

# What Explains the Diversity of Regulatory Reform Outcomes?

FINAL REPORT

Petar Stankov, Aleksandar Vasilev

PROSPER Bulgaria, 18A Shipchenski Prohod Blvd., Apt. 20,

1113 Sofia, Bulgaria

Email: [prosperbulgaria@gmail.com](mailto:prosperbulgaria@gmail.com)

December 29, 2015

## **Abstract**

We set up a tractable general equilibrium (GE) model to study how output of firms of different size grows after entry and labor reforms. We then take the model predictions to the largest global publicly available firm-level data set: the Enterprise Surveys data. The results demonstrate that firms of different size grow differently after identical reforms. Thus, based on the notable differences of firm-size distributions across countries, identical reforms may produce a variety of growth outcomes.

# 1 Introduction

Suppose an identical regulatory reform is adopted simultaneously across a number of countries. Will the reformers be affected identically? This paper argues they will not, and looks for the reasons behind an eventual outcome divergence. The explanation offered here is that regulatory reforms – i.e., the state’s withdrawal from its legal powers to direct pricing, entry and exit on a given market (Winston, 1993) – affects firms of different size differently. Then, if two countries go through identical reforms but their firm size distributions are *ex-ante* different, the two economies will react differently to the reform. Naturally, the argument extends to more than two economies and to more than one regulatory reform. It also produces a variety of reform outcomes across countries and possibly over time. Thus, studying the reform outcomes across firms of different size has important policy implications which is the main reason we look at the reform effects from this angle.

Our approach is to set up a tractable general equilibrium (GE) model and study how output of firms of different size grows after entry and labor reforms. The theoretical results suggest that larger firms would grow faster than smaller firms after a regulatory reform. This turns into the main hypothesis of this paper. We take it to the largest global publicly available firm-level data set to test it, and broadly confirm its validity.

Our theory is in the spirit of Luttmer (2007) and Moscoso Boedo and Mukoyama (2012) who also use GE models to numerically study the effects of entry and labor regulations on productivity and employment but is set apart in two important ways. First, our project still entails a micro-founded GE model but is computationally far less intensive and allows for a tractable analytical solution without losing explanatory virtue. Second, none of the theoretical works considers the different effects of entry and labor regulations

across firm size.

Firm size appears more prominent in the empirical work. Two of the excellent accounts of how the literature evolved since the 1970s are Joskow and Rose (1989) and Winston (1993). Klapper, Laeven, and Rajan (2006) find that more stringent entry regulations hamper the creation of new firms at the benefit of the incumbent firms but also that the incumbent firms grow more slowly due to weaker competition. In addition, Ayyagari, Beck, and Demirguc-Kunt (2007) use data from 76 countries to find a significant effect of entry regulations on the output of small businesses, and that the share of small firms in regimes with lower entry costs is larger.

Apart from entry regulations, labor market reforms could potentially have divergent policy implications across countries based on the evidence of their different effects across firm size. Schivardi and Torrini (2008) identify the effects of labor market regulations on Italian enterprises and conclude that removing some restrictive labor regulations would lead to a modest positive growth of firms, which Gourio and Roys (2012) confirm for France by studying the specifics of the French size-contingent labor regulations. Almeida and Carneiro (2009) also conclude that more stringent labor regulations constrain firm size and output growth in Brazil, while Ahsan and Pagés (2009) reach similar conclusions for India. Kaplan (2009) reinforces the point that more stringent labor regulations would hurt firm output in the small firms segment in a study of 14 developing economies, while Feldmann (2009) and Feldmann (2012) extend the empirical evidence to 73 and 80 economies, respectively, without looking into a size-contingent effect from labor regulations.

The theoretical counterpart focused on developing general equilibrium (GE) models to study how firms grow after a given reform on entry or labor regulations. For example, Antunes and de V. Cavalcanti (2007) develop

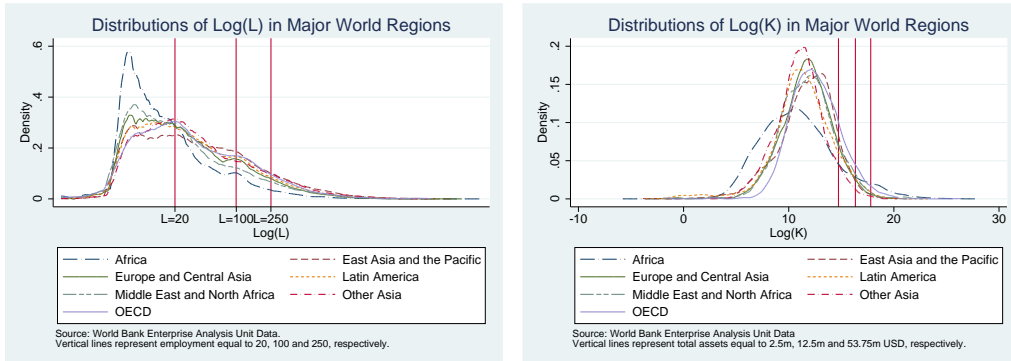
a GE model which numerically solves for the effect of entry barriers and enforcement of regulations on the size of the informal sector. However, their model is not analytically tractable. Moreover, similarly to Bennett and Estrin (2013), they do not account for the differences in the effects of regulatory reforms across firms of different size.

The next section illustrates the differences observed in the firm-size distributions (FSDs) across the globe and argues why those differences matter for delivering different reform outcomes across countries.

## **2 Firm-size Distributions across Countries**

Providing credible evidence of a variety of reform effects across countries hinges on several important questions. First, are there significant differences in the firm-size distributions (FSDs) across countries? If the FSDs are the same, then the reform outcomes across countries would hardly be significantly different, even if small and large firms are found to grow differently after the reform. Second, do reforms influence those distributions? If FSDs are influenced by the reforms over short periods of time, then the FSDs themselves would be endogenous to the reforms. Therefore, it is important to know whether one can take the FSDs as exogenous at least in a cross-sectional setting. Third, are the cross-country growth differences affected by the differences in the FSDs? If they are, then a reform could not only have a different effect on firms of different size but it could also bring aggregate reform implications across countries. This part of the paper addresses each of these questions.

Over recent decades there have been substantial efforts to explain the statistical regularities behind FSDs both within and across countries, and



(a) Log(L)

(b) Log(K)

Figure 1: Firm-Size Distributions of Employment and Assets

over time. Gabaix (2011) reviews the evidence that FSDs in developed countries are found to have a Zipf distribution, at least in their upper tails.<sup>1</sup> However, in some developed countries such as Japan (Kaizoji, Iyetomi, and Ikeda, 2005), and most notably in the developing world, this regularity in FSDs is harder to observe, as the data presented here and additional evidence suggests.<sup>2</sup> In addition, looking at the figures below, it is obvious that there are marked differences in FSDs across major regions of the world, especially in the small-firm segments of the distributions. Those differences are also observed within each of those regions and may be explained by several arguments.

First, many young firms operate in the small-firm segment. The growth of those firms is more volatile (Alexander, 1949; Samuels and Smyth, 1968). They grow faster as well but are also more likely to fail (Dunne, Roberts, and

<sup>1</sup>Following Gabaix (2011), the Zipf distribution in firm size essentially means that the probability of a firm size  $S$  being greater than  $x$  is inversely proportional to  $x$ . More formally,  $P(S > x) \simeq kx^{-\alpha}$ , and in the particular case of Zipf distribution,  $\alpha \simeq 1$ .

<sup>2</sup>For some differences in the FSDs between the developed and the developing world, see Alfaro, Charlton, and Kanczuk (2008).

Samuelson, 1989; Jovanovic, 1982; Mansfield, 1962; Mata, 1994). The snapshots of FSDs in Figure 1(a) and Figure 1(b) capture marked differences in the FSDs across major world regions exactly in the small firms segment [below 20 employees in Figure 1(a) and below USD 2.5m in assets in Figure 1(b)]. Admittedly, the figures are a sample of the universe of firms across countries and global regions. However, the Enterprise Surveys sampling methodology has been designed to be representative of the underlying FSDs.<sup>3</sup>

Second, trade theory produces a well-known proposition that different countries specialize in different industries.<sup>4</sup> If there is a different evolution of FSDs across industries, then the within-country industry specialization would give rise to divergent evolutions of FSDs across countries depending on their industrial structure. Indeed, significant differences in FSDs across industries within a period (Rossi-Hansberg and Wright, 2007) and different evolutions of FSDs across industries have been documented (Lotti and Santarelli, 2004). Lotti and Santarelli (2004) study FSDs of new entrants in several industries and find they vary across their minimum-efficient scale and technological requirements. Technology is also found to be an important factor generating differences in FSDs across industries by Marsili (2005). These facts might explain the differences in FSDs at a point in time across countries.

However, despite the marked cross-country differences in FSDs, and despite the documented underlying evolutionary process towards an equilibrium FSD *within an industry* (Hashemi, 2000), the within-country distributions are relatively stable, as found by Cabral and Mata (2003) and Henly and Sánchez (2009). Cabral and Mata (2003) also note that the FSD of a given cohort of firms changes slowly over time, while Henly and Sánchez (2009)

---

<sup>3</sup>See p.2 in the Sampling Methodology notes on the ES website: <http://www.enterprisesurveys.org/methodology>

<sup>4</sup>See Heckscher-Ohlin and Rybczynski theorems.

add that the within-industry FSD changes over long periods of time and the within-country FSD stays unchanged. Doi and Cowling (1998) assert that in some countries (e.g., Japan) the share of output and employment across size classes is relatively constant over long periods of time, while in others (e.g., the UK) they change only slowly in favor of smaller firms. Axtell (2001) also concludes that FSDs are stable over time, at the same time being robust to the employed definition of firm size. Then, it can be assumed that cross-country FSD differences are stable over relatively long periods of time, and are not affected by reforms in the short-run.

The above exogeneity assumption does not mean the within-country and within-industry FSDs do not evolve.<sup>5</sup> However, it is more likely that the differences in FSDs across countries came from an underlying difference in some fundamental factor rather than a given reform. Lucas (1978) argues that FSD is underlined by a distribution of managerial talent. Thus, different countries end up having different FSDs depending on the international allocation of talent. At the same time, countries with lower quality of institutions and enforcement of property rights have a different allocation of talent into productive and rent-seeking occupations (Murphy, Shleifer, and Vishny, 1991). Thus, it is tempting to explain the observed cross-country differences in FSDs with different underlying institutions and property rights systems which rarely change over short periods of time.

Finally, there are emerging implications in the FSD literature that FSDs are correlated with cross-country income differences (Alfaro et al., 2008; Gabaix, 2011). This evidence contributes to the understanding that FSDs are an important determinant of cross-country differences in the growth effects of reforms.

---

<sup>5</sup>See Sutton (1997, 2007) for extensive discussions on FSD evolution.



In a nutshell, both the firm-level data used here and the size distribution literature point to significant differences in FSDs across countries. However, policy reforms seem to do little to affect the evolution of FSDs over short periods of time within a country. Rather, FSDs are more likely to be driven by fundamentals such as preferences, factor endowments and institutions that affect industry specialization than with policies. Thus, it is legitimate to assume both the FSD within a country and the cross-country differences in FSDs as given, at least in a short panel, and especially in a cross-sectional data setting. However, the variation in the FSDs also affects the cross-country income differences. Thus, it is very intuitive to hypothesize that an identical reform would produce a variety of reform outcomes across countries based on the underlying differences in the FSDs. This hypothesis seems to have evaded both the theoretical and the empirical literature so far.

The next section demonstrates the basic elements of the GE model which is set up to study how an identical regulatory reform produces different effects across various firm sizes. The section also presents the empirical framework.

## **3 Methodology**

### **3.1 Overview of the Model Setup**

There is a representative household that derived utility out of consumption and leisure, where aggregate consumption is a bundle of all varieties available in the economy. There is a unit mass of monopolistically-competitive variety producers, who face regulatory costs of production. More specifically, we extend the framework by Dixit and Stiglitz (1977) by introducing a regulatory cost as in Luttmer (2007), which is going to be measured in terms of time. Lastly, a government regulatory agency (“regulator”) is added, which decides

on the level of regulations, and enforces them through the use of employed bureaucrats, whose wages are paid out of the raised tax revenue.

### 3.2 Household's Problem

In this setup, all varieties  $\{c_i\}_{i=1}^N$  are equally weighted as seen from the household's utility function, which is of the form:

$$\max_{c_i, h_i} \ln\left(\left[\int_0^N c_i^\rho di\right]^{1/\rho}\right) + \ln\left(1 - \int_0^N h_i di\right), \quad (1)$$

where  $\rho \in (0, 1)$ . Note that total time is normalized to unity. In order to generate income to finance consumption of varieties, the household can supply hours  $\{h_i\}_{i=1}^N$  from its time endowment to the firms. The market hourly wage rate is  $w$ , and labor services are assumed to be homogenous across firms (hence the single wage rate prevailing in the economy). Therefore, total labor income is

$$\int_0^N w_i h_i di = w \int_0^N h_i di = wH, \text{ where } H = \int_0^N h_i di \quad (2)$$

In addition, the agent will have claim on all firms' profits ( $\Pi = \int_0^N \pi_i di$ ), where  $\Pi$  and  $\pi(i)$  denote aggregate and individual profits, respectively.<sup>6</sup> The household's budget constraint then becomes

$$\int_0^N p_i c_i di = wH + \Pi - \tau, \quad (3)$$

where  $p_i$  denotes the price of variety  $i$ , and  $\tau$  is the amount of lump-sum taxes owed. Taking  $\{p_i\}_{i=1}^N, w$  as given, the household then chooses  $\{c_i, h_i\}_{i=1}^N$  to

---

<sup>6</sup>With regard to the industry structure, with free entry, there will be only one firm producing a particular variety. With free entry, equilibrium profits will be also shown to be zero.

maximize Eq.(1) s.t. Eqs. (2)-(3). The resulting FOCs are as follows:

$$c_i : \frac{c_i^{\rho-1}}{\int_0^N c_i^\rho di} = \lambda p_i, \quad (4)$$

$$c_j : \frac{c_j^{\rho-1}}{\int_0^N c_j^\rho dj} = \lambda p_j, \quad (5)$$

$$h_i : \frac{1}{1 - \int_0^N h_i di} = \lambda w, \quad (6)$$

$$h_j : \frac{1}{1 - \int_0^N h_j di} = \lambda w. \quad (7)$$

It follows that  $h_i = h_j = h$  as both satisfy the same FOC, and thus  $H = Nh$ . If the going hourly wage rate is the same across firms, and the labor supplied is homogenous, then in equilibrium the household will work the same number of hours in each firm.

Next, the FOCs for  $c_i$  and  $c_j$  are divided side by side to obtain

$$c_i = \left(\frac{p_i}{p_j}\right)^{\frac{1}{\rho-1}} c_j. \quad (8)$$

This expression is then plugged into the household's budget constraint to yield

$$\int_0^N p_i \left(\frac{p_i}{p_j}\right)^{\frac{1}{\rho-1}} c_j di = w \int_0^N h_i di = wH \quad (9)$$

Let  $wH + \Pi - \tau = Y_{MCE}$  (since with free entry there is no profit income). That is, total income equals total real output. Then (taking out terms with index  $j$ ),

$$c_j p_j^{\frac{1}{1-\rho}} \int_0^N p_i^{\frac{\rho}{\rho-1}} di = Y_{MCE} \quad (10)$$

Denote aggregate price aggregator/index as  $P \equiv \int_0^N p_i^{\frac{\rho}{\rho-1}} di$ , then divide output by the price index  $P$ , and call the ratio  $B$  to obtain  $B = Y_{MCE}/P = c_j p_j^{\frac{1}{1-\rho}}$ , or

$$c_j = B p_j^{\frac{1}{\rho-1}} \quad (11)$$

that is, demand for each variety is isoelastic.<sup>7</sup>

### 3.3 Government Regulator's Problem

There is a government regulator, whose objective function is positively monotone in the amount of regulation passed,  $\bar{h}$ , and negatively-related to the amount of lump-sum taxes ( $\tau$ ) that needs to be raised to pay bureaucrats' wages<sup>8</sup>. Taxes are set so that the budget constraint is balanced every period. The utility derived from regulation is assumed to feature positive, but decreasing marginal benefit of regulation, hence  $0 < \theta < 1$ . At the same time, as in Barro (1979), the regulator wants to minimize the distortionary effect of changing taxes, hence the tax term will be described via a convex cost function. The regulator's problem is then to

$$\max_{\bar{h}, \tau} \bar{h}^\theta - \tau^2 \text{ s.t. } w\bar{h} = \tau. \quad (12)$$

Substitute out  $\tau$  from the budget constraint and solve for  $\bar{h}$  to obtain<sup>9</sup>  $\bar{h} = (2/\theta)^{\frac{1}{\theta-2}}$ . Note that computing  $\bar{h}$  does not affect the other parts of the problem. In the model calibration,  $\theta$  parameter will be set in order to match the average value of  $\bar{h}$  in data (where it is pinned down in foregone wages terms).<sup>10</sup>

---

<sup>7</sup>Note that the index is irrelevant, and W.L.O.G. can be substituted with  $i$  in the firm's optimization problem.

<sup>8</sup>It is assumed that bureaucrats are the agents that enforce the regulations created. In this model those bureaucrats do not have any active role, but rather help to close the model.

<sup>9</sup>In the model we can normalize one of the prices, so the wage rate will be set to unity.

<sup>10</sup>Using the specification above, the model with regulator choosing the level of regulation optimally is isomorphic to a setup without a maximizing regulator. The limitation of this analysis is that we are not explicitly solving for the political economy equilibrium, that is, the objective function of the regulator should be an aggregation of voters' preferences for

### 3.4 Variety-Producing Firms

Each of the  $N$  firms in the economy would produce a single variety, which will be differentiated from the other  $N - 1$  goods. The production function of each variety will be Cobb-Douglas in labor and capital, and the total factor productivity (TFP) will enter multiplicatively, *i.e.*

$$c_i = A_i K_i^\alpha (h_i - \bar{h})^{(1-\alpha)}, \quad (13)$$

where  $A_i$  is the productivity shift parameter,  $K_i$  denotes the capital input used by firm  $i$ , and  $\alpha, 1 - \alpha$  denote the capital and labor shares, respectively. Physical capital is assumed to be pre-installed in this setup, and thus will be treated as the fixed input. In the presence of regulatory costs, positive output will be produced only in case hours hired exceed those costs (expressed also in labor terms). Note that such production function features increasing returns to scale: with fixed costs, and capital being a pre-installed input, which does not change in the short-run, charging a price equal to the marginal cost leads to profits, and thus a competitive equilibrium cannot exist. More specifically, given the differentiated nature of their product, each producer will be able to set its price, given a certain demand for the variety.

### 3.5 Symmetric Monopolistically-Competitive Equilibrium

For tractability purposes, the model will focus on symmetric case where it will be assumed that all firms use the same technology and capital, thus TFP can be normalized to unity,  $A_i = 1$ , and  $K_i = K$ . In other words, all 

---

regulation (firms are owned by consumers in the model). Since we have a representative agent, all households will choose the same level of regulation. However, in the general case we would need heterogeneous consumers, which complicates the algebra significantly, and is thus left for future research.

firms will be identical in productivity, capital, and employment. In addition to making the model tractable, the symmetry will allow for a quick check of basic intuition about model inter-relationships through the derivation of comparative static conditions.<sup>11</sup>

From the equation above it is clear that in the presence of regulatory costs, positive output will be produced only in case the hours hired exceed those costs, which are expressed also in terms of labor hours. These costs could be thought of representing some setup costs, such as hiring/firing costs.

Note that such production function features increasing returns to scale (when the capital is taken as a fixed input) and thus the producer cannot be a perfectly-competitive firm. More specifically, with fixed costs, charging a price equal to the marginal cost leads to positive economic profits, which is inconsistent with a (perfectly-)competitive equilibrium outcome. That is why the equilibrium concept will be relaxed to allow for monopolistically-competitive producers and free entry. In that case, the firm's objective is to maximize profit by taking the demand for its product as given, and choosing the price of its variety,  $\{p_i\}$ :<sup>12</sup>

$$\max_{p_i} p_i B p_i^{\frac{1}{\rho-1}} - w h_i \quad (14)$$

Given that

$$c_i = K^\alpha (h_i - \bar{h})^{(1-\alpha)} \quad (15)$$

---

<sup>11</sup>In a Technical Appendix we extend the model to the asymmetric solution, and show that the comparative-statics will depend of the firm-size, as proxied by technology, capital, and employment. However, since the algebra becomes quite messy, we decided to leave this interesting case outside the main body of the paper, and instead present the symmetric equilibrium only. In addition to being tractable, most of the results easily extend to the asymmetric case, where firm-specific characteristics are important.

<sup>12</sup>Note that by optimally setting the price, the firm will optimally set its output as well.

it follows that

$$h_i = c_i^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} + \bar{h} = \left[ B p_i^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} + \bar{h} \quad (16)$$

Plug in the expression for  $h_i$  in firm's optimization problem to obtain

$$\begin{aligned} \max_{p_i} p_i \left[ B p_i^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} - w \left[ \left[ B p_i^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} + \bar{h} \right] = \\ \max_{p_i} \left[ B p_i^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} [p_i - w] - w \bar{h} \end{aligned} \quad (17)$$

From the FOC it follows that

$$p_i = w/\rho > w, \quad (18)$$

or that each price is a fixed mark-up over the wage, hence

$$p = p_i = p_j \quad (19)$$

and also

$$c = c_i = c_j \quad (20)$$

Impose the symmetry into the FOCs for consumption and hours, and divide them side by side to obtain

$$\frac{\frac{c^{\rho-1}}{Nc^\rho}}{\frac{1}{1-H}} = \frac{1}{\rho} \quad (21)$$

or

$$\frac{1-H}{Nc} = \frac{1}{\rho}. \quad (22)$$

Note that aggregate consumption equals

$$Nc = NK^\alpha (h_i - \bar{h})^{(1-\alpha)} = (NK)^\alpha (N h_i - N \bar{h})^{(1-\alpha)} = (NK)^\alpha (H - N \bar{h})^{(1-\alpha)}. \quad (23)$$

Plug this expression in the equation above, and simplify to obtain

$$\rho(1-H) = (NK)^\alpha (H - N \bar{h})^{(1-\alpha)}. \quad (24)$$

We can solve the non-linear equation above for  $H$  as a function of parameters  $\bar{h}, \rho, K, \alpha$ , then obtain hours worked per firm  $h = H/N$ , consumption of each variety, and using free entry condition, determine the number of the firms. Unfortunately, in the general case, there is no closed-form solution, and numerical exercises for different parameters have to be executed to study comparative statics. Only for the special case when  $\alpha = 0$ , i.e., shutting down the physical capital and collapsing the production function to becoming linear in labor, the model can be solved analytically. We will present that particular solution, as then comparative static effects are transparent. In addition to simplifying the algebra, the explicit solution allows us to present the comparative-static effects of interest in a tractable way, without changing the sign of major effects in the model. The transformed model then collapses to

$$\rho + N\bar{h} = (1 + \rho)H \quad (25)$$

$$H = \frac{\rho + N\bar{h}}{1 + \rho} \quad (26)$$

$$h = \frac{\rho + N\bar{h}}{N(1 + \rho)} \quad (27)$$

$$c = h - \bar{h} = \frac{\rho + N\bar{h}}{N(1 + \rho)} - \bar{h} = \frac{\rho(1 - N\bar{h})}{N(1 + \rho)} \quad (28)$$

From the free entry condition ( $\pi = pc - wh = 0$ ), and after some algebra, it follows:<sup>13</sup>

$$N = \frac{1 - \rho}{2\bar{h}}. \quad (29)$$

That is, the total number of varieties will be endogenously-determined, and it will depend on the model parameters. More specifically, the higher the

---

<sup>13</sup>This is so, because for each firm there is a given optimal scale of production. So the firm is not indifferent towards how close it is to its optimal size. And if regulation affects output, then regulation affects how far the firm is from its optimal size. Therefore, even though profits are zero with or without regulation, regulation affects the firm.



setup cost  $\bar{h}$ , the lower the number of varieties  $N$ . Also, since

$$\frac{dN}{d\rho} = -\frac{1}{2\bar{h}} < 0, \quad (30)$$

$$\frac{dN}{d\bar{h}} = -\frac{1-\rho}{2\bar{h}^2} < 0. \quad (31)$$

it follows that the more substitutable the goods (higher  $\rho$ ), the lower the number of varieties. Next, total labor supply can be obtained as:

$$H = \frac{\rho + \frac{1-\rho}{2}}{1+\rho} = \frac{2\rho + 1 - \rho}{2(1+\rho)} = \frac{1+\rho}{2(1+\rho)} = \frac{1}{2}, \quad (32)$$

where the peculiar result is due to the log-log specification for the utility function. Individual hours are then

$$h = \frac{H}{N} = \frac{1}{2N} = \frac{\bar{h}}{1-\rho}, \quad (33)$$

hence positively related to  $\bar{h}$ . In other words, the higher the setup cost, the higher the hours supplied to each firm (in order to produce a positive quantity of a variety). Finally, output for each variety is

$$c_i = c = \frac{\rho(1 - \frac{1-\rho}{2})}{(1+\rho)\frac{(1-\rho)}{2\bar{h}}} = \frac{\rho\bar{h}}{1-\rho} \quad (34)$$

Again, the higher the setup cost  $\bar{h}$ , the higher the output of each variety. This result has important policy implications: any increase in regulations (higher  $\bar{h}$ ) leads to more labor being wasted in setup costs; thus, it is optimal to have less varieties, but to consume more of each existing variety. However, that is not bringing higher utility to the consumer, who prefers to consume a bit of many varieties. Overall, higher levels of regulation decreases consumer's welfare.

The model allows us to derive the following empirical implications from (34): Deregulation (reducing  $\bar{h}$ ) will make each firm produce less, and at the same time will create more varieties to choose from. In addition, if the

elasticity parameter  $\varepsilon = \frac{1}{\rho-1}$  is taken to proxy firm size, then the effect will be stronger for smaller firms which have higher price elasticity of demand. Specifically, after deregulation, smaller firms will reduce output more than larger firms, and will grow slower than larger firms.<sup>14</sup>

### 3.6 Data

There are two main sources of data to feed the model: the Enterprise Surveys data and the World Bank Doing Business data. The Enterprise Surveys data set is produced by the Enterprise Analysis Unit (EAU) at the World Bank. It encompasses firm-level data from 2006 to 2014 in various countries. The data set has about 117 000 firm-level observations from 135 countries and territories, 206 country-years, and 15 major industries in each of those country-years. The firm-level frequency in each of those industries is presented in Table 1 below.

The Enterprise Surveys data set is probably the largest publicly available firm-level data which is suitable for policy analysis. To reduce the number of empty industry-country cells, we drop any industry with less than 1000 observations, and any country with less than 100 observations. We also refrain from using the built-in subjective self-reported evaluations of regulatory performance in the EAU data, as it would present challenges in extracting the exogenous variation in the regulatory performance from those evaluations.

The second data source on regulatory reforms takes advantage of the

---

<sup>14</sup>In the Appendix, we present the asymmetric equilibrium, a case not studied in the literature, where firms will differ in several dimensions, such as their TFP, capital stock, and employment. The algebra is quite tedious, and thus the treatment is laid out in an appendix form. This extension is one of the novelties, as it generates comparative statics that explicitly depend on the size (TFP, capital, employment, sales) of the particular firm.

Table 1: Number of Firm-level Observations at the Industry Level

Industry	No. obs.
Textiles	5,724
Leather	1,014
Garments	7,640
Food	12,476
Metals and machinery	9,907
Electronics	1,810
Chemicals and pharmaceuticals	5,162
Wood and furniture	2,775
Non-metallic and plastic materials	6,930
Auto and auto components	1,038
Other manufacturing	9,412
Retail and wholesale trade	27,296
Hotels and restaurants	6,263
Other services	12,253
Other: Construction, Transportation, etc	6,951
Total	116,651

Notes: The table presents the number of firm-level observations within each of the industries featured in the World Bank Enterprise Analysis Unit data, as of June 26, 2015. We cleaned the data from 369 observations which did not have a definite industry affiliation.

available *objective* measures of regulations across countries. The World Bank Doing Business data base contains numerous regulatory indicators of entry and labor across most countries and territories in the world since 2006. We take 4 measures of entry regulation and one measure of labor regulations to proxy for how easy it is to start a business and hire or fire workers. The four measures of entry reforms are: costs to start up (as a share of annual income); number of days to start up; minimum capital required; and number of procedures. We call these variables *Cost*, *Days*, *MinCap*, and *No.Proc.*, respectively. In addition, we take the labor tax as a gauge of how burdensome the labor regulations in a given country are, and we call that variable *LTax*. To measure the effect of a potential reform on firms of different size, we interact the reform variable with the size of the firm, taken the log(number of employees). The details of the empirical model follow.

### 3.7 Empirical Model

The theoretical model has demonstrated that firms of different size grow differently after identical regulatory reforms. In order to test this prediction, we estimate the following model:

$$\begin{aligned} \Delta \log Y_{ikt} = & \alpha_1 + \alpha_2 \Delta \log L_{ikt} + \alpha_3 \Delta ER_{ikt} S_{ikt} + \alpha_4 \Delta LR_{ikt} S_{ikt} + \\ & + \mathbf{Z}'_{ikt} \alpha + f_s + f_k + f_{st} + f_{kt} + \Delta \varepsilon_{ikt}, \end{aligned}$$

where  $\log Y_{ikt}$  stands for either sales,  $\log SAL_{ikt}$ , or the sales per worker,  $\log SPW_{ikt}$ , of firm  $i$  in country  $k$  in period  $t$ . In addition,  $\log L_{ikt}$  is the number of employees,<sup>15</sup> respectively, to estimate the impact of the main

---

<sup>15</sup>Labor costs is another option for  $L_{ikt}$ . However, the data contains about 20,000 fewer observations on labor costs than number of employees. This is an obvious reason to prefer the latter, especially given the correlation between the two is 0.60 and highly significant.

factors of production;  $ER_{ikt}$  and  $LR_{ikt}$  are the entry and labor regulations, respectively, that firm  $i$  has to deal with in country  $k$  in period  $t$ .

$S_{ikt}$  is the size of the firm measured by either the log-number of employees or by the log-value of assets;  $\mathbf{Z}'_{ikt}$  is a vector of firm observables, including whether the firm has obtained an ISO certification, to capture some differences in the performance of firms with different levels of technology and more sophisticated management procedures, legal structure, age of the firm and top manager experience. Finally,  $f_s$ ,  $f_k$ ,  $f_{st}$  and  $f_{kt}$  are the sector and country fixed and time-varying effects. Including those is motivated by the firm-level evidence by Commander and Svejnar (2011) on the reform outcomes in central and eastern Europe. In this paper, the country fixed effects turn out to be more important than reforms in determining firm performance.

Finally,  $\log K_{ikt}$  is conspicuously missing from the growth equation. As there is no measure of the lagged level of capital in the Enterprise Surveys data (apart from its panel component for Central and Eastern Europe which is very small), we need to assume the growth of sales and sales per worker depends on the growth of labor rather than both the growth of labor and capital. There is a way to include the level of capital in a level equation but the results would not have a clear reform interpretation. Naturally, excluding the change in capital introduces an omitted variable bias (OVB) in all estimates. However, if such a bias indeed exists, it would bias all the estimates in the same way, as capital is missing from all the regression equations. Since we are more interested in the sign of the parameters rather than their magnitudes, we believe the OVB is not crucial in interpreting the results.

## 4 Results

There are two tables presenting the results from testing the conclusions of the theoretical model. The two tables correspond to the two performance indicators: sales and sales per worker. The former is a measure which has implications about the growth of firms in general, while the latter is a better gauge of labor productivity. As both firm-level growth and labor productivity have growth and development implications, we run separate estimations for each of these two performance indicators.

Tables 2 and 3 run the estimations of the benchmark equation in differences. That is, for each of the performance indicators, the difference between the level of that indicator now and 3 years before is taken as the dependent variable, while the explanatory variables are the differences of the interaction between the objective regulatory measures and the size of the firm.

The results demonstrate a somewhat nuanced effect of entry and labor regulations on firms of different size. Both tables 2 and 3 suggest that reducing the costs to start up a firm would make sales and labor productivity grow faster for smaller firms. This result is at odds with our theoretical predictions, which suggested larger firms will grow faster after a reform. However, reducing the number of days to register a company, lowering the minimum capital requirement and reducing the number of procedures would indeed benefit larger firms more, as predicted in our theory.

This result can be explained by the intuition by Aghion, Alesina, and Trebbi (2007). They suggest that entry deregulation would make the incumbent firms innovate more to prevent further entry, especially in technologically advanced industries. Incumbent firms in those industries would therefore grow faster. This means that countries with a high share of large firms will develop faster than countries with a high share of small firms, es-

pecially after an entry reform. The results also demonstrate that firm size is one of the factors behind the divergent regulatory reform outcomes across countries.

Labor reforms, on the other hand, seem to produce no significant differences in the growth of firms across different size classes. Despite the theoretical predictions to the contrary, the Enterprise Surveys data and our empirical methods suggest that labor reforms have little to do with the differences in the development across countries over time.

## 5 Conclusion

The abundance of evidence on the impact of entry and labor regulations motivates this project to build a micro-founded GE model which explains the diversity of reform outcomes across countries by using two notions: first, firms of different size grow differently after a regulatory reform, and second, firm size distributions across countries are distinctly different but also stable over time. The model predicts firms of smaller size would grow slower after a regulatory reform of entry and labor. The model predictions are then tested on the Enterprise Surveys data produced by the World Bank.

The data conforms well with the model predictions when it comes to the impact of some entry and labor reforms. Specifically, both sales and labor productivity of larger firms grow faster than those of smaller firms after reducing the time and the number of procedures to set up a firm and the minimum capital requirements. However, the model predictions are at odds with the data when the reform is thought as reducing the cost to start up an enterprise. Specifically, smaller firms grow faster than larger firms when the costs to start a firm are reduced. In addition, labor reforms seem

irrelevant for the performance differences across firms of different sizes which is an unexpected result, given the model predictions.

The paper extends the recent literature in several ways. First, the tractable micro-founded GE model seems necessary to produce a set of testable predictions. Most of the recent papers rely on previous empirical evidence to seek new evidence. Second, the majority of the literature misses the importance of looking at the divergent effects of reforms across firms of different size. Those differences in the reform outcomes may also produce a variety of reform outcomes across countries, and this avenue for research has been largely underestimated so far. The diverging growth and labor productivity outcomes stem from the notable differences in the size distribution of firms across countries. The latter holds rich policy implications for size-contingent regulatory reforms of entry and, potentially, labor in the developing and emerging economies, especially if more conclusive evidence is found on the variety of labor reform outcomes across firms of different size.

## 6 Acknowledgements

This research was supported by a research grant No. RRC15+04 from the CERGE-EI Foundation under a program of the Global Development Network (GDN). We thank the participants at the GDN Regional Conference in Prague, held in August 2015, for helpful comments and suggestions, especially Randall Filer, Fabio Michelucci, Mario Holzner and Štěpán Juráida. All opinions expressed are those of the authors and have not been endorsed by CERGE-EI or the GDN.



## 7 Tables

Table 2: Regulatory Reforms and  $\Delta \text{Log}(\text{SAL})$  across Firms of Different Size

	Dependent variable: $\Delta \text{Log}(\text{SAL})$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \text{Log}(\text{L})$	.408*** (.017)	.431*** (.018)	.408*** (.017)	.476*** (.024)	.284** (.116)	.766*** (.215)
$\Delta \text{Cost} * \text{Size}$						.002*** (.000)
$\Delta \text{Days} * \text{Size}$		-.001** (.000)				-.006 (.008)
$\Delta \text{MinCap} * \text{Size}$			-.000 (.000)			-.001*** (.000)
$\Delta \text{No.Proc.} * \text{Size}$				-.008*** (.002)		-.039*** (.015)
$\Delta \text{LTax} * \text{Size}$					.004 (.004)	.004 (.005)
Mgr. Exp.	-.001* (.000)	-.001* (.000)	-.001* (.000)	-.001* (.000)	.000 (.001)	.000 (.001)
Firm Age	-.001*** (.000)	-.001*** (.000)	-.001*** (.000)	-.001*** (.000)	-.001* (.001)	-.001** (.001)
Const.	.218 (.180)	.205 (.180)	.219 (.180)	.196 (.180)	.079 (.179)	-.203 (.158)
Observations	55660	55660	55660	55660	11355	10896
Adjusted $R^2$	.152	.152	.152	.152	.473	.057

Notes: The table presents results from OLS estimations of the difference in  $\text{Log}(\text{Sales})$  on the difference in  $\text{Log}(\text{No. of employees})$ , on other observables from the World Bank Enterprise Analysis Unit firm-level data, and on objective reform data, measured by The Doing Business Database interacted with the firm size measured by the  $\text{Log}(\text{No. of employees})$ . All estimations include the age of the firm, its legal status, an indicator of a quality certificate, industry-year and country-year effects. Robust standard errors are given in parentheses. Symbols: \*  $p < .10$ , \*\*  $p < .05$ , \*\*\*  $p < .01$

Table 3: Regulatory Reforms and  $\Delta \text{Log}(\text{SPW})$  across Firms of Different Size

	Dependent variable: $\Delta \text{Log}(\text{SPW})$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \text{Log}(L)$	-.592*** (.017)	-.569*** (.018)	-.592*** (.017)	-.524*** (.024)	-.716*** (.116)	-.234 (.215)
$\Delta \text{Cost}*\text{Size}$						.002*** (.000)
$\Delta \text{Days}*\text{Size}$						
$\Delta \text{MinCap}*\text{Size}$						
$\Delta \text{No.Proc.}*\text{Size}$						
$\Delta \text{LTax}*\text{Size}$						
Mgr. Exp.						
Firm Age						
Const.						
Observations	55660	55660	55660	55660	11355	10896
Adjusted $R^2$	.158	.159	.158	.159	.485	.090

Notes: The table presents results from OLS estimations of the difference in  $\text{Log}(\text{Sales per worker})$  on the difference in  $\text{Log}(\text{No. of employees})$ , on other observables from the World Bank Enterprise Analysis Unit firm-level data, and on objective reform data, measured by The Doing Business Database interacted with the firm size measured by the  $\text{Log}(\text{No. of employees})$ . All estimations include the age of the firm, its legal status, an indicator of a quality certificate, industry-year and country-year effects. Robust standard errors are given in parentheses. Symbols: \*  $p < .10$ , \*\*  $p < .05$ , \*\*\*  $p < .01$

## References

- Aghion, P., A. Alesina, and F. Trebbi (2007, June). Democracy, technology, and growth. Working Paper 13180, National Bureau of Economic Research.
- Ahsan, A. and C. Pagés (2009). Are all labor regulations equal? Evidence from indian manufacturing. *Journal of Comparative Economics* 37(1), 62 – 75. Symposium: Labor Regulation In Developing Countries.
- Alexander, S. S. (1949, August). The effect of size of manufacturing corporation on the distribution of the rate of return. *The Review of Economics and Statistics* 31(3), 229–235.
- Alfaro, L., A. Charlton, and F. Kanczuk (2008, June). Plant-size distribution and cross-country income differences. Working Paper 14060, National Bureau of Economic Research.
- Almeida, R. and P. Carneiro (2009). Enforcement of labor regulation and firm size. *Journal of Comparative Economics* 37(1), 28–46. Symposium: Labor Regulation In Developing Countries.
- Antunes, A. R. and T. V. de V. Cavalcanti (2007). Start up costs, limited enforcement, and the hidden economy. *European Economic Review* 51(1), 203 – 224.
- Axtell, R. L. (2001). Zipf distribution of U.S. firm sizes. *Science* 293(5536), 1818–1820.
- Ayyagari, M., T. Beck, and A. Demirguc-Kunt (2007). Small and medium enterprises across the globe. *Small Business Economics* 29(4), 415–434.
- Bennett, J. and S. Estrin (2013). Regulatory barriers and entry into a new competitive industry. *Review of Development Economics* 17(4), 685–698.
- Cabral, L. M. and J. Mata (2003, September). On the evolution of the firm

- size distribution: Facts and theory. *American Economic Review* 93(4), 1075–1090.
- Commander, S. and J. Svejnar (2011). Business environment, exports, ownership, and firm performance. *Review of Economics and Statistics* 93(1), 309–337. cited By (since 1996)8.
- Dixit, A. K. and J. E. Stiglitz (1977, June). Monopolistic Competition and Optimum Product Diversity. *American Economic Review* 67(3), 297–308.
- Doi, N. and M. Cowling (1998). The evolution of firm size and employment share distribution in Japanese and UK manufacturing: A study of small business presence. *Small Business Economics* 10, 283–292.
- Dunne, T., M. J. Roberts, and L. Samuelson (1989, November). The growth and failure of U.S. manufacturing plants. *The Quarterly Journal of Economics* 104(4), 671–698.
- Feldmann, H. (2009). The unemployment effects of labor regulation around the world. *Journal of Comparative Economics* 37(1), 76 – 90. Symposium: Labor Regulation In Developing Countries.
- Feldmann, H. (2012). Product market regulation and labor market performance around the world. *Labour* 26(3), 369–391.
- Gabaix, X. (2011, 05). The Granular Origins of Aggregate Fluctuations. *Econometrica* 79(3), 733–772.
- Gourio, F. and N. A. Roys (2012, December). Size-dependent regulations, firm size distribution, and reallocation. Working Paper 18657, National Bureau of Economic Research.
- Hashemi, F. (2000). An evolutionary model of the size distribution of firms. *Journal of Evolutionary Economics* 10, 507–521.
- Henly, S. E. and J. M. Sánchez (2009). The U.S. establishment-size distri-

- bution: Secular changes and sectoral decomposition. *Economic Quarterly* 95(4), 419–454.
- Joskow, P. L. and N. L. Rose (1989). The effects of economic regulation. In R. Schmalensee and R. Willig (Eds.), *Handbook of Industrial Organization*, Volume 2, pp. 1449 – 1506. Elsevier. Chapter 25.
- Jovanovic, B. (1982, May). Selection and the evolution of industry. *Econometrica* 50(3), 649–670.
- Kaizoji, T., H. Iyetomi, and Y. Ikeda (2005, December). Re-examination of the size distribution of firms. Quantitative Finance Papers physics/0512124, arXiv.org.
- Kaplan, D. S. (2009). Job creation and labor reform in Latin America. *Journal of Comparative Economics* 37(1), 91 – 105. Symposium: Labor Regulation In Developing Countries.
- Klapper, L., L. Laeven, and R. Rajan (2006). Entry regulation as a barrier to entrepreneurship. *Journal of Financial Economics* 82(3), 591–629.
- Lotti, F. and E. Santarelli (2004, January). Industry dynamics and the distribution of firm sizes: A nonparametric approach. *Southern Economic Journal* 70(3), 443–466.
- Lucas, R. E. (1978, Autumn). On the size distribution of business firms. *Bell Journal of Economics* 9(2), 508–523.
- Luttmer, E. G. J. (2007). Selection, growth, and the size distribution of firms. *The Quarterly Journal of Economics* 122(3), 1103–1144.
- Mansfield, E. (1962, December). Entry, Gibrat’s law, innovation, and the growth of firms. *The American Economic Review* 52(5), 1023–1051.
- Marsili, O. (2005, December). Technology and the size distribution of firms: Evidence from Dutch manufacturing. *Review of Industrial Organization* 27, 303–328.

- Mata, J. (1994). Firm growth during infancy. *Small Business Economics* 6, 27–39.
- Moscoso Boedo, H. J. and T. Mukoyama (2012). Evaluating the effects of entry regulations and firing costs on international income differences. *Journal of Economic Growth* 17(2), 143–170.
- Murphy, K. M., A. Shleifer, and R. W. Vishny (1991, May). The allocation of talent: Implications for growth. *The Quarterly Journal of Economics* 106(2), 503–30.
- Rossi-Hansberg, E. and M. L. J. Wright (2007, December). Establishment size dynamics in the aggregate economy. *American Economic Review* 97(5), 1639–1666.
- Samuels, J. M. and D. J. Smyth (1968, May). Profits, variability of profits and firm size. *Economica, New Series* 35(138), 127–139.
- Schivardi, F. and R. Torrini (2008). Identifying the effects of firing restrictions through size-contingent differences in regulation. *Labour Economics* 15(3), 538–567.
- Sutton, J. (1997, March). Gibrat’s legacy. *Journal of Economic Literature* 35(1), 40–59.
- Sutton, J. (2007). Market structure: Theory and evidence. In M. Armstrong and R. Porter (Eds.), *Handbook of Industrial Organization*, Volume 3 of *Handbook of Industrial Organization*, Chapter 35, pp. 2301–2368. Elsevier.
- Winston, C. (1993, September). Economic deregulation: Days of reckoning for microeconomists. *Journal of Economic Literature* 31(3), 1263–89.

# 1 Technical Appendix: Deregulation effects: the asymmetric case

In this Technical Appendix, the firms will be differentiated by size: First, we will allow the TFP level to differ in order to distinguish between firms of different efficiency:  $A_i$  will be different, so for the same input of labor, output will be different:

$$c_i = A_i K_i^\alpha (h_i - \bar{h})^{(1-\alpha)} \quad (1)$$

However, as seen from the equation above, differences in size might not be triggered by difference in TFP alone; difference in capital input and employment levels can also account for that. Therefore, in the analysis to follow, all those three: labor input and sales will be used as proxies for firm size.

In addition, the consumer side will be slightly amended as well to accommodate the asymmetric solution. In the absence of symmetry, the representative agent should be allowed to supply different number of hours to different firms. In order for such a choice to be optimal, the setup must allow for different wage rates across firms. One simple way to model this consistently is to allow for labor to be heterogeneous. Total labor supply will be then a weighed average of individual hours, rather than just the sum of those, namely

$$H = \int_0^N a_i h_i di, \quad (2)$$

where  $a_i$  will be the weight attached to each the hours supplied to firm  $i$ , with  $a_i > 0$ ,  $\int_0^N a_i di = 1$ . That is, hours are not valued the same by the consumer (or different jobs require different amounts of effort). Attaching different weights  $0 < a_i < 1$  to hours worked in different firms will be taken as a given, as a feature of underlying preferences. The only importance of



this modelling choice is to allow the setup to accommodate different wage rates in different firms. In addition, the specification of the utility of leisure may be rationalized by the fact that certain labor tasks may require different skill level, are performed in hazardous environment to one's health, or lead to excessive amount of stress, and thus decrease the consumer's utility of leisure much faster than other types of labor.

When solving the model extension described above, different wage rates  $w_i$ , and the utility weights attached to hours,  $a_i$ , will show up in the equilibrium expression in a non-linear way, which complicates comparative-static derivations. Therefore, in order to isolate the size from the productivity effect, after taking the FOCs all utility weights will be set equal (thus wages and hours across firms becoming equal), thus only allowing individual firms' TFP to differ. This is a valid approach, as such collapsing of the model is done after every unit in the model has optimized. The comparative static expressions will then simplify greatly, and the sign being contingent on the particular firm TFP level.

**Consumer problem:** As in the symmetric case, the representative consumer problem is to maximize utility subject to the budget constraint, or

$$\max_{c_i, h_i} \ln\left(\left[\int_0^N c_i^\rho di\right]^{1/\rho}\right) + \ln\left(1 - \int_0^N a_i h_i di\right) \quad (3)$$

s.t

$$\int_0^N p_i c_i di = \int_0^N w_i h_i di. \quad (4)$$

FOCs:

$$c_i : \frac{c_i^{\rho-1}}{\int_0^N c_i^\rho di} = \lambda p_i \quad (5)$$

$$c_j : \frac{c_j^{\rho-1}}{\int_0^N c_j^\rho dj} = \lambda p_j \quad (6)$$

$$h_i : \frac{1}{1 - \int_0^N a_i h_i di} = \lambda w_i. \quad (7)$$

$$h_j : \frac{1}{1 - \int_0^N a_i h_i di} = \lambda w_j. \quad (8)$$

Divide the FOCs for  $h_i$  and  $h_j$  to obtain

$$\frac{w_i}{w_j} = \frac{a_i}{a_j} \quad (9)$$

Wages are proportional to the corresponding utility weights attached to hours. Next, divide the FOCs for  $c_i$  and  $c_j$  to obtain

$$c_i = \left(\frac{p_i}{p_j}\right)^{\frac{1}{\rho-1}} c_j \quad (10)$$

Now plug this expression into the budget constraint to obtain:

$$\int_0^N p_i \left(\frac{p_i}{p_j}\right)^{\frac{1}{\rho-1}} c_j = \int_0^N w_i h_i di \quad (11)$$

Let  $\int_0^N w_i h_i di = Y_{MCE}$  (again, with free entry profit income is zero). Then

$$c_j = p_j^{\frac{1}{\rho-1}} \frac{Y_{MCE}}{\int_0^N p_i^{\frac{\rho}{\rho-1}} di}. \quad (12)$$

Analogously to the symmetric case, define

$$P \equiv \int_0^N p_i^{\frac{\rho}{\rho-1}} di. \quad (13)$$

to be the aggregate price index. Also, let  $B = Y/P$  and derive individual demand for variety  $j$  as:

$$c_j = B p_j^{\frac{1}{\rho-1}}. \quad (14)$$

Again, the demand for each variety is isoelastic.

**Firm max:** As in the symmetric case, taking the demand for its variety as given, the firm producing each variety will set its price optimally to maximize profit, or

$$\max_{p_i} K_i^{-\frac{\alpha}{1-\alpha}} [B p_i^{\frac{\rho}{\rho-1}}]^{\frac{1}{1-\alpha}} [p_i - w_i] - w_i \bar{h}. \quad (15)$$

FOC:

$$p_i = w_i/\rho > w_i \quad (16)$$

Prices are again a fixed mark-up over the wage. However, in the asymmetric case the price of variety  $i$  is a mark-up over the wage rate paid for labor services supplied to firm  $i$ . Using the proportionality between prices and wages, it follows that

$$\frac{p_i}{p_j} = \frac{w_i}{w_j} = \frac{a_i}{a_j}, \quad (17)$$

$$p_i = \frac{a_i}{a_j} p_j. \quad (18)$$

Similarly, consumption is also proportional to the utility weights ratio:

$$c_i = \left(\frac{a_i}{a_j}\right)^{\frac{1}{\rho-1}} c_j. \quad (19)$$

Next, construct the marginal rate of substitution between consumption and hours:

$$\frac{c_j^{\rho-2} (1 - \int_0^N a_i h_i di) \left(\frac{a_i}{a_j}\right)}{\int_0^N \left(\frac{a_i}{a_j}\right)^{\frac{1}{\rho-1}} di} = \frac{\lambda p_i}{\lambda w_i}. \quad (20)$$

Simplify to obtain

$$\frac{[A_j K_j^\alpha (h_j - \bar{h})^{(1-\alpha)}] \rho^{-2} (1 - \int_0^N a_i h_i di) \left(\frac{a_i}{a_j}\right)}{\int_0^N \left(\frac{a_i}{a_j}\right)^{\frac{1}{\rho-1}} di} = \frac{1}{\rho}. \quad (21)$$

The equation above can be solved implicitly for  $h_j$  as a function of model parameters  $(A_j, K_j, a_j, \bar{h}, N, \rho)$ . In particular, using the Implicit Function Theorem (IFT) we can compute comparative statics.

To simplify the derivations further, we will normalize all utility weights to unity,  $a_j = a_i = 1$ . That is not a crucial assumption, as the normalization

is done after FOCs are derived.<sup>1,2</sup> With the normalization in place, it follows that  $h_i = h_j = h$ . More specifically, the labor input effect  $h_i - \bar{h}$  in the production function is now isolated, and the factors driving difference in firm size will be the level of TFP and physical capital. The equation of interest then is recast in the following form:

$$F \equiv \rho A_j^{\rho-2} K_j^{\alpha(\rho-2)} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (1 - Nh_j) - N = 0. \quad (22)$$

To obtain the effect of the setup cost (regulation) on labor supplied to individual firm, apply IFT to obtain:

$$\frac{dh_j}{d\bar{h}} = -\frac{F_{\bar{h}}}{F_{h_j}} > 0, \quad (23)$$

since

$$\begin{aligned} F_{h_j} &= \rho A_j^{\rho-2} K_j^{\alpha(\rho-2)} (1 - \alpha)(\rho - 2)(h_j - \bar{h})^{(1-\alpha)(\rho-2)-1} (1 - Nh_j) \\ &\quad + \rho A_j^{\rho-2} K_j^{\alpha(\rho-2)} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (-N) < 0 \end{aligned} \quad (24)$$

$$F_{\bar{h}} = \rho A_j^{\rho-2} K_j^{\alpha(\rho-2)} (1 - \alpha)(\rho - 2)(-1)(h_j - \bar{h})^{(1-\alpha)(\rho-2)-1} (1 - Nh_j) > 0. \quad (25)$$

In other words, the higher the setup cost, the higher the labor supply to an individual firm. The size of the firm does not matter in this case, as the  $A_j^{\rho-2} K_j^{\alpha(\rho-2)}$  term will cancel out.

In turn, given that consumption is monotone in  $h$ , it is easy to show that the higher the setup cost, the higher the output, or:

$$\frac{dc_j}{d\bar{h}} = A_j \left( \frac{dh_j}{d\bar{h}} - 1 \right) > 0. \quad (26)$$

---

<sup>1</sup>If we decide to keep it, we will expect  $dh_i/da_i > 0$  (size effect). We can use the IFT on the equation above to derive that comparative statics.

<sup>2</sup>We are interested in the effect of  $A_j$  not  $a_j$ . In addition,  $a_j$  cannot tell us anything about a firm, since it is a primitive (preference for leisure, or how painful relative to other sectors working in a particular sector is. So it was a modelling trick to accommodate different wages and hours worked in different sectors.

as we shoed above that

$$\frac{dh_j}{d\bar{h}} > 1. \quad (27)$$

Note that the size of the effect will be proportional to the size of the firm (as represented by  $A_j$ ).

The next comparative static to be explored is the dependence between the labor supplied to firm  $i$  and total number of varieties/firms  $N$  (or entry):

$$\frac{dh_j}{dN} = -\frac{F_N}{F_{h_j}} < 0 \quad (28)$$

since

$$\begin{aligned} F_{h_j} &= \rho A_j^{\rho-2} K_j^{\rho-2} (1-\alpha)(\rho-2)(h_j - \bar{h})^{(1-\alpha)(\rho-2)-1} (1 - Nh_j) \\ &\quad + \rho A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (-N) < 0 \end{aligned} \quad (29)$$

$$F_N = \rho A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (-h_j) - 1 < 0 \quad (30)$$

In addition,

$$\frac{dc_j}{dN} = A_j \frac{dh_j}{dN} < 0. \quad (31)$$

The higher the entry, or the larger the number of varieties, the lower the output of each variety. Again, the size of the effect will be proportional to the size of the firm

Next, the effect of the degree of substitutability on labor supplied to individual firm is as follows:

$$\frac{dh_j}{d\rho} = -\frac{F_\rho}{F_{h_j}}, \quad (32)$$

where

$$\begin{aligned} F_{h_j} &= \rho A_j^{\rho-2} K_j^{\rho-2} (1-\alpha)(\rho-2)(h_j - \bar{h})^{(1-\alpha)(\rho-2)-1} (1 - Nh_j) \\ &\quad + \rho A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (-N) < 0 \end{aligned} \quad (33)$$

$$\begin{aligned} F_\rho &= A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (1 - Nh_j) \\ &\quad + \rho(1 - Nh_j) A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} \ln[A_j K_j^\alpha (h_j - \bar{h})^{(1-\alpha)}] \end{aligned} \quad (34)$$

The first term of  $F_\rho$  is positive, the second is unclear. More specifically, the second term can be split in two parts - the first is positive, but the log part is dubious. Ultimately, it all depends on the size of  $A_j K_j^\alpha$ : if  $A_j$ , or/and  $K_j$  is/are large enough, the term will be positive.<sup>3</sup> Hence the effect of  $\rho$  on  $h_j$  is dubious, and so is the effect of  $\rho$  on  $c_j$ . For a large enough firm (sufficiently high enough  $A_i$ , and/or  $K_j$ ), the effects are positive:

$$\frac{dh_j}{d\rho} = -\frac{F_\rho}{F_{h_j}} > 0. \quad (35)$$

Otherwise, both effects are negative. Therefore, the degree of substitutability will produce a dubious result which is conditional on the firm size, as proxied by TFP and capital stock. **To sum up the results from the model**, both the symmetric and the asymmetric cases suggest that identical change in entry deregulation (change in  $\rho$ ) would affect the growth of small firms more than the growth of large firms. In contrast, an identical change in labor deregulation (change in  $\bar{h}$ ) would affect the growth of small firms less than the growth of large firms. This conclusion again holds for both the symmetric and the asymmetric case.

---

<sup>3</sup>Note that  $h_j - \bar{h} > 0$  but small, as total labor supply is much less than 1.