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WORKING PAPER SERIES (ISSN 1211-3298)
Electronic Version

522

Working Paper Series 522
(ISSN 1211-3298)

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Prague, December 2014

ISBN 978-80-7343-327-7 (Univerzita Karlova. Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-319-1 (Akademie věd České republiky. Národohospodářský ústav)

DYNAMICS OF CONSUMPTION AND DIVIDENDS OVER THE BUSINESS CYCLE[i]

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DECEMBER 2014

Abstract

We examine a trivariate time series model that is subject to a regime switch, where the shifts are governed by an unobserved, two-state variable that follows a Markov process. The analysis is performed in a Bayesian framework developed by Albert and Chib (1993), where the unobserved states are treated as missing data and then analyzed via Gibbs sampling. This approach generates the posterior conditional distribution of all the parameters given the hidden states, and the posterior conditional distribution of the states given the parameters. This allows us to obtain the estimated values of all the parameters of interest.

Abstract

Zkoumáme model trojrozměrných časových řad, který podléhá změně režimu, kde jsou změny řízeny pomocí nepozorované dvoustavové proměnné, která sleduje Markovův proces. Analýza je provedena v rámci Bayesovského přístupu vyvinutého Albertem a Chibem (1993), kde je s nepozorovanými stavy zacházeno jako s chybějícími údaji a poté jsou analyzovány pomocí Gibbsova vzorkováni. Tento přístup generuje posteriorní podmíněné rozdělení všech parametrů za daných skrytých stavů a take posteriorní podmíněné rozdělení stavů při daných parametrech. To nám umožňuje získat odhadnuté hodnoty všech parametrů, o které se zajímáme.

Keywords: Asset Pricing, Learning, Consumer Durable Goods, Economic Uncertainty, Business Cycles, Timing Premium

JEL: E13, E21, E27, E32, E37, E44, G12, G14

[i] This study was supported by Charles University Grant Agency (grant no. 578214) and the Czech Science Foundation (project no. P403/12/1394). Helpful comments from Michal Pakoš, Aleš Černý, Michal Kejak, Sergey Slobodyan, Liyousew Gebremedhin Borga, and Dejan Kovač are gratefully acknowledged. Remaining errors are the author's.

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1 Introduction

Economic and financial time series are prone to extraordinary changes in behaviour during periods of financial crises or rapid growth. A number of studies sought to investigate how these time series, such as the GNP, relate to the business cycle. The vast majority of the literature has explored this relationship using either the ARIMA model specification (Beveridge and Nelson, 1981; Nelson and Plosser, 1982; Campbell and Mankiw, 1987), or using the linear unobserved components model (Harvey, 1985; Watson, 1986; Clark, 1987). Other apply and make use of the methodology of Engle and Granger (1987). These ways of modeling, however, imply that the forecasts of variables are a linear function, that is, the optimal forecast of the parameters is a linear function of their lagged values.Hamilton (1989) proposed an alternative modeling of the first differences of time series, by allowing them to follow a nonlinear stationary process. He subjects this process to discrete shifts in regime - when the dynamic behaviour of time series is significantly different. He views the parameters of the time series as the outcome of a discrete-time Markov process, and the states are associated with the business cycle. Hamilton (1989) applies the technique to postwar US data on real GNP and argues that the business cycle is better characterised by shifts between a growth state and a recessionary state.

Albert and Chib (1993) adress the inference issues that arise in the analysis of the Hamilton (1989) type of models by using the simulation tool of Gibbs sampling. This tool (pioneered by Geman and Geman, 1984; Tanner and Wong, 1987; Gelfand et al., 1990; Gelfand and Smith, 1990; Tierney, 1994) has a number of attractive features for financial time series and has been used in many studies (Carlin et al., 1992; Chib, 1993; Chib and Greenberg, 1994; McCulloch and Tsay, 1994; Kim and Nelson, 1999, etc.). These features, for example, allow one to ommit the messy calculations of likelihood function and to obtain the full posterior distributions of all unknown parameters. Albert and Chib (1993) parametrise the mean and the variance of time series in terms of an unobserved state variable that follows a two-state Markov process with unknown transition probabilities. The novelty of their approach is in the fact that the unobserved states are treated as a missing data and are analysed along with other parameters of time series. Moreover, they are able to derive the full posterior distribution of the state variables conditional on all other unknown time series parameters.

In this paper, we analyze a trivarite time series model that includes nondurable goods, expenditure to durable goods, and dividends. The choice of these time series is specific to the DSGE model described in subsection 2.1, which we later use to derive the asset prices. In particular, the novelty of our approach is to model the expenditure to durable goods separately from the nondurable consumption. This way we have an extra term in the stochastic discount factor that magnifies its value (see, for example, Yogo (2006)). We extend the setting of Hamilton (1989) and model the growth rate of nondurable goods, growth rate of expenditure to durable goods, and growth rate of dividends, and we allow the predictable components and the instantaneous volatilities to jointly depend on the hidden two-state Markov chain. This allows us to derive the posterior beliefs about the hidden state and use it as a mechanism that drives the dynamics of asset prices in the economy. We closely follow the procedure of Albert and Chib (1993) and estimate all the parameters of the trivariate time series using the Gibbs sampling procedure. Unlike Albert and Chib (1993), we implement a more efficient approach and simulate the unobserved state variables jointly as a block (as in Kim and Nelson (1999)).

The time series we analyze share a common stochastic trend (see Pakoš, 2004). We extend the sample size of Pakoš (2004) and perform the cointegration test of Johansen to find the number of cointegrating vectors. We find that there indeed exists a single long-run relationship between the time series of interest.

The paper is organized as follows. Section 2 describes the DSGE model and the Gibbs-sampling approach in detail. Section 3 describes the data and performs the cointegration analysis. Section 4 states the results of Gibbs-sampling. Section 5 concludes.

2 A Bayesian Gibbs-Sampling Approach to Estimating Markov-Switching Models

2.1 Preferences and Endowments

Consider an infinitely-lived representative household framework. In each period t, the household purchases C_t units of non-durable consumption goods and I_{t+1} units of durable consumption goods. The non-durable consumption good C_t is non-sortable and is entirely consumed in period t, while the durable good can provide a service flow for more than one period. The household accumulates the stock of durable goods K_t according to the law of motion

$$K_{t+1} = (1-\delta)K_t + I_{t+1},$$

where $\delta \in (0,1)$ is the depreciation rate.

The household's inter-temporal utility is defined recursively as

$$U_t = \left\{(1-\beta)V_t^{\frac{1-\gamma}{\theta}} + \beta \left(\mathsf{E}\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}},$$

where

$$V_t = C_t^{1-\alpha} K_t^{\alpha}$$

is the household's intratemporal utility over the non-durable goods C_t and the service flows K_t from durable goods, with $0 < \alpha < 1$. The parameters of the utility function are the household's subjective discount factor $\beta \in (0, 1)$, the relative risk aversion coefficient $\gamma > 0$, and the EIS $\psi \geq 0$ with $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$.

The household has initial W_0 units of wealth. In every period t, the household invests $B_{i,t}$ units of wealth W_t in one of the $N+1$ available tradable assets in the economy, which realizes the gross rate of return $R_{i,t+1}$ in period $t+1$. The household's budget constraint in period t is given by

$$W_t - C_t - P_t I_t = \sum_{i=0}^{N} B_{i,t},$$

where P_t is the relative price of consumer durable goods in terms of non-durable goods. The $t+1$ period, wealth of the household, is given by

$$W_{t+1} = \sum_{i=0}^{N} B_{i,t} R_{i,t+1}.$$

2.2 Asset Markets and Dividends

We consider two asset classes: equities and bonds. For equities, we additionally distinguish between a non-durable good tree, a durable good tree, and levered equity. These two trees correspond to the standard Lucas tree with which the representative agent is endowed. These are the only assets in positive net supply, and we normalize both to one. The levered equity corresponds to the aggregate equity market, and it may be thought of as a levered consumption claim. For bonds, we consider only purely discount real bonds that pay zero coupons, $D_t^b = 0$. We denote the universe of assets $\mathcal{A} = \{n, d, l, b\}$, where c is the non-durable consumption tree, d represents a durable consumption tree, l is levered equity, and b is a purely discount real bond. We specify the dynamics of the cash flows D_t^a from the asset a (a non-durable consumption tree and levered equity) as a hidden Markov model in logs:

$$g_{t+1}^a \quad = \quad \mu_{S_t}^a + \sigma_{S_t}^a \varepsilon_{t+1}^a, \qquad a \in \{c, l\} \tag{2.1}$$

and the dynamics of the cash flow from the durable consumption tree as a hidden $AR(1)$ Markov model in logs:

$$g_{t+1}^e \quad = \quad \mu_{S_t}^e + \xi(g_t^e - \mu_{S_{t-1}}^e) + \sigma_{S_t}^e \varepsilon_{t+1}^e, \tag{2.2}$$

where

$$\begin{pmatrix} \varepsilon_t^c \\ \varepsilon_t^e \\ \varepsilon_t^l \end{pmatrix} \sim \mathcal{N}(0, I_3).$$

The model is an extension of Hamilton (1989) and Cecchetti et al. (1993). The instantaneous dividend volatilities $\sigma_{S_t}^a$ and the predictable components $\mu_{S_t}^a$ are driven by the common Markov chain S_t with the state space

$$\mathcal{S} = \{1 = \text{expansion}, 0 = \text{recession}\}.$$

The unobservability of the underlying state induces endogenously time-varying uncertainty due to inference problems. All dividend parameters are estimated using the Bayesian framework from postwar U.S. consumption and dividend data.

2.3 Estimation of Model with Markov-Switching Mean

Suppose

$$g_{t+1}^a = \mu_{S_t}^a + \sigma_{S_t}^a \varepsilon_{t+1}^a, \qquad a \in \mathcal{A}$$

where

$$\begin{pmatrix} \varepsilon_t^c \\ \varepsilon_t^e \\ \varepsilon_t^l \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right).$$

The instantaneous volatilities $\sigma_{S_t}^a$ and the predictable components $\mu_{S_t}^a$ are driven by the common Markov chain S_t with the state space

$$\mathcal{S} = \{1 = \text{expansion}, 0 = \text{recession}\}$$

and transition matrix

$$\begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix},$$

where p is the conditional probability of the process being in state 1 next period, given that it is in state 1 this period, that is

$$p = \mathsf{Pr}[S_{t+1} = 1 | S_t = 1]$$

and

$$q = \mathsf{Pr}[S_{t+1} = 0 | S_t = 0]$$

with $p, q \in [0, 1]$. Let

$$\mathbf{y}_t = \begin{pmatrix} g^c_{t+1} \\ g^e_{t+1} \\ g^l_{t+1} \end{pmatrix}, \quad \boldsymbol{\mu}_{S_t} = \begin{pmatrix} \mu^c_{S_t} \\ \mu^e_{S_t} \\ \mu^l_{S_t} \end{pmatrix}, \quad \boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon^c_t \\ \varepsilon^e_t \\ \varepsilon^l_t \end{pmatrix}, \quad \boldsymbol{\sigma}_{S_t} = \begin{pmatrix} \sigma^c_{S_t} & 0 & 0 \\ 0 & \sigma^d_{S_t} & 0 \\ 0 & 0 & \sigma^d_{S_t} \end{pmatrix}.$$

We can then rewrite the model as

$$\mathbf{y}_{t+1} = \boldsymbol{\mu}_{S_t} + \boldsymbol{\sigma}_{S_t}\boldsymbol{\varepsilon}_{t+1}.$$

The idea behind the Bayesian approach is as follows: along with the model's unknown parameters $\boldsymbol{\mu}_1, \boldsymbol{\mu}_0, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, p, q$ we consider S_t for $t = 1, 2, \ldots, T$ as random variables. We seek to derive the joint density

$$f(\tilde{S}_T, \boldsymbol{\mu}_1, \boldsymbol{\mu}_0, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, p, q|\tilde{\mathbf{y}}_T),$$

where $\tilde{S}_T = (S_1 \quad S_2 \quad \ldots \quad S_T)'$ and $\tilde{\mathbf{y}}_T = (\mathbf{y}_1 \quad \mathbf{y}_2 \quad \ldots \quad \mathbf{y}_T)'$. To derive the formula for f, we assume that p and q are independent of model parameters and data $\tilde{\mathbf{y}}_T$. Thus we have:

$$\begin{aligned} f(\tilde{S}_T, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, p, q|\tilde{\mathbf{y}}_T) &= f(\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, p, q|\tilde{S}_T, \tilde{\mathbf{y}}_T) f(\tilde{S}_T|\tilde{\mathbf{y}}_T) = \\ & f(\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, |\tilde{S}_T, \tilde{\mathbf{y}}_T) f(p, q|\tilde{S}_T, \tilde{\mathbf{y}}_T) f(\tilde{S}_T|\tilde{\mathbf{y}}_T) = \\ & f(\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, |\tilde{S}_T, \tilde{\mathbf{y}}_T) f(p, q|\tilde{S}_T) f(\tilde{S}_T|\tilde{\mathbf{y}}_T). \end{aligned}$$

To implement the Gibbs-sampling methodology we start with arbitrary starting values and carry-out the following steps until convergence:

(i) Generate $\tilde{S}_T$ from $f(\tilde{S}_T|\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, , p, q, \tilde{\mathbf{y}}_T.)$.

(ii) Generate the transition probabilities p and q from $f(p, q|\tilde{S}_T)$.

(iii) Generate $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2$ and $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0$, from $f(\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, |\tilde{S}_T, \tilde{\mathbf{y}}_T)$.

2.3.1 Generating $\tilde{S}_T$

We simulate S_t, $t = 1, \ldots, T$, as a block from the joint distribution:

$$f(\tilde{S}_T|\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0, p, q, \tilde{\mathbf{y}}_T).$$

Without loss of generality, we suppress the conditioning on the model's parameters and we derive the joint conditional density:

$$
\begin{aligned}
&f(\tilde{S}_T|\tilde{\mathbf{y}}_T) = \\
&= f(S_1, S_2, \ldots, S_T|\tilde{\mathbf{y}}_T) \\
&= f(S_T|\tilde{\mathbf{y}}_T) f(S_1, S_2, \ldots, S_{T-1}|S_T, \tilde{\mathbf{y}}_T) = \\
&= f(S_T|\tilde{\mathbf{y}}_T) f(S_{T-1}|S_T, \tilde{\mathbf{y}}_T) f(S_1, S_2, \ldots, S_{T-2}|S_T, S_{T-1}, \tilde{\mathbf{y}}_T) = \\
&= f(S_T|\tilde{\mathbf{y}}_T) f(S_{T-1}|S_T, \tilde{\mathbf{y}}_T) f(S_{T-2}|S_T, S_{T-1}, \tilde{\mathbf{y}}_T) \ldots f(S_1|S_T, S_{T-1}, \ldots, S_2, \tilde{\mathbf{y}}_T) = \\
&= f(S_T|\tilde{\mathbf{y}}_T) f(S_{T-1}|S_T, \tilde{\mathbf{y}}_{T-1}) f(S_{T-2}|S_{T-1}, \tilde{\mathbf{y}}_{T-2}) \ldots f(S_1|S_2, \tilde{\mathbf{y}}_1) = \\
&= f(S_T|\tilde{\mathbf{y}}_T) \prod_{t=1}^{T-1} f(S_t|S_{t+1}, \tilde{\mathbf{y}}_t).
\end{aligned}
$$

We further get

$$
\begin{aligned}
f(S_t|S_{t+1}, \tilde{\mathbf{y}}_t) &= \frac{f(S_t, S_{t+1}|\tilde{\mathbf{y}}_t)}{f(S_{t+1}|\tilde{\mathbf{y}}_t)} \\
&= \frac{f(S_{t+1}|S_t, \tilde{\mathbf{y}}_t) f(S_t|\tilde{\mathbf{y}}_t.)}{f(S_{t+1}|\tilde{\mathbf{y}}_t)} \\
&= \frac{f(S_{t+1}|S_t) f(S_t|\tilde{\mathbf{y}}_t)}{f(S_{t+1}|\tilde{\mathbf{y}}_t)} \\
&\propto f(S_{t+1}|S_t) f(S_t|\tilde{\mathbf{y}}_t).
\end{aligned}
$$

We get $f(S_t|\tilde{\mathbf{y}}_t)$ by running a Hamilton (1989) filter. Then we generate S_t in the following way. We first calculate

$$
\mathsf{Pr}[S_t = 1|S_{t+1}, \tilde{\mathbf{y}}_t] = \frac{f(S_{t+1}|S_t = 1) f(S_t = 1|\tilde{\mathbf{y}}_t)}{\sum_{j=1}^{2} f(S_{t+1}|S_t = j) f(S_t = j|\tilde{\mathbf{y}}_t)}
$$

After obtaining $\mathsf{Pr}[S_t = 1|S_{t+1}, \tilde{\mathbf{y}}_t]$, we generate a random number from the uniform distribution. If the generated number is less or equal than $\mathsf{Pr}[S_t = 1|S_{t+1}, \tilde{\mathbf{y}}_t]$, we set $S_t = 1$. Otherwise, S_t is set equal to 0.

2.3.2 Generating p and q

Assume that priors of p and q are independent beta distributions:

$$p \sim Beta(u_{11}, u_{10}),$$

$$q \sim Beta(u_{00}, u_{01}),$$

with

$$f(p,q) \propto p^{u_{11}-1}(1-p)^{u_{10}-1}q^{u_{00}-1}(1-q)^{u_{01}-1},$$

where $u_{ij},\ i,j = 1,0$ are assumed to be known hyperparameters of the priors. The likelihood function for p and q is given by

$$\mathcal{L}(p,q|\tilde{S}_T) = p^{n_{11}}(1-p)^{n_{10}}q^{n_{00}}(1-q)^{n_{01}},$$

where n_{ij} is the number of transitions from state i to j, which can be easily obtained given the simulated $\tilde{S}_T$. The posterior distribution is given by

$$f(p,q|\tilde{S}_T) = f(p,q)\mathcal{L}(p,q|\tilde{S}_T),$$

which implies

$$p|\tilde{S}_T \sim Beta(u_{11}+n_{11}, u_{10}+n_{10}),$$

$$q|\tilde{S}_T \sim Beta(u_{00}+n_{00}, u_{01}+n_{01}),$$

from which we draw p and q.

2.3.3 Generating $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_0$

First suppose that $\boldsymbol{\mu}_{S_t} = \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 S_t$. Substituting the above expression into

$$\mathbf{y}_{t+1} = \boldsymbol{\mu}_{S_t} + \boldsymbol{\varepsilon}_{t+1}$$

we get

$$\mathbf{y}_{t+1} = \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 S_t + \boldsymbol{\sigma}_{S_t}\boldsymbol{\varepsilon}_{t+1},$$

or

$$\mathbf{y}_{t+1} = \underbrace{\begin{pmatrix} 1 & S_t \end{pmatrix}}_{X_t} \underbrace{\begin{pmatrix} \boldsymbol{\mu}_0 \\ \boldsymbol{\mu}_1 \end{pmatrix}}_{\tilde{\boldsymbol{\mu}}} + \boldsymbol{\sigma}_{S_t}\boldsymbol{\varepsilon}_{t+1}.$$

Assume prior distribution for $\tilde{\boldsymbol{\mu}}$ given by

$$\tilde{\boldsymbol{\mu}}|\boldsymbol{\sigma}_{S_t} \sim \mathcal{N}(b_0, B_0)_{\mathbb{1}\{\boldsymbol{\mu}_1>0\}}.$$

Then the posterior distribution is given by

$$\tilde{\boldsymbol{\mu}}|\boldsymbol{\sigma}_{S_t}, \tilde{S}_T, \tilde{\mathbf{y}}_T \sim \mathcal{N}(b_1, B_1)_{\mathbb{1}\{\boldsymbol{\mu}_1>0\}},$$

with

$$b_1 = (B_0^{-1} + \boldsymbol{\sigma}_{S_t}^{-1} X_t' X_t)^{-1}(B_0^{-1} b_0 + \boldsymbol{\sigma}_{S_t}^{-1} X_t' \mathbf{y}_{t+1}), \quad B_1 = (B_0^{-1} + X_t' X_t)^{-1}.$$

We draw $\tilde{\boldsymbol{\mu}}$ from the above distribution, where $\mathbb{1}\{\boldsymbol{\mu}_1 > 0\}$ is an indicator function equal 1 if $\boldsymbol{\mu}_1 > 0$ and 0 otherwise, meaning that we perform rejection sampling, discarding the draws for which $\boldsymbol{\mu}_1 > 0$ does not hold.

2.3.4 Generating $\boldsymbol{\sigma}_1$and $\boldsymbol{\sigma}_0$

Suppose that $\boldsymbol{\sigma}_{S_t}^2 = \boldsymbol{\sigma}_0^2(1 - S_t) + \boldsymbol{\sigma}_1^2 S_t$. We can rewrite the above equation as $\boldsymbol{\sigma}_{S_t}^2 = \boldsymbol{\sigma}_0^2(1 + hS_t)$, where $\boldsymbol{\sigma}_1^2 = \boldsymbol{\sigma}_0^2(1 + h)$ and h is a 3×1 vector. We first generate $\boldsymbol{\sigma}_0^2$ conditional on h, and then generate $1 + h$ conditional on $\boldsymbol{\sigma}_0^2$. We start by dividing both sides of each of the equation

$$g_{t+1}^a = \mu_{S_t}^a + \sigma_{S_t}^a \varepsilon_{t+1}^a, \qquad a \in \{c, d, e\}$$

by $\sqrt{1 + hS_t}$ to get

$$g_{t+1}^{a*} = \mu_0^a x_{0t}^* + \mu_1^a x_{1t}^* + \sigma_0^a \varepsilon_{t+1}^a,$$

where $g_{t+1}^{a*} = g_{t+1}^a / \sqrt{1 + hS_t},\ x_{0t}^* = 1/\sqrt{1 + hS_t}$, and $x_{1t}^* = S_t/\sqrt{1 + hS_t}$.

We assume inverted Gamma distribution as a prior for $(\sigma_0^a)^2$ which we define as

$$(\sigma_0^a)^2 | h, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1 \sim IG(\frac{\nu_0}{2}, \frac{\delta_0}{2}),$$

with δ_0 and ν_0 being known. Then the posterior distribution of $(\sigma_0^a)^2$ is again an inverted Gamma distribution

$$(\sigma_0^a)^2 | h, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1, \tilde{S}_T, \tilde{\mathbf{y}}_T. \sim IG(\frac{\nu_1}{2}, \frac{\delta_1}{2}),$$

where

$$\delta_1 = \delta_0 + \sum_{t=1}^{T} (g_{t+1}^{a*} - \mu_0^a x_{0t}^* + -\mu_1^a x_{1t}^*)^2, \quad \nu_1 = \nu_0 + T.$$

To generate $\bar{h} = 1 + h$ conditional on $\boldsymbol{\sigma}_0^2$, we divide both equations

$$g_{t+1}^a = \mu_{S_t}^a + \sigma_{S_t}^a \varepsilon_{t+1}^a, \qquad a \in \{c, d, e\}$$

by σ_0 to get

$$g_{t+1}^{a**} = \mu_0^a x_{0t}^{**} + \mu_1^a x_{1t}^{**} + \sqrt{1 + hS_t} \varepsilon_{t+1}^a,$$

where $g_{t+1}^{a**} = g_{t+1}^{a}/\sigma_0^a$, $x_{0t}^{**} = 1/\sigma_0^a$, and $x_{1t}^{**} = S_t/\sigma_0^a$.

We assume an inverted Gamma distribution as a prior for $(\sigma_0^a)^2$ which we define as

$$\bar{h}|\boldsymbol{\sigma}_0^2, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1 \sim IG(\frac{\nu_3}{2}, \frac{\delta_3}{2}),$$

with δ_3 and ν_3 being known. Then the posterior distribution of $\bar{h}$ is again an inverted Gamma distribution

$$\bar{h}|\boldsymbol{\sigma}_0^2, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1, \tilde{S}_T, \tilde{\mathbf{y}}_T. \sim IG(\frac{\nu_4}{2}, \frac{\delta_4}{2}),$$

where

$$\delta_4 = \delta_3 + \sum^{N_1}(g_{t+1}^{a*} - \mu_0^a x_{0t}^* + -\mu_1^a x_{1t}^*)^2, \quad \nu_4 = \nu_3 + T_1.$$

N_1 is the set of $S_t = 1$, and T_1 is the number of such $S_t = 1$. Once we have $\bar{h}$ we generate $\boldsymbol{\sigma}_1^2$ by $\boldsymbol{\sigma}_1^2 = \bar{h}\boldsymbol{\sigma}_0^2$.

3 Data

3.1 Source and Construction

We retrieved the personal consumption expenditure (PCE) data from the U.S. National Income and Product Accounts as provided by the Bureau of Economic Analysis. Our measure of non-durable consumption includes food and beverages purchased for off-premises consumption, clothing and footwear, and gasoline and other energy goods. The corresponding seasonally adjusted quarterly quantity index for the sample period 1952:I–2011:IV is from line 8 of Table 2.3.3. (Real Personal Consumption Expenditures by Major Type of Product). Our measure of the stock of consumer durable goods includes motor vehicles, furnishings and durable household equipment, recreational goods and vehicles and other durable goods. The corresponding annual quantity index for the period 1952–2011 is from line 1 of Table 8.2 (Chain-Type Quantity Indexes for Net Stock of Consumer Durable Goods). The relative price of consumer durable goods is constructed as the ratio of the PCE price index for durable goods from line 3 over the PCE price index for non-durable goods from line 8 of Table 2.3.4 (Price Indexes for Personal Consumption Expenditures by Major Type of Product). Note that the BEA reports only the annual series of the net stock of consumer durable goods. The quarterly series must be interpolated by assuming that the depreciation rate is constant within the year and by finding its implied value, which is consistent both with the annual stocks of net consumer durables at the beginning as well as the end of the year, and with quarterly series of PCE expenditures on durable goods. [1] The U.S. population measure used to calculate per-capita quantities covers the period 1952–2011 and may be retrieved from the Bureau of Labor Statistics. The quarterly and annual returns on the common stock market as well as the short-term nominal interest rate for the sample period 1952:I–2011:IV are from the online dataset of the Fama/French Factors. Because non-durable consumption is the numèraire in our analysis we deflate all asset returns with the PCE price index for non-durable goods to obtain real quantities.

3.2 Basic Description of Consumption Data

Table 1 reports descriptive statistics for nondurable and durable goods consumption growth and durable stock growth. Nondurable consumption growth has a mean 0.34% and standard deviation 0.75% per quarter. The expenditure to durable consumption has a mean 1.04% and standard deviation 3.29% per quarter. Durable goods stock growth has mean a 0.93% and standard deviation 0.52% per quarter. The first-order autocorrelations for the nondurable consumption growth, expenditure to durable goods growth and durable goods stock growth are equal to 0.20, -0.02, and 0.99, respectively.

[Table 1 about here.]

3.3 Business Cycle Properties of Nondurable and Durable Consumption

Figure 1 is a plot of the ratio of the stock of durable goods to nondurable consumption ($\frac{K_t}{C_t}$) and the relative price of durables to nondurables. The upward trend in $\frac{K_t}{C_t}$ is consistent with a downward trend of the relative price. The ratio $\frac{K_t}{C_t}$ is pro-cyclical, it rises during booms and falls during recessions. The shaded regions are the recessions as defined by the NBER.

[Figure 1 about here.]

Figure 2 plots (a) the time series of expenditures to durable goods and (b) nondurable goods consumption. Both time series exhibit an upward trend in the sample period and are strongly pro-cyclical.

[Figure 2 about here.]

[1] The law of motion of the consumer durable goods $K_{t+1} = (1-\delta_t) K_t + I_t$ yields after four iterations the equation $K_{t+4} = (1-\delta)^4 K_t + (1-\delta)^3 I_t + (1-\delta)^2 I_{t+1} + (1-\delta) I_{t+2} + I_{t+3}$ that implicitly defines the depreciation rate δ for the given year.

Panel (a) and (b) of Figure 3 plots the corresponding growth rates of nondurable consumption and expenditure to durable goods, respectively, along with (a) growth rate of stock of durable goods and (b) growth rate of relative price of durables. Growth rates of durable stock, expenditure to durable goods and nondurable consumption are strongly pro-cyclical, whereas the growth rate of relative price is strongly coutercyclical. The growth rate of expenditure to durable goods is more pro-cyclical than nondurable consumption, and thus is a good business cycle indicator.

[Figure 3 about here.]

3.4 Unit Roots and Cointegration

One can easily show that the intratemporal first-order condition states that the marginal utility per last dollar spent must be the same across all consumption goods:

$$\frac{V_C(C_t, K_t)}{1} = \frac{V_K(C_t, K_t)}{rc_t},$$

where rc_t is the rental cost for durable goods (taking nondurable goods as a numèraire). The above condition can be rewritten as

$$\frac{V_C(C_t, K_t)}{V_K(C_t, K_t)} = rc_t.$$

On the other hand, the no-arbitrage argument implies the connection between rental cost of durable goods and their relative price, namely

$$rc_t = q_t - (1 - \delta)\mathsf{E}_t[M_{t+1}q_{t+1}],$$

where M_{t+1} denotes the stochastic discount factor. The right hand side of the equation states that one can buy a unit of durable good for q_t and sell it next period for $(1-\delta)q_{t+1}$ (after depreciation). Following a similar argument to Pakoš (2004) one can show that the growth rate of nondurable consumption, the growth rate of stock of durable goods, and their relative price share a single common stochastic trend. As the growth rate of stock of durables is directly related to expenditure on durable goods (see subsection 2.1), this section explores the nature of the long-term relationship between nondurable goods, expenditure on durable goods, and their relative price (we assume that the long-run relationship is of the form $\Delta c_t - \lambda \Delta q_t - \eta \Delta e_t \sim I(0)$). We first test for the presence of unit roots in time series, then use the Johansen Likelihood Ratio test to test for the number of cointegrating vectors, and report the estimated vector error correction model and corresponding estimated cointegrating vector.

Unit Roots We test the null hypothesis that the growth rates of nondurable consumption, expenditures to durable goods, and relative price of durables are difference stationary against the alternative hypothesis of trend stationarity, using the tests ofElliott et al. (1996) and Ng and Perron (2001). In all cases we are unable to reject the hypothesis about difference stationarity (see Table 2).

[Table 2 about here.]

Cointegration Because the time series is trending, we compute the Johansen Likelihood Ratio test assuming an unrestricted constant. Let $H_0(r) : r = r_0$ be the null hypothesis of exactly r_0 cointegrating vectors, and $H_1(r) : r > r_0$ denote the alternative hypothesis of more than r_0 cointegrating vectors. Panel A in Table 3 reports the value of the trace statistics and maximum eigenvalue statistics for the vector of nondurable goods, durable goods expenditure and relative price. Based on the values of trace statistics reported, we cannot accept $H_0(0)$ at 1%, 5% or 10% significance level, but we accept $H_0(1)$. Similarly, the value of maximum eigenvalue statistics suggests that we cannot accept $H_0(0)$ at 5% and 10% significance level, but we accept $H_0(1)$ at 5%. Thus, both test statistics suggest there is exactly one cointegrating vector. Panel B in Table 3 reports the value of the trace statistics and maximum eigenvalue statistics for the vector of nondurable goods, durable goods expenditure and relative price when we impose VAR(2) in levels. Based on the values of trace statistics reported, we cannot accept $H_0(0)$ at 1%, 5% and 10% significance level, but we accept $H_0(1)$ at 5% significance level. Similarly, the value of maximum eigenvalue statistics suggests that we cannot accept $H_0(0)$ at 10% significance level, but we accept $H_0(1)$. We also compute the Johansen Likelihood Ratio test for nondurable goods, durable goods stock and relative price. The values of both trace statistics and maximum eigenvalue statistics (Panel C in Table 3) suggest that we accept $H_0(0)$ at 10%, 5% and 1% significance level. Finally, Panel D in Table 3 reports the values of both trace statistics and maximum eigenvalue statistics of the Johansen Likelihood Ratio test for nondurable goods minus expenditure to durable goods and relative price. In both cases we accept $H_0(0)$ at any convenient significance level.

[Table 3 about here.]

Estimated Vector Error Correction Model As the results in Table 3 indicate, the vector of time series $[\Delta c_t, \Delta e_t, \Delta q_t]'$ follows a cointegrated VAR(2), and hence the lag length for the vector error correction model (VECM) is 1. Table 4 reports the estimated VECM model for the time series. Table 5 reports the estimated cointegrating vector for nondurable goods, expenditure to durable goods, and relative price (Panels A and B) and nondurable goods minus expenditure to durable goods and relative price (Panel C). We can conclude that there is a long-term relationship between nondurable goods, the expenditure to durable goods, and their relative price. At the current stage of research we do not include this relationship in the estimation procedure so as not to fall for the "curse of dimensionality" and overcomplicate the dynamics of the model. As a possible extension for future research we can add the fourth time series (the relative price of durable and nondurable goods) into the model.

[Table 4 about here.]

[Table 5 about here.]

4 Empirical Results

4.1 The Case of an Observable State of the World

We apply the procedures described in section 2.3, first assuming that the state of the world is observable. We run Gibbs-sampling such that the first 2000 draws are discarded and the next 10000 are recorded. We employ uninformative priors for all the model's parameters. Table 6 reports marginal posterior distributions of the transition probabilities p and q, means $\mu_0^c, \mu_1^c, \mu_0^d, \mu_1^d, \mu_0^e, \mu_1^e$, and volatilities $\sigma_0^c, \sigma_1^c, \sigma_0^d, \sigma_1^d, \sigma_0^e, \sigma_1^e$. We find that transition probabilities p and q turn out to equal 0.94 and 0.73, respectively. The growth rates of consumption, dividends, and expenditure to durable goods in recession are equal to -0.35, -0.57, and -0.09 percent, while in expansion they are equal to 0.48, 1.43, and 0.69 percent, respectively. The estimated volatility of consumption in recession and expansion is equal to 0.66 and 0.80 percent, respectively, of dividends to 2.93 and 3.90 percent, and of expenditure to durable goods to 4.84 and 6.37 percent.

[Table 6 about here.]

Figures 4, 5, and 6 display the distribution of all the estimated parameters.

[Figure 4 about here.]

[Figure 5 about here.]

[Figure 6 about here.]

After each iteration of Gibbs-sampling we have a set of filtered probabilities $f(S_t = 0|\tilde{\mathbf{y}}_t)$ and $f(S_t = 1|\tilde{\mathbf{y}}_t)$ for each of 10000 iterations. Figure 7 depicts the probability of expansion and recession.

[Figure 7 about here.]

4.2 The Case of an Unobserved State of the World

We now apply the procedures described in section 2.3, assuming that the state of the world is hidden. Therefore we treat the state of the world as a missing observation. We run Gibbs-sampling such that the first 2000 draws are discarded and the next 10000 are recorded. We employ uninformative priors for all the model's parameters. Table 7 reports marginal posterior distributions of the transition probabilities p and q, means $\mu_0^c, \mu_1^c, \mu_0^d, \mu_1^d, \mu_0^e, \mu_1^e$, and volatilities $\sigma_0^c, \sigma_1^c, \sigma_0^d, \sigma_1^d, \sigma_0^e, \sigma_1^e$. We find that transition probabilities p and q turn out to equal 0.93 and 0.76, respectively. The growth rates of consumption, dividends, and expenditure to durable goods in recession are equal to 0.06, 0.03, and -0.34 percent, while in expansion they are equal to 0.41, 1.27, and 0.77 percent, respectively. The estimated volatility of consumption in recession and expansion is equal to 0.69 and 0.90 percent, respectively, of dividends to 2.97 and 4.12 percent, and of expenditure to durable goods to 5.08 and 5.11 percent.

[Table 7 about here.]

Figures 8, 9, and 10 display the distribution of all the estimated parameters.

[Figure 8 about here.]

[Figure 9 about here.]

[Figure 10 about here.]

After each iteration of Gibbs-sampling we have a set of filtered probabilities $f(S_t = 0|\tilde{\mathbf{y}}_t)$ and $f(S_t = 1|\tilde{\mathbf{y}}_t)$ for each of 10000 iterations. Figure 11 depicts the probability of expansion and recession.

[Figure 11 about here.]

5 Conclusion

In this paper, we jointly model the growth rate of nondurable goods, dividends, and expenditure to durable goods. We separate durable consumption from nondurable consumption in order to increase the magnitude of the stochastic discount factor in economy. The predictable components and the instantaneous volatilities are driven by the common two-state Markov chain. As the result we get the posterior beliefs about the hidden state, and these beliefs drive the dynamic of the asset prices in the model. We estimate all the parameters of interest using the Bayesian framework developed by Albert and Chib (1993) and Kim and Nelson (1999) using postwar US data, where we treat the unobserved state as missing data. This allows us to derive the full posterior distributions for all the parameters of interest. We also test for the presence of the long-term relationship between growth rate of nondurable goods, expenditure to durable goods, and their relative price. We find that these time series have a single common stochastic trend, but we leave out this relationship from the estimation procedure and keep it as a possible extension for the future research.

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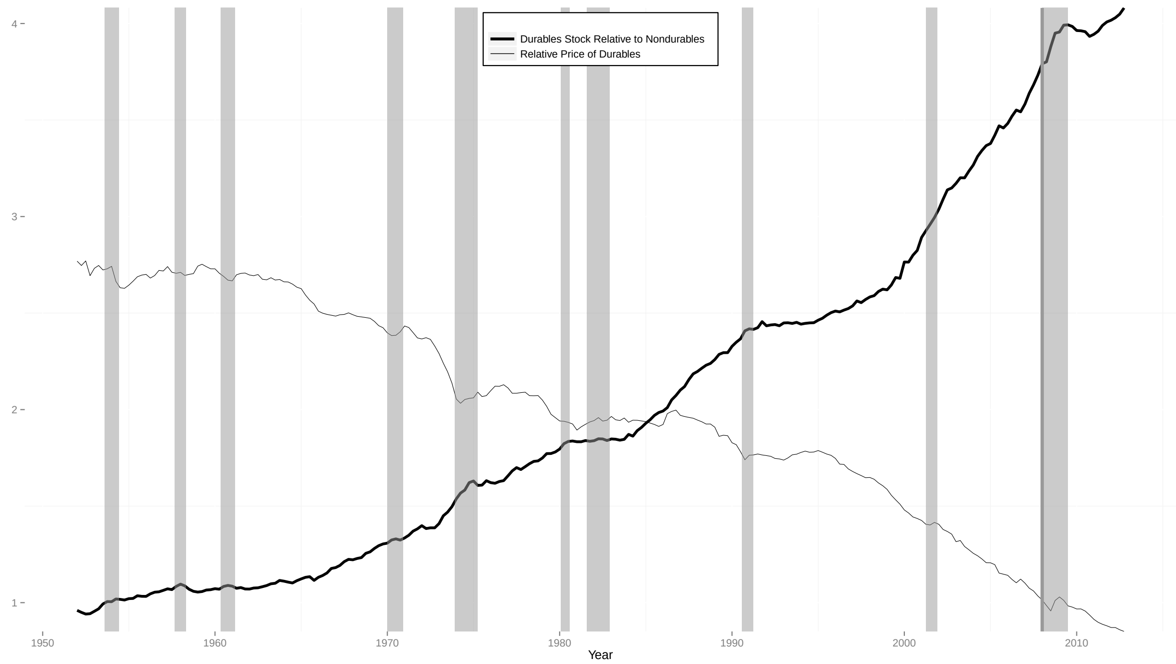

Figure 1: **Price and Stock of Durables Relative to Nondurables.** Time-series plot of the relative price of durables to nondurables and the stock of durables as a ratio of nondurable consumption. The sample period is 1951:I - 2012:IV; the shaded regions indicate NBER recessions.

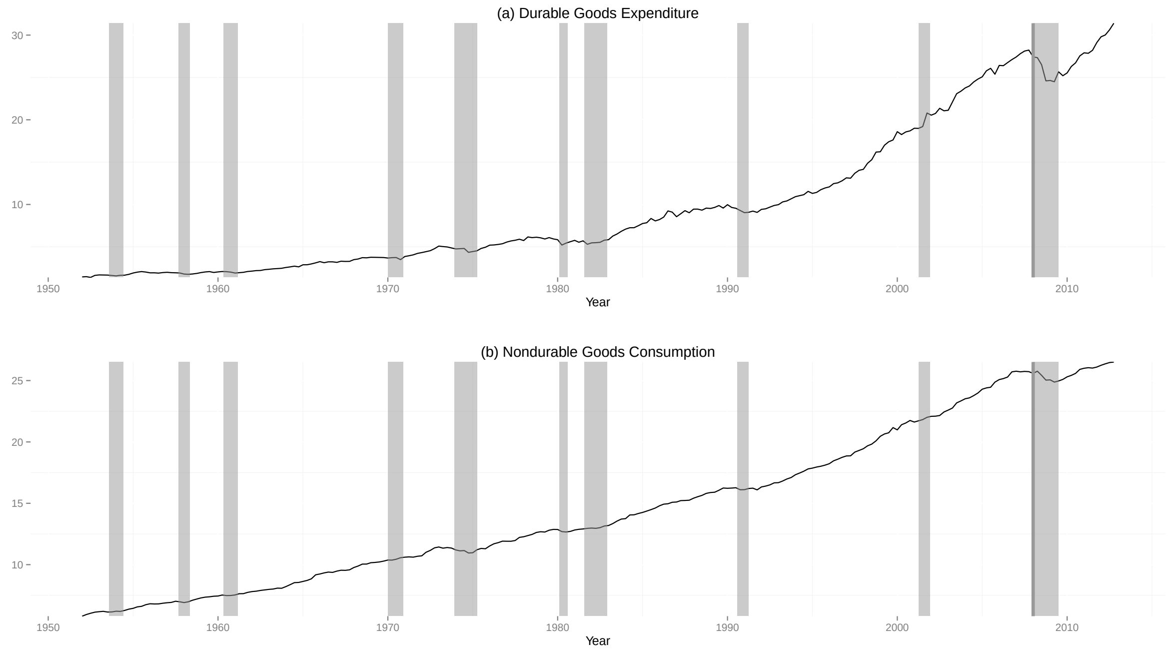

Figure 2: **Durable and Nondurable Goods Expenditure.** Time-series plot of (a) the real durable goods expenditure and (b) the real nondurable goods consumption. The sample period is 1951:I - 2012:IV; the shaded regions indicate NBER recessions.

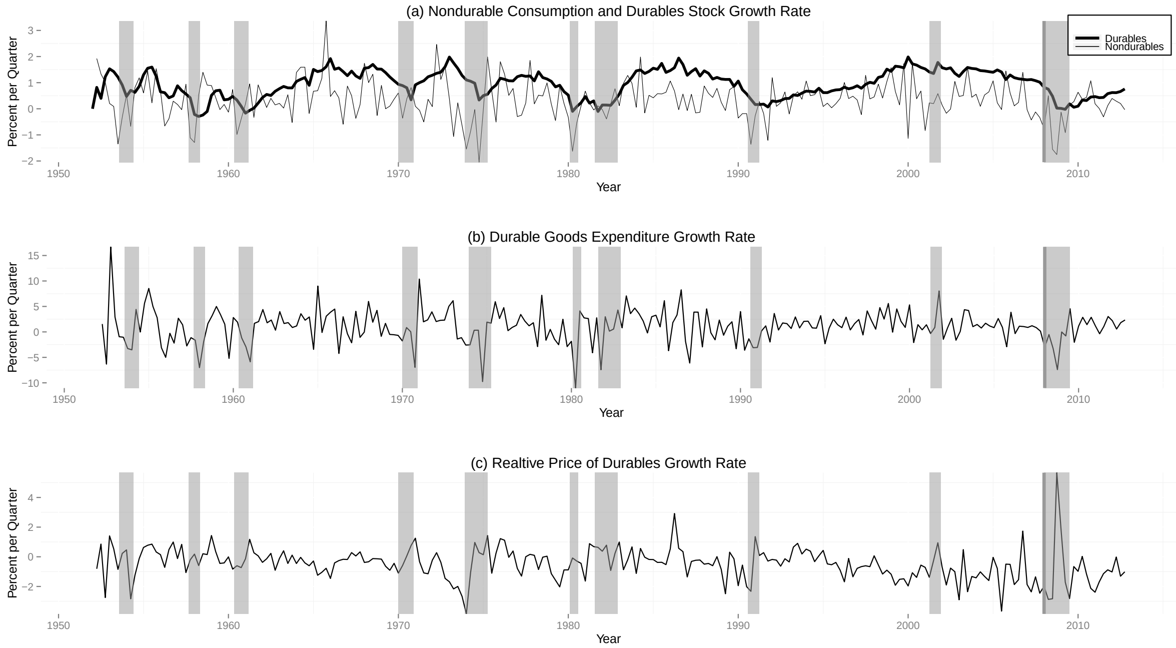

Figure 3: **Growth Rates.** Time-series plot of (a) the real growth rates of the stock of durables and nondurable consumption, (b) the real growth rate of durable goods expenditure, and (c) the growth rate of relative price of durables to nondurables. The sample period is 1951:I - 2012:IV; the shaded regions indicate NBER recessions.

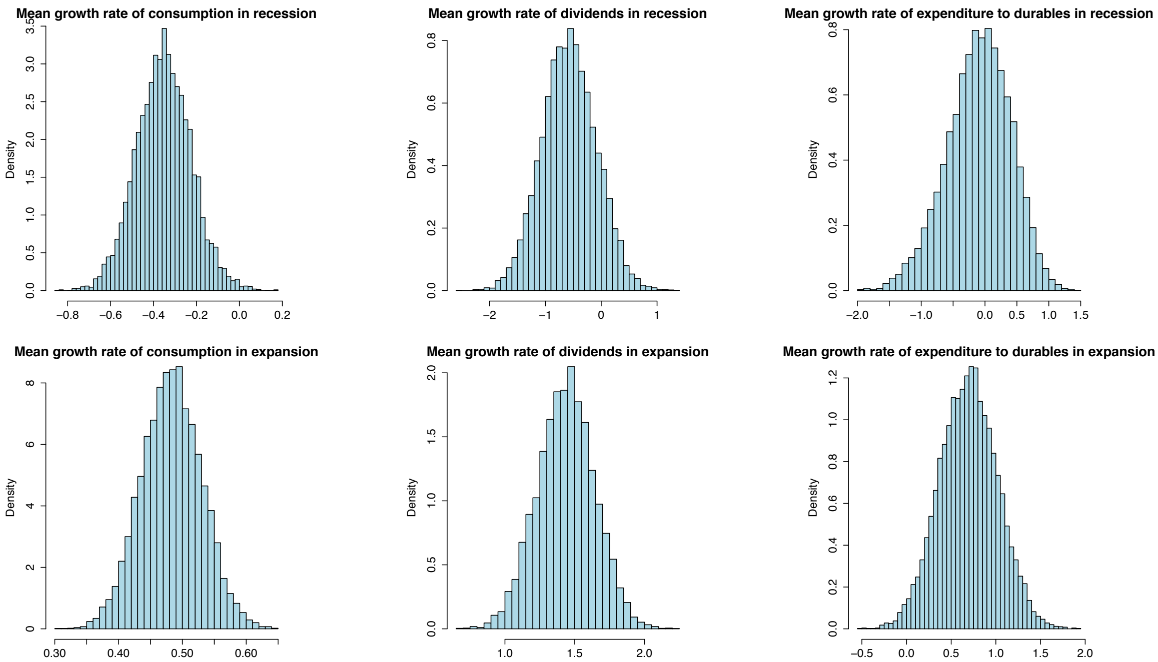

Figure 4: **Posterior distribution of mean.** Figure displays the posterior distribution of mean growth rates of consumption, dividends, and expenditure to durable goods.

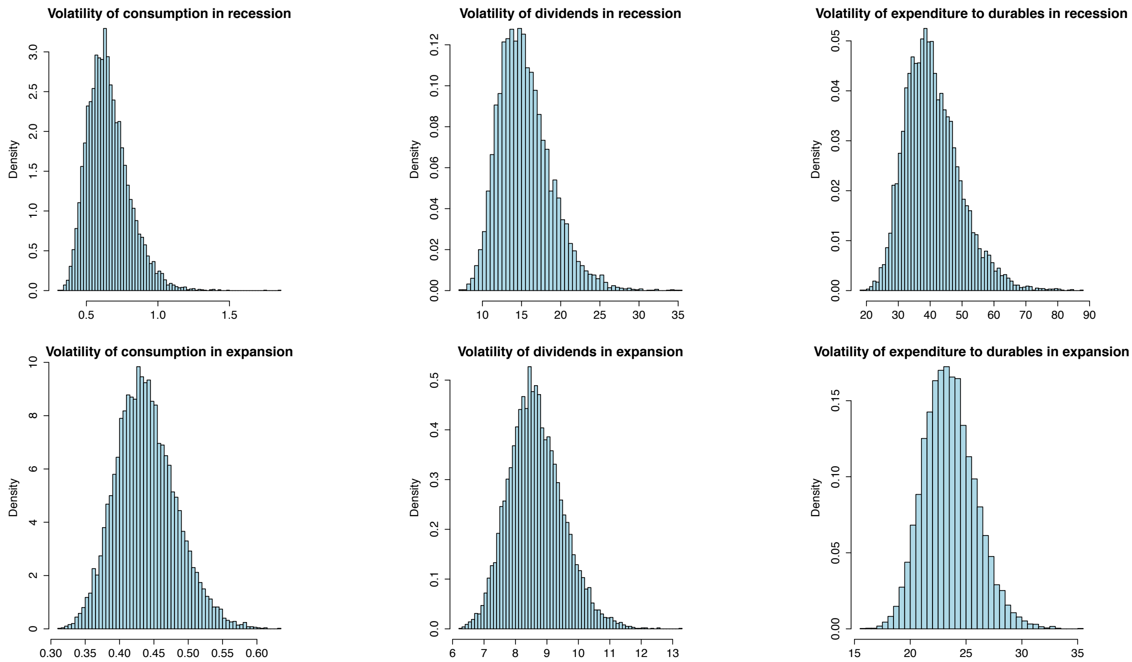

Figure 5: **Posterior distribution of volatility.** Figure displays the posterior distribution of volatility of consumption, dividends, and expenditure to durable goods.

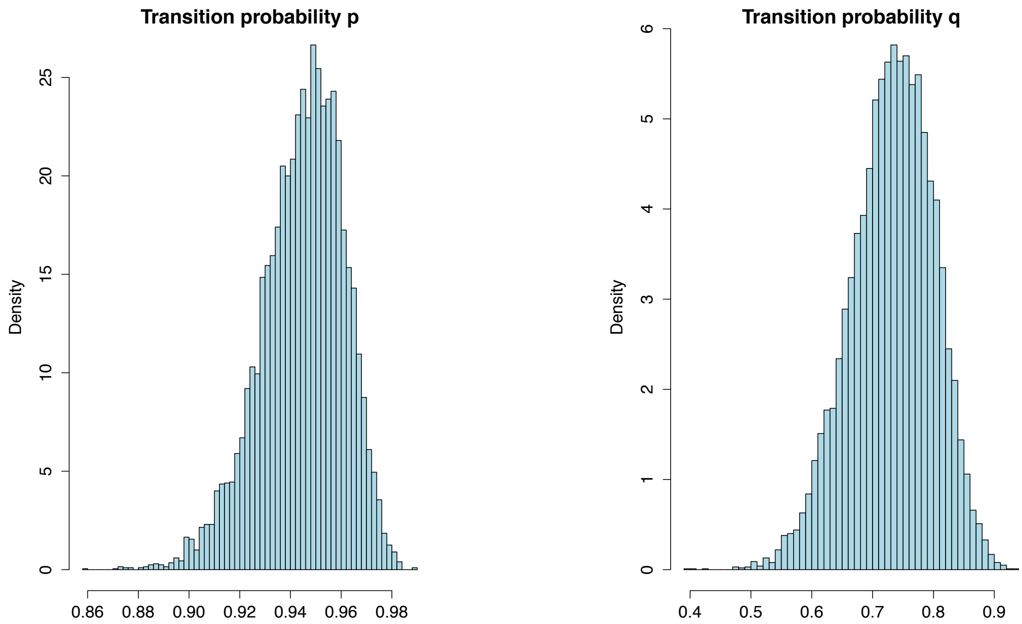

Figure 6: **Posterior distribution of transition probabilities.** Figure displays the posterior distribution of transition probabilities p and q.

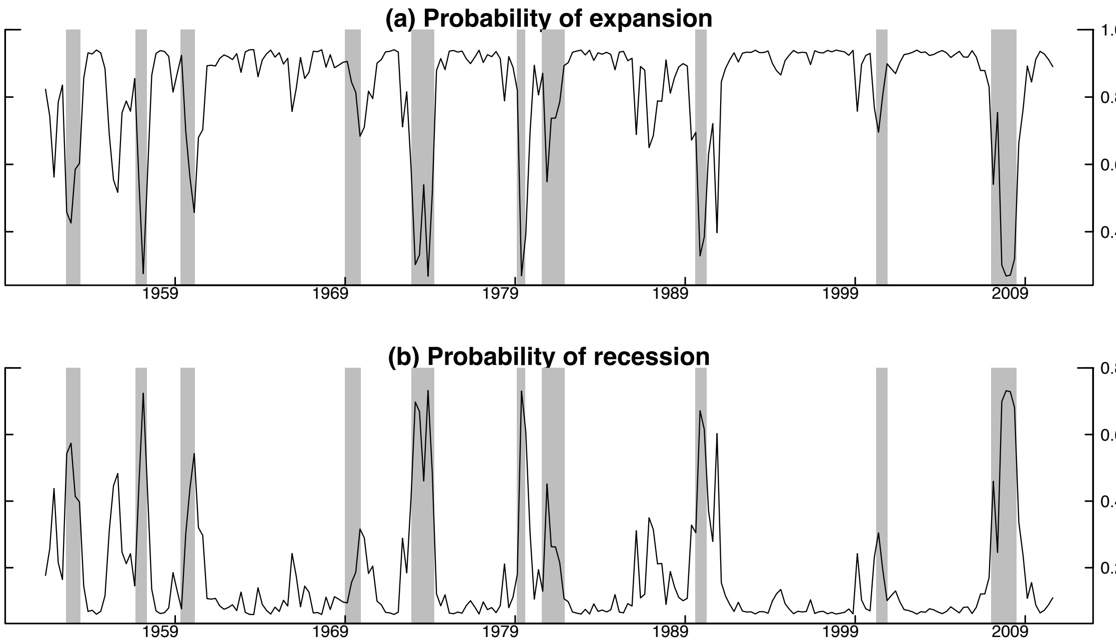

Figure 7: **Probability of expansion and recession.** Figure displays the posterior probabilities of (a) expansion and (b) recession. The sample period is 1951:I - 2012:IV; the shaded regions indicate NBER recessions.

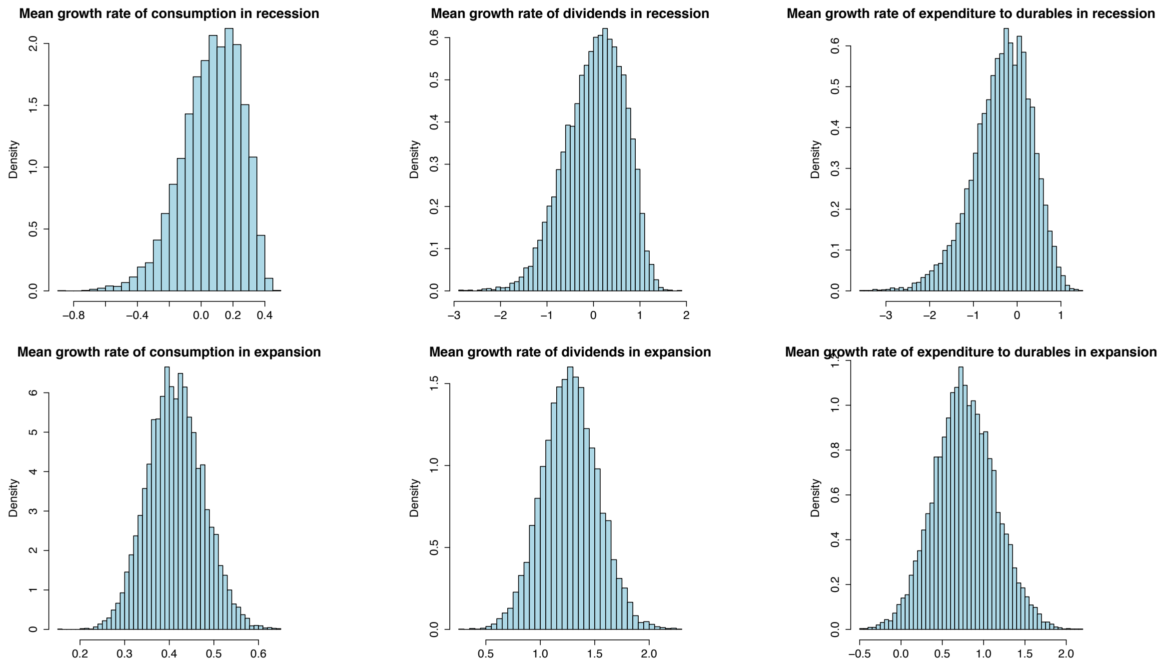

Figure 8: **Posterior distribution of mean.** Figure displays the posterior distribution of mean growth rates of consumption, dividends, and expenditure to durable goods.

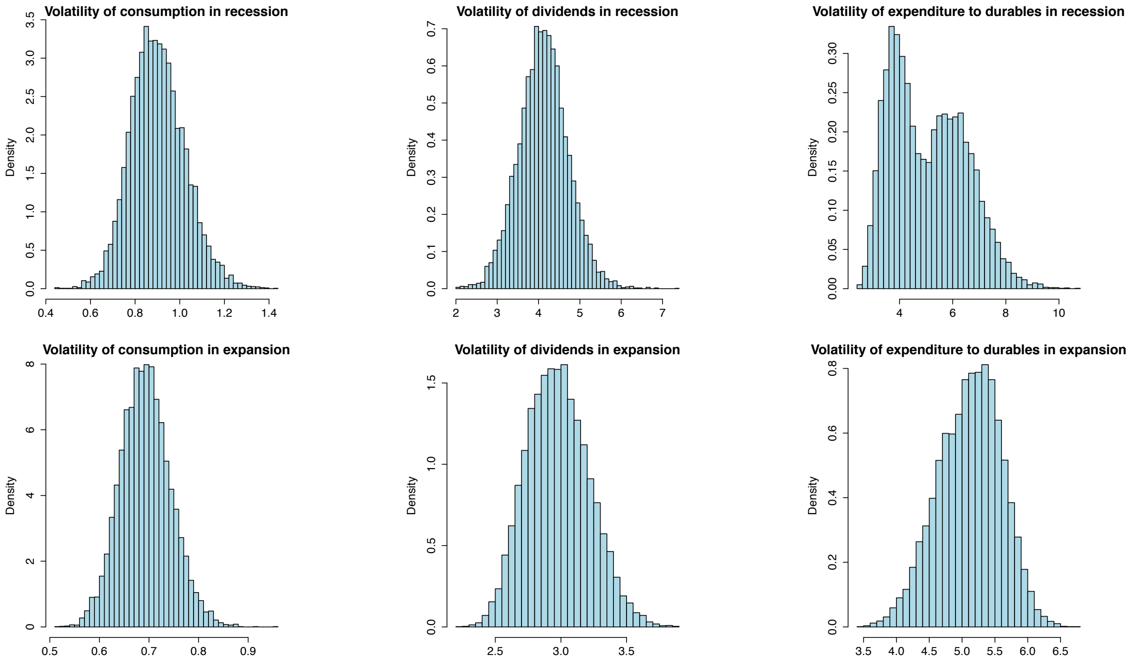

Figure 9: **Posterior distribution of volatility.** Figure displays the posterior distribution of volatility of consumption, dividends, and expenditure to durable goods.

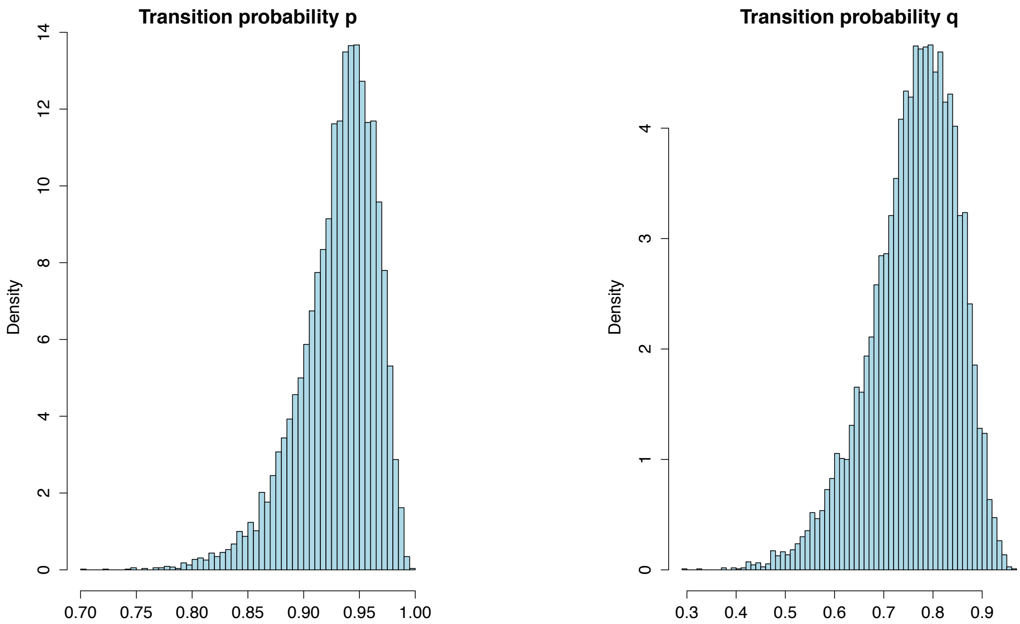

Figure 10: **Posterior distribution of transition probabilities.** Figure displays the posterior distribution of transition probabilities p and q.

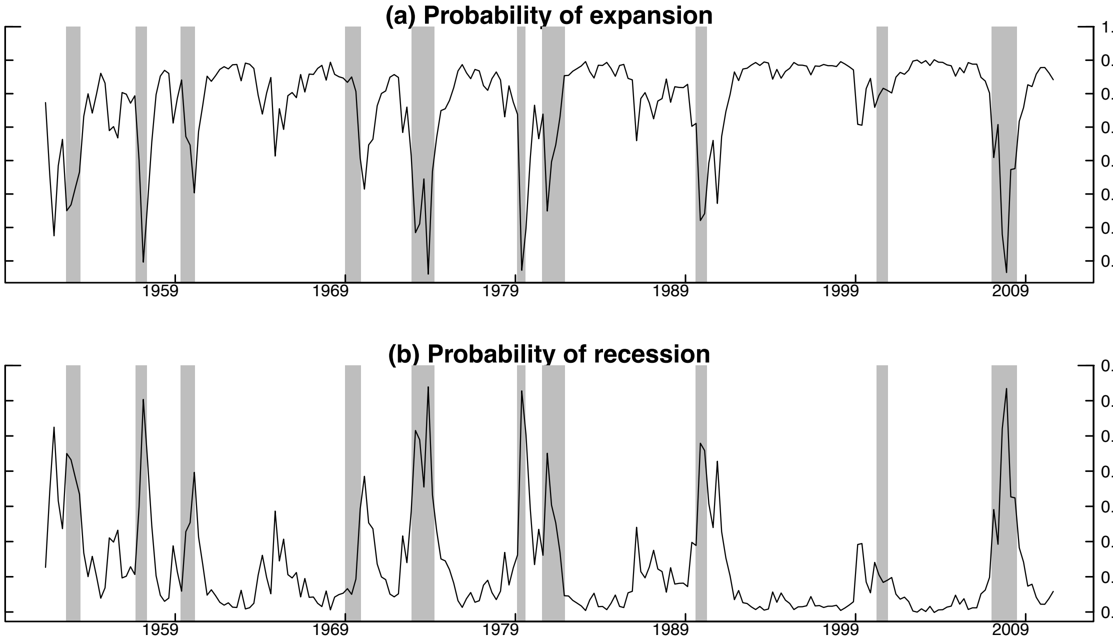

Figure 11: **Probability of expansion and recession.** Figure displays the posterior probabilities of (a) expansion and (b) recession. The sample period is 1951:I - 2012:IV; the shaded regions indicate NBER recessions.

Table 1: Descriptive statistics

Time series	Mean[a] (%)		SD[a] (%)		Autocorrelation	
	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)
Nondurable Goods	0.341	(0.044)	0.747	(0.052)	0.207	(0.059)
Durable Goods Expenditure	1.037	(0.153)	3.290	(0.249)	-0.02	(0.062)
Durable Goods Stock	0.934	(0.086)	0.519	(0.025)	0.991	(0.026)

[a] All variables are in percentage. Standard errors obtained by performing a block bootstrap with each block having geometric distribution with length 32 quarters; 50,000 experiments performed. Sample period is 1952:I-2012:IV.

Table 2: Testing for unit roots [a]

Time Series	ERS	DF-GLS	MPP
In logs			
Nondurable Consumption	9.01	-2.24	-2.24
Durable Good Stock	5.30	-3.12	-3.15
Expenditure to Durable Goods	7.40	-2.55	-2.48
Relative Price of Durable Goods	88.2	0.32	0.33

[a] ERS denotes the test of Elliott et al. (1996) and DF-GLS and MPP denote the modified Phillips-Perron tests of Ng and Perron (2001). Sample period is 1952:I–2012:IV.

Table 3: Testing for cointegration

	Panel A. Nondurable Goods, Durable Good Expenditures and Relative Price [a]						
	Eigenvalue	Trace Stats	90% CV	95% CV	Max Stats	90% CV	95% CV
H(0)++**	0.09	37.57	28.71	31.52	22.47	18.90	21.07
H(1)*	0.06	15.09	15.66	17.95	13.76	12.91	14.90
H(2)	0.01	1.33	6.50	8.18	1.33	8.18	8.18

	Panel B. Nondurable Goods, Durable Good Expenditures and Relative Price [b]						
	Eigenvalue	Trace Stats	90% CV	95% CV	Max Stats	90% CV	95% CV
H(0)++*	0.08	36.63	28.71	31.52	20.27	18.90	21.07
H(1)+**	0.06	16.36	15.66	17.85	15.71	12.91	14.90
H(2)	0.00	0.64	6.50	8.18	0.64	8.18	8.18

	Panel C. Nondurable Goods, Durable Good Stock and Relative Price						
	Eigenvalue	Trace Stats	90% CV	95% CV	Max Stats	90% CV	95% CV
H(0)	0.08	25.87	28.71	31.52	18.63	18.90	21.07
H(1)	0.03	7.24	15.66	17.85	6.51	12.91	14.90
H(2)	0.00	0.73	6.50	8.18	0.73	8.18	8.18

	Panel D. Nondurable Goods Minus Durable Good Expenditures and Relative Price						
	Eigenvalue	Trace Stats	90% CV	95% CV	Max Stats	90% CV	95% CV
H(0)	0.04	10.65	15.66	17.95	10.65	12.91	14.90
H(1)	0.00	0.00	6.50	8.18	0.00	8.18	8.18

[a] Akaike, Bayesian and Hannan-Quinn criteria all suggest VAR(1) in levels.
[b] Imposed VAR(2) in levels.

Table 4: Vector error correction model [a]

	Intercept	$c_t - \eta q_t - \lambda e_t$	Δc_t	Δe_t	Δq_t	R^2
Δc_{t+1}	0.024	-0.015	0.127	0.035	0.074	44.04
	(0.015)	(0.012)	(0.070)	(0.016)	(0.046)	
	[1.558]	[-1.224]	[1.810]	[2.202]	[1.619]	
Δe_{t+1}	-0.097	0.082	1.196	-0.126	0.253	18.59
	(0.067)	(0.052)	(0.308)	(0.070)	(0.201)	
	[-1.436]	[1.565]	[3.881]	[-1.799]	[1.259]	
Δq_{t+1}	-0.075	0.056	-0.112	0.023	0.258	26.66
	(0.022)	(0.017)	(0.099)	(0.023)	(0.065)	
	[-3.447]	[3.333]	[-1.130]	[1.028]	[3.969]	

[a] Asymptotic standard errors in parentheses whereas the t-statistics are in square brackets. Sample period is quarterly 1952:I–2012:IV.

Table 5: Estimated cointegrating vector

Panel A. Nondurable Goods, Durable Good Expenditures and Relative Price [a]			
Coint. Parameter	Estimates	S.E.	t-stats
λ	0.59	(0.03)	[-18.89]
η	0.26	(0.09)	[-2.86]

Panel B. Nondurable Goods, Durable Good Expenditures and Relative Price [b]			
Coint. Parameter	Estimates	S.E.	t-stats
λ	0.60	(0.03)	[-17.95]
η	0.26	(0.10)	[-2.67]

Panel C. Nondurable Goods Minus Durable Good Expenditures and Relative Price			
Coint. Parameter	Estimates	S.E.	t-stats
η	0.90	(0.32)	[2.87]

[a] Akaike, Bayesian and Hannan-Quinn criteria all suggest VAR(1) in levels.
[b] Imposed VAR(2) in levels.

Table 6: Bayesian Gibbs-sampling approach to a two-state Markov-switching model

Parameter	Posterior			
	Mean	SD	95% posterior bands[a]	
p	0.9447	0.0162	(0.9088,	0.9722)
q	0.7322	0.0681	(0.5896,	0.8532)
μ_0^c	-0.3524	0.1277	(-0.6046,	-0.0969)
μ_0^d	-0.5732	0.4903	(-1.5175,	0.3750)
μ_0^e	-0.0972	0.4993	(-1.1574,	0.7938)
μ_1^c	0.4829	0.0469	(0.3913,	0.5755)
μ_1^d	1.4376	0.2051	(1.0328,	1.8334)
μ_1^e	0.6933	0.3239	(0.0630,	1.3243)
σ_0^c	0.8049	0.0875	(0.6532,	0.9918)
σ_1^c	0.6613	0.0328	(0.6005,	0.7300)
σ_0^d	3.9055	0.4209	(3.1787,	4.8255)
σ_1^d	2.9385	0.1463	(2.6708,	3.2404)
σ_0^e	6.3701	0.6601	(5.2182,	7.8007)
σ_1^e	4.8464	0.2397	(4.3998,	5.3380)

[a] All variables are in percentage. Standard errors obtained by performing a block bootstrap with each block having geometric distribution with length 32 quarters; 50,000 experiments performed. Sample period is 1952:I-2012:IV.

Table 7: Bayesian Gibbs-sampling approach to a two-state Markov-switching model

Parameter	Posterior Mean	SD	95% posterior bands[a]	
p	0.9304	0.0348	(0.8447,	0.9799)
q	0.7613	0.0876	(0.5593,	0.9020)
μ_0^c	0.0631	0.1843	(-0.3463,	0.3527)
μ_0^d	0.0317	0.6265	(-1.2749,	1.0680)
μ_0^e	-0.3424	0.6655	(-1.8250,	0.7833)
μ_1^c	0.4137	0.0619	(0.2962,	0.5373)
μ_1^d	1.2746	0.2542	(0.7839,	1.7818)
μ_1^e	0.7719	0.3681	(0.0566,	1.5115)
σ_0^c	0.9018	0.1221	(0.6780,	1.1610)
σ_1^c	0.6933	0.0505	(0.5979,	0.7980)
σ_0^d	4.1218	0.5990	(2.9503,	5.3114)
σ_1^d	2.9775	0.2384	(2.5495,	3.4656)
σ_0^e	5.0850	1.3956	(3.0164,	7.8786)
σ_1^e	5.1181	0.4801	(4.1285,	5.9856)

[a] All variables are in percentage. Standard errors obtained by performing a block bootstrap with each block having geometric distribution with length 32 quarters; 50,000 experiments performed. Sample period is 1952:I-2012:IV.

Working Paper Series
ISSN 1211-3298
Registration No. (Ministry of Culture): E 19443

Individual researchers, as well as the on-line and printed versions of the CERGE-EI Working Papers (including their dissemination) were supported from institutional support RVO 67985998 from Economics Institute of the ASCR, v. v. i.

Specific research support and/or other grants the researchers/publications benefited from are acknowledged at the beginning of the Paper.

Published by
Charles University in Prague, Center for Economic Research and Graduate Education (CERGE)
and
Economics Institute of the ASCR, v. v. i. (EI)
CERGE-EI, Politických vězňů 7, 111 21 Prague 1, tel.: +420 224 005 153, Czech Republic.
Printed by CERGE-EI, Prague
Subscription: CERGE-EI homepage: http://www.cerge-ei.cz

Phone: + 420 224 005 153
Email: office@cerge-ei.cz
Web: http://www.cerge-ei.cz

Editor: Marek Kapička

The paper is available online at http://www.cerge-ei.cz/publications/working_papers/.

ISBN 978-80-7343-327-7 (Univerzita Karlova. Centrum pro ekonomický výzkum
a doktorské studium)
ISBN 978-80-7344-319-1 (Akademie věd České republiky. Národohospodářský ústav)

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Politických vězňů 7
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Czech Republic
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