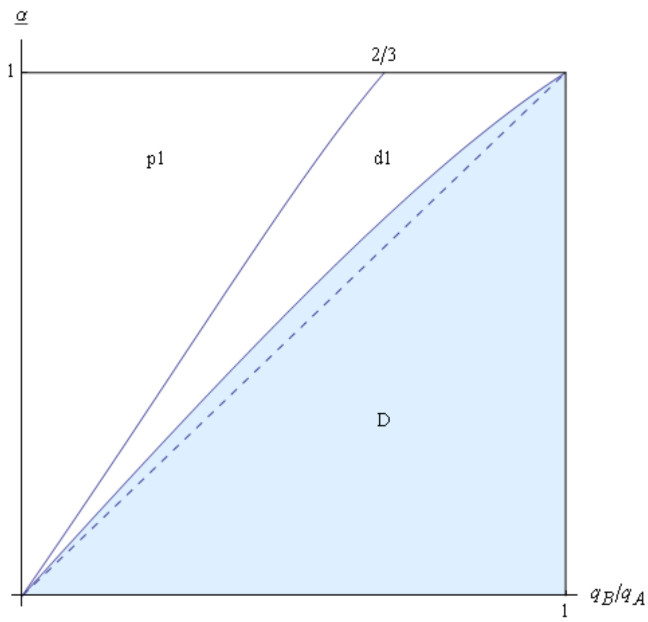
We summarize the above findings in our next proposition:

Proposition 2

There is conflict of interest in Case 1 where the regulator prefers no private (and, conse-

quently) no public protection while the developers prefer the strictest private IPR protection and both developers would like the regulator to set the highest possible public protection. In Case 2, however, the interests of the developers and the regulator are aligned because all of the actors payoff increase with the stricter private protection and with shrinking or eliminating of the lowest quality piracy product.

Finally, our analysis yields the testable hypothesis that the regulator would be less willing to tolerate piracy if the illegal product is of the lowest quality, and would completely eliminate the piracy (and its influence on prices and profits) if the costs of doing that are not "too large".



Graph 2

We conclude the subsection with the graphical presentation of the parameter space of the possible equilibria. The Graph 2 gives the resulting equilibrium market structures of the whole game (provided that $C(\cdot) = 0$ or "slowly" increasing). Note that the upper left triangle (above the 45°dashed line) describes Case 1 as $\alpha > q_B/q_A$ there, and, as we saw, the socially optimal market structure is a piracy duopoly (*p1*) given that i) both $\underline{\alpha}$ and ii) the difference in qualities of the two legal products are "large enough " (or equivalently, $q_B/q_A < 2/3$). The above requirements i) and ii) prevent αq_A to be "close" in value to the low quality product, q_B. When this is not the case (that is, when αq_A is "close" in value to q_B), then the maximum welfare is reached in the constrained non-piracy duopoly region (*d1*) that is located between the 45°dashed line and the lower bound of the piracy region (*p1*). Recall that there are multiple price equilibria in this region, so the regulator cannot precisely anticipate the set of equilibrium prices that the developers would pick in the last stage and the ensuing social welfare will not be under the full control of the regulator. In other words, for the given $\underline{\alpha}$ and for the given X from the constrained non-piracy duopoly (*d1*) interval (that is, $X \in (\ X_{\alpha 1}, X_{\alpha 1+}))$ there would be a range of social welfare values whose magnitudes depend on the chosen pair of equilibrium prices. Nevertheless, it could be shown that the highest possible value of social welfare (supremum) in (*d1)* region (obtained at some X^* such that $X^* \in (X_{\alpha 1}, X_{\alpha 1+}))$ exceeds the maximal social welfare obtained in the piracy regime (*p1*) - see Appendix III[27]. Given that social welfare is a correspondence at the interval $X^* \in (X_{\alpha 1}, X_{\alpha 1+})$, the issue then becomes how the other, lower values of social welfare that are *a priori* equally as likely as the maximal one (associated with the same X^*) compare with the maximal social welfare in the piracy regime. It would be rather hard to do this ranking for each and every value in this set but it would be, instead, interesting and insightful to look at the worst scenario, that is, to look at the lower bounds of each set (infima) for all $X^* \in (X_{\alpha 1}, X_{\alpha 1+})$, and then pick the maximal value among these minimal values. It turns out that the maximal value out of all the lowest values of social welfare is reached at $X_{\alpha 1+}$ when the parameters are in the small region above the dashed line but below the solid line within the (d1) region-see Graph 2. For all other parameter configurations in the (d1) region, the maximal value out of all lowest value of the social welfare is reached at $X_{\alpha 1}$ but maximal social welfare in the piracy regime dominates these values (see Appendix III). Thus if we accepted this "maximin" approach, the regulator would be able to circumvent the problem of multiple equilibria since then the regulator would allow piracy for the most of the *(d1)* region (that is, $X^* \in (0, X_{\alpha 1})$ and, for the small chunk of the parameter space

[27] Recall that the regulator has the option to choose among regimes by setting the size of the expected penalty, and, in the considered situation, it could (for the given α) choose X^* to be in the piracy regime or choose X^* to be in the *d1* regime. In the latter case, however, social welfare is a correspondence so there is a whole range of values associated with X^*.

(just above the 45° dashed line), it would completely eradicate piracy and set $X^* = X_{\alpha 1+}$.

As for the lower right triangle (below the 45°dashed line), it depicts the Case 2 (that is, $\alpha < q_B/q_A$), and the optimal market structure is, as we know, a pure duopoly. Finally, note that if $C(\cdot)$ is "large" and steeply increasing ($C''(\cdot) >> 0$), then the prevailing optimal market structure will be piracy duopoly (both *p1* and *p2*).

We summarize the above findings in the following proposition:

Proposition 3

The equilibrium market structure is always a duopoly. In Case 1 when αq_A and q_B are not "too close" (region p1), there is piracy duopoly while in the Case 2, it is unconstrained duopoly (given zero or low protection costs). Finally, for αq_A and q_B being "too close", the highest possible social welfare could be obtained in (d1) region under the constrained non-piracy duopoly. In this case, however, the regulator cannot fully control the level of social welfare (due to the multiple price equilibria) but can pick the maximal social welfare out of all the smallest realizations of the social welfare. The social welfare maximizing market structure would then be again either piracy duopoly (the upper part of d1 region) or unconstrained duopoly (small region in d1 above the dashed line).

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5.2 The interaction between the optimal private and public IPR protections

After having characterized both the general properties of the optimal public IPR protection, and the particular (benchmark) case of domestic duopoly (and monopoly), we are ready to move to our key question: the interaction between the private and public IPR at the social optimum. Now we can ask what would happen with the optimal public IPR protection if we allow for the private IPR protection at the margin. Technically, this would mean that $\underline{\alpha}$ drops beneath unit by some infinitesimal amount. It turns out the very appearance of small optimal private protection induces positive public protection. We have noted this fact in our last proposition:

Proposition 4

The very appearance of private IPR protection (that is, $\underline{\alpha}$ close to 1) leads (at the margin) to the positive public IPR protection indicating that public and private IPR protections are complements at that point.

Proof: Given $W(X) = G(X) - C(X)$ and $\frac{\partial W}{\partial X} = 0$, by the implicit function theorem $\frac{dX^*}{d\underline{\alpha}} = -\frac{\frac{\partial G^2(X^*)}{\partial X \partial \underline{\alpha}}}{G''(X^*) - C''(X^*)}$ as $C(X)$ does not depend on α. By the second-order condition, $G''(X^*) - C''(X^*) < 0$, and it can be easily shown that $\partial G'(X^*) / \partial \underline{\alpha} < 0$ at $\underline{\alpha} = 1$. Thus, $\frac{dX^*}{d\underline{\alpha}}(\underline{\alpha} = 1) < 0$.

In particular, it is optimal to have no public protection when the pirate product is as good as the highest-quality legal product in the market ($\underline{\alpha} = 1$), but it becomes optimal to introduce positive public protection as soon as $\underline{\alpha} < 1$, because the pirate product becomes, if slightly, inferior to the legal product A. Note that at our point of departure of $\underline{\alpha} = 1$, the only relevant case to consider is Case 1 since $q_B < \alpha q_A$ is the only possibility there. In fact, for X^* interior in the piracy range, $0 < X^* < X_{\alpha 1}$ the optimal solution $X^*(\underline{\alpha})$ can be shown to be concave in $\underline{\alpha}$ with a maximum at some $2/3 \leq \underline{\alpha} < 1$, with $X^* = 0$ when $\underline{\alpha} = 1$.

Thus, for very strong private IPR protection (low $\underline{\alpha}$) private and public IPR protection are again substitutes. The same is always true in Case 2 since it is immediately evident that the optimal solution, $X_{\alpha 2+}$, increases in $\underline{\alpha}$.

Finally, the corollary of Proposition 4 is that it would be socially optimal to give up the public protection in the absence of the private IPR protection (that is, $X^* = 0$ for $\underline{\alpha} = 1$) or alternatively, the zero public IPR protection ceases to be optimal when there is a private IPR protection.

6 Conclusion

In this paper we have studied the economic interaction between two instances of IPR protection in a software market and its positive and normative implications. The first instance is associated with the level of public protection that comes in the form of an expected penalty for violating IPR. The second instance represents the private IPR protection at the level of the developer that appears in the form of restricting additional consumer services for the illegal users. This form of protection discourages illegal usage and makes it less attractive. Thus, we examine the market equilibria with the above form of IPR protections and their social welfare implications.

As for our positive analysis, we show that the expected penalty may affect both market coverage and the corresponding market equilibria in all considered set-ups. In the case of a high expected penalty, where no user has the incentive to use a product illegally, public IPR does not play any role, no matter whether the developers use private IPR protection or not. In the case of medium and low levels of an expected penalty when developers implement private IPR protection, the resulting effect of the expected penalty crucially depends on the framework under consideration. Thus, we show that the illegal users of the product may recruit themselves either from price sensitive users (the low-end of the market) or from the middle part of the market. In the latter case, for instance, (Case 1), the illegal users create a "buffer" between the two groups of legal users, the one with the highest valuation for quality and the other with the lowest preference for quality. In this case, a marginal price change of one developer does not affect the profit of the other developer and, moreover, the high-quality developer generates the same profit as if he were a monopolist constrained only by the size of the expected penalty. In general, when firms protect their IPR by means of service restrictions, the expected penalty has an impact on market conduct and the developer's IPR protection only if it exceeds or goes below a certain threshold. Finally, we also show that private and public IPR protections act as substitutes for each other if the goal is to fully eliminate piracy. That is, introducing private protection enables the regulator to eliminate the illegal usage with a lower (and less costly) expected penalty.

As for the normative analysis, we also studied the welfare maximizing choice of public IPR protection and its economic impacts. We found out that in our benchmark case of both developers being domestic firms, the absence of public IPR protection ceases to be optimal (from the social point of view) when private IPR protection is present. Moreover, the appearance of private IPR protection at the margin leads to the positive public IPR protection in equilibrium indicating that public and private IPR protections are complements at that point. For a private IPR protection at the rather high level, however, the public and private IPR protections may act again as substitutes. The optimal value of the expected penalty is in general positive and depending on the case under consideration and properties of protection costs, may or may not be associated with the presence of the illegal goods in the market.

The equilibrium market structure is always a duopoly and depending on the parameter configurations, there is a piracy duopoly, unconstrained duopoly, or constrained non-piracy duopoly. In the latter case, however, the regulator cannot fully control the level of social welfare due to the multiple price equilibria, but using the heuristic approach ("maximin") can pick the maximal social welfare out of all the smallest realizations of the social welfare in this region. In this case, the welfare maximizing market structure would be, depending on the size of $\underline{\alpha}$ and q_B/q_A , either a piracy duopoly or an unconstrained duopoly.

Finally, the interests of the developers and the regulator are aligned in Case 2, because all of the actors' objective functions increase with stricter private protection and with the shrinking or eliminating of the lowest quality piracy product. In Case 1, however, the interests are opposite: the regulator prefers no private protection and would like to tolerate piracy (recall that zero private protection would be optimally matched with the zero public protection) while the high quality developer prefers the strictest private IPR protection which would benefit the low quality developer too, and both developers would like the regulator to set the public protection as high as possible.

It is important to stress that the primary role of public IPR protection in our set-up is not to restore incentives to invest in R&D and, consequently, generation of higher quality products (since qualities are assumed given in our set-up) but to ensure the presence of a larger share of legal products in the market, boosting the developers' profits and the consumer surplus of legal users, at the expense of the consumers surplus of illegal users.

Our framework could be easily applied to a more specific situation when one of (or both) developers are foreign. As mentioned in the footnote 26, a small Czech developer (firm B) created a word processor called "602 "and tried to compete with Microsoft office (firm A) in the local, Czech market. It is then straightforward to show that when A is a foreign developer, the social optimum is always zero public protection, irrespective of whether B is domestic or foreign and irrespective of whether we are in a Case 1 or Case 2 situation. The reason for this is that in Case 1, the large profit of domestic high quality firm A is essential for social welfare, so its absence cannot be compensated by the positive public protection that would bring about i) the tiny profit of domestic firm B offering the lowest quality, and ii) the government revenue generated from the penalized illegal users. So, it is optimal for the regulator to give up on public IPR protection (or to set minimally required public IPR protection). The situation in Case 2 is a slightly different but with the same outcome of zero public protection. Here the firms produce the two top quality products in the market so the absence of any of them would tilt the balance in favor of no public protection since the loss in profit of any of these firms cannot be compensated by any level of public IPR protection.

Finally, one of the possible modifications and robust checks of our model would be to analyze the interaction between the public and private IPR protection when the latter comes in the form of physical protection. This form (although a bit controversial) is still used to protect software by preventing the illegal users from having any value of a pirated good (this would be equivalent to setting α to zero in our set-up) but, on the other hand, there is no perfect IPR protection so a certain fraction of the skilled users would be always able to circumvent such a protection. So the issue at stake now would be how public protection affects costly private physical protection and what would be the social welfare implications thereof.

APPENDICES

1 Appendix I

1.1 General notes

Most of the calculations in this paper were performed using *Mathematica* and other similar software. The *Mathematica* file is available upon request.

In almost all model situations here, profit functions are concave (quadratic, or, in singular cases, linear) in the respective choice variables, so that an interior solution is always a (local) maximum. In the remaining situations, profit functions are explicitly assumed to be concave in the main text. Thus, second-order conditions always hold in equilibrium, so they are omitted everywhere below.

1.2 Indifferent users

As usual, the notation θ_{YZ}, where Y and Z can be one of $\{0, A, B, I\}$ implies that the users with $\theta < \theta_{YZ}$ strictly prefer Y to Z, and the users with $\theta > \theta_{YZ}$ strictly prefer Z to Y. Throughout this appendix, "product P" refers to the illegal version of product A.

As in the basic model, for thresholds not involving the illegal version of product A,

$$\theta_{0A} = \frac{p_A}{q_A},\ \theta_{0B} = \frac{p_B}{q_B},\ \theta_{BA} = \frac{p_A - p_B}{q_A - q_B}.$$

For thresholds involving product P but not involving product B,

$$\theta_{0P} = \frac{X}{\alpha q_A},\ \theta_{PA} = \frac{p_A - X}{q_A - \alpha q_A}.$$

As for the threshold between B and P, two cases have to be distinguished. First, the quality reduction to illegal users can be relatively low so that P is still better than B, i.e., $q_B < \alpha q_A$. Second, the quality reduction to illegal users can be relatively high so that illegal A becomes worse than B, i.e., $q_B > \alpha q_A$. (If $q_B = \alpha q_A$, then it is impossible that both B and P are in the market, and we concentrate on the cases where all three products are present.) In the first case, users with their sensitivity below the threshold use B whereas those above use P, so we use notation θ_{BP}. In the second case, the situation is the opposite so we use notation θ_{PB}. These are equal to

$$\theta_{BP} = \frac{X - p_B}{\alpha q_A - q_B},\ \theta_{PB} = \frac{p_B - X}{q_B - \alpha q_A}.$$

(Mathematically, these two are identical.)

1.3 Monopoly

The relevant thresholds are θ_{0A}, θ_{0P}, and θ_{PA}. Two cases are possible. First, if $p_A \leq \frac{X}{\alpha}$, then $\theta_{PA} \leq \theta_{0A} \leq \theta_{0P}$ (equality holds everywhere or nowhere) so that P is out of the market and users buy either A or nothing. Second, if $p_A > \frac{X}{\alpha}$, then $\theta_{PA} > \theta_{0A} > \theta_{0P}$ so that both P and A are in the market as in Figure 1 .

The monopolist's profit can be shown to be unimodal, and three outcomes can be distinguished.

First, if $X \geq \frac{1}{2}\bar{\theta}\alpha q_A$, then the unconstrained monopoly price is such that the illegal product is ousted, so that

$$p_A^* = \frac{\bar{\theta} q_A}{2},\ \pi_A^* = \frac{\bar{\theta}^2 q_A}{4}.$$

Second, if $X < \bar{\theta}\alpha q_A \frac{(1-\alpha)}{2-\alpha}$, then both A and P are present so that

$$p_A^* = \frac{X + \bar{\theta} q_A (1-\alpha)}{2},\ \pi_A^* = \frac{1}{4}\frac{\left(X + q_A \bar{\theta}(1-\alpha)\right)^2}{q_A (1-\alpha)}.$$

Third, if $\bar{\theta}\alpha q_A \frac{(1-\alpha)}{2-\alpha} \leq X < \frac{1}{2}\bar{\theta}\alpha q_A$, then while the monopolist has to lower the price due to the possibility of illegal use, this illegal use is still eliminated at the optimum, namely

$$p_A^* = \frac{X}{\alpha},\ \pi_A^* = \frac{X}{\alpha}\left(\bar{\theta} - \frac{X}{\alpha q_A}\right).$$

1.4 Bertrand competition

See *Supplementary Material* for a complete mathematical treatment. The outcomes for all sub-cases except for the constrained non-piracy outcome in Case 1 can be found in the main text. As for the remaining sub-case, it is shown in *Supplementary Material* that in case 1, if $X_{\alpha 1} < X < X_{\alpha 1+}$ then a pair of prices (p_A, p_B) is a constrained non-piracy equilibrium if and only if $\theta_{BP} = \theta_{PA}$ and $p_{A-}(X) \leq p_A \leq p_{A+}(X)$, where the boundaries $p_{A-}(X)$ and $p_{A+}(X)$ are determined in the following way.

$$p_{A-}(X) = \begin{cases} p_{AL}, & X_{\alpha 1} < X \leq X_{\alpha 1L}, \\ p_{Al}, & X_{\alpha 1L} < X < X_{\alpha 1+}, \end{cases} \quad , \quad p_{A+}(X) = \begin{cases} p_{Ah}, & X_{\alpha 1} < X \leq X_{\alpha 1H}, \\ p_{AH}, & X_{\alpha 1H} < X < X_{\alpha 1+}, \end{cases} ,$$

where the symbols used are

$$p_{AL} = \frac{X + \bar{\theta}\left(q_A - \alpha q_A\right)}{2}, p_{Al} = 2X\frac{q_A - q_B}{2\alpha q_A - q_B - \alpha q_B},$$
$$p_{Ah} = X\frac{2\alpha q_A - q_B - \alpha q_B}{2\alpha\left(\alpha q_A - q_B\right)}, p_{AH} = \frac{\left(X + \bar{\theta}\left(q_A - \alpha q_A\right)\right)\left(q_A - q_B\right)}{2q_A - q_B - \alpha q_A},$$
$$X_{\alpha 1L} = \bar{\theta} q_A \frac{(1-\alpha)\left(2\alpha q_A - q_B - \alpha q_B\right)}{(4 - 2\alpha)\, q_A - (3 - \alpha)\, q_B},$$
$$X_{\alpha 1H} = \bar{\theta} q_A \frac{2\alpha\left(q_A - q_B\right)\left(\alpha q_A - q_B\right)}{4\alpha q_A^2 - (2 + 3\alpha)\, q_A q_B + q_B^2}.$$

2 Appendix II: The costly private protection

When it is costly for the developers to undertake private protections, then two basic situations arise: first, if the costs do not increase "too fast" with the degree of private protection, then there will again be a corner solution, $\alpha^* = \underline{\alpha}$, discussed in the main text. Second, if this is not the case, then it would be reasonable to argue that the private IPR cost function, $C(\alpha)$ is convex enough to generate an interior maximum. So we postulate that $C'(\alpha) < 0$ (recall that the costs increase as α falls) and that $C''(\alpha) > 0$. Finally, the optimal α can be again in the range of $[\underline{\alpha}, 1)$ or, alternatively (but less realistically) anywhere in the interval $(0, 1)$.

We now write the net profit function of developer A as a function of α in both cases $i = 1, 2$ as $\Pi_{Ai}^{N}(\alpha) = \Pi_{Ai}^{*} - C(\alpha)$ where the second-order condition $\frac{\partial^2}{\partial\alpha^2}\Pi_{Ai}^{N}(\alpha) < 0$ holds because of the assumption that $C(\alpha)$ is "convex enough" .

Differentiating the first-order condition $\frac{d\Pi_{Ai}^{N}(\alpha[X],X)}{d\alpha} \equiv 0$ with respect to X and solving for $\frac{d\alpha^*}{dX}$ we obtain that $\frac{d\alpha^*}{dX} = -\frac{\frac{\partial^2}{\partial\alpha\partial X}\Pi_{Ai}^{N}(\alpha)}{\frac{\partial^2}{\partial\alpha^2}\Pi_{Ai}^{N}(\alpha)} = sign[\frac{\partial^2}{\partial\alpha\partial X}\Pi_{Ai}^{N}(\alpha)]$.

It is straightforward to show that $\frac{\partial^2}{\partial\alpha\partial X}\Pi_{Ai}^{N}(\alpha) > 0$. For, instance, in the Case 1 where the only the developer A undertakes protection,

$\frac{\partial^2}{\partial\alpha\partial X}\Pi_{A1}^{N}(\alpha) = \frac{X}{2q_A(1-\alpha)^2} > 0$.

For the Case 2, the expression is a little bit more complicated where we write the cross-partial derivative of the producer's A profit as the product of two expressions: $\frac{\partial^2}{\partial\alpha\partial X}\Pi_{A2}^{N}(\alpha) = \frac{4q_A(q_A-q_B)}{(q_A(3\alpha-4)+q_B)^3} \times (-3X - \bar{\theta}(q_B + q_A(2-3\alpha))$. Note that both expressions are negative given the Case 2 requirement that $q_B > \alpha q_A$, yielding $\frac{\partial^2}{\partial\alpha\partial X}\Pi_{A2}^{N}(\alpha) > 0$.Using similar approach also yields $\frac{\partial^2}{\partial\alpha\partial X}\Pi_{B2}^{N}(\alpha) > 0$.

Thus, the private and the public IPR are substitutes even if private IPR protection is obtained as an interior maximum. The only difference compared with the situation of non-costly private IPR protection is that now the marginal change in X affects the optimal private IPR protection. That is, an increase in public IPR protection requires the softening (larger α^*) of the optimal private IPR protection.

3 Appendix III: Optimal Public Protection

3.1 Case 1

In the piracy range of case 1,

$$\frac{\partial W}{\partial X} = \frac{1}{4}\left(\bar{\theta} - X\left(\frac{3-2\alpha}{\alpha(1-\alpha)q_A} + \frac{1}{\alpha q_A - q_B}\right)\right).$$

This is positive at $X = 0$, so the socially optimal level of X is always positive as $C'(0) = 0$ even if $C(\cdot)$ is non-zero. In addition, $\partial W/\partial X < 0$ at $X = X_{\alpha 1}$, so welfare reaches an interior local maximum in the piracy range, which can be shown to be global when $\alpha q_A - q_B$ is not too small. If $C(\cdot) = 0$ or does not increases steeply, then $X^* = \frac{\bar{\theta} q_A (1-\alpha)\alpha(\alpha q_A - q_B)}{q_A(4-3\alpha)\alpha - q_B(3-2\alpha)}$.

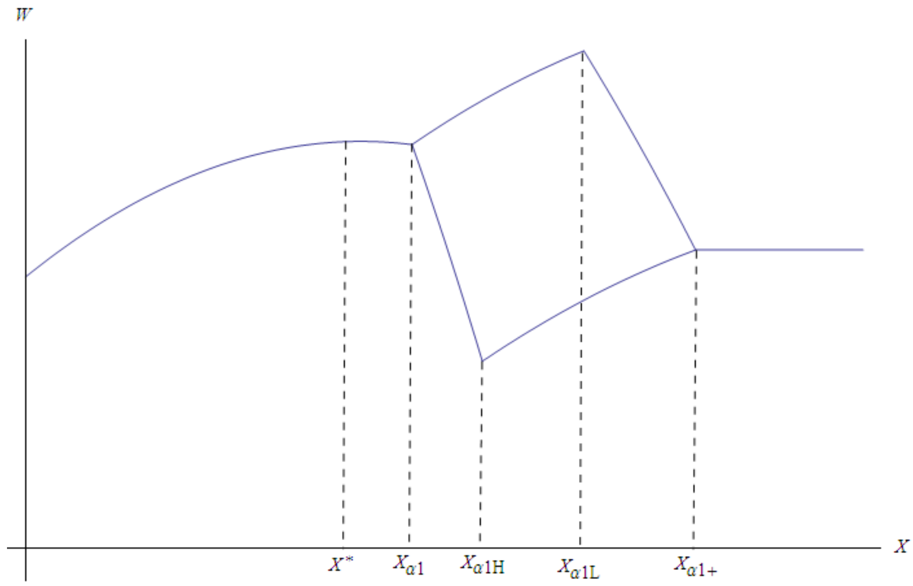
As regards the constrained non-piracy range, which features multiple price equilibria, the value of welfare for the given X is not unique either. However, since welfare is continuous (as a correspondence), it attains its maximum and minimum values for every X, $W_+(X)$ and $W_-(X)$ respectively.

- Recall that each of the multiple equilibria at the given X can be characterized by the value of $p_A \in [p_{A-}(X), p_{A+}(X)]$. It can be shown that equilibrium welfare decreases in p_A in this set for every X, i.e., $W_+(X)$ is reached at $p_A = p_{A-}(X)$ and $W_-(X)$ is reached at $p_A = p_{A+}(X)$.

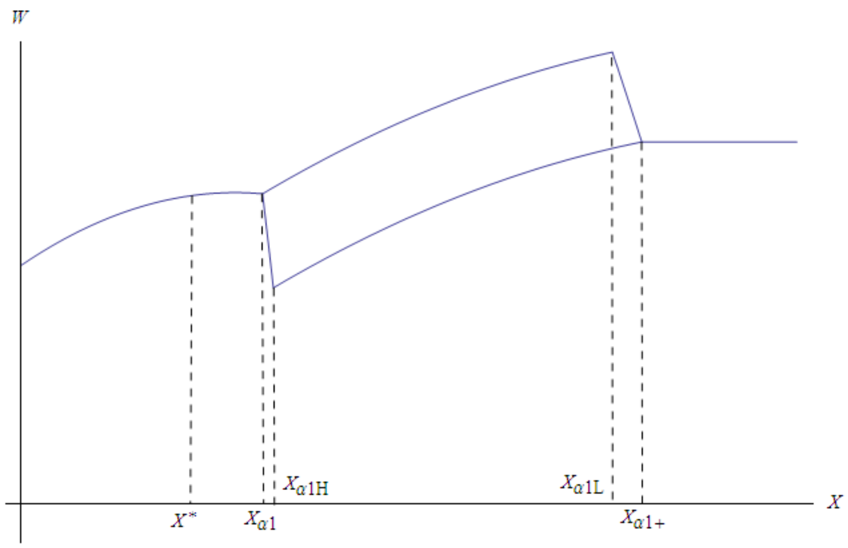
- The maximum welfare $W_+(X)$ increases in X for $X_{\alpha 1} < X < X_{\alpha 1L}$ and decreases in X for $X_{\alpha 1L} < X < X_{\alpha 1+}$ (see Graphs 1a and 1b below). Thus, the supremum welfare in the constrained non-piracy range, $\max_X \{W_+(X)\}$, is reached at $X = X_{\alpha 1L}$ and $p_A = p_{A-}(X_{\alpha 1L})$.

- The minimum welfare $W_-(X)$ decreases in X for $X_{\alpha 1} < X < X_{\alpha 1H}$ and increases in X for $X_{\alpha 1H} < X < X_{\alpha 1+}$. Thus, the maximal guaranteed social welfare ("maximin") in the constrained non-piracy range, $\max_X \{W_-(X)\}$, is reached either at $X = X_{\alpha 1}$ (Graph 1a) or at $X = X_{\alpha 1+}$ (Graph 1b). Note that at the boundaries $X_{\alpha 1}$ and $X_{\alpha 1+}$, the equilibria are unique and coincide with the equilibria in the piracy and unconstrained non-piracy cases, respectively.

Recall that the area (*p1*) denotes the case where the welfare maximum in the piracy regime exceeds $W_+(X_{\alpha 1L})$ (see Graph 2 in the main text), so piracy is unambiguously socially optimal there. In area (*d1*), however, the maximal achievable social welfare, $(W_+(X_{\alpha 1L}))$, is greater than the maximum welfare under piracy but there is no guarantee that this would be an actual outcome if the regulator selects protection level, $X_{\alpha 1L}$. An heuristic way to deal with this situation for the regulator is to choose X such that $\max_X W_-(X)$. It turns out that this approach would extend the piracy equilibrium to the most of the area (*d1*) since there $\max_X W_-(X) = W_-(X_{\alpha 1}) < W(X^*)$ where $X^* \in (0, X_{\alpha 1})$.The only region of $(d1)$ where the $\max_X W_-(X) = W_-(X_{\alpha 1+}) > W(X^*)$ is the tiny area above the 45° dashed line and below the lower bound of $(p1)$ region in Graph 2. Thus this approach results in the partition of $(d1)$ region into the piracy duopoly and unconstrained duopoly market structure.



Graph 1a



Graph 1b

3.2 Case 2

In the piracy range of case 2, $\partial W/\partial X = (\Omega_X X + \Omega_0)/\Omega_D$, where

$$\begin{aligned}\Omega_X &= q_A^3\alpha(12-11\alpha) - q_A^2 q_B(16-11\alpha+2\alpha^2) + q_A q_B^2(8-5\alpha) - q_B^3,\\ \Omega_0 &= 4\alpha(1-\alpha)q_A^2\left(q_A-q_B\right)\left(q_B-\alpha q_A\right)\bar{\theta},\\ \Omega_D &= \alpha q_A\left(q_B-\alpha q_A\right)\left(q_A(4-3\alpha)-q_B\right)^2.\end{aligned}$$

This is clearly positive at $X = 0$, linear in X, and can be shown to be positive at $X = X_{\alpha 2}$. Thus, welfare increases in X in the piracy range of case 2.

In the constrained non-piracy range of case 2,

$$\frac{\partial W}{\partial X} = \frac{q_B\left(\alpha q_A\left(q_A-q_B\right)\bar{\theta} - X\left(4q_A-3q_B\right)\right)}{4\alpha^2 q_A^2\left(q_A-q_B\right)}.$$

This is linear and decreasing in X, and can be shown to be positive at $X = X_{\alpha 2+}$. Thus, welfare increases in X in the constrained non-piracy range of case 2, which completes the proof that if $C(\cdot) \equiv 0$ in case 2, then welfare is maximized at $X = X_{\alpha 2+}$.

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