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# Information and Communication Technologies in a Multi-sector Endogenous Growth Model\*

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## Abstract

This paper investigates the growth impact of Information and Communication Technologies (ICT) in an economy consisting of three sectors, ICT-producing, ICT-using and non-ICT-using. The ICT progress causes falling prices of the consumption and intermediates produced by the ICT-using sector, providing incentives for investment in the sectors using them. Therefore, the non-ICT-using sector benefits indirectly from ICT, while households' utility increases. The magnitude of the growth transmission mechanism relies on the ICT-using sector production shares. Aggregate economy is on a constant growth path, where growth rates differ across sectors. The model predictions are broadly consistent with the U.S. growth experience.

## Abstrakt

Tento článek zkoumá dopad Informačních a Komunikačních Technologí (ICT) na růst v ekonomice, která se skládá ze tří sektorů: ICT produkující sektor, ICT používající sektor a sektor nepoužívající ICT vůbec. Pokrok v ICT způsobuje propad cen spotřebních výrobků a mezi-produktů v sektoru používající ICT, což motivuje další investice. Tudíž sektor nepoužívající ICT profituje z tohoto pokroku nepřímo, zatímco užitek domácností se zvyšuje. Velikost přeneseného vlivu na růst závisí na produkčních podílech sektoru používající ICT. Celková ekonomika je na konstantní růstové dráze, kde se růst liší po sektorech. Předpovědi modelu jsou konzistentní se zkušenostmi z USA.

**JEL classification:** O40, O41

**Keywords:** Multi-sector Economy, Endogenous Growth, Constant Growth Path, Information and Communication Technologies

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# 1 Introduction

Current research on economic growth examines the sources of aggregate growth at a finer level of production. In this spirit, empirical studies of the sources growth for the United States economy, in the post-1970 period (Jorgenson, Ho, and Stiroh 2005, Oliner and Sichel 2002, Stiroh 2002) identify the Information and Communication Technologies (ICT) producing sector as the source of aggregate TFP growth, despite its small value added and employment share.<sup>1</sup> Furthermore, Jorgenson, Ho, and Stiroh (2005) conclude that the most important source of United States growth has been the accumulation of both ICT and non-ICT-capital, especially during the 1990s. Their growth accounting is based on tracing the use of ICT goods (e.g., semiconductors) and flows as capital or intermediates across production units that differ in their inputs' composition.

This paper analyzes in a theoretical framework the role of commodities' flows across production units as a mechanism through which a major technology's growth is spread to the rest of the economy. It shows how the interaction of sectors with different growth potential affects their final output and long-run aggregate economic performance. In relation to ICT, this paper shows that it is important to disaggregate the economy into three sectors, based on the criteria of ICT production and intensive use. The first sector produces ICT. The second sector uses ICT-capital (e.g., computers) to produce intermediate goods (e.g., general purpose machinery or wholesale trade) for itself, and for the third sector, which does not use ICT (e.g., food production or hairdressers).

The growth mechanism is illustrated in the following example. When a new microprocessor is produced, it is embodied in computers. These higher quality computers are used in the production of machinery, that can in turn become available at a lower price. The electric appliances that is part of this production is used by wholesalers, as well as by food producers or hairdressers. Even though the hairdressers do not use directly ICT, they benefit indirectly from its advances because it lowers their costs. Therefore, innovations in ICT provide incentives for capital deep-

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<sup>1</sup>The "ICT-producing" industries produce computer hardware, electronic components, telecommunication equipment and computer services (includes software production).

ening in the entire economy through falling capital and intermediates' costs. This is despite the fact that the direct users of ICT, like wholesalers, face relatively high costs in trading food products. Importantly, the wholesalers need the food producers in their production and vice versa, while consumers derive utility by consuming all goods: food, hairdressers or electric appliances.

This paper shows that this mechanism is endogenously sustained in a long-run constant growth path with constant capital-output and consumption-output ratios, when both the ICT-using and non-ICT-using sectors are essential for the production of both consumption and intermediate goods. On this path, sectorial output growth rates differ. The ICT-producing sector exhibits the fastest growth, followed by the ICT-using one and then the rest of the economy.

The higher is the contribution of the ICT-using sector into the production of intermediates, the higher is the non-ICT-using sector's growth from the accumulation of non-ICT-capital, but the more the ICT-using sector's growth falls short of its full potential (i.e., ICT growth). Also, the higher is the contribution of the ICT-using sector into the production of consumption goods, the higher is aggregate consumption growth due to the decreasing opportunity cost of consuming its goods. The contribution of the ICT-using sector into the production of consumption or intermediates matters not only for the strength of the growth transmission mechanism, but also for the growth engine itself because it affects the allocation of resources in the economy. The results highlight how the ICT-using sector's goods use interacts with the market frictions to have a long-run aggregate impact.

Making use of United States data at the three-digit ISIC level to match the model's sectors, this paper provides supportive evidence for the model's result on the growth ranking across sectors.<sup>2</sup> The model's main parameters are calibrated with United States data for the 1977-2001 period, and its steady-state predictions for sectorial allocations are broadly consistent with the evidence. A short application illustrates the model's ability to account for the transmission of ICT progress to

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<sup>2</sup>As in Jorgenson, Ho, and Stiroh (2005), industries are classified as "ICT-using" or "non-ICT-using" according to their ICT-capital intensity in 1995. See Table C.4 for details regarding the industries in each major sector. EU KLEMS accounts support that this classification is robust to the increase of ICT adoption over time within each industry.

Table 1: United States sector-level ICT-goods share in total capital and intermediates purchases

Use intensity in	ICT-capital	ICT intermediates
ICT-producing	43.2	40.3
ICT-using	40.4	5.1
non-ICT-using	8.7	1.9

Source: BEA, 1997 Benchmark Input Output Use and Capital Flow Tables

the aggregate economy. The model's quantitative performance supports that its parsimonious disaggregation of the economy into the ICT-producing, ICT-using and non-ICT-using sectors can capture the important features of the growth mechanism in the ICT context.

The results highlight that unless these three sectors are accounted, a standard endogenous growth model would not be able to replicate the United States growth experience. The main assumption of the model is that the ICT-using sector's production is the only one that is directly linked with the ICT-producing sector, through its use of ICT-capital. This sector is also assumed to be the only sector using this type of capital goods. Table 1 summarizes the intensity of each sector in capital or intermediate goods produced by the ICT-producing sector. The ICT-using sector is the final good sector that clearly comes out as the intensive user of ICT goods compared to the non-ICT-using one.<sup>3</sup> Besides, the data broadly suggest homogeneity across production units (industries) within each of the model's sectors in terms of ICT-capital use, but important ones across the three sectors that are persistent over time.

Nevertheless, the sector-level disaggregation alone is not sufficient for the model to account for the growth experience in the ICT era. The three-sector environment needs to be enriched with assumptions on these sectors' hierarchical production order and interaction among them. The model makes such assumptions consistent with the observed inter-sector flows of goods in the economy and commodities' use shares.<sup>4</sup> In the model, the "final good" (i.e., consumption) producing sectors are

<sup>3</sup>In particular, the ICT-using sector is three times as intensive in ICT goods' use in its production. Moreover, it uses most of all ICT capital and intermediates (60%) produced for the two final-good sectors.

<sup>4</sup>The commodities use shares for the three sectors are presented in Table C.1.

Table 2: United States inter-sector transactions of intermediates

Shares of intermediates			
produced/used by:	ICT-producing	ICT-using	non-ICT-using
ICT-producing	1.8	1.3	1.3
ICT-using	1.6	14.7	19.4
non-ICT-using	1.1	10.0	48.6

Notes: matrix entries sum up to 100%

aggregate production and use shares sum matrix rows and columns respectively

Source: BEA, 1997 Benchmark Input Output Use Table

Table 3: United States inter-sector transactions of capital

Shares of capital			
produced/used by:	ICT-producing	ICT-using	non-ICT-using
ICT-producing	1.3	8.1	6.7
ICT-using	1.2	5.0	19.6
non-ICT-using	0.6	7.0	50.6

Notes: matrix entries sum up to 100%

aggregate production and use shares sum matrix rows and columns respectively

Source: BEA, 1997 Capital Flow Table

the ICT-using sector and non-ICT-using one, since together they account for 98% of aggregate value added. Tables 2 and 3 suggest that the final goods sectors have an additional role in producing intermediates and capital for each other, while the ICT-producing sector comes out as a virtually purely upstream industry. These evidence also draw attention to the high similarity of the flows of goods as capital or intermediates. The theoretical model also makes no explicit distinction between them.

Furthermore, the ICT-producing sector is modeled as the engine of growth This assumption is consistent with Jorgenson, Ho, and Stiroh (2005)'s finding that it is the source of (almost all) aggregate TFP growth, and studies showing that the ICT-producing sector is highly intensive in R&D and patenting activity (Carlin and Mayer 2003). Figure 1 illustrates the outstanding performance of the ICT-producing sector compared to the rest of the economy in terms of new patents granted by USPTO to the United States non-government institutions. This resulted in the sharp increase in the ICT-producing sector's share in total USPTO granted patents:



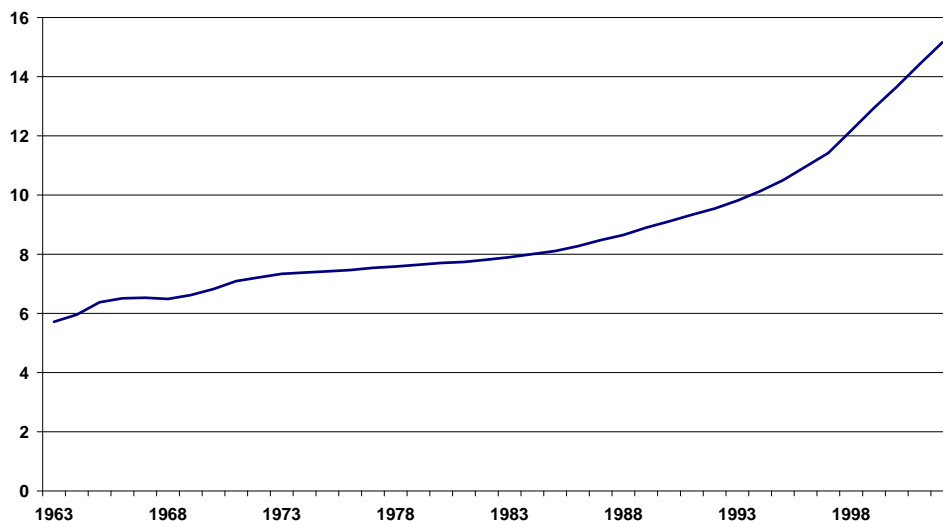


Figure 1: ICT-producing sector's share in total patents granted by the USPTO.

from 7.5 pc. in 1977 to 16 pc. in 2002.<sup>5</sup>

Methodologically, the ICT-producing sector is modeled in the spirit of Romer (1990). This choice is not crucial for the main results, yet it is a closer account for the general purpose or infrastructure nature of these technologies (e.g., circuits and mainframes). The model introduces into a standard economic environment the non-ICT-using sector recognizing that technology adoption is bounded from production features (e.g., hairdressers or agriculture). Finally, it models the production and use of consumption and intermediates, while maintaining assumptions that support the existence of a steady-state path in a multi-sector environment. The interest to focus on the steady-state is driven by United States evidence showing virtually no labour reallocations across the three sectors of the model.<sup>6</sup>

This paper is closely related to the endogenous growth literature that focuses on R&D (e.g., Romer 1990, Aghion and Howitt 1992, Jones 1995), since the incentives for growth on its steady-state path are driven from technological and preference

<sup>5</sup>See also Figure C.1. Patents' data are for non-government institutions and come from Bronwyn H. Hall's website.

<sup>6</sup>The hours shares of the three sectors are virtually constant over the period 1979-2002, and exhibit no clear trends. See Table C.3 for detailed descriptive statistics. In comparison, reallocations across services, manufacturing and agriculture during the same period amount to 10pp.

factors. Its contribution to this literature is to show how inter-sector goods' transactions serve themselves as the means of the technology's growth transmission to the entire economy, when sectors differ in their use of this technology. In this respect, technological and preference factors matter for cross-country sectorial and aggregate growth differences, even when conditioning for the ICT-producing sector's growth.

The paper also relates to the literature that examines the impact of ICT on growth in the context of General Purpose Technologies (GPT).<sup>7</sup> The theoretical and studies in this literature focus on the diffusion process of ICT assuming a broad scope for their adoption (Helpman and Trajtenberg 1998b, Helpman and Trajtenberg 1998a, Jovanovic and Rousseau 2006). This paper contributes to this literature by examining the growth impact of partial adoption of a GPT.

Finally, methodologically the model relates to the multi-sector growth literature (Matsuyama 2005, Ngai and Pissarides 2007, Acemoglu and Guerrieri 2008) that examines the sources of sectorial growth differences and their aggregate implications. This literature largely disregards how inter-sector transactions serve as a means of technology growth transmission.<sup>8</sup>

This paper is organized as follows: Section 2 presents the model. Section 3 analyzes the properties of the unique steady-state, and examines its comparative statics. It also presents in which respects the economy's growth equilibrium differs from the optimal one. Section 4 shows the quantitative performance of the model and discusses its assumptions in view of the evidence. The Section 5 concludes.

## 2 The Model

This section presents the multi-sector economy. There are two sectors that produce distinct final goods, using distinct capital inputs: the ICT-capital using sector (e.g.,

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<sup>7</sup>Economic historians were the first to draw the analogy between ICT and great inventions of the past, such as the combustion engine, electricity and railways, that pioneered the first and second industrial revolutions (David 1991, David and Wright 2003). The features of a GPT, as given by Bekar, Carlaw, and Lipsey (1998), are: "wide scope for improvement and elaboration; applicability across a wide range of uses; potential for use in a wide variety of products and processes; strong complementarities with existing or potential new technologies".

<sup>8</sup>An exception to this is Ngai and Samaniego (2008), who link the production of intermediates to the mechanism transmitting ISTC.

machinery or business services), and the non-ICT-capital one (e.g., food producers or hairdressers). Every variety of ICT-capital variety is developed by the ICT-producing sector (e.g., computers) and manufactured by a monopolist.

The ICT-using and non-ICT-using sectors' final goods are either consumed, or used as intermediates. As consumption goods, they are combined into the "utility basket" that enters the household's preferences. As intermediate goods, they are combined to produce a composite intermediate good. This in turn is the primary input for the production of all capital used in the economy. The production and use of the composite intermediate good is the model-analogue of the input-output cross-sectors' flow of goods.

## 2.1 Consumption side

### 2.1.1 Households

There is a continuum of identical households of size one. The representative household gains utility from a composite consumption good,  $\tilde{C}$ , and its relative risk aversion (CRRA) preferences are

$$\int_0^\infty e^{-\rho t} \frac{\tilde{C}^{1-\sigma} - 1}{1-\sigma} dt; \quad \rho, \sigma > 0. \quad (1)$$

The composite consumption good is a combination ("utility basket") of two distinct consumption goods that are essential for the household: the ICT-using,  $c_1$ , and non-ICT-using one,  $c_0$ ,

$$\tilde{C} \equiv c_0^\theta c_1^{1-\theta}; \quad \theta \in (0, 1). \quad (2)$$

This basket is the numeraire good in this economy.<sup>9</sup>

The labour stock is uniformly distributed across all households, so that each of them offers  $L$  and earns the market wage,  $w_L$ . It also receives the returns on its assets holdings,  $S$ , given the interest rate,  $r_{\tilde{C}}$ . The household uses its resources in order to finance its consumption expenditures,  $C \equiv p_0 c_0 + p_1 c_1$ , and accumulate new

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<sup>9</sup>The choice of the numeraire does not affect any of the basic conclusions of this paper.

assets,  $\dot{S}$ :

$$\dot{S} = r_C S + w_L L - C. \quad (3)$$

## 2.2 Production side

### 2.2.1 Final goods production

#### ICT-using sector

The ICT-using sector uses a fraction,  $u_1$ , of the labour stock and all available varieties  $j$  of ICT-capital,  $x_1(j)$ , where  $j = 0 \dots N$ ;  $N > 0$ , in order to produce the ICT-using good,

$$Y_1 = (u_1 L)^{1-\alpha} \int_0^N x_1^\alpha(j) dj. \quad (4)$$

The ICT-using sector uses exclusively the ICT-capital varieties. The number of available ICT-capital varieties,  $N$ , has the potential to increase over time, due to advances in ICT.

The ICT-using producers maximize profits

$$\max_{L, \{x_1(j)\}_{j \in [0, N]}} \left\{ p_1 Y_1 - w_L L - \int_0^N p_{x_1}(j) x_1(j) dj \right\}, \quad (5)$$

taking their own output unit price,  $p_1$ , wage and ICT-capital varieties' prices,  $\{p_{x_1}(j)\}_{j \in [0, N]}$ , as given.

#### Non-ICT-using sector

The non-ICT-using sector uses a fraction,  $u_0$ , of the labour stock and non-ICT-capital,  $X_0$ , to produce good<sup>10</sup>

$$Y_0 = (u_0 L)^{1-\alpha} X_0^\alpha. \quad (6)$$

The non-ICT-capital good is homogeneous and does not embody any technology. Given its production technology, the non-ICT-using sector has no direct benefit from any ICT advances. The assumption that capital input is sector-specific in its use summarizes the technological differences between the two final good sectors in

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<sup>10</sup>The data support that the two sectors are homogenous in their labour share in production.

their productivity potential. Both the ICT-capital varieties and non-ICT-capital fully depreciate within a period.<sup>11</sup>

The non-ICT-using producers are also competitive. They take their own output unit price,  $p_0$ , the wage and the non-ICT-capital price,  $p_H$ , as given when maximizing profits

$$\max_{L, X_0} \{p_0 Y_0 - w_L L - p_H X_0\}. \quad (7)$$

### 2.2.2 Intermediate goods production

The ICT-using and non-ICT-using sectors' output is used not only as consumption, but also as intermediate good. The intermediates produced by the ICT-using,  $h_1$ , and the non-ICT-using sector,  $h_0$ , are essential inputs in the production of the composite intermediate good,<sup>12</sup>

$$H = h_0^\beta h_1^{1-\beta}; \quad \beta \in (0, 1). \quad (8)$$

The production of the composite intermediate good is perfectly competitive,

$$\max_{L, \{x_1(j)\}_{j \in [0, N]}} \{p_H H - p_1 h_1 - p_0 h_0\}, \quad (9)$$

and its unit price  $p_H$ . The composite intermediate good's production and use summarizes the interaction of the two final good sectors in their production. This is because this good becomes an input in the final goods' production in the form of the ICT and non-ICT-capital.

### 2.2.3 Capital goods production

#### Non-ICT-capital and ICT-capital varieties production

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<sup>11</sup>The assumption that capital goods are non-durables is introduced merely for analytical convenience and wouldn't affect the main results. This implies that capital and intermediates are indistinguishable in the model. Consistent with this, the calibration of the model in Section 4 does the same.

<sup>12</sup>This technology of the composite intermediate good production is common in the literature (Michael 1998, Ngai and Pissarides 2007, Ngai and Samaniego 2008). For a discussion on the assumption regarding the intratemporal preferences and composite intermediate good technology, see Section 4.1.1.

The non-ICT-capital is manufactured in a perfectly competitive environment. Each unit of non-ICT-capital production requires one unit of the composite intermediate good, implying unit price  $p_H$ .

Each ICT-capital variety  $j$  is manufactured by a monopolistic firm that holds the patent providing infinitely-lasting rights on the respective ICT blueprint. For every unit of ICT-capital production, the firm uses one unit of the composite intermediate good. It chooses its production scale and its variety's price,  $p_{x_1}(j)$ , in order to maximize its per-period profits,

$$\pi_1(j) = \max_{p_{x_1}(j), x_1(j)} \left\{ p_{x_1}(j)x_1(j) - p_H x_1(j), \text{ s.t. } p_1 \frac{\partial Y_1}{\partial x_1(j)} = p_{x_1}(j) \right\}, \quad (10)$$

given the variety's demand from the ICT-using sector. There is free-entry in the sector and at any point in time  $t$ , the market value of the firm is

$$V_1(j) = \int_t^\infty e^{-\int_t^\tau r_{\tilde{C}}(s) ds} \pi_1(j)(\tau) d\tau, \quad (11)$$

so that it satisfies the no-arbitrage condition  $r_{\tilde{C}} V_1(j) = \pi_1(j) + \dot{V}_1(t)$ .

### ICT-producing sector

The new ICT blueprints,  $\dot{N}$ , are produced by the ICT-producing sector with the use of a fraction,  $u_N$ , of labour

$$\dot{N} = \lambda N (u_N L). \quad (12)$$

The ICT-producing sector exploits economies of scale due to learning-by-doing: as the number of ICT-capital varieties,  $N$ , increases, more new ICT production ideas and practices become available. The exogenous part of the ICT productivity is  $\lambda$ .<sup>13</sup>

## 2.3 Equilibrium and market clearing conditions

The equilibrium conditions are derived given all competitive prices and interest rate. Details on the analytical solution are found in Appendix A.1. On the consumption

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<sup>13</sup>With this specification, a blueprint stands for a new ICT product (e.g. Pentium III), rather than quality upgrade within a technology class. Allowing for a more general R&D function like in Jones (1995) does not affect the steady-state properties of the model.

side, the household maximizes utility (1)-(2), under its budget constraint (3). Its intratemporal allocation decision is the relative demand of the two consumption goods

$$\frac{c_1}{c_0} = \frac{1 - \theta}{\theta} \frac{p_0}{p_1}, \quad (13)$$

while its intertemporal allocation in units the composite consumption good follows the standard CRRA rule  $\frac{\dot{\tilde{C}}}{\tilde{C}} = \frac{1}{\sigma} (r_{\tilde{C}} - \rho)$ .<sup>14</sup>

On the production side, the ICT-using sector's demand for labour,  $w_L = (1 - \alpha) \frac{p_1 Y_1}{u_1 L}$ , and ICT-capital varieties,  $p_{x_1}(j) = p_1 \alpha (u_1 L)^{1-\alpha} x_1^{\alpha-1}(j)$  for every  $j$ , maximizes its profits (5). Similarly, the non-ICT-using sector demands labour,  $w_L = (1 - \alpha) \frac{p_0 Y_0}{u_0 L}$ , and non-ICT-capital,  $p_H = \alpha \frac{p_0 Y_0}{X_0}$ , to maximize profits in (7). The relative demand for the two final good sectors' intermediates,

$$\frac{h_1}{h_0} = \frac{1 - \beta}{\beta} \frac{p_0}{p_1}, \quad (14)$$

satisfies (9). The composite intermediate good that they produce, is then used as the primary input for the production of all non-ICT-capital and ICT-capital,  $X_1 \equiv \int_0^N x_1(j) dj$ , produced by the monopolists. The monopolists maximize profits in (10), which results in symmetry in their decision to produce  $x_1 = \alpha^{\frac{2}{1-\alpha}} \left( \frac{p_1}{p_H} \right)^{\frac{1}{1-\alpha}} (u_1 L)$  at a price  $p_{x_1} = \frac{p_H}{\alpha}$  above their marginal cost of production. Their profit flow is  $\pi_1 = \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} p_1^{\frac{1}{1-\alpha}} p_H^{\frac{-\alpha}{1-\alpha}} (u_1 L)$ , while the market value of producing an ICT-capital variety is given by (11). The ICT-producing sector's competitive demand for labour equals its marginal product,  $\lambda N$ , valued according to the new blueprint for the new ICT-capital variety monopolist.

The equilibrium price vector  $\{w_L, r_{\tilde{C}}, p_0, p_1, p_H, p_{x_1}\}$  and optimal allocations ensure that all the ICT-using and non-ICT-using sectors' output is used as consumption or intermediates

$$Y_1 = c_1 + h_1, \quad (15)$$

$$Y_0 = c_0 + h_0, \quad (16)$$

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<sup>14</sup>The optimal allocation of resources for the household intratemporally ensures that at any point in time  $p_{\tilde{C}} \tilde{C} = C$ , where  $p_{\tilde{C}} = \Theta p_0^\theta p_1^{1-\theta} \equiv 1$  and  $\Theta = \Theta(\theta)$  is a constant.

the composite intermediates good is used either as input for the non-ICT-capital and ICT-capital,

$$H = X_0 + X_1, \quad (17)$$

all capital,  $X_0 + X_1$ , and labour

$$L = u_1L + u_0L + u_NL, \quad (18)$$

are used in production. The marginal monopolist entering ICT-capital production breaks even, implying that the market value of a new blueprint is  $V_1$  (and thereby  $w_L = \lambda NV_1$  labour demand in ICT-producing sector).

The household owns all monopolistic firms through its assets,  $S = NV_1$ , that give claims on their market value. Hence, the household collects aggregate profits,  $N\pi_1$ , as dividends and its budget constraint (3) is rewritten as the economy's aggregate resource constraint,  $Y = C + p_H H$ , where  $Y \equiv p_0 Y_0 + p_1 Y_1$  is the value of aggregate final good production.<sup>15</sup>

### 2.3.1 Equilibrium prices and labour allocations

The composite intermediate good market clearing condition determines its price

$$p_H = B p_0^\beta p_1^{1-\beta}, \quad (19)$$

where  $B = B(\beta)$  is a constant. This result shows that the price of the composite intermediate good, non-ICT-capital and ICT-capital (since  $p_{x_1} = \frac{p_H}{\alpha}$ ) are functions of the relative prices of the two final goods,  $\frac{p_1}{p_0}$ .<sup>16</sup> Therefore, any changes in  $\frac{p_1}{p_0}$  over time pass through in the prices of intermediate and capital.

The relative prices of the two final good sectors are determined from the labour market clearing condition that requires the value of marginal product of labour is

<sup>15</sup>This uses also the equilibrium labour income, given the value of the equilibrium ICT-using and non-ICT-using sectors' output  $(p_1 Y_1)^* = p_1 \alpha^{\frac{2\alpha}{1-\alpha}} \left(\frac{p_1}{p_H}\right)^{\frac{\alpha}{1-\alpha}} (u_1 L) N$  and  $(p_0 Y_0)^* = p_0 \alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{p_1}{p_H}\right)^{\frac{\alpha}{1-\alpha}} (u_0 L)$  respectively.

<sup>16</sup>This is because  $p_H = B p_0 \left(\frac{p_1}{p_0}\right)^{1-\beta}$ , while  $p_0$  is pinned down by the numeraire,  $\frac{1}{p_0} = \Theta \left(\frac{p_1}{p_0}\right)^{1-\theta}$ .



the same across all three sectors using it.

**Lemma 1** *The final goods' relative prices equal the inverse of their relative labour productivity levels*

$$\frac{p_1}{p_0} = \frac{1}{\alpha^\alpha N^{1-\alpha}}. \quad (20)$$

**Proof.** See Appendix A.1.1. ■

The productivity differential between the two final good sectors derives entirely on the productivity differential of the capital used by either sector. The productivity advantage that the ICT-capital provides comes in the form of a labour augmenting technology  $N^{1-\alpha}$ . However, due to its monopolistic pricing there is a downward bias in its final demand from the ICT-using sector, that implies a downward bias in its level productivity (but not in the rate).<sup>17</sup>

**Corollary 2** *On any equilibrium path with ICT growth,  $g_N \equiv \frac{\dot{N}}{N} > 0$ , prices of the ICT-using and composite intermediate goods fall over time*

$$\frac{\dot{p}_1}{p_1} - \frac{\dot{p}_0}{p_0} = -(1-\alpha)g_N < 0, \quad (21)$$

$$\frac{\dot{p}_H}{p_H} - \frac{\dot{p}_0}{p_0} = -(1-\beta)(1-\alpha)g_N < 0, \quad (22)$$

*relative to the non-ICT-using good's price. The latter increases over time*

$$\frac{\dot{p}_0}{p_0} = (1-\theta)(1-\alpha)g_N > 0. \quad (23)$$

**Proof.** See Appendix A.1.2. ■

The increasing price of the non-ICT-using good reflects its rising consumption opportunity cost in the presence of the falling prices of the ICT-using consumption good. The more important is the non-ICT-using good in the utility basket that the household wants to consume, i.e., the higher is its consumption expenditure share  $\theta$ , the lower is the opportunity cost related to not consuming the cheaper ICT-using good.

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<sup>17</sup>Given (20) see that the equilibrium marginal value of labour and thereby wage is  $w_L = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}p_0^{\frac{1}{1-\alpha}}p_H^{\frac{-\alpha}{1-\alpha}}$ , even though in real terms,  $\frac{\partial Y_1}{\partial u_1 L} > \frac{\partial Y_0}{\partial u_0 L}$ .

The labour market clearing condition also determines the incentives for development of a new ICT blueprint.

**Corollary 3** *On any equilibrium path with ICT growth,  $g_N > 0$ , the rate of returns that a new ICT-capital variety monopolistic firm receives on its value,  $V_1$ , is*

$$r_{\bar{C}} = \alpha \lambda u_1 L - [\alpha \beta + \theta (1 - \alpha)] g_N, \quad (24)$$

**Proof.** See Appendix A.1.3. ■

This result highlights the impact of the multi-sector production structure of the economy and multi-product consumption of the household on the ICT-producing sector's size. The first term in (24), reflects the profit rate received by an ICT-capital variety monopolistic producer that generates the incentives to invest in ICT. This corresponds to a fraction,  $\alpha$ , of the total productivity return of a new variety into the ICT-using sector. The second term in (24), accounts for the change in the value of the monopolistic firm over time,  $\frac{\dot{V}_1}{V_1} = \frac{\dot{\pi}_1}{\pi_1} = \frac{1}{1-\alpha} \frac{\dot{p}_1}{p_1} - \frac{\alpha}{1-\alpha} \frac{\dot{p}_H}{p_H}$ , in terms of the composite intermediate good that enters the household's utility. Given the results of Lemma 2, this corresponds to capital losses. This is because advances in ICT have a twofold impact on the value of the productive assets. There is a direct negative impact from the high productivity of the ICT-capital that induces the falling ICT-using goods' prices. There is also an indirect positive impact that comes from the use of the cheaper ICT-using intermediates and consumption goods. These indirect benefits are not strong enough, because the non-ICT-using intermediates and consumption goods are also essential, reducing the capital gains from the falling ICT-using goods prices.<sup>18</sup>

To conclude with the decentralized economy's equilibrium results, the labour market allocations across the two final good sectors

$$\begin{aligned} \frac{u_1}{u_0} &= \frac{\Sigma}{1 - \Delta}, \\ \Sigma &\equiv (1 - \beta) \alpha + (1 - \theta)(1 - \alpha), \Delta \equiv \alpha^2 \beta + \theta(1 - \alpha^2). \end{aligned} \quad (25)$$

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<sup>18</sup>To see this, (24) may be alternatively expressed in the following three terms:  $r_{\bar{C}} = \alpha \lambda u_1 L + (1 - \alpha)(1 - \theta) g_N - [1 - \alpha(1 - \beta)] g_N$ . Also, as an illustration note that when  $\theta = \beta = 0$ , then  $r_{\bar{C}} = \alpha \lambda u_1 L$ , while for  $\theta = \beta = 1$ , then  $r_{\bar{C}} = \alpha \lambda u_1 L - g_N$ .

is constant, despite the time-varying relative prices (20).<sup>19</sup> This results from equating the marginal rate of transformation between the two final goods to their marginal rate of substitution in the household's utility, when markets clear (15)-(18). The unit intratemporal elasticity of substitution of the two final goods in both intermediates' production and household's utility basket, ensures that the consumption and production substitution patterns of the two goods do not induce reallocation of labour across the two sectors.

### 3 Steady-State Analysis

A Constant Growth Path (CGP) is a steady-state equilibrium path on which the ICT-production stock,  $N$ , value of aggregate output,  $Y$ , aggregate capital,  $X \equiv p_H X_0 + p_H X_1$ , and aggregate consumption,  $C$ , grow at a constant rate.<sup>20</sup>

#### 3.1 Steady-state properties

In view of the falling relative prices of the ICT-using good and the rising ones of the non-ICT-using good (see (21)-(23)), a CGP with  $g_N > 0$  exists because of the unit intratemporal elasticity of substitution between the two final goods as either consumption or intermediate goods. These bilateral assumption excludes cases that the consumption side substitution pattern between the two goods do not match with their production side substitution as intermediates that would violate the market clearing conditions over time.<sup>21</sup>

As a result of these assumptions, the size of the non-ICT-using sector in value terms is never trivial given its constant expenditure share as consumption,  $\theta$ , or intermediate,  $\beta$ , good. Namely, this good is "essential".

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<sup>19</sup>Details on the derivation of (25) and the remaining general equilibrium results are found in Appendix A.1.4.

<sup>20</sup>Given the choice of the numeraire, the CGP in terms of  $C$ , is equivalently in terms of the composite consumption good,  $\tilde{C}$ .

<sup>21</sup>While the specification for the composite intermediates production, (8), is more standard, the one on preferences, (2), is rather non-standard. Appendix A.4, shows that the unit intratemporal elasticity in consumption is a necessary and sufficient condition for the existence of a CGP.

**Proposition 4** *On the CGP, the ICT-producing sector grows endogenously at a rate<sup>22</sup>*

$$g_N^d = \lambda L \frac{\alpha \Sigma - \frac{\rho}{\lambda L} (\Sigma + 1 - \Delta)}{\alpha \Sigma + [1 - (1 - \sigma) \Sigma] (\Sigma + 1 - \Delta)}. \quad (26)$$

*The ICT-producing sector drives capital accumulation, intermediates, output and consumption growth at the sector-level*

$$\frac{\dot{H}}{H} = \frac{\dot{X}_0}{X_0} = \frac{\dot{X}_1}{X_1} = (1 - \beta) g_N^d, \quad (27)$$

$$\frac{\dot{Y}_0}{Y_0} = \frac{\dot{h}_0}{h_0} = \frac{\dot{c}_0}{c_0} = \alpha (1 - \beta) g_N^d, \quad (28)$$

$$\frac{\dot{Y}_1}{Y_1} = \frac{\dot{h}_1}{h_1} = \frac{\dot{c}_1}{c_1} = (1 - \alpha \beta) g_N^d, \quad (29)$$

*and aggregate economy level*

$$\frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{C}}{C} = (1 - \theta) (1 - \alpha) g_N^d + \alpha (1 - \beta) g_N^d. \quad (30)$$

**Proof.** See Appendix A.1.5. ■

The result in (30) shows the engine of aggregate growth are the advances in ICT. It highlights though the importance of the ICT-using sector as a growth transmitting sector, i.e., given  $g_N^d$ . In particular, the second term shows that aggregate growth is driven by the capital deepening induced by the falling ICT-using goods' relative prices (see (27) when capital share is  $\alpha$ ). The first term in (30) accounts for the benefit of the household from consuming the cheaper over time ICT-using good.

The benefits from the ICT-using goods' falling prices are higher the lower are  $\beta$  and  $\theta$ , i.e., the role of the non-ICT-using good as intermediate and consumption respectively. It is worth pointing out that the importance of the non-ICT-using sector's goods matters not only for the strength of the growth transmission mechanism, but also for the engine of growth itself, as (26) is itself a function of  $\beta$  and  $\theta$ . Their impact on the ICT-producing sector is discussed in detail in Section 3.2.

On the CGP path, the aggregate consumption to output and capital to output ratios are constant. The consumption to output and capital to output ratios are

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<sup>22</sup>The sufficient condition for an interior solution is that:  $L > \bar{L}(\theta, \alpha, \lambda, \rho)$ , i.e., the labour stock exceeds a lower bound.

also constant within every final good sector, but different across them, seen in (28) and (29).

**Corollary 5** *At the sector-level, the ICT-producing sector is the fastest growing sector in the economy, followed by the ICT-using sector and lastly by the non-ICT-using sector,*<sup>23</sup>

$$g_N^d > (1 - \alpha\beta) g_N^d > \alpha(1 - \beta) g_N^d.$$

**Proof.** This is straightforward from (28)-(29) for  $\alpha, \beta \in (0, 1)$ . ■

The ICT-producing sector grows due to the positive externalities present in its production. Its output growth would be fully accounted as TFP growth. Both final good sectors grow due to capital accumulation.

In particular, the ICT-using sector is the only final good sector that benefits directly from the ICT-production. Its growth would equal the ICT-producing sector's growth, only if the ICT-using sector was the only capital producing sector in the economy (i.e., for  $\beta \rightarrow 0$ ). The (indirect) use of the relatively expensive non-ICT-using good for the production of capital increases the ICT-capital's cost in terms of the ICT-using good, since  $\frac{\dot{p}_H}{p_H} - \frac{\dot{p}_1}{p_1} = \beta(1 - \alpha) g_N > 0$ . While this discourages capital deepening, the availability of more ICT-capital varieties over time provides with productivity benefits that are sufficiently high to preserve growth driven by capital accumulation.

The productivity benefits of the ICT-capital are absent for the non-ICT-using sector, which implies a constant growth advantage for the ICT-using sector. Nevertheless, this sector benefits indirectly from them and its growth is still driven by capital accumulation due to the falling prices of the non-ICT-capital in terms of the non-ICT-using good, by (22).<sup>24</sup>

To conclude with the steady-state properties, constant real interest rate on this

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<sup>23</sup>However, in units of the composite consumption both final good sectors have the same value growth rate as the aggregate economy's value growth,  $\frac{\dot{p}_0}{p_0} + \frac{\dot{Y}_0}{Y_0} = \frac{\dot{p}_1}{p_1} + \frac{\dot{Y}_1}{Y_1} = \frac{\dot{Y}}{Y}$ .

<sup>24</sup>To illustrate the importance of the ICT-using good as intermediate, note that when  $\beta \rightarrow 1$ , the the growth potential of the non-ICT-using sector is eliminated. The ICT-using sector though still grows, but purely due to the horizontal expansion of its capital varieties.

path ensures that labour allocations are constant in all sectors, with

$$u_1^d = \frac{\Sigma \left[ \frac{\rho}{\lambda L} + 1 - (1 - \sigma)\Sigma \right]}{\alpha \Sigma + [1 - (1 - \sigma)\Sigma] (\Sigma + 1 - \Delta)},$$

and  $u_0^d = \frac{1-\Delta}{\Sigma} u_1^d > 0$  by (25), which ensures that in equilibrium the size of both final good sectors is non-trivial despite the undergoing substitution towards the ICT-using sector's consumption and intermediate goods.

**Corollary 6** *In value terms, the size of the ICT-using sector relative to the non-ICT-using is constant and determined by the respective relative labour allocations,  $\frac{u_1}{u_0}$ , in (25).*

**Proof.** See Appendix A.1.6. ■

This result suggests that in value terms the size of the non-ICT-using sector is relatively bigger, the more important is its output as intermediate (the higher is  $\beta$ ) and/or consumption good (the higher is  $\theta$ ).<sup>25</sup>

## 3.2 Comparative statics

**Lemma 7** *The growth rate of the economy is higher and the labour shares in the two final goods' sectors are lower, the more patient the agents in the economy are, i.e., the lower  $\rho$  is, the higher the intratemporal elasticity of substitution, i.e., the lower  $\sigma$  is, and the more productive the ICT-producing sector is, i.e., the higher  $\lambda$  is. The effect of higher intermediate output elasticity of the non-ICT-using good,  $\beta$ , expenditure share of the non-ICT-using good,  $\theta$ , or higher output elasticity of capital,  $\alpha$ , is ambiguous and depends on the parameters of the model.*

**Proof.** See Appendix A.2. ■

A patient household is more willing to substitute current with future consumption. The additional savings direct resources to the ICT-producing sector. This is because as asset demand increases, interest rate goes down increasing the value of ICT-capital monopolistic firms. This provides incentives for higher ICT-production

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<sup>25</sup>From (25),  $u_0^d > u_1^d$  when  $\beta > \frac{1-[2-\alpha(1+\alpha)]\theta}{\alpha(1+\alpha)}$ . The role of  $\alpha$  is non-linear and depends on  $(\theta, \beta)$ .

by increasing the labour input. An increased productivity in the ICT-producing sector would have similar effect, with both direct productivity and indirect labour allocation increase effect. In either case, there is a level effect of growth, while the relative size of the two final good sectors stays unaffected.

Higher preference towards the non-ICT-using consumption good increases the marginal utility of consumption of the non-ICT-using good, which induces an increase in the labour allocation of this sector (by reducing it in both other sectors). The same would be the outcome of higher importance of the non-ICT-using good in the production of intermediates, since that implies higher marginal product for its intermediates. In either case though, reducing the resources and thereby growth of the ICT-producing sector also reduces the interest rate that consumers receive on their assets (as the price effect becomes weaker). This reduces incentives to direct resources to the non-ICT-using sector, depending on the household's willingness to deviate from consumption smoothing. For unit intertemporal elasticity of substitution, this second effect is eliminated.<sup>26</sup>

The effect of the output elasticity of capital is in principle nonlinear and interacts with  $\beta$  and  $\theta$ . When  $\beta = \theta$ , so that the two final goods are not distinct in their use as consumption or intermediates, then higher output elasticity of capital implies higher growth. This is because this implies lower mark-up and thereby downward distortion for the ICT-capital demand, inducing a positive market effect that increases incentives to direct resources in the ICT-producing sector. However, relaxing the monopolistic friction has no impact on the relative size of the two final goods sectors. This is because relative allocations become then a function only of the total expenditure shares of the two final goods, i.e.,  $\frac{u_1}{u_0} = \frac{1-\beta}{\beta} = \frac{1-\theta}{\theta}$ . Further details on the impact of higher output elasticity of capital are found in Appendix A.2. A more detailed discussion on the role of the monopolistic frictions in the allocation of resources across the final good sectors is in Section 3.3 that follows.

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<sup>26</sup> Also, these results would be complementary, i.e., the higher is  $\theta$ , then the higher is the negative effect of  $\beta$  on growth, and vice versa.

### 3.3 Optimal constant growth path

**Proposition 8** *The social planner maximizes long-run utility and achieves higher long-run growth than the decentralized economy,  $g_N^{SP} > g_N^d$ .*

*The first best allocations differ from the decentralized economy's ones in the following respects: The social planner chooses higher production scale for each ICT-capital variety,  $x_1^{SP} > x_1^d$ , and labour resources in the ICT-producing sector,  $u_N^{SP} > u_N^d$ .*

*The comparison of  $u_0^{SP}$  with  $u_0^d$  and  $u_1^{SP}$  with  $u_1^d$  depends on the model's parameters:*

*If  $\theta = \beta$ , then  $\frac{u_1^{SP}}{u_0^{SP}} = \frac{u_1^d}{u_0^d}$ , while  $u_0^{SP} < u_0^d$  and  $u_1^{SP} < u_1^d$ .*

*If  $\theta > (<) \beta$ , then  $\frac{u_1^{SP}}{u_0^{SP}} > (<) \frac{u_1^d}{u_0^d}$ , because  $u_0^{SP} < u_0^d$  ( $u_1^{SP} < u_1^d$ ), while the comparison of  $u_1^{SP}$  with  $u_1^d$  ( $u_0^{SP}$  with  $u_0^d$ ) is ambiguous and depend on the remaining parameters of the model.*

**Proof.** See Appendix A.3. ■

The result that the social planner maximizes long-run growth is driven from the fact that the social planner acknowledges that the ICT-producing sector is the engine of growth. The frictions in ICT-production and the monopolistic pricing of ICT-capital varieties induce incentives that fail to do so. Therefore, fewer than optimal resources are allocated in the ICT-producing sector.

The difference between the optimal and decentralized economy's long-run growth is driven from the gap in their returns on assets. Since the social planner internalizes the learning-by-doing externalities in the ICT-producing sector, he accounts fully for the returns of a new ICT good on both its production and use:  $1 - u_0$  (see (94) in Appendix A.3). On the contrary, the market economy fails to do so and the returns on assets are only a function of  $u_1$  and are downward biased due to monopolistic frictions (see (24)).

In addition, for any given growth rate there are returns related to the falling value of the productive assets in terms of the non-ICT-using sector over time. The market prices correctly account for the rate of change of these prices (despite the static monopoly distortion). However, the market prices do not fully capture the utility impact of the increasing prices of the non-ICT-using good, i.e., the decreasing over time utility from consuming more of the ICT-using good. Unlike the market



incentives, the social planner fully accounts for the role of ICT-using sector as the transmitter of growth and the impact of this on long-run utility. Also, due to its lower growth, the negative effect of the changing prices on total asset return is alleviated in the decentralized economy. This offsetting secondary effect though is not sufficiently high to induce optimal growth.

The results also show that apart from the distortion in labour allocation in the ICT-producing sector, there can exist a wedge between the optimal and decentralized allocation of labour across the two final good sectors. This wedge is due to the monopolistic frictions in ICT-capital production. The direction and degree of the bias they cause in the relative sizes of the two final good sectors depends on whether each good is used relatively more for consumption or intermediate production. This is because demand affects the production specialization and accordingly input choices. The economy (whether the social planner's or the decentralized one) directs more resources to support the sector producing a good used more intensively as consumption good, given the total supply of intermediates at any point in time.

For example, when the non-ICT-using good demand is stronger as consumption good, i.e.,  $\theta > \beta$ , then its demand for capital is relatively high. The reverse is true for the ICT-using good (since  $1 - \theta < 1 - \beta$ ) and due to the monopolistic distortion in ICT-capital production, its demand is biased downwards disproportionately more than what would be optimal. In turn, the capital input use affects labour input. As a result, there are more than optimal resources into the consumption good intensive non-ICT-using sector. If instead each of the final goods was equally used as consumption or intermediate,  $\theta = \beta$ , then the monopolistic frictions would not distort the relative size of the sectors. Therefore, this distortion in allocations is entirely due to the interaction of frictions with the technology and preferences. Ultimately, it affects long-run growth, because the size of the ICT-using sector provides the market incentives to invest in new ICT-capital varieties.

It is finally worth noting that the comparative statics for the social planner's equilibrium CGP are qualitatively similar to the ones of the decentralized economy, but optimal growth is unambiguously lower in response to higher  $\beta$  or  $\theta$ . This highlights that the social planner's decision to direct resources into ICT production

come from the use of the ICT-using sectors' goods. This is because optimal growth fully accounts for the role of the ICT-using sector in transmitting ICT growth of the entire economy.<sup>27</sup>

## 4 Quantitative Results

As in the theoretical model, the industries are grouped into three major sectors: ICT-producing, ICT-using and non-ICT-using. See Appendix B for precise sources and definitions of the data and methodology applied in aggregation, and Table C.4 for detailed list of the industries in each major sector.

The model predicts an equilibrium CGP that features differential output growth across its three sectors. Table 4 presents the real value added growth for the total economy and ICT-producing, ICT-using and non-ICT-using sectors and its sources (capital, labour and TFP growth).<sup>28</sup> The ranking across sectors in terms of output growth is the one predicted by the model.<sup>29</sup> In addition, Table 4 shows that the value added growth for both the ICT-using and the non-ICT-using sector is driven mostly by capital accumulation supports the model's account for growth.<sup>30</sup> The ability of the model to capture the accounted output growth differentials is discussed in the following Section, which assesses the quantitative performance of the model via calibration.

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<sup>27</sup>Numerical results using the calibrated parameters of Section 4.1, suggest that the gap between the optimal and market outcomes in terms of  $g_N$  decreases for very high levels of  $\beta$  and/or  $\theta$ . For the aggregate economy though, the direct effect of  $\beta$  and  $\theta$  suffice for the gap to be decreasing in  $\beta$  and  $\theta$ .

<sup>28</sup>Calculations are by the author. Any differences to Tables 8.1 and 8.2 of Jorgenson, Ho, and Stiroh (2005) are due to rounding and limitations in the available data. Details on the data and the aggregation method used are in Appendix B.

<sup>29</sup>Gross-output based growth accounting delivers the same pattern and is presented in Table C.2.

<sup>30</sup>Complementary growth accounting exercises (van Ark and O'Mahony 2003, van Ark, Melka, Mulder, Inklaar, and Ypma 2002) investigate the sources of United States and European Union growth by looking at three sectors with the same broad structure as in this paper. They confirm the high productivity growth in the ICT-producing sector and find important gains in productivity that stem from it for all sectors. The benefits are mainly for the ICT-using industries.

Table 4: United States aggregate and sector-level value added sources of growth

	Value added growth	Sources:		
		Capital	Labor	TFP
Total Economy	3.18	1.74	1.17	0.28
ICT-producing	20.42	4.06	3.40	12.97
ICT-using	4.04	2.33	1.68	0.03
non-ICT-using	2.38	1.46	0.92	0.00

Notes: 1977-2000 average growth rate (%)

Source: Jorgenson, Ho, and Stiroh (2005)

Table 5: Calibrated parameters of the model

$\alpha$	$\beta$	$\theta$	$L$	$\lambda$	$\rho$	$\sigma$
0.61	0.643	0.793	1	1.03	0.028	1

## 4.1 Calibration exercise

In what follows, in order to be consistent with the theoretical model, capital and intermediates are grouped together.<sup>31</sup> The analysis that follows treats the entire 1977-2001 period of study as a steady-state. The following discussion argues that this appears to be a good approximation of the United States' growth experience. Table 5 presents the model's parameters that were calibrated to match moments in the data.

In particular, the share of capital for the final good sectors,  $\alpha$ , is calibrated to match the labour input (employees' compensation) share in total inputs used by the ICT-using and non-ICT-using sectors. The share of the non-ICT-using sector in the production of intermediates and capital is used to calibrate parameter  $\beta$  of the model. This is calculated by Table 6, that presents the cross-sector commodity flows, excluding any transactions with the ICT-producing sector. The expenditure share of the non-ICT-using consumption good is matched from the 1979-2001 average consumption expenditure share for this good as calculated by the NIPA Tables, excluding expenditures on ICT-products. The ICT-producing sector's productivity parameter  $\lambda$  is chosen to match the 13pc average annual TFP growth of the ICT-

<sup>31</sup>Focusing only on intermediates makes almost no difference for the calibrated parameters and the final results. This is plausible given that intermediates and capital's share in the gross output of each industry is 50% and 10% respectively.

Table 6: United States final goods' transactions of capital and intermediates

Shares of capital and intermediates produced/used by:			aggregate production share
	ICT-using	non-ICT-using	
ICT-using	14.3	21.4	35.7
non-ICT-using	10.5	53.8	64.3
aggregate use share	24.8	75.2	100

Notes: matrix entries sum up to 100%

aggregate production and use shares sum matrix rows and columns respectively

Source: BEA, 1997 Benchmark Input Output Use and Capital Flow Tables

producing sector.<sup>32</sup> Finally, parameters  $\rho$  and  $\sigma$  are standard (e.g., Attanasio and Weber 1989), while  $L$  is a normalization.

Column (2) of Table 7 presents the model's predictions for allocations of inputs and goods, within and across the two final good sectors for its baseline calibration. The implied cross-sector ranking is in-line with the evidence in column (1).<sup>33</sup> The model performs reasonably well in predicting all remaining allocations, even though it delivers higher intensity in consumption goods' production for both sectors.

Without using of any information on sectorial use of labour in its calibration, the model predicts very closely the relative labour allocations in the two sectors. Its implication for the ranking across all three sectors in terms of their size is also consistent with the evidence. Turning to its predictions for the size of each sector, it predicts reasonably well the ICT-using sector's labour allocation of the ICT-producing sector. However, it predicts too high labour allocation for the ICT-producing sector, which is the outcome of an approximately 10pc. downward bias in the allocations of both final good sectors, given the big scale of the non-ICT-using sector.

Finally, columns (3)-(6) present the sensitivity of the model's predictions to the two key parameters relating to sector-specific features,  $\beta$  and  $\theta$ , when considering 10pp. deviations from their baseline calibration (*ceteris paribus*). The baseline results are shown to be broadly robust to the alternative parameters.<sup>34</sup> Labour

<sup>32</sup>Average growth in patents granted for the ICT-producing sector is also 0.13 in the data. The productivity parameter is calibrated implicitly using the model's steady-state result:  $g_N^d = \lambda L \frac{\alpha \Sigma - \frac{\rho}{\lambda L} (\Sigma + 1 - \Delta)}{\alpha \Sigma + [1 - (1 - \sigma) \Sigma] (\Sigma + 1 - \Delta)}$ .

<sup>33</sup>Employment shares are averages for the 1977-2002 period. All other data are taken from the Benchmark Input-Output and Capital Flows Tables for 1997.

<sup>34</sup>The implied elasticities of allocations with respect to  $\theta$  and  $\beta$  are for most cases below one. The exceptions are the ones of  $c_0/Y_0$ ,  $p_H X_0/p_0 h_0$  and  $h_1/Y_1$ ,  $c_1/h_1$ , where the latter two regard

shares in all sectors are not sensitive to changes in  $\beta$  and  $\theta$ , and this is more so for the ICT-producing sector (elasticity of  $u_N$  is 0.04). This robustness exercise is also suggestive about the scope for variation in  $\beta$  and  $\theta$  to account for sectorial size.

Table 7: Baseline calibration of the United States economy 1970-2000

Sectorial allocations	Data	Model's prediction				
		Baseline	Alternative parameters			
			$\beta = 0.543$	$\beta = 0.743$	$\theta = 0.693$	$\theta = 0.893$
(1)	(2)	(3)	(4)	(5)	(6)	
$c_0/h_0$	0.853	1.045	1.299	0.859	0.953	1.128
$c_1/h_1$	0.445	0.491	0.403	0.648	0.761	0.244
$h_0/Y_0$	0.448	0.489	0.435	0.538	0.512	0.470
$h_1/Y_1$	0.602	0.671	0.713	0.607	0.568	0.804
$p_H X_0/p_0 h_0$	1.170	1.247	1.402	1.134	1.191	1.298
$p_H X_1/p_1 h_1$	0.694	0.555	0.522	0.613	0.655	0.463
$u_1/u_0$	0.401	0.405	0.514	0.307	0.500	0.324
$u_0$	0.70	0.622	0.565	0.685	0.571	0.672
$u_1$	0.28	0.252	0.290	0.210	0.286	0.218
$u_N$	0.02	0.126	0.149	0.104	0.146	0.106

#### 4.1.1 Discussion of modeling assumptions

**The production and use ICT** The systematic over-prediction of the labour allocation for the ICT-producing sector is driven from the calibration of the ICT-producing sector's productivity  $\lambda$ . This also accounts for the model's performance in predicting sectorial output growth rates. In particular, while the model predicts fairly well the non-ICT-using sector's growth at 2.28pc., when average gross output growth is 2.22 (see Table 8), it over-predicts its growth gap from the ICT-using sector and thereby growth of the latter. Nevertheless, its predicted growth for the aggregate economy is broadly in line with the data, being 3.88pc.<sup>35</sup>

The model's predictions for labour allocation and output growth rates would be improved, when accounting for the "effective" ICT growth (i.e., varieties' expansion changes in  $\theta$  only).

<sup>35</sup>There is a vivid discussion in the literature regarding matching multi-sector growth models to the aggregate economy (e.g., Whelan 2003, Ngai and Samaniego 2008). In the theoretical model, aggregate value added growth is matched by consumption growth. It is worth pointing that even when treating  $X$  as intermediates and deriving the "aggregate value added production function" gives the same result, i.e., the production-side modeling of capital as non-durable is not important.

in the model). First, not all innovations in the ICT-producing sector are directly embodied in ICT-capital. Second, the model’s assumption of no innovation in the non-ICT-capital is clearly an abstraction, in view of the evidence reviewed in the introduction. Third, the empirical literature suggests important adjustment and learning costs related to ICT-capital use (e.g., Basu, Fernald, Oulton, and Srinivasan 2003).<sup>36</sup> It is also straightforward that the model’s quantitative performance would improve, if modified to allow both sectors to use both types of capital at different intensities.<sup>37</sup>

**The multi-sector structure** The ability of the model to capture features of the United States’ growth experience critically depends not only on modeling both the ICT-using and non-ICT-using sectors, but also on modeling the use of both their intermediates and consumption goods. Table 8 shows this by contrasting the baseline model’s predicted labour allocations and aggregate growth, row (1), with the ones resulting from alternative assumptions for the multi-sector economy. Rows (2) and (3) show that the non-ICT-using output is distinct as both consumption and intermediate good is a necessary condition for the model to capture features of the United States economy. Row (4) shows the role of the ICT-using sector in transmitting growth. Row (5) shows that the baseline model outperforms the stylized Romer (1990) model, since introducing the non-ICT-using sector is crucial to deliver in the steady-state an ICT-using growth rate lower than the ICT-producing one.

Finally, the first best allocations’ outcomes, row (7), point to two interesting results. First, as common feature of the endogenous growth models, e.g., row (8), they highlight the role of the technology producing sector of the economy (results suggest tenfold reductions in the final good sectors’ allocations). Second, the suggested relative labour allocations across the final good sectors that are not far from the market ones, even though the results point to an allocation biased towards increasing the non-ICT-using sector’s size.

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<sup>36</sup>As an illustration, the average annual growth in new ICT patents was 13pc, while that for non-ICT was 10pc. Taking the extreme approach that all of the latter is for non-ICT-using sector, the implied growth gap between the sectors is only 1pc.

<sup>37</sup>Similarly, the model would be enriched by distinguishing between skilled and unskilled labour.

Table 8: Alternative economic structure assumptions

Alternative modeling assumptions/Data	$u_0$	$u_1$	$u_N$	$u_1/u_0$	$\dot{Y}/Y$
(1) $\beta = 0.643, \theta = 0.793$ ( <i>Baseline</i> )	0.622	0.252	0.126	0.405	0.04
(2) $\beta \rightarrow 0, \theta = 0.793$	0.324	0.450	0.226	1.387	0.09
(3) $\beta = 0.643, \theta \rightarrow 0$	0.265	0.528	0.208	2.540	0.08
(4) $\beta, \theta \rightarrow 1$	0.999	0.001	0.000	0.001	0.00
(5) $\beta, \theta \rightarrow 0$ ( <i>Romer</i> )	-	0.334	0.666	-	0.13
(6) <i>First best</i> ( <i>Baseline</i> )	0.064	0.027	0.911	0.426	0.28
(7) <i>First best</i> ( <i>Romer</i> )	-	0.027	0.973	-	1.00

**The steady-state analysis** As discussed already in the introduction, there is evidence on virtually constant labour allocations across the three sectors of the model (see Table C.3). In the model, the need to account for this fact drives its assumption on unit intratemporal elasticity of substitution in the composite consumption and intermediate goods production. This assumption implies that there is a steady-state, that the expenditure shares on intermediates and consumption of each sector are time-invariant and can be used to calibrate parameters  $\beta$  and  $\theta$ .

To evaluate the validity of the steady-state analysis, Table C.3 presents data on the consumption expenditure shares for the two consumption goods. It shows that the magnitude of changes over time is within the ones in the labour shares, even though the data do show a mild downward trend in the non-ICT-using good's expenditure share. Time-series data for the intermediates' expenditure shares is constrained by the lack of a consistent time-series of Input-Output data. Figure C.2 illustrates  $\beta$  and  $\theta$ , as calculated directly from the 1987, 1992, 1997, 2002 Benchmark Input-Output Tables. This rather rough evidence appear to suggest a stronger downward trend in  $\beta$ , but confirm the calibration result that  $\theta > \beta$ .

To summarize, the data do allow some scope for interpreting the United States's growth path as one associated with structural change or out of steady-state transition dynamics. The model can be extended in a straightforward way to account for either case.<sup>38</sup> Nevertheless, its quantitative results overall support that the

<sup>38</sup>The introduction of a slowly depreciating physical capital through would allow for smooth transition dynamics. That makes the model highly nonlinear and requires the use of numerical solution methods. This case has been explored for an simpler version of this model and its results are available by the author upon request.

steady-state assumption is a reasonable approximation of the United States' growth experience.

**A back-of-the-envelope application** The following exercise illustrates the ability of the model to deliver plausible changes across equilibrium paths in response to a structural change. The mid-1990s, and in particular 1995, is repeatedly documented by the empirical literature for the acceleration in the ICT-producing sector's TFP (in particular semiconductors' production) (e.g., see Jorgenson, Ho, and Stiroh 2005, Oliner and Sichel 2002). Table C.3 presents the main statistics for the ICT-producing sector and other variables for both pre-/post-1995 periods.<sup>39</sup>

In particular, while TFP growth was on average 12pc for the 1977-1995, it increased to 17pc for the 1995-2000 period. The productivity of the ICT-producing sector was calibrated separately for these two periods, in the spirit of the method used in Section 4.1 and given the remaining calibrated parameters. The implied by the model labour reallocations across the two periods are virtually zero: 0.005pp. and are in the direction observed in the data. Its predicted TFP growth acceleration is 0.56pp, which falls only little short of the 0.62pp. TFP acceleration, as documented by Jorgenson, Ho, and Stiroh (2005).<sup>40</sup> To conclude, the model's mechanism with its present parsimony and steady-state focus, may account for how changes in the productivity of the growth engine sector generate an aggregate impact without inducing strong labour reallocations.

## 5 Conclusions

This paper develops a theoretical framework that accounts for growth in the ICT era. It shows that growth is transmitted from the ICT-producing sector to the aggregate economy through the falling prices of the intermediate and consumption goods produced by the sector using capital embodying ICT. As a result, there are

<sup>39</sup>Inspect also the spikes in Figure 1 of the introduction and Figure C.1.

<sup>40</sup>See in Table 8.1. in Jorgenson, Ho, and Stiroh (2005). To derive the model's prediction for aggregate TFP growth, note that TFP accounts for the part of output growth that is not driven from inputs' accumulation. Hence, it is zero for the non-ICT-using sector and  $(1 - \alpha)g_N$  for the ICT-using one. This implies an "aggregate TFP" growth of  $\frac{p_1 Y_1}{p_1 Y_1 + p_0 Y_0} (1 - \alpha)g_N$ .



indirect benefits for both the households and the sector using capital that does not embody ICT because they face lower consumption and investment costs over time. These benefits are stronger the more the ICT-using sector contributes to the production of intermediates and consumption goods.

Consistent with the post-1970s' United States growth experience, the model delivers an aggregate constant growth path, where the growth rates are different across sectors. The ICT-producing sector is the fastest growing sector, while the non-ICT-using sector is the slowest one. Because the ICT-using sector uses the low productivity non-ICT-using intermediate goods in its production, its growth falls short of its full potential, i.e., ICT growth.

The comparative statics and optimal growth analysis show that the ICT-using sector's contribution in the production of consumption and intermediates is important in various respects. First, as highlighted above, it strengthens the growth transmission mechanism and thereby affects the sectorial and aggregate growth. Second, it affects the incentives to direct resources into the ICT-producing sector that is the engine of growth. The impact of market distortions on equilibrium allocations is also sensitive to the relative importance of the ICT-using goods as consumption or intermediates.

These results suggest that differences across economies in terms of how important is the sector using new technologies translates in sectorial and aggregate growth differences. This is even more so in relation to ICT, since this technology is broadly imported from most countries, rather than produced domestically. In view of these, the policy makers would be more concerned about the impact of market distortions upon long-run growth, the lower the intensity of intermediates and final consumption goods in the ICT-using sector's good. Extending the present setting to explicitly account for the role of policy intervention and its scope or interacting with the diffusion of the technology, which is taken as exogenous is left for future research.

As a final note, the present paper's theoretical framework is more general than its selected application. It accounts for growth in a multi-sector environment, when an intensively advancing technology is not used uniformly by all productive units. Its quantitative performance in terms of predicting facts relating to the United States

growth path during the ICT era shows that it is important to acknowledge in such context the inter-sector transactions of goods and the variety in the households' utility basket.

# A Technical Appendix

## A.1 Equilibrium results

Consumer side: The household offers labour  $L$  and makes dynamic decisions relating to  $\tilde{C}$ ,  $S$ , taking  $r_{\tilde{C}}$ , and  $w_L$  as given

$$\mathcal{H} = e^{-\rho t} \frac{\tilde{C}^{1-\sigma}-1}{1-\sigma} + \lambda \left[ r_{\tilde{C}} S + w_L L - \dot{\tilde{C}} \right].$$

The solution to this problem gives the standard condition

$$\frac{\dot{\tilde{C}}}{\tilde{C}} = \frac{1}{\sigma} (r_{\tilde{C}} - \rho). \quad (31)$$

The household decides within every period how to allocate consumption expenditures,  $C \equiv p_0 c_0 + p_1 c_1$ , into the two types of consumption goods,  $c_0$  and  $c_1$ , given their prices,  $p_0$  and  $p_1$  respectively, and unit price of the utility basket  $\tilde{C}$ ,  $p_{\tilde{C}}$ .

$$\max \{ p_{\tilde{C}} c_0^\theta c_1^{1-\theta} - p_0 c_0 - p_1 c_1 \},$$

This optimization problem gives:

$$\frac{c_1}{c_0} = \frac{1-\theta}{\theta} \frac{p_0}{p_1}, \quad (32)$$

$$p_{\tilde{C}} = \Theta p_0^\theta p_1^{1-\theta} \equiv 1, \quad (33)$$

where  $\Theta = [\theta^\theta (1-\theta)^{1-\theta}]^{-1}$ . Because  $\tilde{C}$  is the numeraire, the price index of the "composite consumption good", is normalized to one.

Production side: The final good producers take prices as given in both input and output markets. Therefore, the non-ICT-using sector demands non-ICT-capital at the level where the value of marginal product equals to its price. The same principle determines the demand of the ICT-using sector for every ICT-capital variety. The asymmetry in the capital price that the two sectors face comes from the presence of monopolistic distortions:

$$p_0 \frac{\partial Y_0}{\partial x_0(i)} = p_0 \alpha (u_0 L)^{1-\alpha} X_0^{\alpha-1} = p_H \quad (34)$$

$$p_1 \frac{\partial Y_1}{\partial x_1(j)} = p_1 \alpha (u_1 L)^{1-\alpha} x_1^{\alpha-1}(j) = p_{x_1}(j), \forall j. \quad (35)$$

The intermediate output producer also takes prices as given in both input and output markets. Its input demand for each type of intermediates equates the value of their marginal product to their market price, i.e. the unit price of the final good of either sector:

$$p_H \frac{\partial H}{\partial h_0} = \beta p_H h_0^{\beta-1} h_1^{1-\beta} = p_0, \quad (36)$$

$$p_H \frac{\partial H}{\partial h_1} = (1-\beta) p_H h_0^\beta h_1^{-\beta} = p_1. \quad (37)$$

The implied relative demands and price for the intermediate goods:

$$\frac{\beta}{1-\beta} \frac{h_1}{h_0} = \frac{p_0}{p_1}, \quad (38)$$

$$p_H = B p_0^\beta p_1^{1-\beta}, \quad (39)$$

where  $B = [\beta^\beta (1-\beta)^{1-\beta}]^{-1}$ .

Using (34) the implied demand for non-ICT-capital is

$$X_0 = \alpha^{\frac{1}{1-\alpha}} \left( \frac{p_0}{p_H} \right)^{\frac{1}{1-\alpha}} (u_0 L) \quad (40)$$

The producers of the ICT-capital varieties function under monopolistic competition. In the absence of dynamic decision variables, they maximize their profits by choosing their price and production in every period:

$$\pi_1(j) = \max_{p_{x_1(j)}, x_1(i)} \{p_{x_1(j)} x_1(j) - p_H x_1(j); s.t.(35)\}.$$

The model delivers symmetry across the varieties of each type of ICT-capital goods:

$$x_1 = \alpha^{\frac{2}{1-\alpha}} \left( \frac{p_1}{p_H} \right)^{\frac{1}{1-\alpha}} (u_1 L), \quad (41)$$

$$p_{x_1} = \frac{p_H}{\alpha}. \quad (42)$$

The implied profit flows for every period is:

$$\pi_1 = \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} p_1^{\frac{1}{1-\alpha}} p_H^{\frac{-\alpha}{1-\alpha}} (u_1 L). \quad (43)$$

Hence, aggregate per-period profits are defined as  $\Pi = N\pi_1$ .

The producers of ICT-capital varieties enter the market upon acquiring a blueprint produced by the ICT-producing sector. Free-entry into ICT-capital production implies that the cost that each ICT-capital variety producer assumes for a blueprint is equal to the present discounted value of his entire stream of future profits,

$$V_1(t) = \int_t^\infty \exp \left[ -\int_t^\tau r_{\tilde{C}}(s) ds \right] \pi_1(j)(\tau) d\tau, \quad (44)$$

given  $r_{\tilde{C}}$ . At any point in time the following no-arbitrage condition is true

$$r_{\tilde{C}}(t) V_1(t) = \pi_1(j)(t) + \dot{V}_1. \quad (45)$$

The total value of assets of the households in this economy is  $NV_1(t)$ , given that both types of capital are non-durable.

The demand for labour is competitive, hence each sector demands labour to

equate its marginal product with the market wage , $w_L$ ,

$$p_0 \frac{\partial Y_0}{\partial (u_0 L)} = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} p_0^{\frac{1}{1-\alpha}} p_H^{\frac{-\alpha}{1-\alpha}} = w_L, \quad (46)$$

$$p_1 \frac{\partial Y_1}{\partial (u_1 L)} = (1 - \alpha) \alpha^{\frac{2\alpha}{1-\alpha}} p_1^{\frac{1}{1-\alpha}} p_H^{\frac{-\alpha}{1-\alpha}} N = w_L, \quad (47)$$

$$V_1 \frac{\partial \dot{N}}{\partial (u_N L)} = V_1 \lambda N = w_L. \quad (48)$$

This results uses the the equilibrium output for each final good sector,

$$Y_0 = \alpha^{\frac{\alpha}{1-\alpha}} \left( \frac{p_0}{p_H} \right)^{\frac{\alpha}{1-\alpha}} (u_0 L), \quad (49)$$

$$Y_1 = \alpha^{\frac{2\alpha}{1-\alpha}} \left( \frac{p_1}{p_H} \right)^{\frac{\alpha}{1-\alpha}} (u_1 L) N, \quad (50)$$

which is derived using (40) and (41) respectively.

### A.1.1 Proof of Lemma 1

The labour market clearing condition determines a unique wage that equates (46) and (47). As a result,

$$\frac{p_1}{p_0} = \left( \alpha^{\frac{\alpha}{1-\alpha}} N \right)^{-(1-\alpha)}. \quad (51)$$

Q.E.D.

### A.1.2 Proof of Corollary 2

Applying time derivatives in relative final good prices in (51),

$$\frac{\dot{p}_1}{p_1} - \frac{\dot{p}_0}{p_0} = -(1 - \alpha) g_N. \quad (52)$$

Doing so also in (39) implies that  $\frac{\dot{p}_H}{p_H} = \frac{\dot{p}_0}{p_0} + (1 - \beta) \left( \frac{\dot{p}_1}{p_1} - \frac{\dot{p}_0}{p_0} \right)$ , so that using (52)

$$\frac{\dot{p}_H}{p_H} - \frac{\dot{p}_0}{p_0} = -(1 - \beta) (1 - \alpha) g_N. \quad (53)$$

Finally, the non-ICT-using good price growth

$$\frac{\dot{p}_0}{p_0} = (1 - \theta) (1 - \alpha) g_N, \quad (54)$$

is pinned down by (33) from the implicit numeraire restriction  $\frac{\dot{p}_{\bar{C}}}{p_{\bar{C}}} = 0$ . Q.E.D.

### A.1.3 Proof of Corollary 3

The labour market clearing condition determines a unique wage that equates (47) and (48),

$$V_1 p_1^{\frac{-1}{1-\alpha}} p_H^{\frac{\alpha}{1-\alpha}} = \lambda^{-1} (1 - \alpha) \alpha^{\frac{2\alpha}{1-\alpha}}. \quad (55)$$

In any equilibrium path, because the RHS in (55) is constant, it is necessary that  $\frac{\dot{V}_1}{V_1} = \frac{1}{1-\alpha} \frac{\dot{p}_1}{p_1} - \frac{\alpha}{1-\alpha} \frac{\dot{p}_H}{p_H}$ . Using (52), (53) and (54):

$$\begin{aligned}\frac{\dot{V}_1}{V_1} &= -[\theta(1-\alpha) + \alpha\beta]g_N \\ &= (1-\theta)(1-\alpha)g_N - [1-\alpha(1-\beta)]g_N\end{aligned}$$

Therefore, from (45)

$$V_1 = \frac{\pi_1}{r_{\bar{c}} - (1-\theta)(1-\alpha)g_N + [1-\alpha(1-\beta)]g_N}. \quad (56)$$

Using (43) into (56) and replacing back into (55),

$$r_{\bar{c}} = \alpha\lambda u_1 L + (1-\theta)(1-\alpha)g_N - [1-\alpha(1-\beta)]g_N. \quad (57)$$

Q.E.D.

#### A.1.4 General equilibrium results

This Appendix completes the static equilibrium results, combining those of Appendices A.1-A.1.3. Combining (38) and (32):

$$\frac{p_0}{p_1} = \frac{\beta}{1-\beta} \frac{h_1}{h_0} = \frac{\theta}{1-\theta} \frac{c_1}{c_0}, \quad (58)$$

where relative prices are given from the labour market clearing condition, (51) in A.1.1. The rest of the markets need to also clear out for these prices.

First, the market clearing condition for composite intermediate goods' production is expressed as  $H = h_0^\beta h_1^{1-\beta} = X_0 + X_1 = X_0 \left(1 + \frac{X_1}{X_0}\right)$ . Given the symmetry ICT-capital varieties across varieties  $X_1 = Nx_1 = \alpha^{\frac{2}{1-\alpha}} \left(\frac{p_1}{p_H}\right)^{\frac{1}{1-\alpha}} (u_1 L) N$  by (41). Therefore, by (50) it follows that  $X_1 = \alpha^2 \frac{p_1}{p_H} Y_1$ . Given also (40), and (49) it follows that  $X_0 = \alpha \frac{p_0}{p_H} Y_0$  and  $K_1 = \alpha^2 \frac{p_1}{p_H} Y_1$ . As a result,  $\frac{X_1}{X_0} = \alpha \frac{p_1 Y_1}{p_0 Y_0}$ . Also, from the equation of value of marginal products of labour,  $\frac{p_1 Y_1}{p_0 Y_0} = \frac{u_1}{u_0}$ , so that  $\frac{X_1}{X_0} = \alpha \frac{u_1}{u_0}$ . Therefore, the market clearing for composite intermediate is  $h_0 \left(p_H h_0^{\beta-1} h_1^{1-\beta}\right) = \alpha p_0 Y_0 \left(1 + \alpha \frac{u_1}{u_0}\right)$  giving

$$\frac{h_0}{h_0 + c_0} = \alpha\beta + \alpha^2 \beta \frac{u_1}{u_0}, \quad (59)$$

where the latter follows by using (36), and the market clearing for non-ICT-using good,  $Y_0 = c_0 + h_0$ .

The two final good markets, i.e.,  $Y_0 = c_0 + h_0$  and  $Y_1 = c_1 + h_1$ , need to jointly clear out as well. These are combined into the labour market clearing condition across these sectors,  $\frac{p_1 Y_1}{p_0 Y_0} = \frac{u_1}{u_0} = \frac{p_1(c_1 + h_1)}{p_0(c_0 + h_0)}$ . Using (58) to substitute for  $\frac{c_1}{h_1}$ :

$$\frac{u_1}{u_0} \left(1 + \frac{c_1}{h_0}\right) = \frac{1-\theta}{\theta} \frac{c_0}{h_0} + \frac{1-\beta}{\beta}. \quad (60)$$

For all markets to clear out simultaneously, it is sufficient that conditions (59) and (60) are jointly satisfied. This allows to solve for the consumption to intermediates

shares in the two final goods sectors and relative labour allocations

$$\frac{c_0}{h_0} = \frac{\theta[1-\alpha\beta-\alpha^2(1-\beta)]}{\alpha\beta[\alpha+\theta(1-\alpha)]}, \quad (61)$$

$$\frac{c_1}{h_1} = \frac{(1-\theta)[1-\alpha\beta-\alpha^2(1-\beta)]}{\alpha(1-\beta)[\alpha+\theta(1-\alpha)]}, \quad (62)$$

$$\frac{u_1}{u_0} = \frac{(1-\theta)(1-\alpha)+(1-\beta)\alpha}{\theta(1-\alpha^2)+\beta\alpha^2}, \quad (63)$$

respectively. A well-defined interior solution requires:  $1 - \theta > \alpha(\beta - \theta)$ , which is always satisfied given that  $\beta, \alpha, \theta \in (0, 1)$ .

### A.1.5 Proof of Proposition 4

The CGP definition requires that the state variable and all aggregate control variables grow at constant rates. The CGP condition for constant growth rate for the varieties stock,  $g_N \equiv \frac{\dot{N}}{N}$ , is that the labour allocation in the ICT-producing sector is constant,  $\dot{u}_N = 0$ . That implies  $\dot{u}_1 = -\dot{u}_0$ . Given the result for constant relative allocations across the final good sectors, in (63) of Appendix A.1.4, then  $\dot{u}_1 = \dot{u}_0 = 0$ . Therefore, labour allocations are constant across all sectors.

In order to derive the remaining CGP conditions for  $Y$ ,  $X$  and  $C$  to grow at constant rates, it is necessary to examine in turn the growth of prices and quantities of the different goods on this path. Growth in prices along this path are already summarized in Lemma 2. Turning to the quantities' growth on this path, using (40), (41), relative prices growth from Lemma 2, and that  $\dot{u}_1 = \dot{u}_0 = 0$  on the CGP, the growth rate for each final-good sector reflects only capital deepening

$$\frac{\dot{Y}_0}{Y_0} = \alpha(1 - \beta)g_N, \quad (64)$$

$$\frac{\dot{Y}_1}{Y_1} = (1 - \alpha\beta)g_N, \quad (65)$$

$$\frac{\dot{Y}}{Y} = (1 - \theta)(1 - \alpha)g_N + \alpha(1 - \beta)g_N. \quad (66)$$

The latter shows that growth of aggregate output is equal to growth in the value of the non-ICT-using good. This is because output growth differences are cancelled out by the relative prices' differences of the two final-good sectors. This follows from  $\frac{\dot{Y}}{Y} = \frac{\dot{p}_0}{p_0} + \frac{\dot{Y}_0}{Y_0} + \frac{p_1 Y_1}{p_1 Y_1 + p_0 Y_0} \left( \frac{\dot{p}_1}{p_1} - \frac{\dot{p}_0}{p_0} + \frac{\dot{Y}_1}{Y_1} - \frac{\dot{Y}_0}{Y_0} \right)$ , given (64), (65) and Lemma 2.

Regarding the aggregate capital growth,  $\frac{\dot{X}}{X}$ , given the demand for capital varieties,  $p_H X_0 = \alpha p_0 Y_0$  and  $p_H X_1 = \alpha^2 p_1 Y_1$ , then

$$\frac{\dot{X}_0}{X_0} = (1 - \beta)g_N, \quad (67)$$

$$\frac{\dot{X}_1}{X_1} = (1 - \beta)g_N. \quad (68)$$

As a result, the aggregate capital growth is the same with that of aggregate output,

$$\frac{\dot{X}}{X} = (1 - \theta)(1 - \alpha)g_N + \alpha(1 - \beta)g_N. \quad (69)$$

For completeness, since  $H = h_0^\beta h_1^{1-\beta} = X_0 + X_1$  from the composite intermediate goods' market clearing condition, then  $\frac{\dot{H}}{H} = (1 - \beta)g_N$ . Also, from (38) and the

relative prices' growth on CGP, imply:

$$\frac{\dot{h}_0}{h_0} = \alpha(1 - \beta)g_N, \quad (70)$$

$$\frac{\dot{h}_1}{h_1} = (1 - \alpha\beta)g_N. \quad (71)$$

For the aggregate consumption growth, using (54) and (57) into (31) imply

$$\frac{\dot{\bar{C}}}{\bar{C}} = \frac{1}{\sigma} [\alpha\lambda u_1 L + (1 - \theta)(1 - \alpha)g_N - [1 - \alpha(1 - \beta)]g_N]. \quad (72)$$

However  $\frac{\dot{\bar{C}}}{\bar{C}} = \frac{\dot{C}}{C}$  as well (as this is the numeraire). Using the static equilibrium conditions (61) and (62), and applying time derivatives it follows that:

$$\frac{\dot{c}_0}{c_0} = \frac{\dot{h}_0}{h_0} = \alpha(1 - \beta)g_N, \quad (73)$$

$$\frac{\dot{c}_1}{c_1} = \frac{\dot{h}_1}{h_1} = (1 - \alpha\beta)g_N, \quad (74)$$

As a result, the aggregate value of consumption grows at

$$\begin{aligned} \frac{\dot{C}}{C} &= \frac{\dot{p}_0}{p_0} + \frac{\dot{c}_0}{c_0} + (1 - \theta) \left[ \frac{\dot{h}_1}{h_1} - \frac{\dot{h}_0}{h_0} - (1 - \alpha)g_N \right] \\ &= (1 - \theta)(1 - \alpha)g_N + \alpha(1 - \beta)g_N. \end{aligned} \quad (75)$$

In order to simplify notation in the remaining derivations, let

$$\Sigma \equiv (1 - \theta)(1 - \alpha) + (1 - \beta)\alpha, \quad (76)$$

$$\Delta \equiv \theta(1 - \alpha^2) + \beta\alpha^2. \quad (77)$$

Since (72) and (75) need to be equal, the following condition needs to hold true in the steady-state of the decentralized economy:

$$g_N^d = \frac{\alpha u_1^d \lambda L - \rho}{1 - (1 - \sigma)\Sigma}, \quad (78)$$

Also directly from the ICT-producing production function, in equilibrium  $g_N^d = \lambda L - (u_0^d + u_1^d)\lambda L$ , where the relative labour shares in the two final good sectors are given by (63),  $\frac{u_1^d}{u_0^d} = \frac{\Sigma}{1 - \Delta}$ . Substituting out  $\frac{u_1^d}{u_0^d}$  into  $g_N^d$  gives the second expression in terms of  $g_N^d$  and  $u_1^d$ :

$$g_N^d = \frac{\lambda L \Sigma - \lambda L u_1^d (\Sigma + 1 - \Delta)}{\Sigma}. \quad (79)$$

The system of (78) and (79) may be solved for  $u_1^d$  and  $g_N^d$  in terms of the parameters of the model. This implies that:

$$u_1^d = \frac{\Sigma \left[ \frac{\rho}{\lambda L} + 1 - (1 - \sigma)\Sigma \right]}{\alpha \Sigma + [1 - (1 - \sigma)\Sigma](\Sigma + 1 - \Delta)}, \quad (80)$$

$$u_0^d = \frac{(1 - \Delta) \left[ \frac{\rho}{\lambda L} + 1 - (1 - \sigma)\Sigma \right]}{\alpha \Sigma + [1 - (1 - \sigma)\Sigma](\Sigma + 1 - \Delta)}, \quad (81)$$

$$g_N^d = \lambda L \frac{\alpha \Sigma - \frac{\rho}{\lambda L} (\Sigma + 1 - \Delta)}{\alpha \Sigma + [1 - (1 - \sigma)\Sigma](\Sigma + 1 - \Delta)}. \quad (82)$$

In order to check the conditions for an interior solution, it is sufficient to check



that  $u_1^d > 0$  and  $g_N^d > 0$ . Note that when the denominator in either expression  $\alpha\Sigma + [1 - \Sigma + \sigma\Sigma](\Sigma + 1 - \Delta)$  is strictly positive for all parameter specifications  $\beta, \theta, \alpha \in (0, 1)$ , since all individual terms are strictly positive. For the same reason,  $u_1^d > 0$  as long as  $\rho > 0$ . The only condition for interior solution is  $\alpha\Sigma > \frac{\rho}{\lambda L}(\Sigma + 1 - \Delta)$  or  $L > \frac{\rho(\Sigma + 1 - \Delta)}{\alpha\lambda\Sigma}$ . This provides a strictly positive lower bound on the scale of the economy, which is common in this class of models. It's noteworthy that the condition is not dependent on  $\sigma$ . Q.E.D.

### A.1.6 Proof of Corollary 6

The size of each sector in value terms is fully determined by two factors. First, the labour allocations in the ICT-using and non-ICT-using sectors,  $u_1$  and  $u_0$  respectively. Second, each sector's "final value price", which is  $p_1 \left(\frac{p_1}{p_H}\right)^{\frac{\alpha}{1-\alpha}}$  for the ICT-using sector and  $p_0 \left(\frac{p_0}{p_H}\right)^{\frac{\alpha}{1-\alpha}}$  for the non-ICT-using sector. Because  $p_1 \left(\frac{p_1}{p_H}\right)^{\frac{\alpha}{1-\alpha}} = p_0 \left(\frac{p_0}{p_H}\right)^{\frac{\alpha}{1-\alpha}} \propto N^{(1-\theta)(1-\alpha)+\alpha(1-\beta)}$ , both sectors' output value grows at the same rate and the relative size of the sectors is constant and fully determined by the relative allocations. Q.E.D.

## A.2 Comparative statics results

Let  $D \equiv \alpha\Sigma + (\Sigma + 1 - \Delta)(1 - \Sigma + \sigma\Sigma)$  stand for the denominator in  $g_N^d$  and  $N \equiv \alpha\Sigma - \frac{\rho}{\lambda L}(\Sigma + 1 - \Delta)$  for its numerator, where  $\Sigma$  and  $\Delta$  are already defined in (76)-(77) of Appendix A.1.5. The comparative statics are for parameters that satisfy the condition for interior solution for the decentralized economy, i.e.  $L > \frac{\rho(\Sigma + 1 - \Delta)}{\lambda\alpha\Sigma}$ . The effect of a change in  $\lambda$  is:

$$\begin{aligned}\frac{\partial g_N^d}{\partial \lambda} &= \frac{\alpha\Sigma L}{D} > 0, \\ \frac{\partial(u_1^d/u_0^d)}{\partial \lambda} &= 0.\end{aligned}$$

A change in  $\rho$  implies:

$$\begin{aligned}\frac{\partial g_N^d}{\partial \rho} &= -\frac{(\Sigma + 1 - \Delta)}{D} < 0 \\ \frac{\partial(u_1^d/u_0^d)}{\partial \lambda} &= 0\end{aligned}$$

A change in  $\sigma$  implies:

$$\begin{aligned}\frac{\partial g_N^d}{\partial \sigma} &= \frac{-g_N^d \Sigma (\Sigma + 1 - \Delta)}{D} < 0 \\ \frac{\partial(u_1^d/u_0^d)}{\partial \sigma} &= 0\end{aligned}$$

A change in  $\theta$  implies:

$$\begin{aligned}\frac{\partial g_N^d}{\partial \theta} &= \frac{\lambda L}{D^2} \left\{ -\alpha(1-\alpha) \left( \frac{\rho}{\lambda L} + 1 \right) \mathcal{D} \right. \\ &\quad \left. - (1-\sigma)(1-\alpha) \mathcal{N}(-\alpha\Sigma + \Sigma + 1 - \Delta) \right\} \\ \frac{\partial(u_1^d/u_0^d)}{\partial \theta} &= \frac{-(1-\alpha)}{(1-\Delta)^2} (\alpha\Sigma + \Sigma + 1 - \Delta) < 0\end{aligned}$$

Note that  $-\alpha\Sigma + (\Sigma + 1 - \Delta) \geq 0$  since the reverse implies that  $1 - \Delta < (\alpha - 1)\Sigma < 0$ . The necessary condition for  $\frac{\partial g_N^d}{\partial \theta} < 0$  is  $\alpha \left( \frac{\rho}{\lambda L} + 1 \right) +$

$+g_N^d(1-\sigma)[(1-\alpha)\Sigma + \Sigma + 1 - \Delta] \geq 0$ . A sufficient condition for  $\frac{\partial g_N^d}{\partial \theta} < 0$  is that  $1 - \sigma \geq 0$  or  $\sigma \leq 1$  ( $\Leftrightarrow \frac{1}{\sigma} \geq 1$ ). For  $\sigma > 1$ , then the above condition suggests an upper bound:  $\sigma \leq 1 + \frac{\alpha \left( \frac{\rho}{\lambda L} + 1 \right)}{g_N^d[(1-\alpha)\Sigma + \Sigma + 1 - \Delta]}$ , where the upper bound is itself an increasing

function of  $\sigma$ . Therefore, for a wide range of parameters  $\frac{\partial g_N^d}{\partial \theta} < 0$ .

For  $\sigma = 1$ , note also that

$$\begin{aligned}\left. \frac{\partial g_N^d}{\partial \theta} \right|_{\sigma=1} &= \frac{-\lambda L \alpha (1-\alpha) \left( \frac{\rho}{\lambda L} + 1 \right)}{\mathcal{D}} < 0 \Rightarrow \\ \left. \frac{\partial^2 g_N^d}{\partial \theta \partial \beta} \right|_{\sigma=1} &= -\frac{\lambda L \alpha^2 (1-\alpha) \left( \frac{\rho}{\lambda L} + 1 \right)}{\mathcal{D}} < 0.\end{aligned}$$

A change in  $\beta$  implies:

$$\begin{aligned}\frac{\partial g_N^d}{\partial \beta} &= -\alpha \frac{\lambda L}{D} \left\{ \left[ \alpha - \frac{\rho}{\lambda L} (1-\alpha) \right] \right. \\ &\quad \left. + g_N^d [-1 + (1-\sigma)((1-\alpha)\Sigma + \Sigma - \Delta + 1)] \right\} \\ \frac{\partial(u_1^d/u_0^d)}{\partial \beta} &= \frac{-\alpha(1-\Delta + \alpha\Sigma)}{(1-\Delta)^2} < 0\end{aligned}$$

The necessary condition for  $\frac{\partial g_N^d}{\partial \beta} < 0$  is that  $\alpha - \frac{\rho}{\lambda L} (1-\alpha) - g_N^d +$   
 $+g_N^d(1-\sigma)((1-\alpha)\Sigma + \Sigma - \Delta + 1) \geq 0$ . Noting that  $(1-\alpha)\Sigma + \Sigma - \Delta + 1 > 0$ , while also  $\alpha - \frac{\rho}{\lambda L} (1-\alpha) - g_N^d = \alpha \left( 1 + \frac{\rho}{\lambda L} \right) (\alpha\Sigma + 1 - \Delta) > 0$ , then again a sufficient condition is that  $1 - \sigma \geq 0$ . Otherwise, again that suggests an upper bound for  $\sigma \leq 1 + \frac{\alpha - \frac{\rho}{\lambda L} (1-\alpha) - g_N^d}{g_N^d[(1-\alpha)\Sigma + \Sigma + 1 - \Delta]}$ , which is increasing itself in  $\sigma$ .

For  $\sigma = 1$ , then

$$\begin{aligned}\left. \frac{\partial g_N^d}{\partial \beta} \right|_{\sigma=1} &= -\alpha \frac{\lambda L \left[ \alpha - \frac{\rho}{\lambda L} (1-\alpha) - g_N^d \right]}{\mathcal{D}} < 0 \Rightarrow \\ \left. \frac{\partial^2 g_N^d}{\partial \beta \partial \theta} \right|_{\sigma=1} &= -\frac{\lambda L}{\mathcal{D}} \left( -\left. \frac{\partial g_N^d}{\partial \theta} \right|_{\sigma=1} \right) < 0.\end{aligned}$$

A change in  $\alpha$  implies:

$$\begin{aligned}\frac{\partial g_N^d}{\partial \alpha} &= \frac{\lambda L}{D^2} (\theta - \beta) \left\{ \left[ \alpha - \frac{\rho}{\lambda L} (1-2\alpha) \right] \mathcal{D} \right. \\ &\quad \left. + \mathcal{N}(1-\alpha - (1-\sigma)[(1-2\alpha)\Sigma + \Sigma + 1 - \Delta]) \right\} \\ \frac{\partial(u_1^d/u_0^d)}{\partial \alpha} &= \frac{(\theta-\beta)(1-\Delta+2\alpha\Sigma)}{(1-\Delta)^2}\end{aligned}$$

The results for either the relative allocations or the growth depend on how  $\theta$  and  $\beta$  compare. For  $\theta = \beta$  then unambiguously  $\frac{\partial g_N^d}{\partial \alpha} = \frac{\partial (u_1^d/u_0^d)}{\partial \alpha} = 0$ . Also, for  $\theta > \beta$  then  $\frac{\partial (u_1^d/u_0^d)}{\partial \alpha} > 0$  and for  $\frac{\partial g_N^d}{\partial \alpha} > 0$  it suffices that  $1 \leq \sigma$  and  $\alpha \geq 1/2$ . Q.E.D.

### A.3 Social planner's solution

In what follows recall  $\Sigma$  and  $\Delta$  as already defined in (76)-(77) of Appendix A.1.5. The solution focuses directly on the case of unit intratemporal elasticity of substitution which allows for a CGP. Given the state of the economy,  $N$ , the optimal control problem for the social planner solves for the optimal allocations for  $\{c_0, c_1, X_0, x_1(j), h_0, h_1, u_0, u_1\}$  for  $j \in [0, N]$ :

$$\begin{aligned} \mathcal{H} = & e^{-\rho t} \frac{(c_0^\theta c_1^{1-\theta})^{1-\sigma} - 1}{1-\sigma} + q_0 [(u_0 L)^{1-\alpha} X_0^\alpha - c_0 - h_0] \\ & + q_1 [(u_1 L)^{1-\alpha} \int_0^N x_1^\alpha(j) dj - c_1 - h_1] \\ & + q_H [h_0^\beta h_1^{1-\beta} - X_0 - \int_0^N x_1(j) dj] \\ & + q_N [\lambda L (1 - u_0 - u_1) N] \end{aligned}$$

The Euler conditions with respect to the choice of consumption goods give:

$$\frac{c_1}{c_0} = \frac{1-\theta}{\theta} \frac{q_0}{q_1}. \quad (83)$$

With respect to capital good varieties

$$X_0^{SP} = \left( \frac{q_0}{q_H} \right)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} u_0 L, \quad (84)$$

$$x_1^{SP} = \left( \frac{q_1}{q_H} \right)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} u_1 L, \forall j. \quad (85)$$

and with respect to intermediate goods is

$$\frac{h_1}{h_0} = \frac{1-\beta}{\beta} \frac{q_0}{q_1}. \quad (86)$$

and  $q_H = B p_0^\beta p_1^{1-\beta}$ .

The decision on labour allocations  $q_0(1-\alpha) \frac{Y_0}{u_0 L} = q_1(1-\alpha) \frac{Y_1}{u_1 L} = q_N \lambda N$  together with (83) and (86) implies that

$$\frac{q_0}{q_1} = \frac{\theta}{1-\theta} \frac{c_1}{c_0} = \frac{\beta}{1-\beta} \frac{h_1}{h_0} = \frac{Y_1/u_1}{Y_0/u_0}.$$

In order to complete the static equilibrium results, the above condition is used together with the market clearing conditions in a similar spirit as described in Propo-

sition 1, so that the first-best allocations are:

$$\frac{c_0}{h_0} = \frac{\theta(1-\alpha)}{\alpha\beta}, \quad (87)$$

$$\frac{c_1}{h_1} = \frac{(1-\theta)(1-\alpha)}{\alpha(1-\beta)}, \quad (88)$$

$$\frac{u_1}{u_0} = \frac{(1-\alpha)(1-\theta)+(1-\beta)\alpha}{\alpha\beta+\theta(1-\alpha)}. \quad (89)$$

The relative prices reflect relative productivity differences across sectors. Since the labour market clearing condition requires  $\frac{q_0}{q_1} = \frac{Y_1/u_1}{Y_0/u_0} = N \left( \frac{q_0}{q_1} \right)^{\frac{-\alpha}{1-\alpha}}$  it follows that

$$\frac{q_0}{q_1} = N^{1-\alpha}, \quad (90)$$

so that  $\frac{\dot{q}_1}{q_1} - \frac{\dot{q}_0}{q_0} = -(1-\alpha)g_N$ . The remaining of the relative prices are as follows. Using (86) and the Euler condition  $-q_0 + q_H\beta\frac{H}{h_0} = 0$ , it follows that

$$\frac{q_H}{q_0} = B \left( \frac{q_1}{q_0} \right)^{1-\beta}, \quad (91)$$

which implies that  $\frac{\dot{q}_H}{q_H} = \frac{\dot{q}_0}{q_0} - (1-\beta)(1-\alpha)g_N$ .

Also, using the Euler condition  $-q_N\lambda N + q_0(1-\alpha)\frac{Y_0}{u_0L} = 0$ , and substituting out from (84) and (91)

$$\frac{q_N}{q_0} = \frac{(1-\alpha)(\alpha B)^{\frac{\alpha}{1-\alpha}}}{\lambda} \left( \frac{q_1}{q_0} \right)^{\frac{1-\alpha(1-\beta)}{1-\alpha}}, \quad (92)$$

which implies that  $\frac{\dot{q}_N}{q_N} = \frac{\dot{q}_0}{q_0} - [1-\alpha(1-\beta)]g_N$ . Directly from the Euler conditions, the condition that specifies the growth for the costate variable  $q_N$  provides with

$$-\frac{\dot{q}_N}{q_N} = \lambda L(1-u_0). \quad (93)$$

Therefore, combining the above results

$$-\frac{\dot{q}_0}{q_0} = \lambda L(1-u_0) - [1-\alpha(1-\beta)]g_N, \quad (94)$$

and noting also that  $g_N + \lambda Lu_1 = \lambda L(1-u_0)$  then  $-\frac{\dot{q}_0}{q_0} = \lambda Lu_1 + \alpha(1-\beta)g_N$ . Using these results, it is straightforward to derive all remaining relative prices and respective (relative) growth rates.

The results from (87), (88) and the final output market clearing conditions again imply that along the CGP

$$\begin{aligned} \frac{\dot{c}_0}{c_0} &= \frac{\dot{h}_0}{h_0} = \frac{\dot{Y}_0}{Y_0} = \alpha(1-\beta)g_N, \\ \frac{\dot{c}_1}{c_1} &= \frac{\dot{h}_1}{h_1} = \frac{\dot{Y}_1}{Y_1} = (1-\alpha\beta)g_N. \end{aligned}$$

Finally, from the Euler condition  $e^{-\rho t} (c_0^\theta c_1^{1-\theta})^{1-\sigma} \frac{\theta}{c_0} - q_0 = 0$ , (83) and (90), so that

$$\frac{\dot{c}_0}{c_0} = \frac{1}{\sigma} \left[ -\frac{\dot{q}_0}{q_0} + (1-\sigma)(1-\theta)(1-\alpha)g_N - \rho \right]. \quad (95)$$

The above results allow to solve through for the optimal growth and labour allocations. Given (94), then using (95) and equating with  $\frac{\dot{Y}_0}{Y_0} = \alpha(1-\beta)g_N$ , the optimal growth rates expressed as a function of  $u_1^{SP}$ :

$$g_N^{SP} = \frac{u_1^{SP}\lambda L - \rho}{(1-\sigma)\Sigma}. \quad (96)$$

Also, since from (89):  $\frac{u_0^{SP}}{u_1^{SP}} = \frac{1-\Sigma}{\Sigma}$ , then using this to substitute out relative shares in  $g_N^{SP} = \lambda L - u_1^{SP}(1 + \frac{u_0^{SP}}{u_1^{SP}})\lambda L$ , it follows that

$$g_N^{SP} = \frac{\lambda L \Sigma - \lambda L u_1^{SP}}{\Sigma}. \quad (97)$$

The system of (96) and (97) may be solved for  $u_1^{SP}$  and  $g_N^{SP}$  in terms of the parameters of the model. This gives:

$$u_1^{SP} = \frac{\frac{\rho}{\lambda L} - (1-\sigma)\Sigma}{\sigma}, \quad (98)$$

$$u_0^{SP} = \frac{1-\Sigma}{\Sigma} \frac{\frac{\rho}{\lambda L} - (1-\sigma)\Sigma}{\sigma}, \quad (99)$$

$$g_N^{SP} = \lambda L \frac{\Sigma - \frac{\rho}{\lambda L}}{\sigma \Sigma}. \quad (100)$$

The condition for interior solution with  $g_N^{SP} > 0$  is  $\Sigma > \frac{\rho}{\lambda L}$  or  $L > \frac{\rho}{\lambda \Sigma}$ . For  $u_1^{SP} > 0$  the condition is  $\frac{\rho}{\lambda L} > (1-\sigma)\Sigma$ , which is also the condition for  $u_N^{SP} < 1$ . For the case that  $1-\sigma \leq 0$ , i.e.,  $\frac{1}{\sigma} \leq 1$ , the latter condition is always satisfied. Therefore, it is sufficient that  $\Sigma > \frac{\rho}{\lambda L}$ . For the case that  $0 < \sigma < 1$  then for the solution not to be explosive, it requires that the scale of the economy is bounded from both below and above,  $\frac{\rho}{\lambda \Sigma} < L < \frac{\rho}{\lambda(1-\sigma)\Sigma}$ . This condition cannot be satisfied for  $\sigma \rightarrow 0$ .

The equilibrium results for the social planner (98)-(100) may be contrasted directly with the respective equilibrium conditions (80)-(82) from Proposition 2, that characterize the decentralized economy. The comparison is meaningful when there is interior solution in either case, i.e.  $L > \min \left\{ \frac{\rho}{\lambda \Sigma}, \frac{\rho(\Sigma+1-\Delta)}{\alpha \lambda \Sigma} \right\}$ . However, because  $\Sigma + 1 - \Delta > \alpha \Leftrightarrow \beta - \theta < \frac{1}{\alpha}$  which holds for any values  $\beta, \theta, \alpha \in (0, 1)$ , then it follows that  $\Sigma > \frac{\alpha \Sigma}{\Sigma+1-\Delta}$ . Therefore, the sufficient condition for an interior solution is  $L > \frac{\rho(\Sigma+1-\Delta)}{\alpha \lambda \Sigma}$ .<sup>41</sup>

The guess-and-verify is that the social planner achieves higher growth than the decentralized economy, i.e.,  $g_N^{SP} > g_N^d$ . This is equivalent to  $u_N^{SP} > u_N^d$ , i.e.,

$$\frac{\Sigma - \frac{\rho}{\lambda L}}{\sigma \Sigma} > \frac{\alpha \Sigma - \frac{\rho}{\lambda L}(\Sigma+1-\Delta)}{\alpha \Sigma + [1-(1-\sigma)\Sigma](\Sigma+1-\Delta)},$$

which implies that

$$\frac{\rho}{\lambda L} < \Sigma \frac{[\alpha \Sigma + (\Sigma+1-\Delta)(1-\Sigma) + \sigma \Sigma(\Sigma+1-\Delta) - \alpha \sigma \Sigma]}{\alpha \Sigma + (\Sigma+1-\Delta)(1-\Sigma)}.$$

<sup>41</sup>Note that for  $\sigma \in (0, 1)$  then there is an upper bound in the scale of the economy imposed by the social planner's solution,  $\frac{\rho(\Sigma+1-\Delta)}{\alpha \lambda \Sigma} < L < \frac{\rho}{\lambda(1-\sigma)\Sigma}$  that indirectly provides with a condition on  $\sigma$ :  $\sigma > \frac{\Sigma+1-\Delta-\alpha}{\Sigma+1-\Delta} \equiv \bar{\sigma}$ .

Note that because any interior solution bears the property that  $\frac{\rho}{\lambda L} < \frac{\alpha \Sigma}{\Sigma + 1 - \Delta} < \Sigma$ , then the above condition is always satisfied if the expression in the brackets is strictly above one, which results in the following sufficient condition

$$\sigma \Sigma (\Sigma + 1 - \Delta) - \alpha \sigma \Sigma > 0,$$

which is always true as already noted. Therefore, to the extent that there is an equilibrium in either economy, the social planner would always perform better than the decentralized economy.

Regarding how the respective allocations compare, note that it is already shown that  $u_N^{SP} > u_N^d$  or  $u_0^{SP} + u_1^{SP} < u_0^d + u_1^d$ . The relative allocations,  $\frac{u_1^{SP}}{u_0^{SP}} = \frac{\Sigma}{1 - \Sigma}$  and  $\frac{u_1^d}{u_0^d} = \frac{\Sigma}{1 - \Delta}$ , would be different if and only if  $\Sigma \neq \Delta \iff \theta \neq \beta$ . In particular,  $\Sigma \geq (<) \Delta$  iff  $\theta \geq (<) \beta$ .

For  $\theta = \beta$  then  $\frac{u_1^{SP}}{u_0^{SP}} = \frac{u_1^d}{u_0^d} = \frac{1 - \theta}{\theta}$ , and using this condition in how total final good sectors' labour allocations compare, implies that in this case unambiguously  $u_0^{SP} < u_0^d$  and  $u_1^{SP} < u_1^d$ .

For the case that  $\theta > \beta$  then  $\frac{u_1^{SP}}{u_0^{SP}} > \frac{u_1^d}{u_0^d}$ . Hence, since  $u_0^{SP} + u_1^{SP} < u_0^d + u_1^d$  it follows that  $u_0^{SP} \left(1 + \frac{u_1^{SP}}{u_0^{SP}}\right) < u_0^d \left(1 + \frac{u_1^d}{u_0^d}\right)$ , while  $u_0^{SP} \left(1 + \frac{u_1^d}{u_0^d}\right) < u_0^{SP} \left(1 + \frac{u_1^{SP}}{u_0^{SP}}\right)$ , so that unambiguously  $u_0^{SP} < u_0^d$ . Finally, for  $\theta < \beta$  then  $\frac{u_1^{SP}}{u_0^{SP}} < \frac{u_1^d}{u_0^d}$ . Since  $u_0^{SP} + u_1^{SP} < u_0^d + u_1^d$  it follows that  $u_1^{SP} \left(1 + \frac{u_0^{SP}}{u_1^{SP}}\right) < u_1^d \left(1 + \frac{u_0^d}{u_1^d}\right)$ , while  $u_1^{SP} \left(1 + \frac{u_0^d}{u_1^d}\right) > u_1^{SP} \left(1 + \frac{u_0^{SP}}{u_1^{SP}}\right)$ , so that unambiguously  $u_1^{SP} < u_1^d$ . Now it's the comparison between  $u_0^{SP}$  and  $u_0^d$  that becomes ambiguous.

To conclude, the results show that for  $\theta \leq \beta$ , then unambiguously  $u_1^{SP} < u_1^d$ . The only case that their comparison is ambiguous is for  $\theta > \beta$  (when  $\Sigma > \Delta$ ). The comparison considers different values of  $\sigma$  that would allow for  $u_1^{SP} > u_1^d$ . Given (98) and (80), then the necessary condition for  $u_1^{SP} > u_1^d$  is that

$$\frac{\rho}{\lambda L} > \frac{\sigma \Sigma [1 - (1 - \sigma) \Sigma] + (1 - \sigma) \Sigma [\alpha \Sigma + 1 - (1 - \sigma) \Sigma] (\Sigma + 1 - \Delta)}{\alpha \Sigma + (1 - \Sigma) (\Sigma + 1 - \Delta) + \sigma \Sigma (\Sigma + 1 - \Delta)},$$

while as shown above, for any well defined interior it is true that  $\frac{\rho}{\lambda L} < \frac{\alpha \Sigma}{\Sigma + 1 - \Delta} < \Sigma$ . Consider first the case of  $\sigma = 1$ . Then this condition reduces to  $\frac{\rho}{\lambda L} > \frac{\Sigma}{1 - \Delta + \alpha \Sigma} > \frac{\alpha \Sigma}{\Sigma + 1 - \Delta}$ , which implies that there is no well defined interior solution where  $u_1^{SP} > u_1^d$ . Likewise, for  $\sigma \rightarrow 0$ , then the above condition reduces to  $\frac{\rho}{\lambda L} > \Sigma$ , which again is inconsistent with an interior solution. Therefore, for any  $\sigma \leq 1$ , the conclusion is that when there are interior equilibria for both the decentralized and social planner's economy, it is true that  $u_1^{SP} < u_1^d$ . Finally consider the case that  $\sigma \rightarrow \infty$ . In this case it can be shown that the lower bound's limit is  $-\infty$ , suggesting that  $u_1^{SP} > u_1^d$  is possible for very low levels of intertemporal elasticity if substitution.<sup>42</sup> Q.E.D.

<sup>42</sup>The lower bound can be reduced to the sum of three terms, where the L'Hospital rule is applied in the last two:  $\frac{\Sigma [\alpha \Sigma + (1 - \Sigma) (1 - \Sigma + \Delta)]}{\alpha \Sigma + (1 - \Sigma) (\Sigma + 1 - \Delta) + \sigma \Sigma (\Sigma + 1 - \Delta)} \rightarrow 0$ ,  $\frac{\sigma \Sigma [\Sigma (1 - \Sigma + \Delta) - (1 - \Sigma) (\Sigma - \Delta) - \alpha \Sigma]}{\alpha \Sigma + (1 - \Sigma) (\Sigma + 1 - \Delta) + \sigma \Sigma (\Sigma + 1 - \Delta)} \rightarrow \text{const}$  and  $\frac{-(\sigma \Sigma)^2 (\Sigma - \Delta)}{\alpha \Sigma + (1 - \Sigma) (\Sigma + 1 - \Delta) + \sigma \Sigma (\Sigma + 1 - \Delta)} \rightarrow -\infty$ .

## A.4 General conditions for CGP

This Appendix reviews the decentralized equilibrium results of the baseline model, where household has generalized CES preferences intratemporally, i.e.,  $\tilde{C} \equiv [\theta c_0^\epsilon + (1 - \theta)c_1^\epsilon]^{\frac{1}{\epsilon}}$ . It shows that unit intratemporal elasticity of substitution, i.e.,  $\epsilon = 0$ , is a necessary and sufficient condition for the existence of a CGP.

All production side equilibrium results of Appendix A.1 are the same. The different preferences enter the household's intratemporal decision

$$\max \left\{ p_{\tilde{C}} [\theta c_0^\epsilon + (1 - \theta)c_1^\epsilon]^{\frac{1}{\epsilon}} - p_0 c_0 - p_1 c_1 \right\},$$

and thereby relative demand for the two consumption goods:

$$\frac{c_1}{c_0} = \left( \frac{1 - \theta}{\theta} \frac{p_0}{p_1} \right)^{\frac{1}{1 - \epsilon}}, \quad (101)$$

$$p_{\tilde{C}} = \left[ \theta^{\frac{1}{1 - \epsilon}} p_0^{\frac{-\epsilon}{1 - \epsilon}} + (1 - \theta)^{\frac{1}{1 - \epsilon}} p_1^{\frac{-\epsilon}{1 - \epsilon}} \right]^{-\frac{1 - \epsilon}{\epsilon}} \equiv 1 \quad (102)$$

The latter is the generalized price index of the "composite consumption good". Given the numeraire restriction, this implies that the non-ICT-using good price growth is

$$\frac{\dot{p}_0}{p_0} = (1 - \gamma)(1 - \alpha)g_N, \quad (103)$$

where

$$\gamma \equiv \frac{p_0 c_0}{p_0 c_0 + p_1 c_1} = \frac{\theta c_0^\epsilon}{\theta c_0^\epsilon + (1 - \theta)c_1^\epsilon} = \left[ 1 + \left( \frac{1 - \theta}{\theta} \right)^{\frac{1}{1 - \epsilon}} \left( \frac{p_1}{p_0} \right)^{\frac{-\epsilon}{1 - \epsilon}} \right]^{-1} \quad (104)$$

follows from (101).

In view of the results of Lemma 2 for  $\frac{\dot{p}_1}{p_1}$  growth, on any equilibrium path with  $g_N > 0$  the non-ICT-using good's consumption expenditure share  $\gamma$  is a function of time. Due to (103) and the results of Lemma 2, all absolute prices growth become also time-varying through  $\gamma$ . Time-varying prices growth implies also time-varying capital returns rate on the value of the productive assets,  $\frac{\dot{V}_1}{V_1} = (1 - \gamma)(1 - \alpha)g_N - [1 - \alpha(1 - \beta)]g_N$ . As a result, the interest rate determining incentives to accumulate these assets becomes time-varying as well through  $\gamma$ ,

$$r_{\tilde{C}} = \alpha \lambda u_1 L + (1 - \gamma)(1 - \alpha)g_N - [1 - \alpha(1 - \beta)]g_N, \quad (105)$$

while it is also subject to dynamics in  $\dot{u}_1$ .

Regarding the general equilibrium results, given competitive prices from (51) the substitution between the two goods in household's consumption needs to match their substitution in the composite intermediate's production.

$$\frac{p_0}{p_1} = \frac{\beta}{1 - \beta} \frac{h_1}{h_0} = \frac{\gamma}{1 - \gamma} \frac{c_1}{c_0}. \quad (106)$$

Following the same steps as in Appendix A.1.4, market clearing conditions and

optimal allocations can be shown to imply again (59), while (60) becomes instead

$$\frac{u_1}{u_0} \left(1 + \frac{c_1}{h_0}\right) = \frac{1-\gamma}{\gamma} \frac{c_0}{h_0} + \frac{1-\beta}{\beta}.$$

The resulting consumption to intermediates shares in the two final goods sectors and relative labour allocations

$$\frac{c_0}{h_0} = \frac{\gamma[1-\alpha\beta-\alpha^2(1-\beta)]}{\alpha\beta[\alpha+\gamma(1-\alpha)]}, \quad (107)$$

$$\frac{c_1}{h_1} = \frac{(1-\gamma)[1-\alpha\beta-\alpha^2(1-\beta)]}{\alpha(1-\beta)[\alpha+\gamma(1-\alpha)]}, \quad (108)$$

$$\frac{u_1}{u_0} = \frac{(1-\theta)(1-\alpha)+(1-\beta)\alpha}{\theta(1-\alpha^2)+\beta\alpha^2}, \quad (109)$$

respectively. A well-defined interior solution requires:  $1 - \gamma > \alpha(\beta - \gamma)$ , which is always satisfied given that  $\beta, \alpha \in (0, 1)$  and  $\gamma \in (0, 1)$  in a well-defined interior solution, where both goods are being consumed.

The CGP definition requires that  $N, X, Y$  and  $C$  grow at constant rates. The CGP condition for constant growth rate for the varieties stock,  $g_N \equiv \frac{\dot{N}}{N}$ , is that the labour allocation in the ICT-producing sector is constant,  $\dot{u}_N = 0$ . That implies  $\dot{u}_1 = -\dot{u}_0$  or  $\frac{\dot{u}_1}{u_1} = -\frac{u_0}{u_1} \frac{\dot{u}_0}{u_0}$ .

Turning to output growth, using (40), (41) and the growth rates of relative prices, the implied growth rate for each final-good sector reflects its productivity and potential labour accumulation. Output growth differences across the two final good sectors are cancelled out by their relative prices' growth differences and labour reallocations aggregate to zero:

$$\frac{\dot{Y}_0}{Y_0} = \alpha(1-\beta)g_N + \frac{\dot{u}_0}{u_0}, \quad (110)$$

$$\frac{\dot{Y}_1}{Y_1} = (1-\alpha\beta)g_N + \frac{\dot{u}_1}{u_1}, \quad (111)$$

$$\frac{\dot{Y}}{Y} = (1-\gamma)(1-\alpha)g_N + \alpha(1-\beta)g_N. \quad (112)$$

The latter follows from  $\frac{\dot{Y}}{Y} = \frac{\dot{p}_0}{p_0} + \frac{\dot{Y}_0}{Y_0} + \frac{p_1 Y_1}{p_1 Y_1 + p_0 Y_0} \left( \frac{\dot{p}_1}{p_1} - \frac{\dot{p}_0}{p_0} + \frac{\dot{Y}_1}{Y_1} - \frac{\dot{Y}_0}{Y_0} \right)$ , given (110), (111), Lemma 2,  $\frac{p_1 Y_1}{p_1 Y_1 + p_0 Y_0} = \frac{u_1}{u_1 + u_0}$ , and  $\frac{\dot{u}_1}{u_1} - \frac{\dot{u}_0}{u_0} = -\frac{\dot{u}_0}{u_0} \frac{u_1 + u_0}{u_1}$ . It is worth pointing out that this expression is a function of constants and a unique time varying variable, which is the non-ICT-using goods' expenditure share  $\gamma$ , in (104).

Regarding the growth of aggregate capital,  $\frac{\dot{X}}{X}$ , given the demand for capital varieties,  $p_H X_0 = \alpha p_0 Y_0$  and  $p_H X_1 = \alpha^2 p_1 Y_1$ , then along the CGP:

$$\frac{\dot{X}_0}{X_0} = (1-\beta)g_N + \frac{\dot{u}_0}{u_0}, \quad (113)$$

$$\frac{\dot{X}_1}{X_1} = (1-\beta)g_N + \frac{\dot{u}_1}{u_1} \quad (114)$$

Note that now  $\frac{\dot{X}}{X} = \frac{\dot{p}_0}{p_0} + \frac{\dot{Y}_0}{Y_0} + \frac{X_1}{X_1 + X_0} \left( \frac{\dot{p}_1}{p_1} - \frac{\dot{p}_0}{p_0} + (1-\alpha)g_N + \frac{\dot{u}_1}{u_1} - \frac{\dot{u}_0}{u_0} \right)$ , for  $\frac{X_1}{X_1 + X_0} = \frac{\alpha u_1}{\alpha u_1 + u_0}$ . Hence the last term in  $\frac{\dot{X}}{X}$  which captures the effect of relative shares changes



is now  $-\frac{\dot{u}_0}{u_0} \frac{\alpha(u_1+u_0)}{\alpha u_1+u_0}$ . To conclude, the growth of aggregate capital is

$$\frac{\dot{X}}{X} = (1 - \gamma)(1 - \alpha)g_N + \alpha(1 - \beta)g_N + (1 - \alpha) \frac{\dot{u}_0}{u_0} \frac{u_0}{\alpha u_1 + u_0} \quad (115)$$

Applying (109) to substitute out for  $\frac{u_1}{u_0}$  in (115),

$$\frac{\dot{X}}{X} = (1 - \gamma)(1 - \alpha)g_N + \alpha(1 - \beta)g_N + (1 - \alpha) \frac{\dot{u}_0}{u_0} \frac{\alpha^2\beta + \gamma(1 - \alpha^2)}{\alpha + \gamma(1 - \alpha)}. \quad (116)$$

This is a function of two time-varying variables,  $u_0$  and  $\gamma$ . It may be reduced to an expression that is only a function of  $\gamma$ . This is done by exploring the dynamics implied by the static optimization condition (109), which needs to be satisfied within every period and over time. Therefore, by applying time derivatives in both sides of (109) and using the steady state requirement that  $\frac{\dot{u}_1}{u_1} = -\frac{u_0}{u_1} \frac{\dot{u}_0}{u_0}$ , provides with an expression linking  $\frac{\dot{u}_0}{u_0}$  with the non-ICT-using goods' expenditure share growth,  $\frac{\dot{\gamma}}{\gamma}$ :

$$\frac{\dot{u}_0}{u_0} = \frac{\dot{\gamma}}{\gamma} \frac{\gamma[1 - \alpha\beta - \alpha^2(1 - \beta)]}{1 - \alpha\beta - \gamma(1 - \alpha) + \alpha^2\beta + \gamma(1 - \alpha^2)}. \quad (117)$$

Using this expression to substitute out for  $\frac{\dot{u}_0}{u_0}$  in (116) proves that the aggregate capital growth is a function of a only of parameters of the model and  $\gamma$ .

For completeness, since  $H = h_0^\beta h_1^{1-\beta} = X_0 + X_1$  from the market clearing condition for composite intermediate goods, then  $\frac{\dot{H}}{H} = (1 - \beta)g_N + (1 - \alpha) \frac{\dot{u}_0}{u_0} \frac{u_0}{\alpha u_1 + u_0}$ . Also, from (38) and the relative prices' growth on CGP, imply:

$$\frac{\dot{h}_0}{h_0} = \alpha(1 - \beta)g_N + (1 - \alpha) \frac{\dot{u}_0}{u_0} \frac{u_0}{\alpha u_1 + u_0}, \quad (118)$$

$$\frac{\dot{h}_1}{h_1} = (1 - \alpha\beta)g_N + (1 - \alpha) \frac{\dot{u}_0}{u_0} \frac{u_0}{\alpha u_1 + u_0}, \quad (119)$$

where the second term in either expression may be expressed in terms of  $(\dot{\gamma}, \gamma)$  and the models' parameters, using (109) and (117) in a similar spirit as above.

For the aggregate consumption growth, note first that (54) and (105) into (31) imply

$$\frac{\dot{C}}{C} = \frac{1}{\sigma} [\alpha\lambda u_1 L + (1 - \gamma)(1 - \alpha)g_N - [1 - \alpha(1 - \beta)]g_N]. \quad (120)$$

However  $\frac{\dot{C}}{C} = \frac{\dot{C}}{C}$  as well. Using the static equilibrium conditions (107) and (108), and applying time derivatives it follows that:

$$\frac{\dot{c}_0}{c_0} = \frac{\dot{h}_0}{h_0} + \frac{\dot{\gamma}}{\gamma} \frac{\alpha}{\alpha + \gamma(1 - \alpha)}, \quad (121)$$

$$\frac{\dot{c}_1}{c_1} = \frac{\dot{h}_1}{h_1} - \frac{\dot{\gamma}}{\gamma} \frac{\gamma}{(1 - \gamma)[\alpha + \gamma(1 - \alpha)]}, \quad (122)$$

where the growth rates of either type of intermediates is given by (118) and (119). Finally, using all information available to derive the growth of aggregate consumption:

$$\frac{\dot{C}}{C} = \frac{\dot{p}_0}{p_0} + \frac{\dot{c}_0}{c_0} + (1 - \gamma) \left[ \frac{\dot{h}_1}{h_1} - \frac{\dot{h}_0}{h_0} - (1 - \alpha)g_N - \frac{\dot{\gamma}}{\gamma} \frac{1}{1 - \gamma} \right] \quad (123)$$

$$= (1 - \gamma)(1 - \alpha)g_N + \frac{\dot{h}_0}{h_0} - \frac{\dot{\gamma}}{\gamma} \frac{\gamma(1 - \alpha)}{\alpha + \gamma(1 - \alpha)}. \quad (124)$$

Using (118) and (109) into the last expression provide with:

$$\frac{\dot{C}}{C} = (1 - \gamma)(1 - \alpha)g_N + \alpha(1 - \beta)g_N + (1 - \alpha)\frac{\dot{u}_0}{u_0}\frac{\alpha^2\beta + \gamma(1 - \alpha^2)}{\alpha + \gamma(1 - \alpha)} - \frac{\dot{\gamma}}{\gamma}\frac{\gamma(1 - \alpha)}{\alpha + \gamma(1 - \alpha)},$$

where as discussed above, (117) can be used to rewrite the above expression fully in terms of  $\gamma$ .

To conclude, the necessary and sufficient condition for constant  $\frac{\dot{Y}}{Y}$  is that  $\dot{\gamma} = 0$ . Since from (104) it holds over time that  $\dot{\gamma} = -\frac{\epsilon}{1 - \epsilon}\gamma(1 - \gamma)(1 - \alpha)g_N$ , then the necessary and sufficient condition for a well-defined interior solution of a CGP with  $g_N > 0$  is  $\epsilon = 0$ .

Note that this condition is sufficient for constant  $\frac{\dot{X}}{X}$  and  $\frac{\dot{C}}{C}$  since it eliminates all time-variant terms. This is because for  $\epsilon = 0$  then  $\gamma = \theta$  and  $\dot{u}_0 = \dot{u}_1 = 0$ .

The proof continues in order to show that  $\epsilon = 0$  is also the necessary condition for constant  $\frac{\dot{X}}{X}$  and  $\frac{\dot{C}}{C}$ . The necessary condition for constant  $\frac{\dot{X}}{X}$  is that

$$(1 - \gamma)(1 - \alpha)g_N + (1 - \alpha)\frac{\dot{u}_0}{u_0}\frac{u_0}{\alpha u_1 + u_0} = \xi,$$

where  $\xi$  is some constant. At the same time, the necessary condition for constant  $\frac{\dot{C}}{C}$  is

$$(1 - \gamma)(1 - \alpha)g_N + (1 - \alpha)\frac{\dot{u}_0}{u_0}\frac{u_0}{\alpha u_1 + u_0} - \frac{\dot{\gamma}}{\gamma}\frac{\gamma(1 - \alpha)}{\alpha + \gamma(1 - \alpha)} = \zeta,$$

for some other constant  $\zeta$ . These two conditions need to be mutually consistent along any equilibrium path, implying that

$$\xi = \frac{\dot{\gamma}}{\gamma}\frac{\gamma(1 - \alpha)}{\alpha + \gamma(1 - \alpha)} + \zeta,$$

which provides indirectly a second condition that describes the dynamics of  $\gamma$ . Full consistency requires that

$$\frac{\dot{\gamma}}{\gamma} = (\xi - \zeta)\frac{\alpha + \gamma(1 - \alpha)}{\gamma(1 - \alpha)} = -\frac{\epsilon}{1 - \epsilon}(1 - \gamma)(1 - \alpha)g_N,$$

which is inconsistent with constant  $g_N$ . Therefore, this is also the necessary condition for constant  $\frac{\dot{X}}{X}$  and  $\frac{\dot{C}}{C}$  rates. It is worth emphasizing that the result is independent of the value of the intertemporal elasticity of substitution,  $1/\sigma$ . This proves the original claim that preferences with  $\epsilon = 0$  is the only case that permit a well-defined CGP, given technologies in the production side of the economy. *Q.E.D.*

## B Data Sources and Methodology

### B.1 Data sources

The data on average value added and Domar shares, value added, and TFP growth for the 1977-2000 period for 44 industries, are taken from Table 8.6 in Jorgenson et al. (2005). Table 7.1 provides the decomposition of the output growth for these 44 industries into the contribution of capital, labour, intermediate materials, and TFP for the 1977-2000 period. ICT-capital intensity in 1995 for each of the 44 industries come from Table 4.2. All data are based on the three-digit SIC 1987 industry

classification. Details on the sources and methodology for a detailed industry growth accounting are found in Jorgenson et al. (2005), Chapter 4.

The data on employment (in hours), value added, and value added, deflators for 36 industries at the second and third-digits of the United States economy are taken from the EU KLEMS, is available from the Groningen Growth and Development Center and is a project financed by the European Commission. The data cover the period 1970-2005 (SIC version March 2007) and are based on the three-digit NACE Rev. 1 industry classification. The data for the United States economy are based on the annual industry accounts provided by the Bureau of Economic Analysis (BEA). The sources for nominal and volume measures regarding the Inter-industry accounts come from the National Accounts. For the period 1960-2000 the data are taken from Dale Jorgenson. The breakdown of his 44-industry level into the industry detail of EU KLEMS database is made on the basis of weights based on benchmark Input-Output (I-O) tables from BEA. Details on the mapping to NACE for the United States economy is found in the country notes details of the dataset.

The data on the use and production shares of the commodities and inter-industry transactions are from the "Use Table" of the "Benchmark 1997 Input-Output Table" (after redefinitions) and the "1997 Capital Flow Table", both available from the BEA. The 1997 Benchmark I-O and capital flow accounts use the classification system that is based on the North American Industry Classification System (NAICS). The BEA also provides with the Benchmark I-O Tables for 2002 (NAICS 2002), 1992 (1987 SIC based), for 1987 (1987 SIC based). Historical Benchmark I-O Tables are also available, however the time-series consistency is faulty.

The data on "Personal Consumption Expenditures by Type of Expenditure" are taken from NIPA Table 2.5.5. available from BEA. The data on "Real Personal Consumption Expenditures by Type of Expenditure, Quantity Indexes" are taken from NIPA Table 2.5.3. available from BEA. NIPA Tables from BEA are consistent with the NAICS basis used in their I-O Tables.

Since different data sources rely on different systems of industry classification, the mapping of every industry is only approximate across the different databases. The original classification tables for NAICS 1997, NAICS 2002, SIC 1987, ISIC Rev. 3.1., NACE Rev. 1 were checked together with the correspondence tables provided by the United Nations and U.S. Census Bureau. The time-series data from the Benchmark Tables is the one most susceptible to being problematic.

To illustrate the consistency across the different data sources, the following Table B1 at the end of the appendix summarizes the main variables' values across the different sources, while Table B2 provides descriptive statistics of the main variables used.

#### Variables

Value added is current gross value added measured at producer prices or at basic prices, depending on the valuation used in the national accounts. It represents the contribution of each industry to total GDP.

Value added deflator is the change in the value added deflator. It can be combined with current value added to derive quantity indices of real value added at industry level<sup>43</sup>.

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<sup>43</sup>The official data were readily adjusted into using a hedonic deflator system, so as to

Hours refers to average annual hours worked per employee or per person engaged. Personal consumption expenditures are the goods and services purchased by persons<sup>44</sup>.

#### Aggregation Method

In each dataset, the industries are grouped into three aggregate sectors: ICT-producing, ICT-using and non-ICT-using. Any transactions with abroad are not taken into consideration.

The Information and Communication Technology sector (ICT) producing sector is defined as in Jorgenson (2005) to include (SIC 1987 codes in parentheses) Computers and Office equipment (357), Electronic Components (367), Communications equipment (36 x 366-367) and Computer Services (737)<sup>45</sup>.

Following Jorgenson et al. (2005), the criterion for classifying an industry as ICT using is its degree of ICT capital intensity in 1995. In particular, the share of the ICT capital out of total capital compensation for an industry in 1995 needs to exceed the 15%<sup>46</sup>. Details on the mapping of the EU KLEMS data industries in each aggregate sector are provided below.

The aggregation is straightforward for the hours and consumption expenditures, intermediates and value added at current prices data. The direct aggregation across industries follows the "Aggregate Production Possibility Frontier" approach as first developed by Jorgenson (1966) and employed in recent growth accounting studies (Jorgenson et. al., 2005, van Ark et al., 2003). A Törnqvist index was applied to obtain value added deflators and value added growth rates for each of the three sectors<sup>47</sup>. The Domar weights were used for the aggregation of the contributions of capital, labour and TFP growth in aggregate value added.

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account better for the benefits arising from the ICT production and use. The deflators provided in the GGDC database come from official BEA data (harmonising of the deflators for other countries in the dataset does not affect USA data) and are based on the double deflation procedure for the ICT related industries. For an overview of the literature regarding hedonic deflators, see OECD "Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes: Special Application to Information Technology Products", Triplett J. (2004).

<sup>44</sup>In the national income and product accounts (NIPAs), persons consist of individuals, nonprofit institutions that primarily serve individuals, private noninsured welfare funds, and private trust funds.

<sup>45</sup>Compared to the OECD definition of the ICT sector that is followed in other studies (e.g. O' Mahony et. al, 2003, Van Ark et. al. 2003), Jorgenson's ICT-producing definition excludes the manufacturing industries ISIC Rev. 3. 1, (3312) and (3313), while it only includes the services industry ISIC rev. 3.1, (72).

<sup>46</sup>Alternative definitions for both the ICT-producing and ICT-using sectors were used, as well as the exclusion of the government sectors. The results presented in the paper are relatively robust to these alternative measures. The particular application was preferred because of its implied TFP data availability and its straightforward comparison to already found results.

<sup>47</sup>The Törnqvist aggregation method is based on weighting each industry's exponential annual growth rate with a two-period average of its share in aggregate value added. After computing the growth rate, the implied quantity index was derived, with the normalization that it is equal to 100 in 1995.

## C Additional Figures and Tables

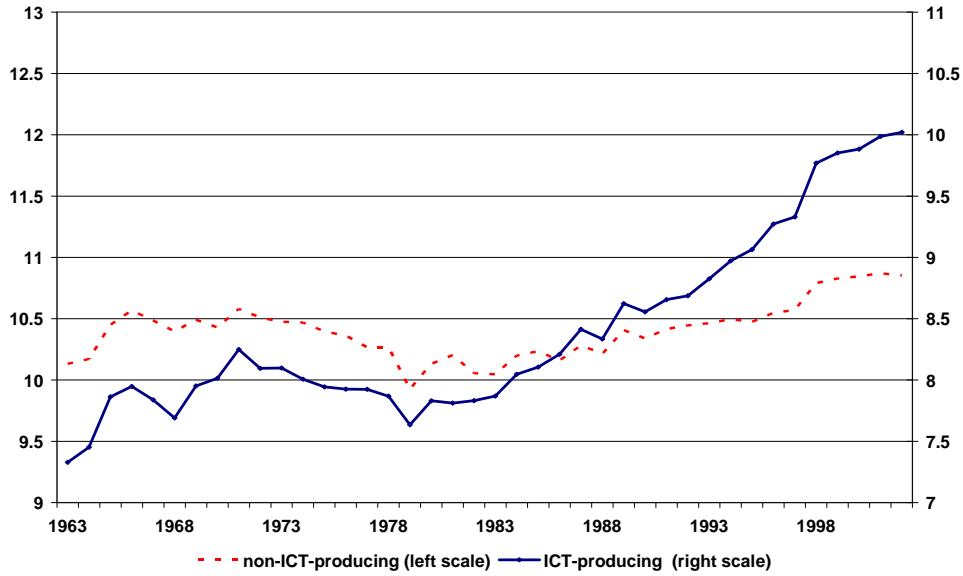


Figure C.1: Number of new patents granted by the USPTO for the ICT-producing and all other sectors (logs).

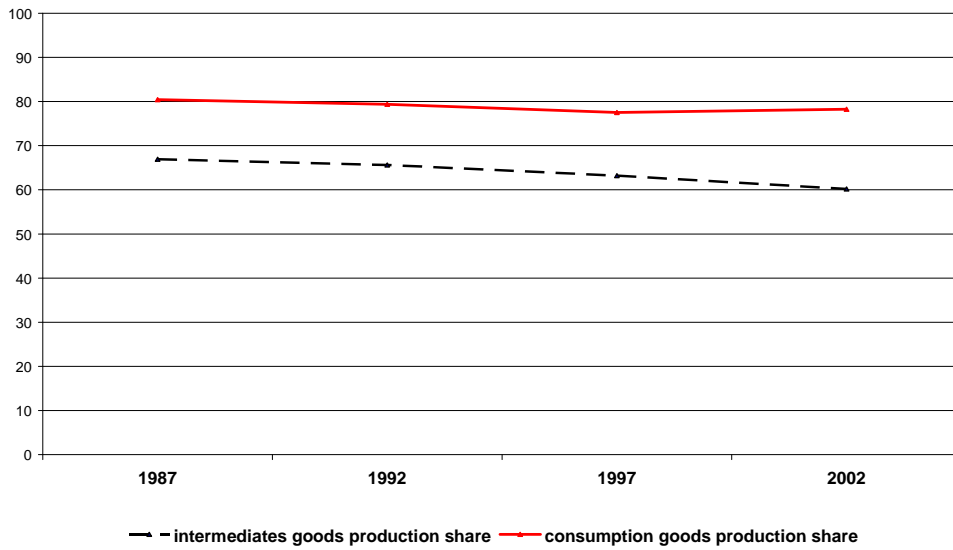


Figure C.2: United States non-ICT-using sector's intermediates and consumption production shares.

Table C.1: United States sector-level production shares by commodity use.

Producing sector/ Commodity use	Intermediates	Capital	Consumption
ICT-producing	4.4	16.1	0.9
ICT-using	35.8	25.7	22.3
non-ICT-using	59.8	58.2	76.8

Notes: columns sum up to 100%

Source: BEA, 1997 Benchmark Input Output Use and Capital Flow Tables

Table C.2: United States sector-level gross output sources of growth

	Gross output growth	Sources:			
		Capital	Labor	Intermediates	TFP
ICT-producing	16.76	1.98	1.66	6.33	6.79
ICT-using	3.76	1.33	0.96	1.45	0.02
non-ICT-using	2.24	0.62	0.33	0.59	0.00

Notes: 1977-2000 average growth rate (%)

Source: Jorgenson, Ho, and Stiroh (2005)

Table C.3: Basic data statistics

		Average			St. Deviation		
		1977-01	1977-95	1995-01	1977-01	1977-95	1995-01
share of total hours worked (in percent)	ICT-producing	1.9	1.7	2.4	0.4	0.2	0.3
	ICT-using	28.0	27.8	28.7	0.9	0.9	0.3
	non-ICT-using	70.1	70.5	68.9	1.2	1.0	0.6
share of value added (in percent)	ICT-producing	2.3	1.9	3.1	0.6	0.3	0.4
	ICT-using	29.3	28.5	31.7	2.1	1.6	1.1
	non-ICT-using	68.5	69.6	65.2	2.7	1.9	1.4
real value added growth rate (in percent)	ICT-producing	18.50	18.34	19.71	7.76	8.01	7.18
	ICT-using	3.92	3.75	4.39	2.75	2.95	1.93
	non-ICT-using	2.17	2.10	2.19	1.99	2.25	0.93
nominal value added growth rate (in percent)	ICT-producing	10.57	10.98	9.83	6.22	6.33	5.86
	ICT-using	7.73	8.31	6.00	3.82	3.93	2.81
	non-ICT-using	6.06	6.73	3.95	3.35	3.55	1.04
expenditure shares (in percent)	ICT-using	20.7	20.0	22.4	1.61	1.32	0.50
	non-ICT-using	79.3	80.0	77.6	1.61	1.32	0.50
real relative expenditure growth rate (in percent)	ICT-using (relative to non-ICT-using)	1.27	1.10	1.68	1.48	1.21	2.07
Source: see Section B							

Table C.4: Aggregate sectors in the EU KLEMS database.

ICT-producing sector		ICT-using sector		ICT-using sector	
Industry name	NACE code	Industry name	NACE code	Industry name	NACE code
Office, accounting and computing machinery	30	Printing, publishing and reproduction	22	AGRICULTURE, HUNTING, FORESTRY AND FISHING	A1B
Insulated wire	313	MACHINERY, NEC	29	MINING AND QUARRYING	C
Radio, television and communication equipment	32	Other electrical machinery and apparatus nec	31x	FOOD - BEVERAGES AND TOBACCO	1516
Computer and related activities	72	Medical, precision and optical instruments	33	TEXTILES, TEXTILE, LEATHER AND FOOTWEAR	1719
		Other transport equipment	35	WOOD AND OF WOOD AND CORK	20
		MANUFACTURING NEC; RECYCLING	36i37	Pulp, paper and paper	21
		Gas supply	402	CHEMICAL, RUBBER, PLASTICS AND FUEL	23i25
		Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel		OTHER NON-METALLIC MINERAL	26
		Wholesale trade and commission trade, except of motor vehicles and motorcycles	50		
		POST AND TELECOMMUNICATIONS	51	BASIC METALS AND FABRICATED METAL	2728
		FINANCIAL INTERMEDIATION	64	Motor vehicles, trailers and semi-trailers	34
		Renting of machinery and equipment	J	Electricity supply	40x
		Research and development	71	WATER SUPPLY	41
		Other business activities	73	CONSTRUCTION	F
			74	Retail trade, except of motor vehicles and motorcycles; repair of household goods	52
				HOTELS AND RESTAURANTS	H
				TRANSPORT AND STORAGE	6063
				Real estate activities	70
				COMMUNITY SOCIAL AND PERSONAL SERVICES	LtQ



Table C.5: Consistency across data sets.

source: variable/ period of comparison:	Jorgenson (2005) 1977-2000	EUKLEMS (SIC) 1977-2000	BEA, I-O 1997	BEA, C-F 1997	BEA, NIPA 1997
value added growth (in percent)					
	3.08	3.02			
ICT-producing	20.09	18.50			
ICT-using	3.89	3.92			
non-ICT-using	2.31	2.17			
shares in value added (in percent)					
	2.1	2.3	3.0	3.5	
	26.1	29.3	31.3	31.6	
	71.8	68.5	65.7	64.9	
capital producing shares (in percent)					
			15.6	16.1	
			23.7	25.7	
			60.8	58.2	
expenditure shares (in percent)					
			22.5		22.3
			77.5		77.7
Source: see Section B					

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