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Abstract

People are partially time inconsistent and many have difficulties committing to a detailed schedule for a project. I study optimal interim deadlines and how they affect the behavior and resulting welfare of the present-biased agent. I consider a model in which there are three types of agent in terms of how the agent understands her present bias: naïve, sophisticated, and partially-sophisticated. For each type, there is a unique design for an exogenous interim deadline that maximizes the agent's welfare. However, only the sophisticated agent would self-impose an optimal interim deadline, while the naïve agent would not apply a self-imposed deadline at all. The partially-sophisticated agent sets a nonoptimal self-imposed deadline and can even decrease her own welfare by imposing it. The main result is that the partially-sophisticated agent who is relatively less present-biased would decrease her own welfare by using a self-imposed deadline, and the partially-sophisticated agent who is relatively more present-biased would increase her welfare.

Keywords: Time-Inconsistent Preferences, Present Bias, Deadlines, Procrastination, Self-Control

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1 Introduction

Procrastination is common and by definition[1] suboptimal. Many of those who have engaged in procrastination would have preferred in hindsight to stick to their original project plan to produce better outcomes. In this paper, I analyze the behavior of a procrastinator under differently designed interim deadlines, describe the optimal deadline that maximizes an agent's welfare, and study how the agent would self-impose an interim deadline. In real life, different situations are considered procrastination. In this paper, I focus on the present-biased agent who works on a project for several periods. In each period, the agent chooses how much effort to invest in the project. The amount of effort invested is associated with immediate costs and increases the reward the agent receives at the end of the project. The setup is simple and ubiquitous, so most life activities can be connected to the framework. For example, any job can be divided into intervals in which we work on a project every working day in a month, and then we receive a salary based on the effort we have invested. Students study every day at school during a semester to receive the final grades at the end, and runners train daily to achieve the best possible results in a competition. In all these situations, people invest effort across several periods and earn an outcome at the end based on their accumulated effort expended.

An agent who is a procrastinator invests less effort than she planned in advance. Persons who experience self-control problems[2] use different tools, such as deadlines, commitment contracts, and self-control penalties,[3] to overcome procrastination. I describe and analyze the present-biased agent's behavior under an interim deadline and shed new light on how the partially-sophisticated agent affects her welfare by setting a self-imposed deadline. I investigate the effect of an interim deadline on the agent's behavior and welfare, leaving aside other instruments and incentives to overcome procrastination. I consider an interim deadline to be a unique instrument that can be used to improve the agent's performance and welfare. In my model, the deadline specifies the goal and the period.[4] To meet the deadline, the agent has to achieve a goal by this period. Deadlines are common in modern life: students' homework, preparing for exams, and job contracts all involve deadlines. Often, we restrict our future selves by setting deadlines on

[1]I follow the definition provided by Ericson & Laibson, 2019 (page 29): "we will define someone as procrastinating if, when completing a costly task, there is a delay that appears suboptimal from their own perspective."

[2]To describe this behavior, I assume the agent has time-inconsistent preferences. In other words, the agent's preferences change over time, which can lead to the difference between planned effort for future periods and actual expended effort when that period comes. The agent's behavior is suboptimal from the current period perspective (current preferences) but differs from the optimal behavior from the general perspective. Because the agent's preferences change from period to period, optimal agent's behavior also changes across periods and the agent evaluates her overall performance differently in different periods. For models similar to the one in this paper, it is standard to take so-called "long-run" preferences to evaluate the agent's performance (e.g., Herweg & Müller, 2011; O'Donoghue & Rabin, 1999a; O Donoghue, Rabin, et al., 2006). The idea is to consider the situation when the entire project is in the future and the agent evaluates different periods in the project similarly. Typically, the optimal agent's behavior in this situation coincides with the behavior of the time-consistent agent.

[3]See Giné, Karlan, and Zinman (2010); Houser, Schunk, Winter, and Xiao (2018); Trope and Fishbach (2000).

[4]For the formal definition, see Definition 3.

purpose to increase our performance or to achieve a certain goal. Thus, the questions of how an imposed deadline will affect us and how to set deadlines optimally are important and relevant in the modern world.

A deadline can be a powerful restriction on future actions and the effect of a deadline on the agent's welfare can be negative. While some experimental papers document a positive effect of deadlines on agents' performance and welfare (e.g., Ariely & Wertenbroch, 2002), others document no effect (e.g., Bisin & Hyndman, 2020) or even a negative effect (e.g., Burger, Charness, & Lynham, 2011). In this paper, first, I investigate how the design of an interim deadline affects the agent's behavior and the resulting welfare, and characterize the optimal interim deadline that maximizes the agent's welfare. Second, I study how the agent would self-impose a interim deadline and how this would affect her resulting welfare.

The agent can be one of three types based on how she understands her present bias: naïve, sophisticated, and partially-sophisticated. The naïve agent is not aware of her time inconsistency and does not realize that she will suffer from self-control problems in the future (believes she is time-consistent), while sophisticated and partially-sophisticated agents are aware. The sophisticated agent correctly knows her present bias and fully predicts her future behavior. The partially-sophisticated agent underestimates her present bias, but cannot correctly predict her future. This is the framework in which I study how the agent would self-impose the deadline and how this deadline would affect her welfare. The analysis is divided into two parts.

Firstly, I investigate how the imposed deadline affects the agent's behavior and welfare. I consider the situation when the interim deadline is imposed exogenously for the purpose of maximizing the agent's welfare. Because my aim is to analyze how the imposed deadline affects the agent, I focus on the situation when the agent is restricted to satisfying the deadline condition, leaving aside the question of who the person might be who is interested in maximizing the agent's welfare, and assuming that the agent has relatively significant losses when she does not meet the deadline.[5] In this setup, I find that the agent continually procrastinates from the beginning of the project to the deadline. In other words, the agent postpones effort from earlier periods to later periods that are closer to the deadline. As a result, the later interim deadline can accumulate a larger amount of effort in the last periods and even decrease the agent's welfare. Thus, depending on the design, the interim deadline may increase or decrease the agent's welfare. I find that there exists a unique interim deadline that maximizes the agent's welfare and that the design for this deadline depends only on the degree of the agent's present bias. Under the same interim deadline, the naïve agent would postpone more effort for future periods compared to sophisticated and partially-sophisticated agents. Thus, the naïve agent's

[5] One might think about specific principal-agent problems or the social planner problem. While I study how the imposed deadline affects the agent and focus on when the agent has to meet the deadline, the results are valid when the agent has a zero outside option. Because all results correspond to the agent's behavior that yields positive welfare, it is sufficient to set the reward to zero when the agent misses the deadline to motivate the agent to meet the deadline.

welfare will be the lowest. The sophisticated agent would behave more like the time-consistent agent than naïve and partially-sophisticated agents, and would gain the highest welfare. The partially-sophisticated agent will fall between naïve and sophisticated agents. As a result, the optimal deadline for the naïve agent is in the earlier period, for the sophisticated agent it is in the later period, and for the partially-sophisticated agent it is in between.

Second, I study how the agent would self-impose the interim deadline and how the deadline would affect the agent's welfare compared to when the agent behaves with no deadline. Because the naïve agent believes she is time-consistent, she has no incentive to impose a deadline on herself and she would not set any self-imposed deadline. By contrast, both sophisticated and partially-sophisticated agents are aware of their self-control problems and would use a self-imposed deadline to affect their future selves. While the sophisticated agent always uses the self-imposed deadline that improves her welfare and sets the deadline optimally, I find that the self-imposed deadline may increase or decrease the agent's welfare (compared to no deadline) depending on the combination of the agent's present bias and sophistication level.[6] Fixing the sophistication level, the agent who is relatively less present-biased would be worse off with a self-imposed deadline, while the agent who is relatively more present-biased would be better off.

The results contribute to the existing literature on behavioral economics, time-inconsistent preferences, and deadlines. To my knowledge, this is the first theoretical paper that considers how an interim deadline affects the agent's effort choice across several periods, and the resulting welfare. This paper characterizes the agent's behavior under the imposed deadline and the optimal design for the interim deadline that maximizes the agent's welfare. The paper also presents a novel finding regarding the impact of a self-imposed deadline on the welfare of a partially-sophisticated agent. The paper's findings are useful for understanding how deadlines affect behavior and welfare.

The rest of the paper is organized as follows. Section 2 describes related literature and contextualizes this paper into the existing research. Section 3 lays out the model of effort choice with an interim deadline for all agent types. Section 4 analyzes the agents' behavior across periods with and without a deadline. Section 5 provides the results on the optimal exogenous interim deadline for different types. Section 6 describes the results of the self-imposed interim deadline. Section 7 discusses generalizations, limitations, and possible extensions. Section 8 concludes.

[6]The partially-sophisticated agent's underestimation of her present bias can be different. Under the sophistication level, I understand the parameter that describes how close the agent's beliefs about her present bias are to reality. For the formal definition, see Definition 2

2 Related Literature

This paper contributes to the literature on time-inconsistent behaviors starting from Strotz (1955),[7] in particular on the quasi-hyperbolic model[8] ((β, δ) - preferences) originally suggested by Phelps and Pollak (1968) and developed by Laibson (1994, 1997) and O'Donoghue and Rabin (1999a, 1999b). I use a finite-horizon model with discrete time and present-biased preferences similar to Herweg and Müller (2011) and Kaur, Kremer, and Mullainathan (2015).[9]

Strotz (1955) and Pollak (1968) distinguish naïve and sophisticated agents, according to how a present-biased agent understands her bias. A naïve agent believes she will behave according to her current preferences in the future, and is not aware of her self-control problems, while a sophisticated agent is fully aware of her present bias and perfectly predicts her future behavior. Initially, in subsequent work, most researchers assume that the agent has either naïve or sophisticated beliefs,[10] however, recent studies tend to compare the behavior of naïve, sophisticated, and partially-sophisticated agents (e.g., Herweg & Müller, 2011; Hyndman & Bisin, 2022; Kaur et al., 2015). The partially-sophisticated agent is aware of her self-control problem, but underestimates her present bias and cannot correctly foresee her future behavior. In this paper, I analyze and compare the behavior of all three types of agents under exogenous and self-imposed interim deadlines. I characterize the optimal interim deadline for each type and shed new light on how the partially-sophisticated agent would affect her own welfare by using a self-imposed deadline.

In the related literature, deadlines are considered a tool or a commitment device to overcome procrastination which improves the agent's performance. Substantial work has been done on the experimental side, and while the results document a significant demand for the deadlines,[11] the effect on performance is different. Ariely and Wertenbroch (2002) show a positive effect on the agent's performance, and claim that the participants are partially-sophisticated. Burger et al. (2011) document a negative effect, and Bisin and Hyndman (2020) find no effect on the performance. Bisin and Hyndman (2020) studied the effect of deadlines on students who have to complete a single task. The agents must choose the optimal time to exert the immediate costly effort before the deadline. They find that students do not set deadlines optimally and as a result, their deadlines can negatively affect them. This is in line with my results on how

[7] Ericson and Laibson (2019) provides an exhaustive review on the topic of intertemporal choice models.

[8] Researchers aim to describe and explain time-inconsistent behavior by proposing general models and models with specific functional forms. Several researchers have used general models to describe time-inconsistent behavior (e.g., Goldman, 1979, 1980; Peleg & Yaari, 1973; Phelps & Pollak, 1968; Pollak, 1968; Yaari, 1977), while others propose specific functional forms (e.g., Ainslie, 1991, 1992; Ainslie & Haslam, 1992; Ainslie & Herrnstein, 1981; Chung & Herrnstein, 1967; Loewenstein & Prelec, 1992). This paper contributes to the second stream of literature.

[9] The model has undergone many variations and generalizations over past decades (e.g., Cao & Werning, 2016; Harris & Laibson, 2012).

[10] For example, Akerlof (1991) assume naïve beliefs, but Fischer, 2001; Laibson, 1994, 1996, 1997 assume sophisticated beliefs.

[11] This can be interpreted as a support for the fact that the agents are sophisticated or partially-sophisticated.

partially-sophisticated agents set self-imposed deadlines. Because the partially-sophisticated agent underestimates her present bias, she might set a self-imposed deadline so that it decreases her welfare.

In addition to studies of deadlines, many researchers have documented demand for other types of commitment devices (e.g., Giné et al., 2010; Houser et al., 2018; Trope & Fishbach, 2000), which supports the assumption that agents are aware of their self-control problems, and emphasizes the importance of studying how commitment devices affect the agent. Kaur et al. (2015) conduct a one-year field experiment to study the workers' behavior under the possibility of taking the dominated contract which elicits future efforts. They found a significant demand for dominated contracts and a positive effect of paydays on effort. They also use the model to predict the agent's behavior. While their model of the agent is similar to mine, they use the model to predict the agent's choice between two contracts. In this paper, I extend the model to describe and analyze the agent's effort choice under the deadline in every period and to analyze the welfare effects.

From the theoretical literature, the Phelps and Pollak (1968) and Laibson (1994) model includes several generalizations and is used in different studies. Harris and Laibson (2012) introduce a continuous time version, and Hyndman and Bisin (2022) provide an adaptation for studying the optimal stopping time problem. Hsiaw (2013) studies the interaction between goal and self-control and Grenadier and Wang (2007) model investment-timing decisions with time-inconsistent preferences. Cao and Werning (2016) generalize the Harris and Laibson (2012) model to study flexible dynamic savings in continuous time. My model is similar to Herweg and Müller (2011), who study the agent's effort choice in two projects during two periods. Instead of two projects and two periods, I consider an N-period model with an agent pursuing a single project.

Deadlines in theoretical literature are often presented as a time when the agent loses the possibility to complete a task with immediate costs and delayed reward. The problem then is in finding the optimal stopping time when the agent has to stop waiting and complete the task instantly (e.g., Hyndman & Bisin, 2022; O'Donoghue & Rabin, 1999b). Other papers consider the agent who invests a certain level of effort in the task during several periods and then is rewarded according to the accumulated efforts after the deadline. Herweg and Müller (2011) study a two-period model with a time-inconsistent agent and compare the behavior of the agents with naïve and sophisticated beliefs. Instead of imposing deadlines, Kaur et al. (2015) provide a choice between two contracts that specify the pay rates based on whether the agent achieves the goal by the predefined moment. Their contracts can be considered as a deadline that specifies the goal and time when the agent is incentivized to achieve his goal. In this paper, I define the deadline in a similar way: the interim deadline specifies the period and goal the agent is supposed to achieve by that period.

The results of the experimental papers suggest that deadlines can differently affect performance and agents' welfare, which is in line with the results of this paper. Most of the existing theoretical literature considers situations different from the setup in this paper, while a few papers study similar setups but different extensions. This paper is a logical continuation of the existing studies. Its results can be used in future research and its conjectures in different fields, including behavioral economics, management, and contract design.

3 Model

3.1 Agent

The agent has to perform a project for several (N) periods. In each period $t \in \{1, ..., N\}$, she chooses an effort level $e_t \geq 0$ that she expends in the current period. In period t, she also chooses the effort plan for the future periods $n \in \{t+1, ..., N\}$: $e_n \geq 0$. The expended effort level e_t in the current period is associated with the immediate costs $c(e_t)$ and the reward after the end of the project for the agent. The cost function is assumed to be the same for every period and is taken in a quadratic form:

$$c(e_t) = \frac{e_t^2}{2} \tag{1}$$

The agent is rewarded after the end of the project in period $N+1$ according to the reward function $R\left(\sum_{t=1}^{N} e_t\right)$. The reward function is assumed to be in an additive form:

$$R\left(\sum_{t=1}^{N} e_t\right) = \sum_{t=1}^{N} e_t \tag{2}$$

The convex cost function represents the fact that the agent becomes tired when she works more hours on a given day. The quadratic form is the simplest and most popular to model the convex costs. However, the agent is rested and faces the same marginal cost at the beginning of any day. At the end of the project, the agent is paid according to her total expended effort. It is assumed that one unit of expended effort is associated with one utility unit at the end of the project. This motivates the choice of the additive linear reward function.

I study the agent's behavior with time-inconsistent preferences with quasi-hyperbolic discounting or so-called (β, δ) – preferences (Laibson, 1997; O'Donoghue & Rabin, 1999a; O'Donoghue

& Rabin, 1999b).[12] Precisely, the agent's intertemporal preferences at the chosen period $t \in \{1, ..., N\}$ can be represented by intertemporal utility function U_t:

$$U_t = u_t + \beta \left[\sum_{n=t+1}^{N+1} \delta^{n-t} u_n \right] \tag{3}$$

u_t represents the agent's instantaneous utility from period t; $\delta \in (0, 1]$ – is a standard discount factor; and $\beta \in (0, 1]$ – is a present bias parameter. The present bias parameter here is crucial and presents the time inconsistency in the model. In any fixed period t, the agent weights the current period more than the future when $\beta < 1$.[13] In order to focus on the agent's procrastination problem, I abstract away from the standard exponential discounting and set $\delta = 1$.[14] Because the agent faces the cost functions in each period $t \in \{1, ..., N\}$ and the reward function after the project at period $N + 1$, her preferences in period t can be presented by intertemporal utility U_t:

$$U_t = -c(e_t) + \beta \left[- \sum_{n=t+1}^{N} c(e_n) + R\left(\sum_{n=1}^{N} e_n \right) \right] = -\frac{e_t^2}{2} + \beta \left[- \sum_{n=t+1}^{N} \frac{e_n^2}{2} + \sum_{n=t}^{N} e_t \right] \tag{4}$$

The agent is modeled as a sequence of intertemporal selves at each period $t \in \{1, ..., N\}$. At any chosen period t, the agent observes past efforts and maximizes her intertemporal utility U_t by choosing the effort level for the current period e_t and the effort plan for future periods, $\left\{ e_n^{(t)} \right\}_{n=t+1}^{N}$. The superscript denotes the period t for the intertemporal agent's self who max-

[12] (β, δ) – preferences in period t can be presented as the following utility function:

$$U_t = u_t + \beta\delta u_{t+1} + \beta\delta^2 u_{t+2} + \beta\delta^3 u_{t+3} + \beta\delta^4 u_{t+4} + ...$$

Here β is a present bias parameter and δ is a long-run discount factor, U_t is total utility, and u_t is the utility in period t. Under these preferences, the agent chooses the current and future consumption/effort in period t maximizing total utility U_t. Moving to period $t + 1$, the agent's preferences change and can be described with:

$$U_{t+1} = u_{t+1} + \beta\delta u_{t+2} + \beta\delta^2 u_{t+3} + \beta\delta^3 u_{t+4} + ...$$

Again, the agent chooses current and future consumption/effort in period $t + 1$ maximizing the total utility U_{t+1}. In period $t + 1$, the intertemporal substitution between periods $t + 1$ and any further period has changed compared to period t. Specifically, under assumption $0 < \beta < 1$, the agent prefers to consume more and puts in less effort in period $t + 1$ compared to her plan for the period $t + 1$ in period t; in other words, the agent procrastinates.

[13] β is a present bias parameter that describes how differently the agent weights the future compared to the current moment (today). $\beta = 1$ describes the time-consistent agent, while $0 < \beta < 1$ corresponds to the agent who weighs today relatively more than the future. That agent would prefer to work less and enjoy leisure more today and work more and enjoy leisure less tomorrow.

[14] In the Analysis section, I discuss how the results change when $\delta < 1$.

imizes the intertemporal utility U_t. In other words, in period t, the agent behaves according to the optimal *intertemporal* strategy $\left\{\hat{e}_\tau^{(t)}\right\}_{\tau=t}^{N}$. This strategy is the solution of the following utility maximization problem (UMP):

$$\left\{\hat{e}_\tau^{(t)}\right\}_{\tau=t}^{N} \in \arg\max_{\{e_\tau\}_{\tau=t}^{N}} \left\{-\frac{e_t^2}{2} + \beta\left[-\sum_{n=t+1}^{N}\frac{e_n^2}{2} + \sum_{n=t}^{N} e_t\right]\right\} \tag{5}$$
$$s.t.: \quad \{e_n\}_{n=1}^{t-1} \ \ are \ \ given$$

Definition 1: *The agent's* ***optimal intertemporal strategy*** *at period t is the profile of the optimal actions for current and future periods from the perspective of the agent's intertemporal self at period t.*

At every period $t \in \{1, ..., N\}$, the agent behaves according to the current optimal *intertemporal* strategy and invests $\hat{e}_t^{(t)}$ into the project. Therefore, the resulted agent's action profile during the project consists of the current actions from the optimal *intertemporal* strategies: $\left\{\hat{e}_t^{(t)}\right\}_{t=1}^{N}$.

3.2 Agent Types

The agent can be one of three types, based on how she understands her present bias (β): naïve, sophisticated, or partially-sophisticated. The naïve agent is not aware of her time inconsistency and believes she is time-consistent. The sophisticated agent correctly knows her present bias and fully predicts her future behavior. While the partially-sophisticated agent is aware of her self-control problems, she underestimates her present bias.

In the current period t, the agent invests only the effort level $\hat{e}_t^{(t)}$ into the project. Because the naïve, the sophisticated, and the partially-sophisticated agents differ in terms of how they understand their present bias, they have different beliefs about their future behavior. The naïve agent believes she will behave according to the current optimal *intertemporal* strategy $\left\{\hat{e}_\tau^{(t)}\right\}_{\tau=t+1}^{N}$ in the future or, in other words, that β will disappear in the next period. On the contrary, the sophisticated agent fully understands her present bias and correctly predicts the behavior of her future selves. In period t, she knows that she will face the same problem (5) in every future period. The partially-sophisticated agent anticipates her future behavior, however, she underestimates her present bias and in period t and believes she will face a similar UMP to (5) in period $t+1$, but with the present bias parameter equal to $\bar{\beta}$ instead of β, where $\beta < \bar{\beta} < 1$:

$$\left\{\hat{e}_{\tau}^{(t+1)}\right\}_{\tau=t+1}^{N} \in \arg\max_{\{e_\tau\}_{\tau=t+1}^{N}} \left\{-c(e_{t+1}) + \bar{\beta}\left[-\sum_{n=t+2}^{N} c(e_n) + R\left(\sum_{n=1}^{N} e_n\right)\right]\right\} \tag{6}$$
$$s.t.: \quad \{e_n\}_{n=1}^{t-1} \;\; are \;\; given$$

Thus, the partially-sophisticated agent's choice of future effort plan depends on her sophistication level γ.

Definition 2: *A* ***sophistication level*** *is the parameter γ which characterizes how incorrectly the agent estimates her present bias parameter β and is defined by:*

$$\gamma = \frac{1-\bar{\beta}}{1-\beta} \tag{7}$$

3.3 Welfare Criteria

Because the agent's preferences change over the periods, the agent evaluates her overall performance differently in different periods. The same agent's action profile gives different utility levels for different intertemporal selves during the project. Therefore, to analyze the agent's welfare, it is standard to take the so-called "long-run" preferences to evaluate the agent's performance:

$$U_0 = \left[-\sum_{t=1}^{N} c(e_t) + R\left(\sum_{t=1}^{N} e_t\right)\right] \tag{8}$$

This approach is in line with O'Donoghue and Rabin (1999a), O Donoghue et al. (2006) and Herweg and Müller (2011). Additionally, these preferences represent the agent who considers the entire project in advance (how she evaluates the performance in a 0-period). The difference then is in multiplication by the constant β.

3.4 Interim Deadline

In this paper, I study the agent's behavior under an exogenously imposed deadline to investigate the effect of the deadline on the agent's behavior and welfare. Then I study how the agent sets the self-imposed deadline and how this deadline affects her welfare.[15] However, in both cases, the interim deadline is an instrument used to maximize the agent's welfare; the agent's "long-run" utility (8).

Definition 3: *An **interim deadline** (ID) is the constraint on the agent's behavior defined by two parameters: **timing** $k \in \{1, ..., N\}$ and **goal** $A \geq 0$. The agent is restricted to investing the total level of effort greater or equal to A by the end of period k.*

In other words, if the agent faces an interim deadline $ID = (A, k)$, then her intertemporal selves face the following constraint in every period $1, ..., k$:

$$\sum_{t=1}^{k} e_t \geq A \tag{9}$$

In the case of the exogenous deadline, the agent is informed about the interim deadline before the project starts and has to meet it. Similarly, in the case of the self-imposed interim deadline, the agent has the possibility to choose the interim deadline $ID = (A, k)$ before the project starts. After the interim deadline is chosen, the agent has no possibility to adjust or cancel it, and must satisfy the deadline as in the endogenous case. Because the agent chooses the interim deadline before the project starts, she chooses the $ID = (A, k)$ to maximize her "long-run" utility (8). Thus, the deadlines are used to maximize the same objective in both cases.

Under the interim deadline, the agent can affect her future selves through her choice of the current effort, and this choice depends on her effort history. Because the different types of agent have different beliefs about their future selves' behavior, the behavior of naïve, sophisticated, and partially-sophisticated agents will be different. After the interim deadline is met at period k, the agent behaves according to the optimal *intertemporal* strategies when no deadline is imposed (5).

[15] Additionally, in the case of the exogenous deadline, the results of this paper can potentially be useful in different setups. For example, one might think about the situation when there is a principal who designs a deadline contract before a project starts. The agent then has to sign it if she is interested in the project. An incentive for the agent to strictly meet the deadline could be very significant losses (or penalties) if the deadline is not met. For example, a student's cost of one more hour of study needed to pass an exam is incomparable to the loss of failing a course and potentially losing their place on a study program.

4 Analysis

The analysis is made in the following order. Firstly, I identify the optimal behavior for the agent with "long-run" preferences, or in other words, I analyze how the time-consistent agent would behave. This would be the first-best outcome, which is not achievable for a time-inconsistent agent. Second, I analyze the behavior of the time-inconsistent agent under no deadline and compare the results with those of a time-consistent agent to measure the present bias effect. Further, I study how the agent would change her behavior under the exogenous interim deadline (9). Based on the agent's response to the imposed deadline, I characterize the parameters (A, k) which maximize the agent's welfare. Finally, I study how the partially-sophisticated agent would set the self-imposed deadline and analyze how the deadline would affect the resulting behavior and welfare.

4.1 Time-Consistent Agent

The time-consistent agent (with $\beta = 1$) would behave according to her "long-run" preferences because the optimal strategy would not change across different periods. Thus, the agent behaves according to the solution for the UMP with objective (8), cost function (1), and reward functions (2):

$$\max_{\{e_t\}_{t=1}^{N}} \left\{ -\sum_{t=1}^{N} \frac{e_t^2}{2} + \sum_{t=1}^{N} e_t \right\} \tag{10}$$

The first-order conditions are:

$$-e_t + 1 = 0 \quad , \forall\, t \in \{1, ..., N\} \tag{11}$$

As a result, the time-consistent agent expends 1 effort at every period during the entire project, $\{\hat{e}_t\}_{t=1}^{N} = \{1, ..., 1\}$. The optimal intertemporal strategy for the agent with the "long-run" preferences (agent's self at period 0) then is $\left\{\hat{e}_t^{(0)}\right\}_{t=1}^{N} = \{1, ..., 1\}$. This behavior maximizes the agent's "long-run" preferences and the corresponding utility level $U_0\left(\left\{\hat{e}_t^{(0)}\right\}_{t=1}^{N}\right) = \frac{N}{2}$ is the upper bound and the unachievable target for the problem with a time-inconsistent agent and an interim deadline.

4.2 Time-Inconsistent Agent Under No Deadline

When no deadline is imposed, sophisticated and partially-sophisticated agents have no commitment devices to affect their future selves.[16] Thus, the resulting agents' action profiles coincide for different agent types.

In period t, the naïve agent solves the UMP (5) and her optimal *intertemporal* strategy is to expend β effort in the current period t and 1 effort in every future period: $\left\{\hat{e}_\tau^{(t)}(ND)\right\}_{\tau=t}^{N} = \{\beta, 1, ..., 1\}$. ND denotes here that No Deadline is imposed. The sophisticated agent correctly predicts her future behavior and is aware that she will face the same UMP (5) in every future period. Thus, her optimal *intertemporal* strategy is to expend β effort in every period: $\left\{\hat{e}_\tau^{(t)}(ND)\right\}_{\tau=t}^{N} = \{\beta, \beta, ..., \beta\}$. The partially-sophisticated agent predicts her future behavior incorrectly and believes that she will face the UMP (6) in every future period. Thus, her optimal *intertemporal* strategy in period t is to expend β effort in the current period and to expend $\bar{\beta}$ effort in every future period: $\left\{\hat{e}_\tau^{(t)}(ND)\right\}_{\tau=t}^{N} = \{\beta, \bar{\beta}, ..., \bar{\beta}\}$.

All agent types expend β afford in the current period t. Because there is no instrument to commit to future behavior, the resulting agent's action profile is $\{\beta, \beta, ..., \beta\}$ regardless of the agent type. Because $\beta < 1$, she expends less effort at every period. This behavior is not optimal from the perspective of the agent with "long-run" preferences. The agent expends less effort in every period than the time-consistent agent would expend. This behavior represents procrastination. The resulting agent's welfare then is $U_0\left(\left\{\hat{e}_t^{(t)}(ND)\right\}_{t=1}^{N}\right) = \beta(2-\beta)\frac{N}{2}$, which is lower than the time-consistent agent's welfare $\left(\frac{N}{2}\right)$ for any $\beta < 1$.

4.3 Time-Inconsistent Agent Under an Exogenous Interim Deadline

When the exogenous interim deadline $ID = (A, k)$ is imposed, the agent faces the constraint (9) at every period $1, ..., k$. Therefore, the agent's UMP in period $t \in \{1, ..., k\}$ can be written as:

$$\max_{\{e_\tau\}_{\tau=t}^{N}} \left\{-\frac{e_t^2}{2} + \beta\left[-\sum_{n=t+1}^{N}\frac{e_n^2}{2} + \sum_{n=1}^{N} e_n\right]\right\} \tag{12}$$
$$s.t.: \quad \sum_{\tau=t}^{k} e_\tau \geq A - \sum_{n=1}^{t-1} e_n$$

[16] Generally, the agent can affect the behavior of her future self through the reward function by changing the marginal benefit in future periods when choosing effort in the current period. Herweg and Müller (2011) show that the agent would decrease her effort in the current period to incentivize her future self to invest more effort in the next period under their assumptions on cost and reward functions. However, in this paper, I focus on the effects of deadlines and choose the additive structure of the reward function, so the agent cannot affect the marginal benefit in future periods by choosing the current effort.

$$\{e_n\}_{n=1}^{t-1} \ \ are \ \ given$$

I assume that the *goal* A is chosen such that the deadline constraint binds from the first period and consider the situation when the deadline does not bind in the first period further in this section. This means that the agent's optimal *intertemporal* strategy in period 1 with No Deadline, $\left\{\hat{e}_t^{(1)}(ND)\right\}_{t=1}^{N}$, does not satisfy the deadline constraint:

$$\sum_{t=1}^{k} \hat{e}_t^{(1)}(ND) < A \tag{13}$$

In Appendix I, I show that if the deadline constraint binds at some period t, then it binds at any future period $t+1, ..., k$.

Under the interim deadline, the agent's behavior depends on her type, because now the agent can affect her future selves' behavior by investing more or less effort in the current period. However, the naïve agent would not do that, because she always believes that she will stick to the current effort plan in future periods.

4.3.1 Naïve Agent

The naïve agent believes that her preferences are time-consistent and that she will behave according to the current effort plan. Then, she behaves in the current period $t \leq k$ according to the optimal *intertemporal* strategy as a function of imposed ID (9) and her effort history:

$$\left\{\hat{e}_\tau^{(t)}\left(ID, \left\{\hat{e}_n^{(n)}\right\}_{n=1}^{t-1}\right)\right\}_{\tau=t}^{N} \in \arg\max_{\{e_\tau\}_{\tau=t}^{N}} \left\{-\frac{e_t^2}{2} + \beta\left[-\sum_{n=t+1}^{N}\frac{e_n^2}{2} + \sum_{n=1}^{N} e_n\right]\right\} \tag{14}$$

$$s.t.: \quad \sum_{\tau=t}^{k} e_\tau \geq A - \sum_{n=1}^{t-1} \hat{e}_n^{(n)}$$

$$\left\{\hat{e}_n^{(n)}\right\}_{n=1}^{t-1} \ \ are \ \ given$$

ID here denotes that the agent solves for the optimal intertemporal strategies under the interim deadline. I omit the dependence on the effort history further in the notations so as not to create complications in the equations. I write $\hat{e}_\tau^{(t)}(ID)$ instead of $\hat{e}_\tau^{(t)}\left(ID, \left\{\hat{e}_n^{(n)}\right\}_{n=1}^{t-1}\right)$ keeping in mind that the agent's choice of effort always depends on her effort history under the interim deadline.

Because the naïve agent is not aware of her time inconsistency and does not anticipate her future behavior, the agent behaves according to the first-order conditions for UMP (14) and keeps the effort ratio the same for the optimal *intertemporal* strategy in every period t:

$$e_t = \beta e_n \quad , \forall\, n \in \{t+1, ..., k\} \tag{15}$$

However, in different period t, the agent has to redistribute different total amounts of effort $\left(A - \sum_{n=1}^{t-1} e_n\right)$ across periods. Note that the naïve agent has the same effort ratio (15) for her optimal *intertemporal* strategy when she behaves under no deadline.

Proposition 1: *When the naïve agent faces the interim deadline $ID = (A, k)$, she behaves in periods $\{1, ..., k\}$ as follows:*

- *the agent postpones a certain amount of effort to later periods;*
- *the agent expends greater effort in every subsequent period;*
- *the expended effort satisfies equation (16);*

$$\hat{e}_t^{(t)}(ID) = \begin{cases} \beta\alpha\Pi_{l=0}^{t-1}\left(\frac{k-l}{k-l-(1-\beta)}\right) & , \quad t \in \{1, ..., k-1\} \\ \alpha\Pi_{l=0}^{k-2}\left(\frac{k-l}{k-l-(1-\beta)}\right) = \frac{1}{\beta}\hat{e}_{k-1}^{(k-1)}(ID) & , \quad t = k \\ \beta & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{16}$$

Where $\alpha = A/k$.

Parameter α here corresponds to the average level of effort expended in every period under the interim deadline during periods $1, ..., k$. I provide the proof for Proposition 1 in Appendix II. The choice of A is equivalent to the choice of α. I assume that goal $A = k$[17] (or $\alpha = 1$), because α is a common coefficient for all effort from period 1 to k. I call this interim deadline *natural*. First, I consider the agent's behavior under the *natural* interim deadline and then discuss how the choice of α (or A) affects the agent's behavior and welfare later in this section.

[17] When A is set to be equal to k, the agent is restricted to expend at least 1 effort in every period on average before the deadline. Thus, on average, the total effort expended coincides with what the time-consistent agent would expend (first-best behavior). Note, when A is chosen to be equal to k, the deadline constraint binds in period 1, because $\beta < 1$.

Definition 3: *An interim deadline $ID = (A, k)$ is called* ***natural*** *when goal A is chosen such that the agent is restricted to expend at least the same amount of total effort by period k that a time-consistent agent would expend.*

The interim deadline imposed in period k does not affect the agent's behavior in periods $k+1$ to N, thus, I focus only on the first k periods in the project further in this subsection. In the first period, the agent expends $\hat{e}_1^{(1)}(ID) = \beta\frac{k}{k-(1-\beta)}$, which is greater than what the agent would expend without the imposed deadline $\left(\hat{e}_1^{(1)}(ND) = \beta\right)$. However, $\hat{e}_1^{(1)}(ID) < 1 = \alpha$ for any $k \geq 2$. This means that the agent still procrastinates at the beginning of the project. On the other hand, the agent has to satisfy the deadline (expend on average $\alpha = 1$ level of effort per period during periods $\{1, ..., k\}$), thus, expending a lower level of effort than 1 at the beginning of the project means that she must invest greater effort ($> \alpha$) in the periods close to k. In other words, the agent continues to procrastinate and then accumulates effort just prior to the deadline. Note that the expended effort in period $t \in \{1, ..., k-1\}$ is the multiplication of $\beta\alpha$ and t constants: $\frac{k}{k-(1-\beta)}$, $\frac{k-1}{k-1-(1-\beta)}$, ... , $\frac{k-t+1}{k-t+1-(1-\beta)}$. Each of these constants is greater than 1, because $\frac{k-l}{k-l-(1-\beta)} > 1$ for any $l \in \{1, ..., k-1\}$. Precisely, the effort expended in period $t+1$ is equal to the effort expended in period t, but multiplied by $\frac{k-t}{k-t-(1-\beta)} > 1$. Thus, in every subsequent period, the agent expends more effort than in the prior period.

The agent's present bias parameter β continues to play the main role in the procrastination process before the deadline. When $\beta \to 1$ (agent is close to time-consistent), the effort expended in every period tends to 1. In case $\beta \to 0$, the agent expends 0 effort in every period before k and $\alpha k = A$ effort in period k, which is the most inefficient behavior from the "long-run" perspective.

The interim deadline $ID = (k, k)$ increases the expended effort in total from period 1 to period k according to the first-best: the agent now expends $\sum_{t=1}^{k} \hat{e}_t^{(t)}(ID) = k$ effort in total by period k instead of $\sum_{t=1}^{k} \hat{e}_t^{(t)}(ND) = \beta k$. However, the interim deadline allows the agent to redistribute her effort across periods $\{1, ..., k\}$ according to the time-inconsistent preferences. At every period $t \in \{1, ..., k\}$, the agent's optimal intertemporal strategy satisfies the F.O.C.s:

$$\hat{e}_n = \frac{1}{\beta}\hat{e}_t \quad , \; n \in \{t+1, ..., k\} \tag{17}$$

That is, the agent always expends less (times β) effort today than she will in future periods closer to the deadline (period k). In the current period, she plans to expend equal effort in every future period. However, she does less when the next period comes and again postpones some effort to future periods. If the interim deadline $ID = (k, k)$ is imposed at a relatively later period, it increases the total effort expended relatively more. However, the later interim deadline gives more space for the agent to procrastinate. If k increases, the agent expends less effort in the first period and accumulates a relatively greater degree of effort is necessary in periods closer to the deadline. When $k \to \infty$ (assuming $N \to \infty$, too), the agent expends $\hat{e}_1^1(ID) \to \beta$ in period 1, which is equal to what the time-inconsistent agent under no deadline would expend. If k decreases, the agent expends a relatively higher (closer to 1, as the time-consistent agent would expend) level of effort in period 1 and postpones relatively less effort for future periods.

Coming back to the effect of A on the agent's behavior, the deadline binds if A satisfies (13). If the opposite, the interim deadline $ID = (A, k)$ does not affect the agent's behavior in period 1. Thus, the agent behaves according to the optimal intertemporal strategy when No Deadline is imposed:

$$\left\{\hat{e}_t^{(1)}(ND)\right\}_{t=1}^{N} = \{\beta, 1, ..., 1\} \tag{18}$$

As a result, the agent expends $\hat{e}_1^{(1)}(ID) = \hat{e}_1^{(1)}(ND) = \beta$ in period 1 and moves to the second period. In period 2, the agent still faces the interim deadline constraint, but now she has already expended $\hat{e}_1^{(1)}(ID) = \beta$ effort in period 1, and the interim deadline constraint changes to:

$$\sum_{t=2}^{k} e_t \geq A - \beta \tag{19}$$

The expression $A - \beta$ can be denoted as a new interim deadline *goal* $A' = A - \beta$ and the problem comes down to the previous one in period 1. If the deadline binds in period 2, then the agent behaves according to (16) in periods $\{2, ..., k\}$, but with new $A' = A - \beta$ and $k' = k - 1$. If the deadline again does not bind in period 2, then the agent behaves according to the optimal *intertemporal* strategy in period 2 under No Deadline and moves to period 3, and so on. Therefore, if the interim deadline does not bind for the first n periods and binds in period $n+1$, the agent behaves as a procrastinator during these n periods and then according to (16) with $A' = A - n\beta$ and $k' = k - n$:

$$\hat{e}_t^{(t)}(ID) = \begin{cases} \beta = \hat{e}_t^{(t)}(ND) & , \quad t \in \{1, ..., n\} \\ \beta\alpha' \Pi_{l=0}^{t-n-1} \left(\frac{k'-l}{k'-l-(1-\beta)}\right) & , \quad t \in \{n+1, ..., k-1\} \\ \alpha' \Pi_{l=0}^{k'-2} \left(\frac{k'-l}{k'-l-(1-\beta)}\right) = \frac{1}{\beta}\hat{e}_{k-1}^{(k-1)}(ID) & , \quad t = k \\ \beta & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{20}$$

The resulting behavior is equivalent to the situation in which the agent behaves under the interim deadline $ID = (A', k')$ but with permutations: the agent procrastinates n periods, then expends greater effort from period $n+1$ to k, and continues to procrastinate from period $k+1$ till period N, which is equivalent to expending greater effort from period 1 to $k' = k - n$ and procrastinating from period $k'+1$ till period N. Thus, further in the paper, I abstract from the cases when the interim deadline does not bind in period 1 and assume A satisfies the condition (13).[18]

4.3.2 Sophisticated Agent

The sophisticated agent (SA) is aware of her time inconsistency and correctly predicts her future behavior. In period $t \leq k$, the agent solves the UMP by backward induction and considers future efforts as functions of the effort choice for the current period. Because the sophisticated agent correctly estimates her present bias, she correctly predicts her future behavior and follows her optimal *intertemporal* strategy from the beginning of the projects. In other words, her optimal intertemporal strategy does not change from period to period:

$$\hat{e}_t^{(t)}(ID) \in \arg\max_{e_t} \left\{ -\frac{e_t^2}{2} + \beta \left[-\sum_{n=t+1}^{N} \frac{e_n^2(e_t)}{2} + \sum_{n=1}^{N} e_n(e_t) \right] \right\} \tag{21}$$

$$s.t.: \quad \sum_{\tau=t}^{k} e_\tau \geq A - \sum_{n=1}^{t-1} \hat{e}_n^{(n)}$$

$$\left\{ \hat{e}_n^{(n)} \right\}_{n=1}^{t-1} \quad are \;\; given$$

As a result, the sophisticated agent's optimal intertemporal strategy in period t consists of $\hat{e}_t^{(t)}(ID)$ and $\left\{ \hat{e}_n\left(\hat{e}_t^{(t)}(ID)\right) \right\}_{n=t+1}^{k}$. The problem is complicated and there is no analytical solution, however, the resulting agent's action profile can be described as follows:

Proposition 2: *If the sophisticated agent faces the binding interim deadline $ID = (A, k)$ in period t and has to expend at least $A' = A - \sum_{n=1}^{t-1} e_n$ effort till period k, she expends $\omega_t^{SA} A'$ effort in the current period t. The agent expends less effort than the time-consistent agent would expend and postpones some amount of effort to future periods (share ω_t^{SA} is lower than $\frac{1}{k-t+1}$). The share ω_t^{SA} can be found by:*

[18] In the general case with $\delta < 1$, the agent would prefer to expend effort closer to the end of the project. However, the time-consistent agent would change the behavior and expend less effort at the beginning of the project and more effort in the final periods before the deadline.

$$\omega_t^{SA} = \underbrace{\Phi\left(\Phi\left(\Phi\left(...\Phi\left(1\right)...\right)\right)\right)}_{k-t \quad times} \tag{22}$$

$$where \quad \Phi(x) = \frac{1}{1 + \frac{1}{x(1-(1-\beta)x)}} \tag{23}$$

I provide the proof for Proposition 2 in Appendix III. In period 1, the agent expends $\hat{e}_1^{SA}(ID) = \omega_1^{SA} A$ and then, in period 2, she expends $\hat{e}_2^{SA}(ID) = \omega_2^{SA}(1-\omega_1^{SA})A$. Thus, in period $n \leq k$, the agent expends $\hat{e}_n^{SA}(ID) = \omega_n^{SA}(1-\omega_{n-1}^{SA})...(1-\omega_2^{SA})(1-\omega_1^{SA})A$. However, this is correct if the interim deadline binds in period 1. In other words, the sophisticated agent would affect her future behavior by the choice of the current effort when the *goal* A is greater than the total effort expended under no deadline:

$$k\beta < A \tag{24}$$

Because the agent perfectly anticipates her future behavior, the deadline does not bind in every period if it does not bind in the first period. If the deadline binds and A satisfies equation (24), the agent affects the behavior of her future selves by expending effort according to Proposition 2. However, the sophisticated agent would expend $e_1 = \beta$ if $\hat{e}_1^{SA}(ID) = \omega_1^{SA} A \leq \beta$. Similar to the naïve agent case, the *goal* in the second period adjusts to $A' = (A - \beta)$ and the situation repeats: the agent behaves as though she is facing the interim deadline $ID = (A', k-1)$ and expends $e_2 = \beta$ if $\hat{e}_2^{SA}(A', k-1) \leq \beta$. As a result, the sophisticated agent expends $e_t = \beta$ while the effort calculated according to Proposition 2 is less than or equal to β. Because $A > k\beta$, the average effort per period that the agent has to expend is increasing in every subsequent period and there is a period $n+1$ when the agent switches her behavior to conform with Proposition 2:

$$\hat{e}_t^{SA}(ID) = \begin{cases} \beta & , \quad t \in \{1, ..., n\} \\ \left[\omega_{t-n}^{SA} \Pi_{l=1}^{t-n-1}\left(1-\omega_l^{SA}\right)\right](A - n\beta) & , \quad t \in \{n+1, ..., k\} \\ \beta & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{25}$$

As a result, the sophisticated agent does better than the naïve agent under the same interim

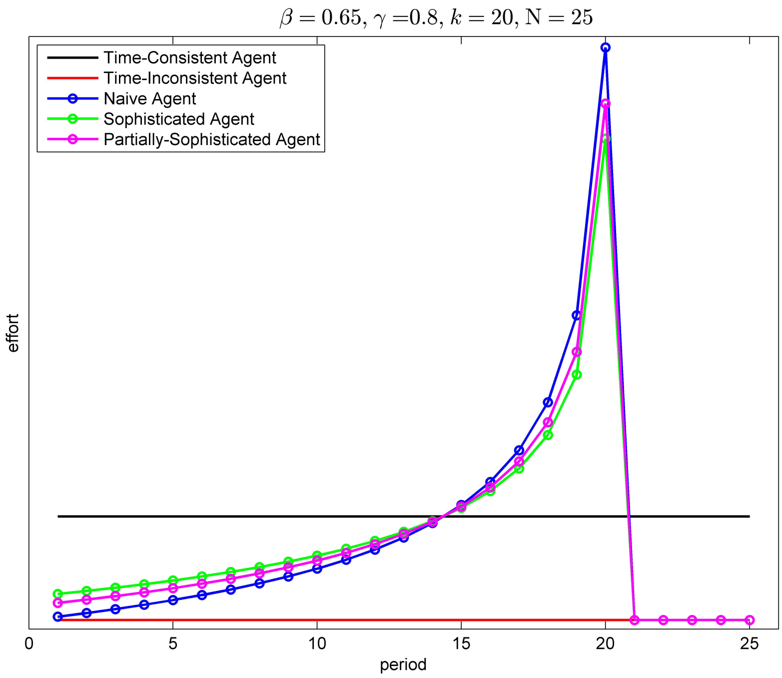

Figure 1: The comparison of behavior for different types of agent under the same interim deadline $ID = (20, 20)$ with present bias parameter $\beta = 0.65$ and sophistication level $\gamma = 0.8$ for the partially-sophisticated agent. The black and red lines present the behavior of time-consistent and time-inconsistent agents. The blue line presents the naïve agent's behavior, the green presents the sophisticated agent's behavior, and the magenta line presents the partially-sophisticated agent's behavior.

deadline $ID = (A, k)$. Because the agent anticipates the effect of her current choice of expenditure of effort on future behavior, she postpones relatively less effort to later periods than does the naïve agent.

Figure 1 presents a comparison of the behavior of different agent types over the entire project, given that the other parameters and interim deadline are fixed. The green line with cycles describes the sophisticated agent's behavior. The blue line depicts the behavior of the naïve agent under the same interim deadline and with the same present bias parameter β. The sophisticated agent invests more effort in the earlier periods and less in the later periods. While both agents expend more effort than without any interim deadline (red line on the graph), their behavior significantly differs from that of the time-consistent agent (black line on the graph). Thus, the sophisticated agent effectively uses the interim deadline to affect her future selves and performs better overall than the naïve agent. However, the sophisticated agent still procrastinates and postpones effort to the final periods before the deadline.

4.3.3 Partially-Sophisticated Agent

The partially-sophisticated agent (PSA) is aware of her time inconsistency but predicts her future behavior incorrectly. She believes that her future selves will behave according to the present bias parameter $\bar{\beta} = 1 - \gamma(1-\beta)$. Thus, the partially-sophisticated agent would behave similarly to a sophisticated and solve the problem by backward induction, but with a slightly different algorithm for calculating coefficient ω_t^{PSA} in period t. The agent's belief about how her current choice of effort affects future decisions depends on her sophistication level γ. This affects both her choice regarding the current effort and her plan for future efforts. In period $t \leq k$, the partially-sophisticated agent solves for $e_t^{(t)}(ID, \gamma)$:

$$\hat{e}_t^{(t)}(ID, \gamma) \in \arg\max_{e_t} \left\{ -\frac{e_t^2}{2} + \beta \left[-\sum_{n=t+1}^{N} \frac{e_n(e_t, \gamma)}{2} + \sum_{n=1}^{N} e_n(e_t, \gamma) \right] \right\} \tag{26}$$

$$s.t.: \quad \sum_{\tau=t}^{k} e_\tau \geq A - \sum_{n=1}^{t-1} \hat{e}_n^{(n)}$$

$$\left\{ \hat{e}_n^{(n)} \right\}_{n=1}^{t-1} \quad are \;\; given$$

As a result, the partially-sophisticated agent's choice of current effort depends on her effort history, interim deadline, and sophistication level γ. Then, the partially-sophisticated agent's optimal *intertemporal* strategy in period t consists of the current period effort $\hat{e}_t^{(t)}(ID)$ and her effort plan on future periods $\left\{ \hat{e}_n \left(\gamma, \hat{e}_t^{(t)}(ID) \right) \right\}_{n=t+1}^{k}$. Because the agent's beliefs about her future behavior are incorrect, she expends $\hat{e}_t^{(t)}(ID)$ in the current period, but her optimal *intertemporal* strategy in the next period does not coincide with the current one. The resulting agent's action profile can be described as follows:

Proposition 3: *If the partially-sophisticated agent faces the binding interim deadline $ID = (A, k)$ in period t and has to expend at least $A' = A - \sum_{n=1}^{t-1} e_n$ effort till period k, she expends $\omega_t^{PSA} A'$ effort in the current period t. There ω_t^{PSA} can be found as:*

$$\omega_t^{PSA} = \Phi \left(\underbrace{\bar{\Phi}\left(\bar{\Phi}\left(\ldots\bar{\Phi}(1)\ldots\right)\right)}_{k-t-1 \quad times} \right) \tag{27}$$

$$where \quad \bar{\Phi}(x) = \frac{1}{1 + \frac{1}{x\left(1-\left(1-\bar{\beta}\right)x\right)}} \tag{28}$$

The function $\Phi(x)$ is the same as for the sophisticated agent and is defined by equation (23). Since $\beta < \bar{\beta} < 1$, the partially-sophisticated agent expends a lower share of A' compared to the sophisticated agent under the same interim deadline. In period t, the partially-sophisticated agent believes she will expend $\bar{\beta}$ effort in every subsequent period under no deadline. Thus, the deadline binds in period 1 if:

$$\beta + (k-1)\bar{\beta} < A \tag{29}$$

Because the agent has incorrect beliefs about her future behavior, the deadline may not bind in the first n periods and binds from period $n+1$:

$$\begin{cases} \beta + (k-n)\bar{\beta} \geq A - (n-1)\beta \\ \\ \beta + (k-n-1)\bar{\beta} < A - n\beta \end{cases} \tag{30}$$

From the period when the interim deadline begins to bind, the partially-sophisticated agent behaves according to Proposition 3 if her calculated effort is greater than β. Similar to a sophisticated agent, the deadline might not affect the agent's behavior during the first m periods:

$$\hat{e}_t^{PSA}(ID) = \begin{cases} \beta & , \quad t \in \{1, ..., m\} \\ \\ \left[\omega_{t-m}^{PSA} \Pi_{l=1}^{t-m-1} \left(1 - \omega_l^{PSA}\right)\right] (A - m\beta) & , \quad t \in \{m+1, ..., k\} \\ \\ \beta & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{31}$$

The magenta line in Figure 1 presents the behavior of the partially-sophisticated agent in comparison to the naïve and sophisticated agents. As expected, the partially-sophisticated agent performs somewhere in between naïve and sophisticated agents. The partially-sophisticated agent uses the interim deadline to affect her future selves as a sophisticated agent does, but due to her incorrect estimation of present bias, she does it less efficiently than a sophisticated agent.

For all types of time-inconsistent agents, the interim deadline significantly affects the agent's behavior. However, the agent behaves differently from a time-consistent agent, and postpones effort to the final periods before the deadline. Obviously, from the "long-run" perspective, such behavior is not optimal. Thus, while the interim deadline increases the total amount of effort expended, the effect on the agent's welfare is not clear.

5 Optimal Interim Deadline

The interim deadline in period k does not affect the agent's behavior in periods $k+1$ to N. For periods 1 to k, the agent expends a greater degree of effort in each period than she would with no deadline. However, the agent continues to procrastinate and postpones a certain amount of effort to future periods according to the present-bias parameter β. As a result, the interim deadline in period k increases the effort spent on the project in periods 1 to k, while an accumulated greater amount of effort is expended in the final periods before the deadline. I begin the analysis with the *natural* interim deadline $ID = (k, k)$ and then discuss how the choice of parameter α (choice of *goal* A) affects the agent's welfare.

5.1 Natural Interim Deadline

In the past, it was generally agreed that deadlines help agents to minimize or avoid procrastination and therefore increase agent welfare, because procrastination lowers it. As discussed above, an imposed deadline increases effort expended in the periods before the period k and accumulates the effort in the last periods before the period when imposed. On the one hand, the increase in effort helps to overcome procrastination and positively affects the agent's welfare due to greater effort expended on the project. I call this effect the "increase in effort" effect. Under the *natural* interim deadline the "increase in effort" would only positively affect the agent's welfare if the increase in effort were the same for every period $\{1, ..., k\}$. On the other hand, the agent distributes the effort across periods not equally: less than optimally in the beginning and greater than optimally in the final periods before k. Because the agent is restricted to expend, on average, $\alpha = 1$ effort in every period, unequal distribution of effort across periods $\{1, ..., k\}$ decreases the agent's welfare. I call this the "redistribution of effort" effect.

The interim deadline $ID = (k, k)$ increases the total effort expended from $\sum_{t=1}^{N} e_t = \beta N$ to $\sum_{t=1}^{N} e_t = \beta N + (1 - \beta)k$. During periods $\{1, ..., k\}$, the agent expends k effort, and it would be optimal to expend 1 effort per period from the "long-run" perspective. However, the agent procrastinates and due to the "redistribution of effort" effect expends less effort in earlier periods and more effort in later periods. This effect does not affect the reward function, because the agent expends the same effort in total, but it does change the total costs. While less effort in the earlier periods decreases costs in those periods, greater effort in later periods increases costs relatively more because the cost function is convex.

Proposition 4: *There is a unique period $\hat{k}$ such that the agent's behavior induced according to Proposition 1, 2, 3 (depending on her type) under the natural interim deadline $ID = (\hat{k}, \hat{k})$ yields the highest agent welfare (among all possible natural interim deadlines $ID = (k, k)$, $k \in \{1, ..., N\}$). The number of this period $\hat{k}$ depends only on the present-bias parameter β.*

I provide the proof in Appendix IV, and here I describe the general intuition and logic. Generally, the later interim deadline helps the agent to expend greater effort overall. Thus, it would be optimal to impose the interim deadline in the last period if the agent distributes effort equally across all periods. From one side, the increase in reward function ("increase in effort" effect) is the same when the interim deadline is moved from the first period to the second or from the 11th to the 12th. However, the agent accumulates greater effort for the final period before the deadline for the later interim deadline. In other words, the "redistribution of effort" effect increases in k. The logic is that it is profitable to postpone the interim deadline to the later periods while the "increase in effort" effect is greater than the "redistribution of effort" effect. Because the first effect is constant in k and the second is increasing in k, there is $k = \hat{k}$ after which the later interim deadline decreases the agent's welfare. This *natural* interim deadline $ID = (\hat{k}, \hat{k})$ can be described as one under which the agent's welfare is higher than with the interim deadline imposed in periods $\hat{k} - 1$ or $\hat{k} + 1$:

$$\begin{cases} U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}, \hat{k})\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}+1, \hat{k}+1)\right\}_{t=1}^{N}\right) > 0 \\ U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}-1, \hat{k}-1)\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}, \hat{k})\right\}_{t=1}^{N}\right) < 0 \end{cases} \tag{32}$$

Any *natural* interim deadline $ID = (k, k)$ affects the agent's behavior only in periods $\{1, ..., k\}$, but in later periods, the agent expends effort equal to β. Thus, the optimal period $\hat{k}$ does not depend on the length of the project N[19] and inequalities (32) contain only parameter β and $\hat{k}$. Therefore, the optimal interim deadline $ID = (\hat{k}, \hat{k})$ is defined only by the present-bias parameter β.

Figure 2 presents the dependence of the agent's welfare on the period k when the interim deadline $ID = (k, k)$ is imposed for all three agent types. Similar to Figure 1, the other parameters are fixed.[20] The red line shows the agent's welfare when no deadline is imposed and the agent expends β effort in every period. The blue, green, and purple lines present how the welfare of the naïve, sophisticated, and partially-sophisticated time-inconsistent agent correspondingly depends on the choice of k when the agent behaves under the *natural* interim deadline $ID = (k, k)$.

Moving the interim deadline from the first period to the second significantly increases the agent's welfare, while setting it in later periods increases welfare less and less and eventually

[19] If N is large enough. If the optimal $\hat{k}$ according to (32) is greater than N, then it is optimal to impose the interim deadline in the last period or $\hat{k} = N$.

[20] The meaning of the present bias parameter β is chosen such all the necessary effects are observed on the graph for the 25-period project for all types of agents. Empirical studies provide mixed results of estimations of the present-bias parameter β. Laibson, Repetto, and Tobacman (2007) estimate that the parameter β is equal to 0.50, while DellaVigna and Paserman (2005) find $\beta = 0.9$. Paserman (2008) estimates $\beta = 0.5$ for low-income workers and $\beta = 0.9$ for high-income workers. For the exhaustive review on the topic of measuring time preferences see Cohen, Ericson, Laibson, and White (2020).

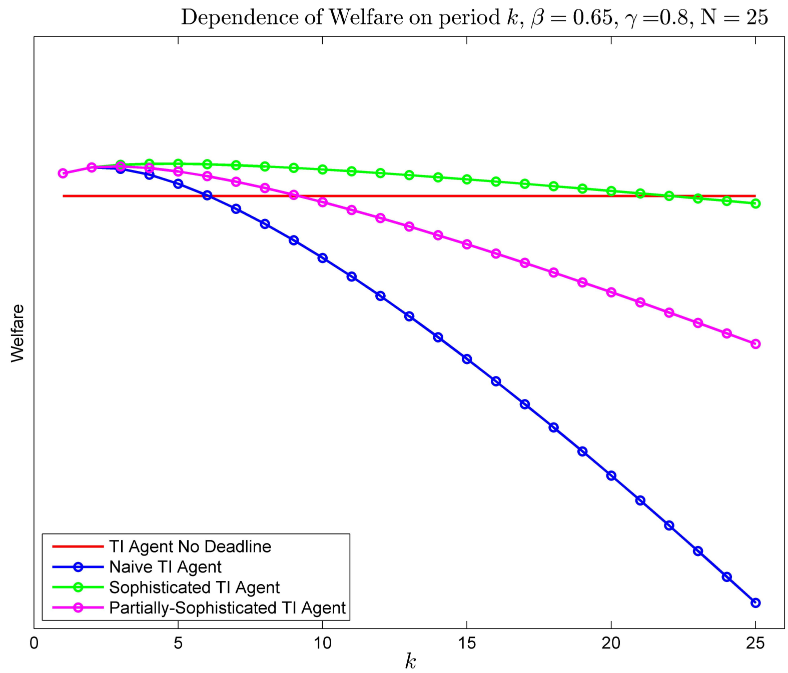

Figure 2: The dependence of the agent's utility level on the period k when the *natural* interim deadline $ID = (k, k)$ is imposed. The blue, green, and magenta lines present the dependence for naïve, sophisticated, and partially-sophisticated agents respectively. The red line presents the utility level of the time-inconsistent agent (regardless of the agent's type) pursuing the project under no deadline.

begins to decrease it. Further relocation of the *natural* interim deadline to later periods only decreases the agent's welfare. The interim deadline can even cause lower welfare than in a situation without any deadline. In Figure 2, this occurs after the intersections with the red line (in different periods for different types). These intersections of colored lines with the red line are defined by inequalities similar to (32) and depend only on the agent's present-biased parameter β. I discuss this specifically in Appendix IV.

As a result, when the agent's present-bias parameter β is known, it is always possible to impose a *natural* interim deadline $ID = (\hat{k}, \hat{k})$ that will maximize the agent's welfare. Such $\hat{k}$ can be found from the inequalities (32). In case $\hat{k} \geq N$, it is optimal to impose the interim deadline in the last period N.

5.2 Choice of Goal A

Next, I consider how the choice of *goal* A ($\neq k$) affects the agent's behavior and welfare. When interim deadline $ID = (\alpha k, k)$ is imposed in period k, the agent's welfare is:

$$U_0\left(\left\{\hat{e}_t^{(t)}(\alpha k,k)\right\}_{t=1}^{N}\right)=\sum_{t=1}^{k}\left[\alpha\hat{e}_t^{(t)}(k,k)-\frac{\left(\alpha\hat{e}_t^{(t)}(k,k)\right)^2}{2}\right]+(N-k)\frac{\beta(2-\beta)}{2} \tag{33}$$

Taking the first-order condition, the optimal $\hat{\alpha}$ as a function of k is defined by:

$$\hat{\alpha}(k)=\frac{k}{\sum_{t=1}^{k}\left[\hat{e}_t^{(t)}(k,k)\right]^2}<1 \tag{34}$$

The optimal $\hat{\alpha}$ characterizes how inefficiently the agent redistributes efforts across periods. By the definition of the interim deadline, $\sum_{t=1}^{k}\hat{e}_t^{t}(k,k)=k$. It is optimal to distribute effort equally across periods with a convex cost function and expend 1 effort in each period (as the time-consistent agent would do). Thus, the maximum $\hat{\alpha}$ is for the time-consistent agent and equal to 1, while $\hat{\alpha}$ is increasing in β (a higher β corresponds to a less present-biased agent) for the time-inconsistent agent. Plugging the optimal $\hat{\alpha}(k)$ back into the agent's welfare (33):

$$U_0\left(\left\{\hat{e}_t^{(t)}(\hat{\alpha}k,k)\right\}_{t=1}^{N}\right)=\frac{1}{2}\hat{\alpha}(k)k+(N-k)\frac{\beta(2-\beta)}{4}=\frac{1}{2}\tilde{A}(k)+(N-k)\frac{\beta(2-\beta)}{4} \tag{35}$$

Where $\tilde{A}(k)=\hat{\alpha}(k)k$. The optimal *goal* or $\hat{\alpha}$ is a function of the chosen period k and the problem comes down to the choice of k. Thus, both parameters of optimal interim deadline $ID=(\hat{\alpha}\hat{k},\hat{k})$ are defined by the agent's present-bias parameter β, because the optimal *timing* $\hat{k}$ is defined by β.

However, the choice of α according to equation (34) does not guarantee that the interim deadline will bind in the first periods. For the naïve agent, this may occur when β is low and the agent postpones too much effort for the last periods, that $\hat{\alpha}$ is low enough and $\tilde{A}$ does not satisfy condition (13). On the other hand, the optimal $\hat{k}$ is increasing in β[21] and the optimal period $\hat{k}$ is earlier for the lower present-bias parameter β. Thus, $\hat{\alpha}$ must be even lower to create the situation in which the deadline does not bind in the first period with earlier $\hat{k}$. For the sophisticated and partially-sophisticated agents, the deadline constraint always binds if it

[21] When $\beta\to 1$, the agent is close to time-consistent and postpones less effort for future periods. Thus, the variance of effort across periods $\{1,...,k\}$ is low and it is optimal to move k to later periods. When $\beta\to 0$, the agent postpones almost all effort to the future and it is optimal to set $k=1$, otherwise, the agent would invest 0 effort in periods $\{1,...,k-1\}$ and A effort in period k.

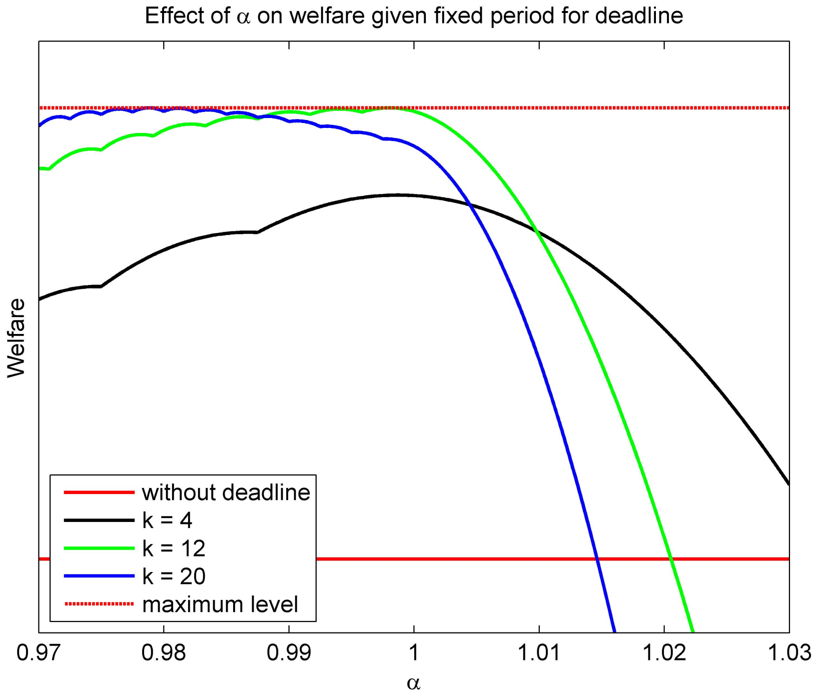

Figure 3: The dependence of the naïve agent's welfare on the value of parameter α for different fixed periods for the interim deadline. The black, green, and blue lines present the dependence for cases when the interim deadline is imposed in periods 4, 12, and 20 respectively. The red line presents the utility level of the time-inconsistent agent (regardless of the agent's type) pursuing the project under no deadline, while the dotted red line corresponds to the maximum possible utility level.

binds for the naïve agent. The sophisticated agent would affect her future selves if the *goal* A is lower or equal to the total effort under no deadline: $k\beta \geq A$. The partially-sophisticated agent believes she will behave according to the present-bias parameter $\bar{\beta}$ in future periods, thus, she would affect her future selves if $\beta + (k-1)\bar{\beta} < A$. Because $k\beta < \beta + (k-1)\bar{\beta} < \beta + (k-1)$, the deadline constraint binds for sophisticated and partially-sophisticated agents when it binds for the naïve agent. In case, when the interim deadline does not bind in the first several periods, the agent behaves according to equations (20), (25), and (31) as was described in the previous section.

In real life, it is easy to imagine a situation in which the period for the interim deadline is fixed or predefined. For example, the end of a semester for students or pre-specified deadlines in worker contracts. In that case, only the choice of *goal* A can affect the agent's welfare. When the period k is lower than the optimal period $\hat{k}$, it is optimal to set α according to (34), and no improvement is possible. However, when the interim deadline is imposed in period $k > \hat{k}$, α can be chosen such that the deadline would not bind for the first $(k - \hat{k})$ periods. Thus, the agent will behave according to (20), (25), or (31), and her behavior will be equivalent to her behavior under the interim deadline in period $\hat{k}$. For example, the deadline constraint (13) binds for any α such that $(1-\alpha)k < (1-\beta)$. In the opposite case, the naïve agent expends $e_1^{(1)}(ID) = \beta$ and moves to the next period. In the second period, the condition on α changes to $(1-\alpha)k < 2 * (1-\beta)$ and so on with condition $(1-\alpha)k < n * (1-\beta)$ in the nth period. Thus, to induce the naïve agent's behavior equivalent to that under the interim deadline in period $\hat{k}$, α can be chosen such that the deadline does not bind in the first $(k - \hat{k})$ periods and binds in period $(k - \hat{k} + 1)$:

$$\begin{cases} (1-\alpha)k \geq (k-\hat{k})(1-\beta) \\ (1-\alpha)k < (k-\hat{k}+1)(1-\beta) \end{cases} \tag{36}$$

Figure 3 shows how the naïve agent's welfare depends on the parameter α in cases with different periods when the interim deadline is imposed. The red line at the bottom shows the welfare when there is no deadline imposed and the agent expends β effort in every period. The green line demonstrates how the naïve agent's welfare depends on α when the interim deadline is imposed in period $\hat{k}$ ($\hat{k} = 12$ with chosen parameters). Finally, black and blue lines present the dependence of the agent's welfare on α when the interim deadline is imposed in an earlier ($k = 4$) or a later ($k = 20$) period than $\hat{k}$.

It is optimal to set $\alpha = \hat{\alpha}$ when the interim deadline is imposed in period $\hat{k}$ or earlier, and making it higher or lower meaning cannot increase the agent's welfare (green and black lines on Figure 3). However, it is possible to increase the agent's welfare by inducing the agent's procrastination in the first several periods with lower α according to (36) when the interim deadline is imposed in period $k > \hat{k}$. The blue line in Figure 3 illustrates this case: the interim deadline is imposed in period $k = 20$ and it is optimal to set α such the deadline constraint would not bind for the first eight periods. Then the agent behaves in the same way as under the interim deadline imposed in period $\hat{k} = 12$ from the 9th period till the 20th period, which results in the greatest welfare. The periodic waves in Figure 3 show when parameter α becomes too small such the condition $(1-\alpha)k < n * (1-\beta)$ does not hold and the deadline constraint does not bind for one more period.

When the interim deadline is imposed in period $\hat{k}$ or before it, decreasing α such that the deadline constraint would not bind in the first period yields lower welfare. However, the case when the interim deadline is set after $\hat{k}$ can be reduced to the case with optimal $\hat{k}$ by choosing a lower α: after procrastinating in several periods (several waves to the left from $\alpha = 1$) the maximum point of the blue line on Figure 3 yields the same welfare as the maximum point of the green line. As a result, it is always possible to find the optimal $\hat{\alpha}(k)$ given that the interim deadline is imposed in period k.

Proposition 5: *There exists a unique goal $\hat{A}(k)$ (or $\hat{\alpha}(k)$) such that, given fixed timing for the interim deadline, k, the interim deadline with goal $\hat{A}(k)$ maximizes the agent's welfare. The value of goal $\hat{A}(k)$ depends only on the present-bias parameter β and on period k when the interim deadline is imposed (and sophistication level γ for the partially-sophisticated agent).*

The fact that $\hat{A}(k)$ depends only on β and k strictly follows from the equation (34) and inequalities (36), (30), and because the number of periods that the sophisticated and partially-sophisticated agents procrastinate are defined by functions $\Phi(\cdot)$ and $\bar{\Phi}(\cdot)$. Consequently, combining this fact with Proposition 4, the optimal period and the corresponding optimal *goal* depend only on the present-bias parameter β.

Because the agent's behavior when the interim deadline is imposed in period k and binds is equivalent to her behavior when the interim deadline is imposed in period $k+n$ and does not bind for the first n periods (with specifically chosen *goal* A), the optimal interim deadline which maximizes the agent welfare is not unique. There exists the earliest period k^* such that the interim deadline $ID = (\hat{A}(k^*), k^*)$ maximizes the agent's welfare. As discussed above, the same welfare maximum is achievable by choosing optimal *goal* $A = \hat{A}(k^* + m)$ when the interim deadline is imposed in period $k^* + m \leq N$. Based on simulations, the earliest period k^* either coincides with the period $\hat{k}$ for the optimal *natural* interim deadline $ID = (\hat{k}, \hat{k})$, or equal to the next one, $k^* = \hat{k} + 1$, depending on the present bias parameter β. It is never optimal to impose the interim deadline in an earlier period than k^* (and $\hat{k}$) because it would yield lower welfare.

While the agent's behavior under interim deadline $ID = (\hat{A}(k^*), k^*)$ is equivalent to the behavior under any later interim deadline $ID = (\hat{A}(k^* + m), k^* + m)$ when the standard discount factor δ is assumed to be equal to 1, the situation differs when $\delta < 1$. With $\delta < 1$, the agent strictly prefers to invest greater effort later (or closer to the delayed rewards). Thus, only the latest interim deadline is optimal, $ID = (\hat{A}(N), N)$. In other words, it is optimal to set the interim deadline in the last period and adjust the *goal* such that the agent would procrastinate during the optimal number of periods at the beginning of the project.

As a result, it is optimal to set the interim deadline in the last period (N) of the project for all types of agents. The optimal deadline induces behavior as follows. The agent procrastinates at the beginning of the project (expends β effort in earlier periods) and postpones significant effort for later periods. Starting from some period ($N - k^* + 1$), the deadline binds and the agent begins to expend greater effort in each subsequent period. However, while it is optimal to impose the deadline in the last period for all types, the optimal *goal* $\hat{A}(N)$ (and the resulting performance) is different for different types: the lowest for naïve and the highest for sophisticated. On the other hand, imposing the same deadline on all types leads to different welfare for different types. The sophisticated agent always does better than naïve and partially-sophisticated and naïve always does worse than other types.

6 Self-Imposed Interim Deadline

In this case, the agent chooses the interim deadline $ID = (A, k)$ before the project starts (in period 0) and then has to satisfy the deadline constraint. The agent has no possibility to change, adjust, or cancel the interim deadline in later periods during the project. In this framework, the naïve agent believes she will behave according to current preferences and has no incentive to impose any restrictions on her future self, while the sophisticated and partially-sophisticated agents do. Thus, the naïve agent would choose not to impose any interim deadline.[22]

In contrast to the naïve agent, the sophisticated agent anticipates her future behavior and would prefer to use a self-imposed deadline to affect her future self. Since the sophisticated agent correctly predicts her future behavior, she can set the interim deadline optimally and imposes it according to the previous section. Therefore, the most interesting case is when the agent is partially-sophisticated and underestimates her present bias. She is aware of her time inconsistency and would use a self-imposed deadline to affect her future self, however, she cannot use the interim deadline as optimally as a sophisticated agent can.

In period 0, the partially-sophisticated agent with sophistication level γ believes she will behave as a sophisticated agent with a present bias parameter equal to $\bar{\beta}^{PSA} = 1 - \gamma(1 - \beta^{PSA})$. Thus, the partially-sophisticated agent would set her self-imposed deadline equal to the optimal interim deadline for the sophisticated agent with $\beta^{SA} = \bar{\beta}^{PSA}$. Given the results of the previous section, the agent set her self-imposed deadline in the last period and set the *goal* $\hat{A}$ according to the period k^* (or period $\hat{k}$). Thus, I focus on the *natural* interim deadlines in further analysis. Because the optimal period for the *natural* interim deadline is increasing in β for all types of the time-inconsistent agent and $\bar{\beta}^{PSA} > \beta^{PSA}$, the partially-sophisticated agent would set the interim deadline later than the optimal one. As a result, depending on parameters β^{PSA} and γ, the agent sets her self-imposed deadline such that it might increase or decrease her welfare.

Figure 4 presents the regions of pairs of parameters (β^{PSA}, γ) for the partially-sophisticated agent where the self-imposed deadline set by the agent with these parameters would increase or decrease the agent's welfare. The white region presents the pairs of parameters with which the agent sets the self-imposed deadline such it will decrease her welfare. Respectively, the blue region presents the pair of parameters with which the agent sets the self-imposed deadline so that it will increase her welfare.

For the partially-sophisticated agents with low sophistication levels, the self-imposed deadline would decrease the agent's welfare for all possible present bias parameters β. However, for the agents with higher sophistication levels (e.g., $\gamma = 0.75$), the self-imposed deadline would decrease

[22] Strictly speaking, in period 0, the naïve agent is indifferent between not imposing the interim deadline and any *natural* interim deadline because she believes she will behave as a time-consistent agent. I assume that the agent prefers not to impose the interim deadline if she is indifferent between imposing the interim deadline or not.

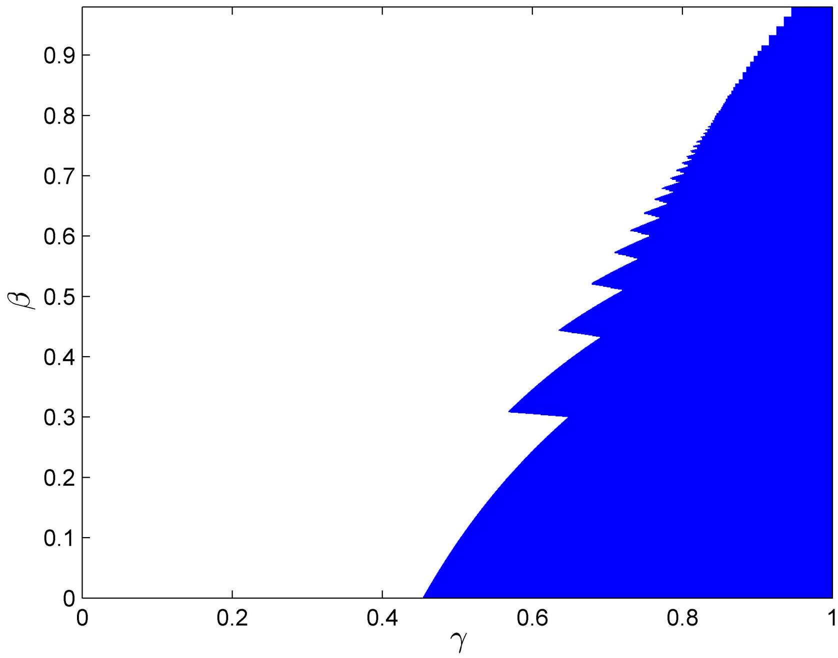

Figure 4: The regions of pairs $\beta - \gamma$ with which the agent would increase or decrease her welfare by setting the self-imposed deadline. The white area corresponds to the parameters' pairs with which the agent would decrease her welfare by setting the self-imposed deadline, while she would increase her welfare with parameters from the blue area.

the welfare for agents with high present bias parameters and increase the welfare for agents with low present bias parameters. In other words, even for agents with high sophistication levels, the self-imposed deadline is a useful instrument only for agents with serious self-control problems. The agents who are close to time-consistent may suffer from setting the self-imposed deadline. The intuition here is in the fact that the same underestimation of present bias leads to different delays in the interim deadline compared to the optimal deadline for partially-sophisticated agents with different present bias parameters. The optimal interim deadline for the agent with a lower present bias parameter is in the earlier period, while for the agent with a higher present bias parameter, it is in the later period. When setting a self-imposed deadline, the agent with a lower present bias parameter sets the deadline relatively closer to the optimal one, as the agent with a higher present bias parameter does. As a result, the larger delay in the imposed deadline leads to higher induced costs in the final periods before the deadline, due to the the "redistribution effect". Thus, the partially-sophisticated agent with a larger present bias parameter β suffers relatively more from setting the self-imposed deadline.

7 Discussion

One of the main limitations of this paper is the specific choice of cost and reward functions. However, it can be shown that the results obtained for the naïve agent are valid for the larger class of the reward and cost functions $R(\cdot)$ and $c(\cdot)$. Because I focus on the effects of the deadline on the agent's behavior and welfare and chose the reward function such that the agent cannot affect her future self through it, the general agent's UMP in period t would look like:

$$\max_{\{e_l\}_{l=t}^{k}} \left\{ -c\left(e_l\right) - \beta \sum_{l=t+1}^{k} c\left(e_l\right) + \beta \sum_{l=t}^{k} u\left(e_l\right) \right\} \tag{37}$$

Where $c(\cdot)$ is strictly increasing, strictly convex, and $c(0) = 0$; $R(e_1, ..., e_N) = \sum_{t=1}^{N} u(e_t)$; $u(\cdot)$ is increasing, concave, $u(0) = 0$, and $u'(0) > c'(0)$. The problem is equivalent to:

$$\max_{\{e_l\}_{l=t}^{k}} \left\{ -f\left(e_l\right) - \beta \sum_{l=t+1}^{k} f\left(e_l\right) + \beta \sum_{l=t}^{k} e_l \right\} \tag{38}$$

Where the new "cost function" $f(\cdot)$ is chosen such that:

$$f'(\cdot) = \frac{c'(\cdot)}{u'(\cdot)}, \quad f(0) = 0 \tag{39}$$

Under the assumptions on $c(\cdot)$ and $u(\cdot)$, the function $f(\cdot)$ is monotone, strictly increasing, and strictly convex. Thus, the solution exists and is unique. One further assumption on function $f(\cdot)$ is needed to ensure that the results are valid in the general case:

$$\beta f'(e_t) = f'(\beta e_t) \tag{40}$$

Then the naïve agent's behavior under the binding *natural* interim deadline is:

$$\hat{e}_t^{(t)} = \begin{cases} \beta \left(f'\right)^{-1}(1) \frac{\Pi_{l=0}^{t-1}(k-l)}{\Pi_{l=1}^{t}(k-(l-\beta))} & , 1 \leq t < k \\ \left(f'\right)^{-1}(1) \frac{\Pi_{l=0}^{k-2}(k-l)}{\Pi_{l=1}^{k-1}(k-(l-\beta))} & , t = k \\ \left(f'\right)^{-1}(\beta) & , k < t \leq N \end{cases} \tag{41}$$

For the naïve agent's behavior according to (41), all the propositions hold, because the difference is in constants. Generalization of the results for sophisticated and partially-sophisticated agents is complicated and requires further research. However, their behavior is similar to that of the naïve agent and the simulations suggest that it is reasonable to believe that the results would hold for the general case.

Another limitation is in the assumption that the agent has to satisfy the deadline constraint. Several papers consider penalty functions as an incentive for the agent to meet the deadline (e.g., Ariely & Wertenbroch, 2002; El-Tannir, 2019). In other words, if the agent expends less overall effort than A by the end of period k, she faces the additional costs $D(\cdot)$. In this paper, the assumption that the agent has to satisfy the deadline constraint is the extreme case when the additional cost function $D(\cdot)$ instantly grows to an incomparable level to the costs if the deadline is not met. In my research, the general additional cost function would only complicate the first-order condition and create an additional trade-off for the agent postponing effort to later periods. However, it is enough to assume that $D'(\cdot)$ when the deadline is missed (the marginal costs of missing the deadline) is greater than $c'(e_k)$ (the marginal costs in the period where the higher effort is accumulated) to induce the same agent behavior.[23]

An additional open question is who sets the exogenous interim deadline to maximize the agent's welfare. In this paper, I focus on the agent's behavior and resulting welfare, leaving aside the discussion of who this entity might be. However, for the exogenous interim deadline case, it is the social planner who is informed about the agent's self-control problem. As a real-life example, this might be a schoolteacher who aims to find a good balance between giving too much homework and keeping students engaged in the study process or parents who care about their children and push them to do exercises. These examples can motivate subsequent research when the social planner is aware of the agent's self-control problems but cannot correctly estimate her present bias. Another interesting extension would be to consider the principal-agent problem. The principal aims to maximize the total effort expended and so imposes an interim deadline. However, the principal can be not fully informed about the agent's present bias, can estimate it incorrectly, or has only restricted options for setting a deadline. This setup is closer to the examples when the agent is an employee and the principal is an employer who suggested a contract to an agent which she can accept or reject.

All the limitations and possible extensions I discussed above are potential spaces for subsequent research.

8 Conclusion

In this paper, I study how the design of exogenous and self-imposed interim deadlines affect an agent's behavior and welfare. The agent can be one of three types based on how she understands

[23]Under the assumption that $D(\cdot)$ grows faster than $c(\cdot)$.

her present bias: naïve, sophisticated, and partially-sophisticated.

Firstly, I consider the situation in which an interim deadline is imposed exogenously with the purpose of maximizing the agent's welfare, and when the agent is restricted to meeting the deadline. I find that the agent continues to procrastinate from the beginning of the project to the deadline, and postpones her efforts to the final periods before the deadline. A later interim deadline increases her overall expended effort, while allowing the accumulation of greater effort in the final periods, and can decrease the agent's welfare. Thus, an interim deadline may increase or decrease the agent's welfare depending on the design. Further, I find that a unique interim deadline exists that maximizes the agent's welfare and that the design of this deadline depends only on the level of the agent's present bias (and sophistication level for the partially-sophisticated agent). Under the same interim deadline, the naïve agent would postpone more effort to future periods than would sophisticated and partially-sophisticated agents. Thus, the naïve agent's welfare will be the lowest. The sophisticated agent would act more like a time-consistent agent than would naive and partially-sophisticated agents and would gain the greatest welfare. The partially-sophisticated agent will fall in between naïve and sophisticated agents. As a result, the optimal deadline for the naïve agent is in an earlier period of the project, for the sophisticated agent, it is in a later period, and for the partially-sophisticated, it is in between.

Second, I study the behavior of a present-biased agent under a self-imposed interim deadline. The naïve agent has no incentive to impose any restrictions on herself and does not set a self-imposed deadline. While a sophisticated agent always sets the self-imposed deadline optimally, the partially-sophisticated agent imposes the deadline such it increases or decreases her welfare depending on the combination of the agent's present bias and sophistication level. An agent with a low sophistication level ($\gamma \lesssim 0.5$) decreases her welfare by setting a self-imposed deadline for all possible β. However, given the same high sophistication level ($\gamma \gtrsim 0.5$), the agent with a relatively high present bias parameter (close to the time-consistent agent) would decrease her welfare while the agent with a relatively low present bias parameter (far from the time-consistent agent) would increase her welfare by setting a self-imposed deadline.

The results contribute to the existing economic literature on the topics of behavioral economics, time-inconsistent preferences, and deadlines. To my knowledge, this is the first theoretical paper that considers how an interim deadline affects the agent's effort choice across several periods and impacts the resulting welfare. An additional novel result is about how the partially-sophisticated agent would affect her welfare by using a self-imposed deadline. The paper's findings are useful for the understanding of how deadlines affect our behavior and welfare.

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Appendix I

For the naïve agent, the interim deadline $ID = (A, k)$ binds in period t' if the optimal intertemporal strategy without a deadline does not satisfy the deadline constraint:

$$\sum_{t=t'}^{k} \hat{e}_t(ND) = \beta + (k - t') < A - \sum_{t=1}^{t'-1} e_t \tag{42}$$

On average, the agent has to expend $\alpha' = \frac{A'}{k-(t'-1)} = \frac{A-\sum_{t=1}^{t'-1} e_t}{k-(t'-1)}$ during periods $\{t', ..., k\}$. However, the agent expends less than average in period t' because her optimal intertemporal strategy in period t' has to satisfy first-order condition $\beta e_{t'} = e_t$, $t \in \{t'+1, ..., k\}$. Thus, the agent has to expend on average in periods $\{t'+1, ..., k\}$ more than on average in periods $\{t', ..., k\}$ and the deadline constraint also binds in period $t'+1$ as well:

$$\sum_{t=t'+1}^{k} \hat{e}_t(ND) = \beta + (k - t' - 1) < A - \sum_{t=1}^{t'} e_t \tag{43}$$

To show this formally, I rewrite the inequalities (42) and (43) in average terms. Because the inequality (42) is for $k - t' + 1$ periods, and inequality (43) is for $k - t'$ periods, they can be rewritten as:

$$\begin{gathered} \frac{\beta+(k-t')}{k-t'+1} < \frac{A-\sum_{t=1}^{t'-1} e_t}{k-t'+1} \\ \frac{\beta+(k-t'-1)}{k-t'} < \frac{A-\sum_{t=1}^{t'} e_t}{k-t'} \end{gathered} \tag{44}$$

The right side of the bottom inequality becomes greater than the right side of the top inequality due to the first-order conditions. However, the left side obviously becomes lower since $\beta < 1$. Thus, the inequality (43) always holds, when the inequality (42) holds, and the deadline constraint binds in every subsequent period (till period k) if it binds in the current period.

When the interim deadline binds or does not for the sophisticated and partially-sophisticated agents is discussed in the corresponding parts of the Analysis section. However, the agents behave according to Proposition 2 and Proposition 3 when the deadline binds. As discussed in Appendix III, the agent then expends less effort than she has to on average during the remaining periods before the deadline. Precisely, the agent's behavior is defined by function $\Phi(x)$[24].

[24]For a formal definition, see Proposition 2

Because $x(1-(1-\beta)x) < x$ for all $x \in (0,1]$, function $\Phi(x)$ is such that if the agent has to expend at least A total effort during the remaining n periods and interim deadline binds, the agent expends less than $\frac{A}{n}$ effort in the current period. Thus, the deadline binds in the next period and in all subsequent periods (till period k).

Link back to subsection 4.3.

Appendix II: Proof of Proposition 1

Proof. I prove Proposition 1 (equation (16)) using the induction method. Under the imposed deadline (A, k), in period t, the agent is considered to be her current self and behaves according to the optimal intertemporal strategy $\left\{\hat{e}_\tau^{(t)}(ID)\right\}_{\tau=t}^{N}$. This strategy can be found as a solution to the corresponding agent's UMP at period t:

$$\left\{\hat{e}_n^{(t)}(ID)\right\}_{n=t}^{N} = \arg \max_{\{e_n\}_{n=t}^{N}} \left\{-\frac{e_t^2}{2} + \beta\left[-\sum_{n=t+1}^{N}\frac{e_n^2}{2} + \sum_{n=1}^{N} e_n\right]\right\} \tag{45}$$

$$s.t.: \quad \sum_{n=t}^{k} e_n \geq A - \sum_{n=1}^{t-1} e_n$$

$$\{e_n\}_{n=1}^{t-1} \;\; are \;\; given$$

<u>Base</u> Requires the agent to behave according to (16) in period $t = 1$:

$$\hat{e}_1^{(1)}(ID) = \beta\alpha \frac{\Pi_{l=0}^{t-1}(k-l)}{\Pi_{l=1}^{t}\left(k-(l-\beta)\right)}\Bigg|_{t=1} = \beta\alpha\frac{k}{k-(1-\beta)} \tag{46}$$

In period 1, the agent's UMP (45) is:

$$\max_{\{e_n\}_{n=1}^{N}} \left\{-\frac{e_1^2}{2} + \beta\left[-\sum_{n=2}^{N}\frac{e_n^2}{2} + \sum_{n=1}^{N} e_n\right]\right\} \tag{47}$$

$$s.t.: \quad \sum_{n=1}^{k} e_n \geq A$$

The deadline constraint binds by the assumption and the first-order conditions give:

$$\begin{cases} e_t = \frac{1}{\beta}e_1 & , \quad t \in \{2, ..., k\} \\ \sum_{n=1}^{k} e_n = A & \\ -e_t + 1 = 0 & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{48}$$

Then the solution for the UMP (46) is:

$$\hat{e}_t^{(1)}(ID) = \begin{cases} \beta\frac{A}{k-(1-\beta)} & , \quad t=1 \\ \frac{A}{k-(1-\beta)} & , \quad t \in \{2,...,k\} \\ 1 & , \quad t \in \{k+1,...,N\} \end{cases} \tag{49}$$

This is the optimal intertemporal strategy at period 1 and the agent expends $\hat{e}_1^{(1)}(ID)$ in the first period:

$$\hat{e}_1^{(1)}(ID) = \beta\frac{A}{k-(1-\beta)} = \beta\frac{A}{k}\frac{k}{k-(1-\beta)} = \beta\alpha\frac{k}{k-(1-\beta)} \tag{50}$$

<u>Assumption</u> Next step is to assume that (16) is correct for all periods from 1 to some $t' < k$:

$$e_t^{(t)}(ID) = \beta\alpha\frac{\Pi_{l=0}^{t-1}(k-l)}{\Pi_{l=1}^{t}(k-(l-\beta))} \; , \quad t \in \{1,...,t'\} \tag{51}$$

<u>Induction Step</u> A further step is to show that (16) will be correct for $t = t'+1 < k$. In period $t'+1$, the agent faces the following UMP:

$$\max_{\{e_n\}_{n=t'+1}^{N}} \left\{ -\frac{e_{t'+1}^2}{2} + \beta\left[-\sum_{n=t'+2}^{N}\frac{e_n^2}{2} + \sum_{n=1}^{N} e_n \right] \right\} \tag{52}$$
$$s.t.: \quad \sum_{n=t'+1}^{k} e_n \geq A - \sum_{n=1}^{t'} e_n$$
$$\{e_n\}_{n=1}^{t'} \; are \; given \; according \; to \; (51)$$

The deadline constraint binds and the first-order conditions give:

$$\begin{cases} e_t = \frac{1}{\beta} e_{t'+1} & , \quad t \in \{t'+2, ..., k\} \\ \sum_{n=t'+1}^{k} e_n = A - \sum_{n=1}^{t'} e_n \\ -e_t + 1 = 0 & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{53}$$

Then the deadline equality in (53) can be rewritten as:

$$\hat{e}_{t'+1}^{(t'+1)}(ID) = \frac{\beta\alpha}{k - (t'+1-\beta)} \left(k - \sum_{t=1}^{t'} \left[\beta \frac{\Pi_{l=0}^{t-1}(k-l)}{\Pi_{l=1}^{t}(k-(l-\beta))} \right] \right) \tag{54}$$

After reduction to a common denominator, the denominator in the expression for $\hat{e}_{t'+1}^{(t'+1)}(ID)$ transforms to $\Pi_{l=1}^{t'+1}(k-(l-\beta))$. The nominator then transforms to $\Pi_{l=0}^{t'}(k-l)$ and the optimal action for the current period $t'+1$ is:

$$e_{t'+1}^{(t'+1)}(ID) = \beta\alpha \frac{\Pi_{l=0}^{(t'+1)-1}(k-l)}{\Pi_{l=1}^{t'+1}(k-(l-\beta))} \tag{55}$$

Then the agent's optimal intertemporal strategy at period $t'+1$ is:

$$\hat{e}_{t'+1}^{(t'+1)}(ID) = \begin{cases} \beta\alpha \frac{\Pi_{l=0}^{(t'+1)-1}(k-l)}{\Pi_{l=1}^{t'+1}(k-(l-\beta))} & , \quad t = t'+1 \\ \frac{1}{\beta}\hat{e}_{t'+1}^{(t'+1)}(ID) = \alpha \frac{\Pi_{l=0}^{(t'+1)-1}(k-l)}{\Pi_{l=1}^{t'+1}(k-(l-\beta))} & , \quad t \in \{t'+2, ..., k\} \\ 1 & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{56}$$

In period $t'+1$, the agent expends $\hat{e}_{t'+1}^{(t'+1)}(ID)$, which satisfies (16). However, this works only for $t'+1 < k$. When $t'+1 = k$, the agent has to expend exactly the planned effort in period $t' = k-1$ to meet the deadline.

<u>Period k</u> To meet the deadline, in period k, the agent must expend effort at the same level as she planned in the previous period. In other words, the agent does not procrastinate only in period k. In period $k-1$, the agent's optimal intertemporal strategy is:

$$\hat{e}_t^{(k-1)}(ID) = \begin{cases} \beta\alpha \frac{\Pi_{l=0}^{(k-1)-1}(k-l)}{\Pi_{l=1}^{k-1}(k-(l-\beta))} & , \quad t = k-1 \\ \\ \frac{1}{\beta} e_{k-1}^{(k-1)}(ID) = \alpha \frac{\Pi_{l=0}^{(k-1)-1}(k-l)}{\Pi_{l=1}^{k-1}(k-(l-\beta))} & , \quad t = k \\ \\ 1 & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{57}$$

That is, in period $k-1$, the agent planes to expend $\hat{e}_k^{(k-1)}(ID)$ in period k. Therefore, the agent expends exactly $\hat{e}_k^k(ID) = \hat{e}_k^{(k-1)}(ID)$ in period k. Thus, the agent behaves according to (16) in period k.

$\square$

Link back to Proposition 1.

Appendix III: Proof of Proposition 2

Proof. In period t, the agent has to expend $A' = A - \sum_{n=1}^{t-1} e_n$ effort during the remaining periods when facing the interim deadline $ID = (A, k)$. Thus the agent solves the following maximization problem in period t:

$$\max_{\{e_n\}_{n=t}^{k}} \left\{ -\frac{e_t^2}{2} + \beta \left[-\sum_{n=t+1}^{k} \frac{e_n^2}{2} + \sum_{n=t}^{k} e_n \right] \right\} \tag{58}$$

$$s.t.: \quad \sum_{n=t}^{k} e_n \geq A'$$

Because the agent is sophisticated and correctly anticipates her future behavior, in period t, she knows how her future selves will behave in future periods. For future periods $\{t+1, ..., k\}$, the agent will have to expend $(A' - e_t)$. Denote by ω_n^{t+1} the part of total effort left $(A' - e_t)$ after expended effort e_t in period t which agent will expend in period $n \in \{t+1, ..., k\}$. Then the effort expended by the agent in the future period n can be rewritten as $\omega_n^{t+1}(A' - e_t)$ and the agent UMP is:

$$\max_{\{e_n\}_{n=t}^{k}} \left\{ -\frac{e_t^2}{2} + \beta \left[-\frac{(A' - e_t)^2}{2} \sum_{n=t+1}^{k} \left(\omega_n^{t+1}\right)^2 + A' \right] \right\} \tag{59}$$

The first-order condition gives the effort expended by the agent in period t:

$$e_t = A' \frac{\beta \sum_{n=t+1}^{k} \left(\omega_n^{t+1}\right)^2}{1 + \beta \sum_{n=t+1}^{k} \left(\omega_n^{t+1}\right)^2} = A' \frac{1}{1 + \frac{1}{\beta \sum_{n=t+1}^{k} \left(\omega_n^{t+1}\right)^2}} \tag{60}$$

Then the ω_t^t is defined and shares for future periods can be recalculated:

$$\begin{cases} \omega_t^t = \frac{1}{1 + \frac{1}{\beta \sum_{n=t+1}^{k} \left(\omega_n^{t+1}\right)^2}} \\ \omega_n^t = (1 - \omega_t^t)\omega_n^{t+1} \qquad , \ n \in \{t+1, ..., k\} \end{cases} \tag{61}$$

The sum of squared future shares can be rewritten as:

$$\sum_{n=t+1}^{k}\left(\omega_n^{t+1}\right)^2=\left(\omega_{t+1}^{t+1}\right)^2+\left(1-\omega_{t+1}^{t+1}\right)^2\sum_{n=t+2}^{k}\left(\omega_n^{t+1}\right)^2$$

And using the first part of the equation (61) but for the next period:

$$\sum_{n=t+1}^{k}\left(\omega_n^{t+1}\right)^2=\left(\omega_{t+1}^{t+1}\right)^2+\left(1-\omega_{t+1}^{t+1}\right)\frac{\omega_{t+1}^{t+1}}{\beta}$$

Plugging this relation into the equation (61), the share ω_t^t can be represented as a function of share ω_{t+1}^{t+1}:

$$\omega_t^t\left(\omega_{t+1}^{t+1}\right)=\frac{1}{1+\frac{1}{\omega_{t+1}^{t+1}\left(1-(1-\beta)\omega_{t+1}^{t+1}\right)}} \tag{62}$$

Or

$$\omega_t^t\left(\omega_{t+1}^{t+1}\right)=\Phi\left(\omega_{t+1}^{t+1}\right) \tag{63}$$

Where

$$\Phi(x)=\frac{1}{1+\frac{1}{x(1-(1-\beta)x)}} \tag{64}$$

Because $x(1-(1-\beta)x)<x$ for all $x\in(0,1]$, the agent expends less effort than the time-consistent agent would do. That is, in period t, the agent expends $e_t<\frac{A'}{k-t+1}$ and postpones some amount of effort to future periods.

□

Link back to Proposition 2.

Appendix IV: Proof of Proposition 4

Proof. To show that such an interim deadline exists and is unique, I consider how the reward and total cost functions change when the principal imposes the interim deadline $ID = (k+1, k+1)$ instead of the interim deadline $ID = (k, k)$. When the deadline is imposed one period later, the total expended effort increases by $1-\beta$, because the agent is restricted to expend $\beta N+(1-\beta)(k+1)$ instead of $\beta N + (1-\beta)k$. Then, the increase in total expended effort does not depend on period k and is constant when moving the interim deadline from the 10th period to the 11th or from the 110th period to the 111th. Thus, moving the interim deadline from period k to $k+1$ increases the agent's welfare only through the reward function $R\left(\sum_{t=1}^{N} e_t\right)$ and this increase in reward function does not depend on k.[25]

The total cost function $C\left(\{e_t\}_{t=1}^{N}\right) = \sum_{t=1}^{N} c(e_t)$ is a convex function, because $c(\cdot)$ is a convex function. Thus, in contrast to the increase in the reward function, the increase in costs is growing with k when the interim deadline is moved from period k to $k+1$. Therefore, given the linearly increasing reward function and convex increasing cost function, there is an intersection between reward and cost functions when the interim deadline is moved further (increasing total effort). When the increase in the total cost function is lower than the increase in the reward function, it is optimal to move the interim deadline to the next period. However, when they become equal, the total cost function grows faster and it is not profitable to move the interim deadline to the next period. This intersection of reward and total cost functions is unique, because the agent's present-bias parameter β defines the distribution of total effort across periods according to Proposition 1, or Proposition 2, or Proposition 3. Thus, the interim deadline which maximizes the agent welfare is unique.

From the "long-run" perspective, the interim deadline $ID = (\hat{k}, \hat{k})$ is optimal when this deadline causes the highest possible utility level. This period $\hat{k}$ can be characterized as follows: the agent's behavior under the interim deadline $ID = (\hat{k}, \hat{k})$ yields greater welfare than the agent's behavior under interim deadlines $ID = (\hat{k}+1, \hat{k}+1)$ and $ID = (\hat{k}-1, \hat{k}-1)$:

$$\begin{cases} U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}, \hat{k})\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}+1, \hat{k}+1)\right\}_{t=1}^{N}\right) > 0 \\ U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}-1, \hat{k}-1)\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}, \hat{k})\right\}_{t=1}^{N}\right) < 0 \end{cases} \tag{65}$$

Because the interim deadline in period k does not affect the agent's behavior in periods $\{k+1, ..., N\}$, the agent behaves identically in periods $\{k+2, ..., N\}$ in all three cases. Thus, the inequalities (65) can be rewritten as following:

[25] Because the reward function is linear.

$$\begin{cases} \sum_{t=1}^{\hat{k}+1}\left[\hat{e}_t^{(t)}(\hat{k},\hat{k}) - \frac{\left(\hat{e}_t^{(t)}(\hat{k},\hat{k})\right)^2}{2}\right] - \sum_{t=1}^{\hat{k}+1}\left[\hat{e}_t^{(t)}(\hat{k}+1,\hat{k}+1) - \frac{\left(\hat{e}_t^{(t)}(\hat{k}+1,\hat{k}+1)\right)^2}{2}\right] > 0 \\ \sum_{t=1}^{\hat{k}+1}\left[\hat{e}_t^{(t)}(\hat{k}-1,\hat{k}-1) - \frac{\left(\hat{e}_t^{(t)}(\hat{k}-1,\hat{k}-1)\right)^2}{2}\right] - \sum_{t=1}^{\hat{k}+1}\left[\hat{e}_t^{(t)}(\hat{k},\hat{k}) - \frac{\left(\hat{e}_t^{(t)}(\hat{k},\hat{k})\right)^2}{2}\right] < 0 \end{cases} \tag{66}$$

Or

$$\begin{cases} \sum_{t=1}^{\hat{k}+1}\left[\left(e_t^{(t)}(\hat{k}+1,\hat{k}+1)\right)^2 - \left(e_t^{(t)}(\hat{k},\hat{k})\right)^2\right] > 2(1-\beta) \\ \sum_{t=1}^{\hat{k}}\left[\left(e_t^{(t)}(\hat{k},\hat{k})\right)^2 - \left(e_t^{(t)}(\hat{k}-1,\hat{k}-1)\right)^2\right] < 2(1-\beta) \end{cases} \tag{67}$$

Under the *natural* interim deadline, the agent's behavior in periods $1, ..., k$ is defined by the period k and present bias parameter β according to Proposition 1, Proposition 2, and Proposition 3 (and on sophistication level γ for the partially-sophisticated agent). Thus, the conditions (67) depends only on present-bias parameter β, because $e_t^{(t)}(\hat{k},\hat{k})$ and $e_t^{(t)}(\hat{k}-1,\hat{k}-1)$ are the functions of $\hat{k}$ and β (and on sophistication level γ for partially-sophisticated agent). Therefore, the agent's present-bias parameter β defines the period $\hat{k}$ for the optimal interim deadline among all interim deadlines $ID = (k, k)$. In other words, regardless of the length of the project (assuming it is long enough), there is a period $\hat{k}$ such that the agent's welfare is higher when the agent unrestrictedly procrastinates after this period $\hat{k}$ than when the *natural* interim deadline is moved to the next period.

□

Moving the *natural* interim deadline in further periods only decreases the agent's welfare. Moreover, every time the deadline is moved to the next period will decrease welfare even more due to the convex cost function. Since the increase in costs is growing with k there is a period $\underline{k}$ such that the *natural* interim deadline imposed in any later period leads to lower agent's welfare than under no deadline. In other words, any *natural* interim deadline $ID = (k, k)$ imposed in period $k > \underline{k}$ decreases the agent's welfare (compared to the situation when the agent behaves under no deadline). This period $\underline{k}$ can be described with the following inequalities:

$$\begin{cases} U_0\left(\left\{\hat{e}_t^{(t)}(\underline{k},\underline{k})\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(ND)\right\}_{t=1}^{N}\right) > 0 \\ U_0\left(\left\{\hat{e}_t^{(t)}(\underline{k}+1,\underline{k}+1)\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(ND)\right\}_{t=1}^{N}\right) < 0 \end{cases} \tag{68}$$

Using the same logic as in proof, the inequalities can be rewritten as:

$$\begin{cases} \sum_{t=1}^{\underline{k}} e_t^{(t)}(\underline{k}, \underline{k}) < \underline{k}(2 - 2\beta + \beta^2) \\ \\ \sum_{t=1}^{\underline{k}+1} e_t^{(t)}(\underline{k} + 1, \underline{k} + 1) > (\underline{k} + 1)(2 - 2\beta + \beta^2) \end{cases} \tag{69}$$

Thus, similar to $\hat{k}$, the period $\underline{k}$ is defined only by the present bias parameter β (and sophistication level γ for partially-sophisticated agent). The period $\underline{k}$ is increasing in β and $\underline{k}^{NA} < \underline{k}^{PSA} < \underline{k}^{SA}$. This period and the corresponding *natural* interim deadline $ID = (\underline{k}, \underline{k})$ show how much someone can push the agent to expend more effort without making the agent worse off. This can be interesting in the setup in which the person who imposes an interim deadline is the principal. The principal uses the interim deadline to maximize the agent's welfare, however, for example, the agent then has the outside option and the principal cannot push the agent to expend as much effort as the principal wishes her to.

Link back to Proposition 4.

Abstrakt

Lidé jsou částečně časově nekonzistentní, a mnozí mají potíže se zavázat k podrobnému harmonogramu projektu. Studuji optimální průběžné termíny, a jak ovlivňují chování a výsledný blahobyt prezenčně vychýleného agenta. Vytvářím model, ve kterém existují tři typy agentů z hlediska toho, jak agent chápe své prezenční vychýlení: naivní, sofistikovaný a částečně sofistikovaný. Pro každý typ existuje jedinečný design pro exogenní průběžný termín, který maximalizuje blahobyt agenta. Pouze sofistikovaný agent by si však sám stanovil optimální průběžný termín, zatímco naivní agent by ho sám neuplatnil vůbec. Částečně sofistikovaný agent si sám stanoví neoptimální termín a může dokonce snížit svůj vlastní blahobyt tím, že si ho uloží. Hlavním výsledkem je, že částečně sofistikovaný agent, který je relativně méně prezenčně vychýlený, by snížil svůj vlastní blahobyt tím, kdyby použil termín, který si stanovil sám, a částečně sofistikovaný agent, který je relativně více prezenčně vychýlený, svůj blahobyt zvýší.

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