THE IMPLICATIONS OF FINANCIAL FRICTIONS AND IMPERFECT KNOWLEDGE IN THE ESTIMATED DSGE MODEL OF THE U.S. ECONOMY

Yuliya Rychalovska

CERGE

EI

Charles University
Center for Economic Research and Graduate Education
Academy of Sciences of the Czech Republic
Economics Institute

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The Implications of Financial Frictions and Imperfect Knowledge in the Estimated DSGE Model of the U.S. Economy*

Yuliya Rychalovska†
CERGE-EI ‡

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Abstract

In this paper I study how alternative assumptions about expectation formation can modify the implications of financial frictions for the real economy. I incorporate a financial accelerator mechanism into a version of the Smets and Wouters (2007) DSGE model and perform a set of estimation and simulation exercises assuming, on the one hand, complete rationality of expectations and, alternatively, several learning algorithms that differ in terms of the information set used by agents to produce the forecasts. I show that the implications of the financial accelerator for the business cycle may vary depending on the approach to modeling the expectations. The results suggest that the learning scheme based on small forecasting functions is able to amplify the effects of financial frictions relative to the model with Rational Expectations. Specifically, I show that the dynamics of real variables under learning is driven to a significant extent by the time variation of agents' beliefs about financial sector variables. During periods when agents perceive asset prices as being relatively more persistent, financial shocks lead to more pronounced macroeconomic outcomes. The amplification effect rises as financial frictions become more severe. At the same time, a learning specification in which agents use more information to generate predictions produces very different asset price and investment dynamics. In such a framework, learning cannot significantly alter the real effects of financial frictions implied by the Rational Expectations model.

JEL classification: E52, E44,E30,C11

Keywords: DSGE models, financial accelerator, adaptive learning

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†Email: yuliya.rychalovska@cerge-ei.cz

‡A joint workplace of the Center for Economic Research and Graduate Education, Charles University, Prague, and the Economics Institute of the Academy of Sciences of the Czech Republic. Address: CERGE-EI, P.O. Box 882, Politických vězňů 7, Prague 1, 111 21, Czech Republic

Abstrakt

V tomto článku studuji jak alternativní předpoklady ohledně formování očekávání můžou pozměnit implikace finančních frikcí pro reálnou ekonomiku. Začleňuji finanční akcelerační mechanismus do verze DSGE modelu od autorů Smets a Wouters (2007) a provádím sadu odhadů a simulačních cvičení za předpokladu kompletně racionalních očekávání, na jedné straně, a několika algoritmů učení které se liší informační množinou používanéou agenty k produkování prognóz, na straně druhé. Ukazuji, že důsledky finančního akcelerátoru pro hospodářký cyklus se můžou lišit v závislosti na způsobu modelování očekávání. Výsledky naznačují, že učení na základě malých prognózových funkcí může vést k amplifikaci účinků finančních frikcí relativně oproti modelu s racionálními očekáváními. Konkrétně ukazuji, že dynamika reálných proměnných za předpokladu učení je převážně daná časovou variací očekávání agentů ohledně finančních proměnných. Během period, kdy agenti vnímají ceny aktiv jako relativně persistentní, finanční šoky vedou k výraznější makroekonomických odezvám. Zhoršení finančních frikcí vede k zesílení amplifikačního efektu. Současně, specifikace mechnismu učení, ve které agenti používají více informací k vytváření prognóz produkuje velmi odlišnou dynamiku cen aktiv a investic. V tomto rámci učení nemůže významně pozměnit reálné účinky finančních frikcí vyplývajících z modelu s racionálními očekáváními.

1 Introduction

Economists generally admit that the public's expectations greatly influence actual macroeconomic outcomes and thus may impact the ability of the central bank to maintain a stable monetary and financial environment as well as high employment. Therefore, when forming macroeconomic projections and policy reactions, policymakers constantly monitor and analyze the dynamics of expectations formed by different market participants such as households, professional economists, credit institutions, and businesses, who produce their own perceptions of future market developments. The effective expectation channel, which implies the ability of the bank to anchor the public's expectations, may decrease the costs of implementing the policy in the presence of trade-offs between several competing objectives and prevent the economy from following a self-fulfilling path leading to periods of high inflation or prolonged recessions.

Despite the significant implications for actual macroeconomic dynamics, the process of modeling the expectation formation mechanism has not received sufficient attention in the literature. The current generation of macroeconomic models is based on the strongest form of rationality, which implies that agents possess complete knowledge about the economy (the model and its parameters) and therefore rely on "true" forecasts in their decision-making process. At the same time, modern economies face various uncertainties and feature unstable and constantly evolving structures. In particular, the dynamic growth of financial markets and the implementation of more sophisticated financial instruments, which requires instant analysis and adjustment to new information, have complicated the task of efficient and up-to-date pricing and credit decisions. In other words, in reality, agents possess only limited information about the economy and have to rest their choices on the basis of forecasts produced in an environment with incomplete information. Therefore, allowing the public to learn the underlying economic structure is more realistic and enables generating more reasonable conclusions about the factors affecting the evolution of the public's predictions, the way the expectations may affect actual economic activity and how they, in turn, are influenced by policy actions and communications. Models with more realistic forms of rationality could add to a better understanding the economic linkages and risks originating from an uncertain environment with imperfectly efficient financial markets.

In this paper, I contribute to studying the macro-financial linkages by focusing on adaptively formed expectations as a mechanism that can potentially amplify and propagate shocks to the real economy and introduce additional challenges to the policy conduct. The rationale for combining the "expectation" and "macro-financial" factors is twofold. First of all, the latest financial turmoil has demonstrated that the impact of imbalances in the financial sector on the real economy and wealth can be far more influential than many economists have anticipated. Even the most recent studies that analyze DSGE models with financial frictions have documented the problems in replicating the observed boom-bust cycle, explaining the "surprising" origin of the crisis and its propagation channels. Therefore, it might be useful to consider other features of the transmission mechanism that might interact with/affect the dynamics of the financial and real sectors. Secondly, expectations play an important role in driving asset prices, risk premia, and investments – the key financial-market variables. Thus, two frameworks can be naturally combined.

More specifically, I add to the existing literature in two aspects. Firstly, I incorporate adaptively formed expectations (Evans and Honkapohja, 2001) into a version of the Smets and Wouters (2007) DSGE model with a financial accelerator. I estimate the model using Bayesian methods and assess the joint role of financial frictions and the departure from the complete rationality assumption for the U.S. business cycle. I evaluate and compare the model fit, estimated parameters, and the transmission mechanism in models with Rational Expectations (RE) and adaptive learning (AL). Several AL schemes that differ in terms of the information set used by agents to form their expectations are considered. Discussing the estimation results, I evaluate the role of alternative sources of inertia: structural rigidities (such as habit formation, Calvo pricing, indexation etc.) and learning in propagating financial and non-financial shocks. Secondly, on the basis of the estimated model as well as simulation exercises, I assess the ability of alternative learning algorithms to modify the transmission mechanism relative the RE model with financial frictions and generate additional macroeconomic fluctuations in line with real data. To my best knowledge, this paper is the first one that evaluates the effects of financial frictions under adaptive learning within the *estimated* model. Soto et al. (2010) studies how a financial accelerator mechanism combined with adaptive learning influences the business cycle fluctuations in a *calibrated* model. Another important difference of their paper from mine is the information set that learning agents are assumed to use in order to form their predictions. The results of Soto et al. (2010) are derived for so-called "MSV" learning. This means that agents use the full set of endogenous and exogenous variables in their forecasting functions. The same set of variables is used to form forecasts under RE. In this paper I assume that agents may use a very limited information set. In fact, I compare the results for alternative information sets and demonstrate that the learning scheme is an important determinant of the effects of the financial accelerator for the real economy.

1.1 Related literature

In the recent literature, the "financial accelerator" represents the most common approach to incorporate financial frictions into DSGE models. This framework implies that endogenous developments in credit markets work to amplify and propagate shocks to the real economy. Depending on the origin/type of such an acceleration mechanism, two main strands in the literature can be distinguished. The first one implies capturing the firms' balance sheet effects on investment by relying on a one-period stochastic optimal debt contract with costly state verification (Bernanke and Gertler, 1989; Carlstrom and Fuerst, 1997; Cespedes et al., 2004). The key aspect is that such a framework allows modeling of an endogenous, positive interest rate spread. The second approach emphasizes another aspect of many possible frictions – the role of endogenous collateral constraint that links the credit capacity of borrowers to the value of their asset holdings (Kiyotaki and Moore, 1997; Iacoviello, 2005; Iacoviello and Neri, 2008).

In this paper, I follow the first approach and incorporate financial frictions in the form of the financial accelerator from Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999). They introduce the agency problem with asymmetric information in order to model a positive interest rate spread, i.e. an "external finance premium" defined as the difference between the cost of external sources of funding and the opportunity cost of funds internal to the firm. Due to the agency problem in lending, the external finance premium depends inversely on the borrowers' net wealth and thus will be countercyclical, enhancing swings in real variables and amplifying the effects of monetary and financial shocks. Bernanke, Gertler and Gilchrist (1999) incorporate an "external finance premium" into a dynamic New Keynesian model with nominal rigidities to study how credit market frictions may influence the transmission of monetary policy. They show that under a reasonable parameterization of the model, the financial accelerator significantly amplifies the effects of shocks to the economy. In terms of its empirical relevance, recent research has found that for the Euro Area and for the U.S. the financial accelerator plays a relevant role in amplifying shocks that move prices and output in the same direction (e.g. monetary policy shocks) as well as in explaining the business cycle (Christiano et al., 2007). De Graeve (2008) estimates the external finance premium for the U.S. economy incorporating a financial accelerator into the Smets and Wouters (2003) model. He finds that a model-consistent estimate of this unobservable financial variable has substantial realistic content (the estimate strongly comoves with the proxies for the premium). Another important result of his study is that incorporating financial frictions improves the empirical performance of an otherwise standard DSGE model.

In modeling departures from complete rationality assumptions, I follow the most influential contributions in the adaptive learning literature such as Evans and Honkapohja (2001), Milani (2007) and Orphanides and Williams (2007). In particular, I assume that agents know the structure but they are uncertain about the parameters of the model. To learn the parameters, they formulate models based on their economic perceptions and re-estimate these models as soon as new information arrives. A number of studies have demonstrated that adaptive learning can improve the fit of macroeconomic models. In particular, Milani (2007, 2008) and Sargent, Williams and Zha (2005) have shown that introducing adaptive learning can generate the levels of persistence observed in U.S. data. Slobodyan and Wouters (2008, 2010) incorporate less-than-rational beliefs into the Smets and Wouters (2007) model and find that impact of the adaptive learning on macro dynamics is more pronounced when the agents' information set is more restrictive than the one implied by rational expectations. In small forecasting models learning can explain episodes of inflation dynamics in the U.S. and lowers the persistence of some of the exogenous shocks. Rychalovska and Slobodyan (2010) estimate a set of DSGE models of various complexity for the Euro Area. They also find that assuming adaptive expectations results in better model fit than if RE is used, especially when the agents use very little information to form their beliefs. Therefore, the conclusion that adaptive learning based on small forecasting models outperforms MSV and RE models seems to be a robust one, at least for U.S. and European data. In this paper, I follow Slobodyan and Wouters (2010) and assume that agents' forecasts can be based on very small forecasting models, in particular on a model where expected value of a forward-looking variable depends on a constant and two lags of this variable. Agents estimate and update simple forecasting models using the Kalman filter algorithm. Thus, the learning represents an alternative source of endogenous inertia and influences the degree of economic persistence through the time variation in agents' beliefs.

The rest of the paper is organized as follows: in Section 2 I present the model; Section 3 contains the estimation methodology and results; Section 4 describes the effects of financial frictions on the transmission mechanism in the model with adaptive learning, and Section 5 concludes.

2 The model with a financial accelerator under learning

In this paper, I incorporate financial frictions á la Bernanke et al. (1999) combined with an imperfectly rational expectation formation mechanism into a medium-scale DSGE model based on Smets and Wouters (2007). The model contains a number of nominal and real rigidities such as monopolistic competition on goods and labor markets, Calvo price and wage stikiness, habit formation in consumption and capital adjustment costs. Following the seminal contributions of Smets and Wouters (2003), (2007) and Christiano et al. (2005), these structural rigidities have become widely used in order to match the observed properties of the main macroeconomic series. The model also incorporates a financial accelerator mechanism based on a costly state verification problem between borrowers and lenders, originally proposed by Bernanke et al. (1999) and extensively explored in the recent literature.[1]

The economy consists of households, final and intermediate goods producers, a monetary authority and a financial sector. Intermediate-sector firms are monopolistically competitive. They produce differentiated goods, decide on labour and capital input and set prices according to the Calvo (1983) model. Households supply homogenous labour to an intermediate labour union, which differentiates the labour services. Since there is a certain monopoly power over labour, unions can set wage rates. I assume that unions face Calvo (1983)-type frictions in setting the wages. In addition, nominal rigidities in wage and price setting are augmented by the assumption that prices that are not re-optimised are partially indexed to past inflation rates.

The financial sector is represented by capital good producers, a financial intermediary (bank) and entrepreneurs. Capital producers accumulate new capital and sell it to entrepreneurs. Entrepreneurs borrow from the bank in order to finance capital purchases and rent capital stock to intermediate firms. Financial-market imperfections are set up in the form of assymmetric information between the entrepreneurs and the banks. Due to this friction, the optimal financial contract, which maximizes the payoff of the entrepreneur subject to the required rate of retrun of lenders, implies the existence of an endogenous external finance premium that depends on the entrepreneur's leverage ratio. As in Kiyotaki and Moore (1997), the financial frictions of Bernanke et al. (1999) are based on the idea that asset price variability affects the entrepreneurial financial position and therefore drives credit-market imperfections. Introducing adaptive learning and incomplete information into such a framework brings in additional volatility in asset prices and the external finance premium. As a result, imperfectly rational beliefs become one of the driving forces behind the fluctuations of financial markets and modify the impact of financial frictions on the real economy.

The model is detrended with a deterministic trend γ that represents a labor-augmenting growth rate in the economy. The non-linear system is then linearised around the stationary steady state of the detrended variables. Lower-case variables denote detrended variables expressed in real terms. In this section, I outline the main features and present a log-linearized version of the model (for a more detailed description of micro-foundations see the original papers).

[1] The most relevant examples include De Graeve (2008), Christensen and Dib (2008) and Chtistiano et al. (2003).

2.1 Households and Labour Markets

Household j chooses consumption, hours worked and savings so as to maximize a utility function, non-separable[2] in two arguments – a CES basket of consumption-good varieties and labour services:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma_c} (C_{t+s}(j) - \eta C_{t+s-1})^{1-\sigma_c} \right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_{t+s}(j)^{1+\sigma_l} \right), \tag{1}$$

where σ_c and σ_l are preference paremeters and η is an external habit-formation parameter, which introduces the dependance of the household consumption on the lagged aggregate consumption. Households can save by depositing funds in the bank and by buying government bonds. These assets (denoted, in total, as AT) are perfect substitutes and earn the same riskless nomial interest rate R^n. Households also obtain dividends from owing intermediate and capital goods producers as well as from labor unions. Therefore, the budget constraint of the representative household takes the form:

$$C_{t+s}(j) + \frac{AT_{t+s}(j)}{\varepsilon_t^b R_{t+s}^n P_{t+s}} - T_{t+s} = \frac{W_{t+s}(j) L_{t+s}(j)}{P_{t+s}} + \frac{AT_{t+s-1}(j)}{P_{t+s}} + \frac{Div_{t+s}}{P_{t+s}}, \tag{2}$$

where c_t is exogenous premium on the bonds' return, W_{t+s}^h is the nominal wage, T_{t+s} are lump-sum taxes or subsidies and Div_{t+s} are dividend payments.

The first-order conditions with respect to consumption and assets result in the Euler equation, which after model detrending and log-linearization takes the following form:

$$\begin{aligned} \widehat{c}_t = {} & \frac{1}{(1+(\eta/\gamma))} E_t \left[\widehat{c}_{t+1} \right] + \frac{(\eta/\gamma)}{(1+(\eta/\gamma))} \widehat{c}_{t-1} \\ & - \frac{(1-\eta/\gamma)}{\sigma_c(1+(\eta/\gamma))} (\widehat{b}_t + \widehat{R}_t^n - E_t[\widehat{\pi}_{t+1}]) - \frac{(\sigma_c - 1)(w_*^h L/c_*)}{\sigma_c(1+(\eta/\gamma))} (E_t \left[\widehat{L}_{t+1} \right] - \widehat{L}_t). \end{aligned} \tag{3}$$

The backward-looking term arises in the consumption equation due to the assumptions of external habit formation captured by the parameter η.Therefore, current consumption ($\widehat{c}_t$) depends on a weighted average of past and expected future consumption. The consumption process is also affected by the expected growth in hours worked ($E_t \left[\widehat{L}_{t+1} \right] - \widehat{L}_t$) (due to the non-separable in consumption and labour form of the utility function), the ex–ante real interest rate ($\widehat{R}_t^n - E_t[\widehat{\pi}_{t+1}]$) and a disturbance term $\widehat{b}_t$. γ is the deterministic trend, which arises as a result of model detrending[3]. $\widehat{b}_t$ is assumed to follow a first–order

[2] A sensitivity check demonstrated that the use of the separable (in consumption and labor) form of the utility function, employed in Smets and Wouters (2003), does not significantly affect the estimation results or the conclusions of the paper.

[3] Detrended real variables are obtained by dividing the nominal variables by a deterministic trend: $c_t = C_t/\gamma^t$, $w_t = W_t/(\gamma^t P_t)$ etc.

autoregressive process with an iid–normal error term: $\widehat{b_t} = \rho_b \widehat{b}_{t-1} + \epsilon_t^b$. Variables with stars denote the steady-state values.

As in Smets and Wouters (2007), labour markets consist of labour unions, who allocate and differentiate labour supplied by households, and labour packers, who buy labour from the unions, package it into a Kimball (1995) composite aggregator L_t that is resold to intermediate goods producers. Unions have market power over labour services and set wages that are subject to nominal rigidities á la Calvo. Every period only $(1-\xi_w)$ fraction of intermediate labour unions can readjust wages. The chosen wage rate set by the union maximizes the stream of future (discounted) wage incomes for all the time periods when the union is stuck with that wage in the future. The first-order conditions to problems 1 and 2 with respect to hours worked combined with the solution to the profit-maximization problem of the intermediate labour union and the law of motion of the aggregate wage result in the following wage equation:

$$\begin{aligned}\widehat{w}_t \; &= \frac{1}{(1+\overline{\beta}\gamma)}(\widehat{w}_{t-1} + \overline{\beta}\gamma E_t\left[\widehat{w}_{t+1}\right] - (1+\overline{\beta}\gamma\iota_w)\widehat{\pi}_t + \iota_w\widehat{\pi}_{t-1} + \overline{\beta}\gamma E_t\left[\widehat{\pi}_{t+1}\right] \qquad (4)\\ &+\frac{(1-\xi_w\overline{\beta}\gamma)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}[\frac{1}{1-\eta/\gamma}\widehat{c}_t - \frac{\eta/\gamma}{1-\eta/\gamma}\widehat{c}_{t-1} + \sigma_l\widehat{L}_t - \widehat{w}_t\;] + \widehat{\lambda_{w,t}}\;,\end{aligned}$$

where $\overline{\beta} = \beta/\gamma^{\sigma_c}$ and β is a discount factor applied to households. Due to nominal wage stickiness and the partial indexation of wages to inflation, real wages adjust only gradually to the desired wage mark–up. ξ_w is a wage stickiness parameter. Parameter ι_w measures the degree of indexation. If wages are perfectly flexible ($\xi_w = 0$), the real wage is a constant mark–up over the marginal rate of substitution between consumption and leisure. When wage indexation is zero (ι_w), real wages do not depend on the lagged inflation. In addition to wage stikiness, the speed of adjustment to the desired mark-up depends on the demand elasticity for labour, which is a function of the steady-state labour market mark-up $(\phi_w - 1)$ and the curvature of the Kimball labour-market aggregator ε_w. The wage–mark up disturbance $(\widehat{\lambda}_{w,t})$ is assumed to follow an ARMA (1,1) process with an iid–normal error term: $\widehat{\lambda}_{w,t} = \rho_w \widehat{\lambda}_{w,t-1} - \mu_w \epsilon_{w,t-1} + \epsilon_t^w$.

2.2 Production sector: Firms

The production sector consists of final- and intermediate-good producers. Final-good producers buy intermediate goods $Y_t(i)$, aggregate them into a composite final good Y_t and resell to consumers in a perfectly competitive market. The solution to the profit-maximization problem of these firms is standard and determines the demand function for intermediate inputs $Y_t(i)$. Intermediate-good producers, who operate under monopolistic competition, rent capital from entrepreneurs at the rate R_t^k, hire labor from labour packers and use a typical Cobb-Douglas production function augmented with fixed costs:

$$Y_t(i) = \varepsilon_t^a K_t^S(i)^{\alpha} \left[\gamma^t L_t(i)\right]^{1-\alpha} - \gamma^t \Phi, \tag{5}$$

where $K_t^S(i)$ is capital services used in production, $L_t(i)$ is aggregate labour input, α is the share of capital in production and Φ is a fixed cost. γ^t respresents the labour-augmenting deterministic growth rate in the economy and ε_t^a is total factor productivity. The log-linearized aggregate supply equation 5 takes the form:

$$\widehat{y}_t \;= \Phi(\; \alpha(\widehat{k^S}_t) \;+\; (1-\alpha)\widehat{L}_t + \widehat{A}_t), \tag{6}$$

where the total factor productivity ($\widehat{A}_t$) is assumed to follow a first-order autoregressive process: $\widehat{A}_t \;\; = \rho_a \widehat{A}_{t-1} + \epsilon_t^a$. The solution to the cost-minimization problem yields the conditions that determine the labour demand function in the following log-linear form:

$$\widehat{L}_t = \widehat{k^S}_t \;-\; \widehat{w}_t \;+\; \widehat{r}_t^k. \tag{7}$$

Equation 7 implies that the rental rate of capital is negatively related to the capital–labour ratio and positively to the real wage (both with unitary elasticity). The marginal cost is the same for all firms and represenetd by the following relation:

$$\widehat{mc}_t \;\; = \;\; (1-\alpha)\; \widehat{w}_t + \alpha \; \widehat{r}_t^k - \widehat{A}_t. \tag{8}$$

Similar to wages, each period only a fraction of firms $(1-\xi_p)$ can re-optimize prices. In the environment of price rigidities, the optimal price will maximize the expected discounted stream of future firm's profits for all states of nature when the firm cannot reset the price optimally. Thus the current inflation rate will be a function of current *and* future expected marginal costs. Non-reoptimized prices are partially indexed to past inflation, which gives rise to the backward-looking term in the inflation equation. Profit maximization by price–setting intermediate firms gives rise to the following New–Keynesian Phillips curve:

$$\widehat{\pi}_t = \frac{1}{(1+\overline{\beta}\gamma\iota_p)}(\iota_p\widehat{\pi}_{t-1} + \overline{\beta}\gamma E_t\left[\widehat{\pi}_{t+1}\right] + \frac{1}{((\phi_p-1)\varepsilon_p+1)}\frac{(1-\xi_p\overline{\beta}\gamma)(1-\xi_p)}{\xi_p}(\widehat{mc}_t)) + \widehat{\lambda_{p,t}}, \tag{9}$$

where ι_p denotes the indexation coefficient. The inflation equation demonstrates that the speed of adjustment to the desired mark-up depends on the degree of price stickiness ξ_p, the curvature of the Kimball goods market aggregator ε_p and the steady state mark-up $(\phi_p - 1)$. The price mark–up disturbance $(\widehat{\lambda}_{p,t})$ is assumed to follow an ARMA(1,1) process: $\widehat{\lambda}_{p,t} = \rho_p\widehat{\lambda}_{p,t-1} - \mu_p\epsilon_{p,t-1} + \epsilon_t^p$, where ϵ_t^p is an iid–Normal price mark–up shock.

2.3 Financial sector

2.3.1 Capital-good producers

Capital-good producers, owned by households, produce new capital goods which are sold to entrepreneurs at price Q_t. Capital-good producers are competitive and take the price as given. They combine investment goods, purchased from the final good producers, with the existing capital stock, rented from the entrepreneurs, to produce new capital goods, K_{t+1}. Following the setup of Bernanke et al. (1999), it is assumed that the rental rate for the existing capital is zero, since the operation takes place within one period. Capital-good producers are subject to quadratic adjustment costs specified as function $S(\frac{I_t}{I_{t-1}})$, with $S''(.) > 0$. In addition, the capital production technology is affected by an investment-specific shock ε_t^i. The optimization problem of capital-good producers, in real terms, consists of choosing the level of investment I_t to maximize the real expected profits:

$$\max_{I_t} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[Q_{t+s} I_{t+s} \varepsilon_{t+s}^i - I_{t+s} - Q_{t+s} I_{t+s} \varepsilon_{t+s}^i S(\frac{I_{t+s}}{I_{t+s-1}}) \right] \right\}, \tag{10}$$

where λ_t denotes the marginal utility of the real income of the household. The solution to the problem is:

$$\varepsilon_t^i Q_t \left(1 - S(\frac{I_t}{I_{t-1}}) \right) = 1 + \varepsilon_t^i Q_t S'(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}} - E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \varepsilon_{t+1}^i Q_{t+1} S'(\frac{I_{t+1}}{I_t}) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}. \tag{11}$$

Equation 11 relates the price of capital to investment and adjustment costs. In the absence of adjustment costs Q_t is constant and equal to one. The presence of investment adjustment costs mitigates the response of investment to different shocks, which affects the price of capital. Therefore, introducing volatility in Q_t, investment adjustment costs is one of the factors that contributes to the dynamics of entrepreneurial net worth. After detrending and log-linearization of 11, the dynamics of investment is given by:

$$\widehat{i}_t = \frac{1}{(1+\overline{\beta}\gamma)} (\widehat{i}_{t-1} + (\overline{\beta}\gamma)\widehat{i}_{t+1} + \frac{1}{\gamma^2 S''} \widehat{Q}_t \ ) + \widehat{q}_t, \tag{12}$$

where S'' is the steady-state elasticity of the capital adjustment cost function and $\overline{\beta} = (\beta/\gamma^{\sigma_c})$ where β is the discount factor applied by households. As in CEE (2005), a higher elasticity of the cost of adjusting capital reduces the sensitivity of investment $(\widehat{i}_t)$ to the real value of the existing capital stock $(\widehat{Q}_t)$. Finally, $\widehat{q}_t$ represents a disturbance to the investment–specific technology process and is assumed to follow a first–order autoregressive process with an iid–normal error term: $\widehat{q}_t \ = \rho_q \widehat{q}_{t-1} + \epsilon_t^i$.

The evolution of the capital stock is represented by the following expression:

$$\widehat{k}_t = (1 - \frac{i_*}{\overline{k}_*})\, \widehat{k}_{t-1} \; + \; \frac{i_*}{\overline{k}_*}\widehat{i}_t + \frac{i_*}{\overline{k}_*}(1+\overline{\beta}\gamma)\gamma^2 S''\widehat{q}_t. \tag{13}$$

2.3.2 Entrepreneurs and banks

In the original Smets and Wouters (2007) model, financial markets do not incorporate endogenous forms of inefficiency. In particular, households can borrow in any quantity at the rate that might exceed the risk-free rate R_t set by the central bank due to the exogenous premium ε_t^b. Modeling endogeous credit imperfections requires distinguishing between borrowers and lenders and the existence of a conflict between the two parts. Therefore, new types of agents have to be introduced. In this paper, I follow the financial accelerator framework of Bernanke et al. (1999) in modeling the financial frictions. Entrepreneurs, who are risk neutral and survive until the next period with probability $\varkappa$, use their own funds (the net worth, N_{t+1}) and loans from the bank (B_{t+1}) to finance capital that is rented to the production sector. Competitive banks finance the loans by accepting deposits from the households at the risk-free rate. The financial intermediation between the households, banks and entrepreneurs is subject to friction based on the agency problem, which leads to the existence of the interest rate premium. In particular, after the purchase of the capital stock, each entrepreneur receives a productivity shock that affects the return on capital holdings (R^K_{t+1}) and can be costlessly observed. Banks have to pay a "state verification" (monitoring) cost to infer the realized return. As a result, entrepreneurs have to pay an external finance premium over the riskless rate in order to borrow funds.

At the end of period t, entrepreneurs purchase capital K_{t+1} from capital-goods producers at price Q_t. Thus, the amount of borrowed funds is given by $B_{t+1} = Q_t K_{t+1} - N_{t+1}$. After obzerving the $t+1$ shock, the entrepreneur decides on the degree of capital utilization (U_{t+1}) and rents a part of the capital services to intermediate-good firms at rate $\widehat{r}^k_{t+1}$. A non-depreciated capital stock is then sold at price Q_{t+1}. As newly installed capital only becomes effective with a one–quarter lag, current capital services used in production is a function of capital installed in the previous period (K_t) and the degree of capital utilization (U_{t+1}). Therefore, the amount of effective capital that entrepreneurs can rent to firms is $K^S_{t+1} = U_{t+1}K_t$. The income from renting capital services is $r^k_{t+1}U_{t+1}K_t$,while the (real) cost of changing capacity utilization is $a(U_{t+1})K_t$, where a is a convex function with $a', a'' > 0$. The entrepreneur chooses U_{t+1} to solve:

$$\max_{U_{t+1}} \left[r^k_{t+1}U_{t+1} - a(U_{t+1}) \right] K_t. \tag{14}$$

The solution implies that

$$r^k_{t+1} = a'(U_{t+1}). \tag{15}$$

The log linearized relation for capital services is given by

$$\widehat{k^S}_{t+1} \;=\; \widehat{u}_{t+1} \;+\; \widehat{k}_t. \tag{16}$$

The average (aggregated over all the entrepreneurs) real return to capital purchased at time t is given by

$$E_t R^k_{t+1} = E_t \left[\frac{r^k_{t+1} U_{t+1} - a(U_{t+1}) + Q_{t+1}(1-\tau)}{Q_t} \right], \tag{17}$$

where τ is the depreciation rate. Expression 17 equates the marginal return of capital, given by the right-hand-side terms, to the real expected interest rate on external funds. The log-linerized relation that describes the dynamics of the average expected real return to capital is given by

$$E_t \widehat{R}^K_{t+1} = \frac{1-\tau}{\overline{R}^K} E_t \widehat{Q}_{t+1} + \frac{\overline{r}^k}{\overline{R}^K} E_t \widehat{r}^k_{t+1} - \widehat{Q}_t, \tag{18}$$

where $\overline{R}^K$ denotes the steady-state return to capital and $\overline{r}^k$ is the steady-state rental rate.

The equilibrium condition on financial markets is derived from the optimal-debt contract problem, which maximizes the welfare of the entrepreneur, combined with the zero-profit condition of the bank. The details of the financial contract specification and derivations can be found in appendix A of Bernanke et al. (1999). The optimality condition, which determines the link between the external financing costs, capital purchases and entrepreneurial financial position, is given by

$$E_t R^k_{t+1} = E_t \left[s\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right) \varepsilon^b_t R_t \right]. \tag{19}$$

Equation 19 indicates that the cost of external financing is composed of the premium for borrowed external funds represented by a function $s(\frac{N_{t+1}}{Q_t K_{t+1}})$, the risk-free interest rate and an exogenous shock that describes fluctuations in the risk premium not captured by the financial frictions of Bernanke et al. (1999). Therefore, in this model, the financial accelerator mechanism consists of both endogenous and exogenous components. The log-linearized version is represented by the following equation:

$$E_t \widehat{R}^K_{t+1} = -el \left\{ E_t \left[\widehat{N}_{t+1} - \widehat{Q}_t - \widehat{k}_{t+1} \right] \right\} + \widehat{R}_t + \widehat{b}_t, \tag{20}$$

where $\widehat{R}_t = (\widehat{R}_t^n - E_t[\widehat{\pi}_{t+1}])$ is the risk-free real interest rate and el represents the elasticity of the external finance premium to the change in the financial conditions. The equation above indicates that in equilibrium an entrepreneur purchases capital up to the point where the expected real return to capital is equal to the marginal cost of external finance. The higher the fraction of the project value financed by the entrepreneur's internal funds (the higher the net worth N relative to the gross value of capital QK), the lower the capital market friction and the lower the corresponding risk premium. The absence of financial frictions implies the case when entrepreneurs have sufficient net worth to finance the demand for capital stock. In such a situation, the risk of default associated with borrowing external funds vanishes, the risk-free rate and the real return to capital coincide, and the model reduces to the model of Smets and Wouters (2007).

The law of motion for the aggregate financial wealth of entrepreneurs is given by

$$N_{t+1} = \varkappa V_t + W_t^e, \tag{21}$$

where $\varkappa$ is the entrepreneurial survival rate and W_t^e is the transfer to all the entrepraneurs who are in business in period t. The aggregate net worth of surviving entrepreneurs V_t is equal to the difference between the revenue from capital holding in time t and the cost of borrowing carried over from the previous period (the rate of interest paid by entrepreneurs on loan contracts B_t signed in time $t-1$), averaged across all the entrepreneurs:

$$V_t = \left[R_t^K Q_{t-1} K_t - E_{t-1} R_t^K \left(Q_{t-1} K_t - N_t\right)\right]. \tag{22}$$

The log-linerization of combined equations 21 and 22 leads to the expression of entrepreneurial net worth in the form of the following accumulation equation:

$$\widehat{N}_{t+1} = \varkappa \overline{R}^K \left[\frac{\overline{K}}{\overline{N}}\left(\widehat{R}_t^K - E_{t-1}\widehat{R}_t^K\right) + E_{t-1}\widehat{R}_t^K + \widehat{N}_t\right], \tag{23}$$

where $\overline{K}/\overline{N}$ is the steady-state ratio of capital to net worth, i.e. the inverse of the leverage ratio. Equation 23 demonstrates that, in general terms, the endogenous variations in the next period net worth come from the unexpected changes in the real return to capital. In this model, the variability of asset prices is one of the main sources of such volatility, especially if firms are leveraged. Combining equations 20 and 23, the net worh can be expressed as a function of the risk-free interest rate and the exogenous and endogenous finance premia:

$$\widehat{N}_{t+1} = \varkappa \overline{R}^K \left[\frac{\overline{K}}{\overline{N}}\widehat{R}_t^K - \left(\frac{\overline{K}}{\overline{N}} - 1\right)\left(\widehat{R}_{t-1} + \widehat{b}_{t-1}\right) - el\left(\frac{\overline{K}}{\overline{N}} - 1\right)\left(\widehat{k}_t + \widehat{Q}_{t-1} - \widehat{N}_t\right) + \widehat{N}_t\right]. \tag{24}$$

The values of the parameters $\varkappa$, $\overline{K}/\overline{N}$ and el determine the impact of financial frictions on the real economy. The higher the entrepreneurial survival rate and the capital to net worth steady-state ratio, the more persistent the evolution of the net worth will be. Combined with the higher elasticity of the external finance premium, this would imply a stronger response of the wedge between the expected return to capital and the risk-free rate. Therefore, shocks affecting entrepreneurial net worth would have greater real effects.

2.4 Monetary policy and equilibrium

Finally, the model is completed by adding the following empirical monetary policy reaction function:

$$\begin{aligned} \widehat{R}_t^n &= \rho_R \widehat{R}_{t-1}^n + (1-\rho_R)(r_\pi \widehat{\pi}_t + r_y \widehat{ygap}_t) \\ &\quad + r_{\Delta y}(\widehat{ygap}_t - \widehat{ygap}_{t-1}) + r_t. \end{aligned} \tag{25}$$

The monetary authority follows a generalized Taylor rule responding to inflation and the output gap terms (current and lagged). The latter is defined as the difference between actual and potential output. The output gap is approximated by $\widehat{ygap}_t = \widehat{y}_t - \widehat{A}_t$. The parameter ρ_R captures the degree of interest rate smoothing. I assume that the monetary policy shock (r_t) follows a first–order autoregressive process with an iid–Normal error term: $\widehat{r}_t = \rho_r \widehat{r}_{t-1} + \epsilon_t^r$.

The aggregate resource constraint is given by

$$Y_t = a(U_t)K_{t-1} + \mu_t^{bank} + C_t^e + C_t + I_t + G_t. \tag{26}$$

The first term of equation 26 captures the capital utilization costs; μ_t^{bank} measures the bank monitoring cost (small under reasonable parametrization and therefore typically neglected); $C_t^e = \Theta(1-\varkappa)V_t$ corresponds to the consumption of the $(1-\varkappa)$ entrepreneurs, who exit the economy in period t, where V_t is their net worth. In practice, Θ is normally set to zero. the log-linear representation of 26 is given by

$$\widehat{y}_t = \frac{r_*^k k_*}{y_*}\widehat{u}_t + \widehat{\mu}_t^{bank} + \frac{c_*}{y_*}\widehat{c}_t + \frac{i_*}{y_*}\widehat{i}_t + \widehat{g}_t , \tag{27}$$

where $\widehat{g}_t$ is exogenous government spending, which is assumed to follow a first–order autoregressive process with an iid–normal error term and is also affected by the productivity shock as in Smets and Wouters (2007): $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \rho_{ga}\epsilon_t^a + \epsilon_t^g$. The relation between domestic productivity and government spending is motivated by the fact that in estimation the exogenous spending component also includes net exports, which may be affected by domestic productivity developments.

2.5 Introducing adaptive learning

I depart from the assumption of rational expectations and assume that agents possess incomplete knowledge about the economic environment (model structure and parameters). Therefore they are unable to produce model-consistent predictions of the path of forward-looking variables and have to form their own beliefs on the basis of the information they observe. As in Marcet and Sargent (1989) and Evans and Honapohja (2001), agents gradualy learn the "true" parameters of the model by apdating their expectations with the use of a certain learning algorithm. In this section I present a general description of a Kalman filter learning setup. For more details on the implementation of this learning algorithm see Slobodyan and Wouters (2010).

The model described in Section 2 can be represented in the following structural form:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+1} + B_0 \epsilon_t = 0, \tag{28}$$

where the vector y_t includes endogenous variables of the model and w_t is an exogenous process, which follows an AR(1) process:

$$w_t = \Gamma w_{t-1} + \Pi \epsilon_t. \tag{29}$$

The rational expectation solution (28) is given by the following expression:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu + T \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R\epsilon_t. \tag{30}$$

The vector y contains state variables y^s that appear with a lag, forward variables y^f that appear with a lead, and the so–called static variables.[4] T and R are time-invariant matrices, which are functions of the model structural parameters. Deviation from the rational equilibrium assumption implies that agents do not have sufficient information about the model parameters to form model-consistent expectations of $E_t y_{t+1}$. Instead agents formulate the so-called Perceived Law of Motion (PLM), which relates the values of the lead variables and endogenous model variables using a linear function:

$$y^f_{j,t} = \beta_{j,t-1} X_{j,t-1} + \widetilde{R}_{j,t-1} \epsilon_{j,t-1}. \tag{31}$$

[4] y^f and y^s could intersect.

The agents then use the linear model (31) for forecasting. Data matrix X_j includes a set of variables that are used to form predictions about forward-looking variable j. In particular, X_j may consist of all the state variables of the model. In my estimations, such a specification would correspond to the "all states" learning model. Simpler forms of forecasting equations may imply the presence of the subset of endogenous variables, for example only one or two lags of the corresponding forward-looking variable on the RHS (as in the "AR(2)" model). In addition, X_j may also incorporate a constant. The error term in (31) represents different linear combinations of true model errors with variance-covariance matrix Σ. In all learning specifications considered in this paper, I assume that agents do not access values of exogenous process parameters when forming the predictions.

Kalman filter learning consists of two steps:

1. Prediction. According to agents' perceptions, the coefficients β follow a vector autoregressive process:

$$\left(\beta_t - \overline{\beta}\right) = F \cdot \left(\beta_{t-1} - \overline{\beta}\right) + v_t, \tag{32}$$

where F is a diagonal matrix with $\rho \leq 1$ on the main diagonal and v_t are i.i.d. errors with variance-covariance matrix V . Parameter ρ measures the intensity of updates and therefore is referred to the learning "gain" parameter, which is estimated. Using (32), the forecast of the evolution of β can be obtained as follows: $\left(\beta_{t+1/t} - \overline{\beta}\right) = F \cdot \left(\beta_{t/t} - \overline{\beta}\right)$. The predicted estimate covariance is given as: $P_{t+1/t} = FP_{t/t}F' + V$.

2. Update. The Kalman filter is used to obtain the updated estimates of the vector of beliefs β and the error covariance matrix P:

$$\begin{aligned} \beta_{t/t} &= \beta_{t/t-1} + K_t \widetilde{z}_t, \\ P_{t/t} &= \left(I - K_t X_{t-1}\right) P_{t/t-1}, \end{aligned} \tag{33}$$

where the innovation or measurement residual $\widetilde{z}_t = y_t^f - \beta'_{t/t-1} X_{t-1}$, the innovation (or residual) covariance $S_t = \Sigma + X'_{t-1} P_{t/t-1} X_{t-1}$; and the optimal Kalman gain $K_t = P_{t/t-1} X_{t-1} S_t^{-1}$. Updating of the beliefs at any t depends on the data (best estimates of the variables at time $t-1$) and on the initial beliefs. Following the standard assumption in the learning literature, I assume that initial beliefs are consistent with the REE. Thus initial values in the Kalman filter for the vector of beliefs β and variance-covariance matrix are derived on the basis of correlations between the model variables implied by the rational expectations equilibrium. Specifically, $\beta_{1|0}$ is given by the projection of X on y:

$$\beta_{1|0} = E\left[X'X\right]^{-1} \cdot E\left[X'y\right].$$

Given $\beta_{1|0}$, the variance-covariance matrix is calculated as:

$$\Sigma = E\left[\left(y_t^f - X_{t-1}\beta_{1|0}\right)\left(y_t^f - X_{t-1}\beta_{1|0}\right)^T\right].$$

The variance-covariance matrix of shocks V and an initial guess for $P_{1|0}$ are taken to be proportional to the generalized least square estimate of the variance of β : $V = \sigma\left(X'\Sigma^{-1}X\right)^{-1}$and $P_{1|0} = \gamma\left(X'\Sigma^{-1}X\right)^{-1}$, where scaling parameters σ and γ can be calibrated or estimated.

The beliefs generated in the Kalman filter step are then used to generate expectations of forward–looking variables according to forecasting equation (31). Substituting these expectations into the structural representation of the model (28), we obtain the Actual Law of Motion (ALM) of the system in a purely backward–looking form:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t\epsilon_t. \tag{34}$$

Introducing adaptive learning does not affect the initial steady state of the system, i.e. at time $t = 0$ we start from the RE equilibrium solution given by equation (30). Following the shock, agents start the learning process iterating over (33). Therefore, the simulation of the system's dynamics under adaptive learning reduces to calculating a time–varying transmission mechanism determined by μ_t, T_t and R_t, which are the functions of the model structural parameters given by matrices A_0, A_1, A_2 and B_0 as well as beliefs β and $\widetilde{R}$. The intensity of time-variation determines to which extent the equilibrium path under learning differs from the RE equlibrium. The values of μ_t, T_t and R_t are then used to form expectations of the next period model variables in the main Kalman filter step and are used to calculate the model likelihood. The time-varying procedure makes T_t a complicated function of the data, current parameters, and beliefs that could easily become unstable for one or several periods. Such discontinuities in the evolution of beliefs lead to numerical problems during estimation and the deterioration of estimation results. In particular, allowing T_t to be explosive for some periods leads to an increase in forecasting errors and thus to a much worse likelihood. In this paper, I have to deal with explosive dynamics of T_t for some time periods when estimating the DSGE model under non-MSV learning (when agents use a limited information set to form predictions). This problem seems to be more important for estimations of a model with a financial accelerator. In particular, financial frictions introduce additional volatility, which in turn may lead to more frequent and sizable adjustments of beliefs. Thus the probability of the eigenvalues of T_t to jump outside of the unit circle also increases. However, in all the estimations I performed, the number of periods with unstable eigenvalues does not exceed 5. As is common in the learning literature, I use a projection facility that skips updating in such cases.

3 Estimation strategy and results

I estimate several model specifications. In particular, I estimate versions with and without financial frictions in order to assess the empirical validity of the financial accelerator mechanism. In addition, I estimate each model under the assumption of RE and with learning. Thus I have two dimensions of comparison – the effect of financial frictions and the impact of expectations. Moreover, when assessing the effects of the financial accelerator under learning I experimented with alternative adaptive learning schemes, which differ in terms of the variables used by agents to form forecasts. The log-linearized versions of the models are estimated using Bayesian methods. These methods combine a likelihood function of the data with a prior density to derive the posterior distribution of the structural parameters. The prior density contains information about the model parameters from other sources (microeconometric and calibration evidence). The estimation procedure included: first, the estimation of the mode of the posterior distribution by maximizing the log posterior function and second, the Metropolis–Hastings algorithm was used to compute the posterior distribution and to evaluate the marginal likelihood of the model. Typically, from 300,000 to 500,000 MCMC draws were performed, using three chains. For more details on Bayesian estimation of DGSE models, see An and Schorfheide (2007). In order to speed up the convergence, I employed the so-called Adaptive Metropolis–Hastings algorithm. This method was proposed by Haario, Saksman and Tamminen (2001). They show that in some cases, the performance of the Adaptive MH is significantly better than the standard random-walk Metropolis Hastings when dealing with DSGE models, in the sense that it explores the posterior distribution more efficiently and accurately.

I choose priors following Smets and Wouters (2003, 2007). These papers present a careful description of the estimation methodology as well as the justification for the choice of priors. The priors for additional parameters related to the financial frictions are based on calibration exercises and previous literature (Bernanke et al., 1999; De Graeve, 2008). In particular, $\overline{R}^K \sim Normal(1.0149, 0.002)$. $\varkappa$, $\overline{k}/\overline{N}$, and el are assumed to have Uniform priors with sufficient standard deviations. The choice of the flat and disperse priors enables checking whether macroeconomic time series used in the estimation contain information about financial frictions parameters.

3.1 Data and measurement equations

The model is estimated using seven key macro economic quarterly U.S. time series as observable variables: real GDP, real consumption, real investment, real wage, hours worked, GDP deflator and the federal funds rate. Nominal variables are deflated by the GDP deflator. Aggregate variables are expressed in per capita terms. All variables except hours, inflation, and interest rate are taken in first differences. Thus the data set is the same as in Smets and Wouters (2003, 2007). Financial variables are not included in the set of observables in order to facilitate comparison with the results from the previous studies. Moreover, De Graeve (2008) points out that it is rather problematic to find a proxy for net worth or an external finance premium that would be consistent with the model dynamics. I estimate the model for the sample period 1954:1 - 2008:3. The long data sample is chosen in order to assess the importance of time variation in the model parameters introduced by adaptive learning. The sample ends in 2008 because the (linear) model framework is not designed to capture the impact of non-linearities, caused by the interest rate zero lower bound and unconventional policy measures implemented during that period. The estimated model is augmented with a set of the following measurement equations:

$$\begin{bmatrix} dlGdp_t \\ dlCons_t \\ dlInv_t \\ dlWage_t \\ lHours_t \\ dlP_t \\ FedFundsR_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma_y} \\ \overline{\gamma_c} \\ \overline{\gamma_i} \\ \overline{\gamma_w} \\ \bar{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} \widehat{y}_t - \widehat{y}_{t-1} \\ \widehat{c}_t - \widehat{c}_{t-1} \\ \widehat{i}_t - \widehat{i}_{t-1} \\ \widehat{w}_t - \widehat{w}_{t-1} \\ \widehat{l}_t \\ \widehat{\pi}_t \\ \widehat{R}_t^n \end{bmatrix}, \tag{35}$$

where l and dl stand for log and log difference, respectively. Unlike Smets and Wouters (2007), I estimate separately the trends for output, consumption, investment and wages growth rates, instead of imposing a common trend on these variables. $\overline{\pi} = 100(\Pi_* - 1)$ is the quarterly steady–state inflation rate and $\overline{r} = 100(\overline{\gamma}^{\sigma_c}\Pi_*/\beta - 1)$ is the steady–state nominal interest rate. Given the estimates of the average trend growth rate and the steady–state inflation rate, the latter will be determined by the estimated discount rate. Finally, $\bar{l}$ is steady–state hours worked.

3.2 Bayesian estimation under adaptive learning

Adaptive learning is implemented within the Dynare 3.064 Matlab toolbox which is used to estimate and simulate DSGE models. I use the toolbox developed by Slobodyan and Wouters (2009, 2010). Agents learn the model parameters using the Kalman filter algorithm. The alternative, widely used, learning method is the constant gain Recursive Least Squares (RLS). Sargent and Williams (2005) demonstrate that both learning methods mentioned above are asymptotically equivalent on average. However, their transitory behavior may differ significantly. In particular, the Kalman filter tends to result in a much faster adjustment of agents' beliefs. Therefore, I opt for the Kalman filter and estimate several adaptive learning specifications, which differ in terms of the information sets used by agents to form their beliefs about the forward-looking variables:

- "AR(2)+constant": the forecasting equation for every forward-looking variable includes two lags and a constant. Thus, agents form and update their beliefs about the persistence and expected mean of endogenous variables.
- "AR(2)": the forecasting equation for every forward-looking variable includes only two own lags (without a constant).
- "All states": the forecasting equation for every forward-looking variable includes all the state variables. Therefore, functional form of the relationship between forward and state variables is very similar to MSV Rational Expectation Equilibrium (REE) reduced form.

3.3 Estimation results

3.3.1 Model fit

The fit of a model estimated using Bayesian methods can be ascertained using marginal data density, defined as

$$p\left(Y|\mathcal{M}\right)=\int\mathcal{L}\left(\theta|Y\right)p(\theta)d\theta,$$

where $\mathcal{L}\left(\theta|Y\right)$ is the likelihood function of the data Y given parameters of the model θ, and $p(\theta)$ is the prior density. This measure allows a straightforward comparison of several models estimated on the same data with respect to a reference model. The posterior odds ratio, a measure of how much more likely a model $\mathcal{M}_1$ is when compared to model $\mathcal{M}_2$, is given by

$$\frac{\pi\left(\mathcal{M}_1\right)}{\pi\left(\mathcal{M}_2\right)}\cdot\frac{p\left(Y|\mathcal{M}_1\right)}{p\left(Y|\mathcal{M}_2\right)},$$

where $\pi\left(\mathcal{M}_i\right)$ represents the prior probability of model $\mathcal{M}_i$. The first term in the above expression is known as prior odds and the second as the Bayes factor. Usually, the prior probabilities are taken to be equal, and thus a posterior odds ratio equals the corresponding Bayes factor. For more details on model comparison, consult An and Schorfheide (2007).

Table 1 reports the logarithms of marginal data densities for the various specifications I have estimated. I compare the results for models with RE and AL. In addition, each version was estimated with the financial accelerator mechanism (FA) and without ($noFA$).

Table 1: Model Comparison in Terms of Marginal Likelihood

Model specification	*FA*	*noFA*
REE	-1207.8	-1231.52
Kalman Filter AL :		
AR(2)+constant	-1198.39	-1209.8
AR(2)	-1201.44	-1212.46
All states (near MSV)	-1206.11	-1234.55

Notes: Log marginal data densities for the models with and without financial accelerator. Learning specifications vary depending on the information set used to form predictions; initial beliefs are REE-consistent. Bayes factor — a relative probability of one model over another equals exp of the difference between the corresponding log densities.

The estimation results suggest that the REE model with a financial accelerator fits the data much better compared to the version that does not incorporate financial frictions (similar to the findings of De Graeve, 2008 and Christensen and Dib, 2008). The ability of AL to improve the data fit can be clearly observed for "$noFA$" specifications. In particular, Table 1 indicates that the RE hypothesis in this case is definitely restrictive. Relaxing the rationality assumption through the introduction of a Kalman filter AR(2) learning significantly improves the fit of the model. Such a result can imply that additional volatility and time variation introduced by adaptive learning can correct for some of the model misspecification. The performance of the adaptive learning model based on more complicated forecasting functions (i.e. "all states") is essentially the same as the RE model. Table 1 also demonstrates that the improvement in the data fit under learning is lower once financial frictions are introduced. In order to shed more light on this result, I analyze the relative likelihood (evaluated at the posterior mode) of alternative model specifications as a function of time. I would like to find out how introducing a financial accelerator mechanism changes the relative performance of RE and AL models over time, i.e. for which time periods the "best" learning model outperforms the RE version with financial accelerator and why. Firstly, I will illustrate graphically how the departure from the RE hypothesis affects the data fit in models with and without a financial accelerator. Figure 1 shows the cumulative likelihood for "AR(2)+constant" learning models with and without a financial accelerator *relative* to the corresponding RE models.[5] The upward

[5] I compute the difference in likelihood for AL and RE models, and plot the cumulative sum of this difference.

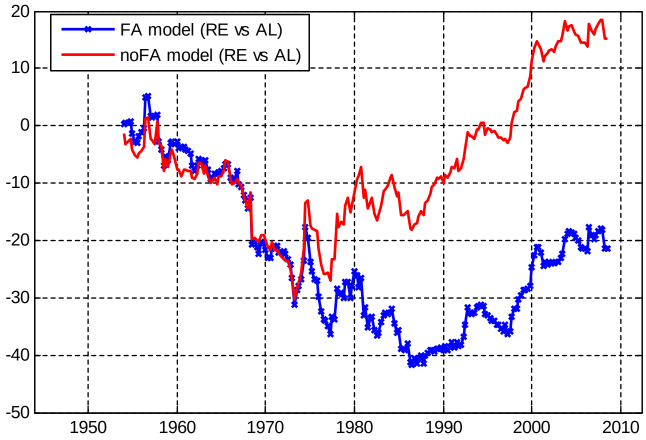

Figure 1: Cumulative likelihood for the "AR(2)+const." learning specification with and without a financial accelerator relative to the corresponding RE model.

trend of the cumulative difference line indicates that on average the likelihood of the learning model on this time interval is better relative to the RE one.

Figure 1 indicates that the RE and learning models estimated with or without a financial accelerator delivered very similar relative data fit before the beginning of the 1970s. In particular, on average the RE model fits the data better than the learning model. At the same time, the model with adaptive expectations and frictionless financial markets significantly outperformed the corresponding RE model in 1973-1974, the late 1970s, and the beginning of the 1980s. The aptitude of the learning model to describe the data generating process has been improving gradually but surely since the late 1980s. The graphs presented in Figure 1 also show that the relative performance of the RE and learning models in "FA" and "noFA" specifications differ the most during the middle 1970s and 1980s. This period was characterized by two severe recessions accompanied by an increased volatility of inflation, consumption, investment growth and labor hours. In the version without a financial accelerator, the time variation introduced by AL enables capturing varying economic processes and thus contributes to explaining the increased instability. In other words, learning can partially substitute for the absence of financial market inefficiencies. The inclusion of financial frictions considerably improves the performance of the RE model especially during the middle 1970s and the beginning of the 1980s. Thus, the relative gain from modeling the time varying transmission is somewhat reduced. Figure 1 also demonstrates that learning introduced in a model with a financial accelerator steadily adds to improved data fit after the late 1980s. The overall gain is, however, more modest than for the "noFA" model.[6]

[6] In the next subsections we will compare the implied persistence of the main macroeconomic variables under assumptions RE and AL and will add to explaining the results presented above.

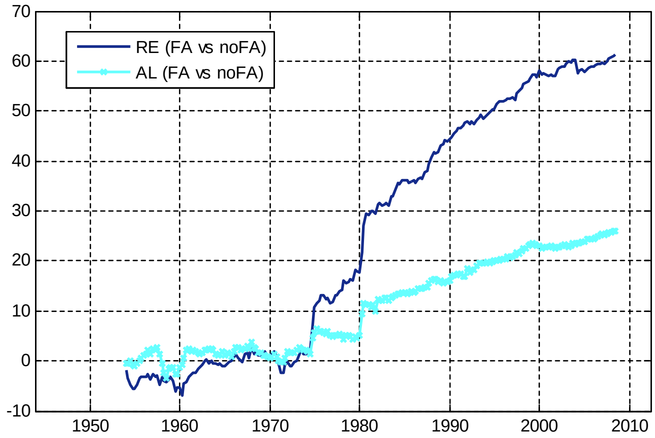

Figure 2: Cumulative likelihood for RE and AL models with "FA" relative to the corresponding "noFA" models.

In order to illustrate the contribution of the financial accelerator mechanism to the data fit under alternative expectation assumptions, I calculate the cumulative likelihood for the RE model with a financial accelerator relative to the RE version which does not incorporate financial frictions. The same cumulative difference in likelihood is computed for the model with the "AR(2)+const." adaptive learning scheme. Figure 2 compares the results. The upward trend of the likelihood differences indicates that the "FA" specification (with RE or AL) fits the data better than the "noFA" model. Figure 2 indicates that integrating financial frictions into the DSGE model with either rational or adaptive expectations improves the data fit. The highest gain in likelihood is observed in the middle 1970s and the beginning of the 1980s and then gradually increases. The overall gain is much greater for the model with rational expectations, whose performance appears to be more sensitive with respect to the inclusion of financial frictions.

Forecasting performance is an important criterion in model selection. In fact, a likelihood function can be interpreted as a measure of a one-period-ahead prediction error. Figure 2a compares the one-step-ahead forecasting performance of the RE and the best performing AL model. In particular, I contrast the forecasts and the actual dynamics of inflation, output growth and investment growth. Moreover, the graph contains the results up to the latest period, i.e. 2011q3. The analysis of the model performance on the data interval covering the recent financial crisis is of particular interest. However, this period is not included in the baseline estimation sample due to the fact that “crisis” observations produce too adverse model dynamics and significantly distort the estimation results. Therefore, in order to check the forecasting performance on the period starting from 2008 q3, I fix the parameters at the corresponding posterior mode values and run the forecast forward without a systematic update. Figure 2a illustrates that the AL model was more successful in predicting inflation in the middle 1970s and the beginning of the 1980s. Output and investment growth was better predicted by the AL model during the recessions at the beginning of the 1980s and 1990s. In addition, the learning model is particularly successful in capturing the most recent economic downturn, characterized by a significant and persistent drop in the real sector.

Therefore, the main results of this section can be summarized as follows. Introducing both adaptive learning and the financial accelerator mechanism into the otherwise standard DSGE model can contribute to capturing the properties of real data on certain time intervals, especially in the second half of the data sample.

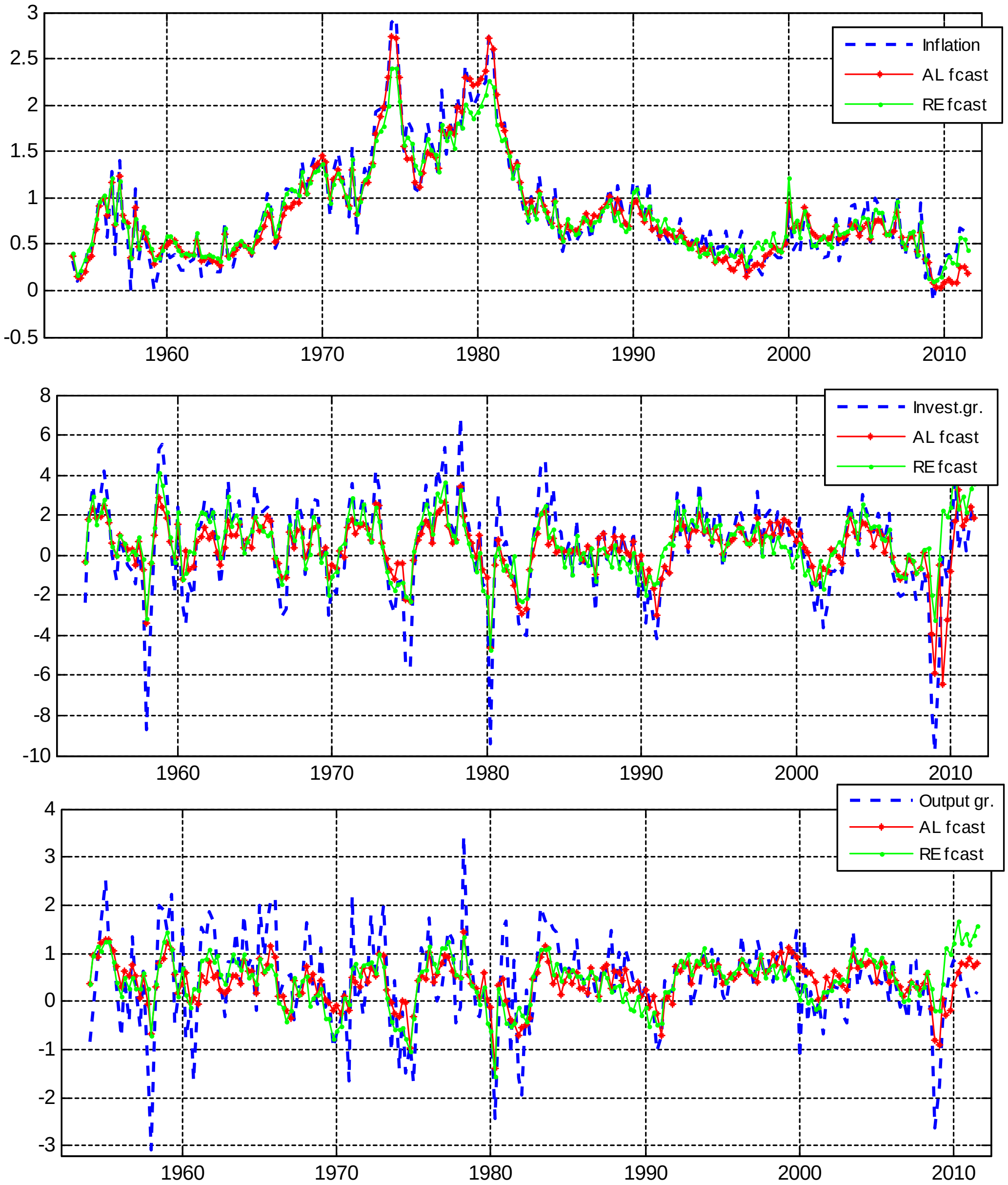

Figure 2a: One-step-ahead forecasting performance under RE and AL.

3.3.2 Parameter estimates

In this section I will present the MCMC results for the RE and "AR(2)+constant" learning models estimated with and without financial frictions. I will also report the results from the posterior maximization of all adaptive learning models and contrast them with RE. The results of the model comparison in terms of the estimated parameters are presented in Tables 2 and 3. Apart from the financial sector, the model is based on Smets and Wouters (2007) and Slobodyan and Wouters (2010). Therefore, in a version without a financial accelerator, I can observe a very similar pattern in deviations of the parameters estimated under "AR(2)+const" learning from the parameters obtained under RE. In particular, I find that some of the estimated structural rigidities and shock persistence decrease. More specifically, autoregressive components of exogenous processes for price and wage mark up shocks fall significantly. The decline in the persistence of the shock to investment technology is not so dramatic, but still notable. The confidence bounds for this parameter clearly shift left and do not overlap with the range of possible values implied by the posterior distribution of the RE model. In addition, the variance of the investment shock declines under learning. These results imply that learning is helpful in explaining inflation, wage and investment dynamics. Modeling adaptive expectations of these variables introduces "endogenous" persistence, which has an empirically appealing economic interpretation. At the same time, the autoregressive coefficient of the exogenous equity premium shock, which can be viewed as a proxy for financial-market inefficiencies, tends to increase under learning. This may imply that learning cannot substitute for financial frictions. The parameters of structural rigidities do not show a consistent change. The degree of price rigidities, wage indexation and to some extent wage stickiness decline. For the AL model without a financial accelerator, a significant decline in the investment adjustment cost parameter is observed. The degree of habit formation is also estimated at a somewhat lower level. At the same time, price indexation tends to increase under learning. We may conclude that learning is an important source of endogenous inertia, but it can only partially substitute for the "mechanical" source of rigidities and the persistence of some of the disturbances.

Analyzing the estimated parameters of the model that incorporate financial frictions, I would like to highlight several interesting details. Three parameters – capital to net worth ratio, entrepreneurial survival rate and the elasticity of the external finance premium are jointly responsible for the financial accelerator effects in the model. The higher value of these parameters strengthens the impact of financial frictions on the real economy. Comparing the results for RE and AL models presented in Table 2, I can see that the estimated capital-to-net worth ratio tends to increase under learning from 2.89 to 3.02 at the posterior mean (the confidence bounds, however, very much overlap). The confidence bounds of the elasticity of the risk premium also rise slightly under learning. Thus, it appears that there might be a tendency for these financial parameters to trend up under learning. The entrepreneurial survival rate stays essentially the same. The posterior distributions for the financial parameters have nice shapes (uni-modal). This suggests that the data is quite informative about the degree of financial market frictions. Our estimates of the elasticity (0.0186 in the posterior mean for the RE model and 0.0196 for AL) are somewhat lower compared to the regression and calibration results from the previous literature.[7] In particular, Bernanke et al. (1999) calibrates $el = 0.05$ based on a realistic value of monitoring costs and bankruptcy rates. Christensen and Dib (2008) estimate this parameter at 0.042. However, they calibrate the remaining financial parameters at a lower level. De Graeve (2008) reports a value of elasticity of 0.1047. At the same time, his estimated k/N ratio is twice lower compared with my results. Therefore, the estimated overall impact of financial frictions has comparable magnitude across different studies.

the results presented in Tables 2 and 3 demonstrate that introducing a financial accelerator influences some of the structural parameters. Comparing the regression results for RE models I notice that investment adjustment costs increase slightly in the specification with financial frictions; the autoregressive parameter of the exogenous equity premium shock declines. The latter fact may suggest that introducing a financial accelerator captures some of the persistence in the fluctuations of the external risk premium. However, the variability of the exogenous premium shock is not falling, which implies that the type of financial friction considered in this paper cannot fully explain the equity premium dynamics. In AL specifications, introducing a financial accelerator leads to an even more pronounced increase in the investment adjustment cost parameter (φ). The autoregressive component of the exogenous risk premium shock shows some decline compared to its value estimated in models without FA. I can explain the rise in k/N and φ as follows. Equation (11) demonstrates that the external finance premium can be decomposed into endogenous and exogenous components: $(E_t\widehat{R}^K_{t+1} - \widehat{R}_t) = -el \left\{ E_t \left[\widehat{N}_{t+1} - \widehat{Q}^k_t - \widehat{k}_{t+1} \right] \right\} + \widehat{b}_t$. Modeling adaptive expectations of asset prices, rental rate and inflation (all are forward-looking variables) lead to more persistent evolution of $\widehat{R}^K$ (see equation 18) and $\widehat{R}$ and thus of the risk premium. At the same time, in this very simple form of financial frictions, the endogenous component of the risk premium does not incorporate a sufficient persistence mechanism related either to expectations formation or other financial- market features. Agents do not form predictions about the net worth or capital (both are state variables). Stronger (on impact) and more persistent response of these variables can be achieved only via an increase of "mechanical" factors: adjustment costs (for capital) and k/N (for net

[7] In fact, we can compare only with estimation results for models with rational expectations.

worth). It would be interesting to check how adaptive learning affects the parameters in the model with more elaborated types of financial frictions.

Table 2: Comparison of RE and AR(2) learning models in terms of the estimated parameters

Parameters		Prior distribution			Posterior, RE model			Posterior, AL model		
		Type	Mean	St.err	5%	Mean	95%	5%	Mean	95%
Shocks										
exo.risk prem.	σ_b	I.Gam	0.1	2	2.3479	2.9965	3.5874	1.4758	2.0498	2.7821
investment	σ_q	I.Gam	0.1	2	0.4536	0.5396	0.6221	0.3752	0.4309	0.4972
price markup	σ_p	I.Gam	0.1	2	0.1344	0.1598	0.1844	0.1654	0.1849	0.2054
wage markup	σ_w	I.Gam	0.1	2	0.1962	0.2245	0.2547	0.2042	0.2269	0.2476
AR.coeff-s										
exo.risk prem.	ρ_b	Beta	0.5	0.2	0.0288	0.1253	0.2174	0.2175	0.3243	0.4441
investment	ρ_q	Beta	0.5	0.2	0.5812	0.6709	0.7615	0.3003	0.4030	0.5272
price markup	ρ_p	Beta	0.5	0.2	0.8372	0.9002	0.9634	0.0357	0.2018	0.3522
wage markup	ρ_w	Beta	0.5	0.2	0.8985	0.9315	0.9699	0.2964	0.5279	0.7721
pr.markup,ma	μ_p	Beta	0.5	0.2	0.6458	0.7752	0.9020	0.2763	0.4260	0.5637
w.markup,ma	μ_w	Beta	0.5	0.2	0.7944	0.8566	0.9303	0.1551	0.4146	0.6833
Str.params										
adjust. cost	φ	Norm	4	1.5	4.7261	6.3475	7.9322	4.3979	5.9552	7.4496
habit	η	Beta	0.7	0.1	0.6258	0.6873	0.7510	0.6850	0.7335	0.7967
Calvo wages	ξ_w	Beta	0.5	0.1	0.6737	0.7633	0.8500	0.6974	0.7500	0.8043
Calvo prices	ξ_p	Beta	0.5	0.1	0.6255	0.6914	0.7579	0.5259	0.5904	0.6455
index. wages	ι_w	Beta	0.5	0.15	0.2534	0.4582	0.6378	0.0921	0.1914	0.2912
index. prices	ι_p	Beta	0.5	0.15	0.0772	0.1933	0.3052	0.2382	0.4139	0.5849
int.rate smooth	ρ_r	Beta	0.75	0.1	0.8352	0.8603	0.8859	0.8982	0.9193	0.9392
pol rule, inflat.	r_π	Norm	1.5	0.25	1.6977	1.9354	2.1733	1.4630	1.7521	2.0222
cap./net worth	k/N	U	1	3.5	2.3413	2.8907	3.5000	2.4914	3.0231	3.5000
survival rate	$\varkappa$	Norm	0.9	1	0.9142	0.9451	0.9787	0.9039	0.9466	0.9672
elast.risk prem	el	Norm	0	0.5	0.0031	0.0186	0.0318	0.0039	0.0196	0.0351
gain (AL)	g	Beta	0.5	0.289	-	-	-	0.9751	0.9842	0.9931

4 Financial frictions under learning. Time variation and transmission mechanism

4.1 Evolution of agents' beliefs and implied persistence

Adaptive learning can affect model fit in several ways. First, the time variation of beliefs allows the model to become time varying 34. This could improve the model fit if the process that generates the time series of the observed variables is itself time-varying. On the other hand, if the beliefs-updating process is too volatile, parameter uncertainty could

Table 3: Comparison of RE and alternative learning models in terms of the estimated parameters (Posterior Mode)

Parameters / Poster.Mode		RE		AR(2)+const		AR(2)		All states	
		FA	noFA	FA	noFA	FA	noFA	FA	noFA
St.er shocks									
exo.risk prem.	σ_b	2.9133	2.6325	1.7918	1.3622	1.8087	1.6461	3.7498	2.4496
investment	σ_q	0.5283	0.4761	0.3959	0.4452	0.4041	0.4525	0.9005	0.911
price markup	σ_p	0.1624	0.1642	0.1852	0.1926	0.188	0.1895	0.1347	0.1198
wage markup	σ_w	0.2255	0.2279	0.2277	0.2238	0.2279	0.2233	0.2564	0.2631
AR. coeff-s									
exo.risk prem.	ρ_b	0.1091	0.2197	0.3881	0.4515	0.3897	0.4293	0.2318	0.368
investment	ρ_q	0.6753	0.6887	0.4265	0.4303	0.4192	0.3924	0.4233	0.4633
price markup	ρ_p	0.9006	0.8783	0.3206	0.1698	0.1628	0.1637	0.1518	0.1514
wage markup	ρ_w	0.944	0.9681	0.5702	0.5357	0.553	0.7273	0.8625	0.7815
pr.markup,ma	μ_p	0.7881	0.757	0.5020	0.3886	0.4123	0.3919	0.4447	0.4937
w.markup,ma	μ_w	0.8896	0.9234	0.4520	0.4265	0.4334	0.594	0.7772	0.6359
Str. params									
adjust. cost	φ	6.0523	5.2727	5.9774	3.299	5.7383	3.8392	5.6242	4.4685
habit	η	0.6765	0.7477	0.7194	0.6631	0.7201	0.6895	0.8347	0.7855
Calvo wages	ξ_w	0.7807	0.7825	0.7329	0.7498	0.7375	0.7373	0.8262	0.8452
Calvo prices	ξ_p	0.6947	0.6822	0.5756	0.5603	0.575	0.5546	0.7295	0.7041
index. wages	ι_w	0.4472	0.424	0.2020	0.2015	0.1933	0.2014	0.3695	0.4453
index. prices	ι_p	0.1822	0.1727	0.4184	0.4316	0.4501	0.4529	0.5504	0.5057
int.rate smooth	ρ_r	0.8580	0.8603	0.9209	0.9216	0.9198	0.9241	0.8814	0.8814
pol rule, inflat.	r_π	1.9251	1.8771	1.6692	1.7382	1.6609	1.6409	1.6396	1.6545
cap./net worth	k/N	3.4950	-	3.4525	-	3.499	-	3.50	-
survival rate	$\varkappa$	0.937	-	0.9447	-	0.9431	-	0.90	-
elast.risk prem	el	0.013	-	0.0197	-	0.0187	-	0.0047	-
gain (AL)	g	-	-	0.9881	0.9868	0.9872	0.9917	0.8458	0.8896

lead to the deterioration of the fit. Another channel through which adaptive learning operates is through a change in the transmission mechanism. Even when beliefs are consistent with a REE and are not time-varying, the changes in the information set used by the agents to form the expectations (which will introduce differences from the MSV set) will lead to a divergence of the transmission mechanism from that under RE. From the estimation results I can see that both factors mentioned above matter. Some of the parameters in adaptive learning models differ from those obtained under RE. Moreover, there is significant divergence in the parameters estimated under alternative learning schemes (both versions of AR(2) and "all states"). In particular, the elasticity of the external finance premium parameter in the "all states" model is several times lower than in the "AR(2)+const" model. The discrepancy is also observed across the structural parameters. As a result, financial-market frictions may have very different macroeconomic implications depending on the assumptions about the rationality of expectations and the information set.

Figure 3 plots the time variation of agents' beliefs given by the coefficients of the forecasting functions for the "AR(2)+constant" adaptive learning model. I present the evolution of the autoregressive component, given as a sum of AR(1) and AR(2) coefficients, and a constant. In other words I plot agents' Perceived Law of Motion (PLM) given by 31.

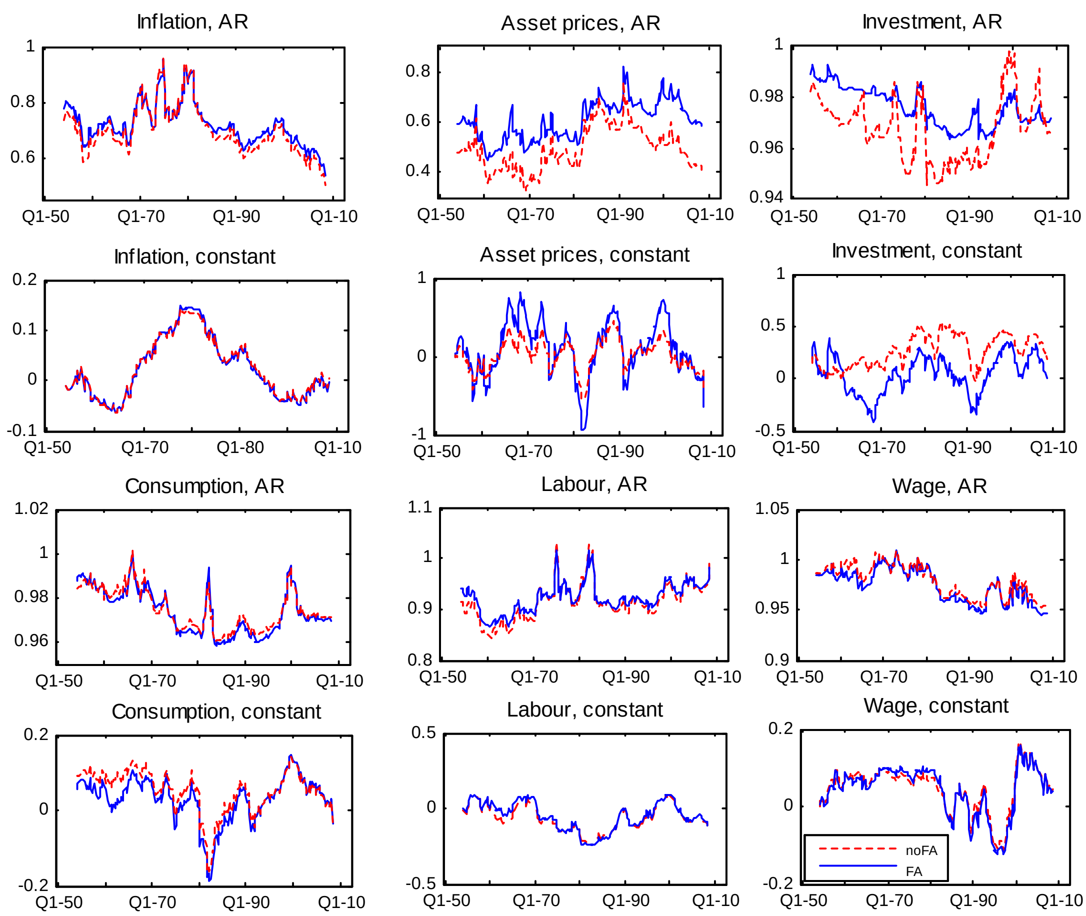

Figure 3: Beliefs about key macroeconomic variables: Persistence and constant

Figure 3 illustrates that agents perceive real consumption, labour, wages and investment as highly persistent processes with relatively stable autoregressive parameters. At the same time, Kalman filter learning introduces significant time variation in agents beliefs about inflation and asset prices. The perceived inflation persistence displayed peaks around the middle 1970s and again around 1980, then gradually declined to a level around 0.6 since the middle 1980s. The perceived asset price persistence evolved in the range 0.4-0.8, with noticeable spikes in the 1980s, 1990s and 2000-s. In the model with a financial accelerator, the perceived asset price persistence is higher compared with the persistence estimated in the version without financial frictions. The perceived inflation persistence is not significantly affected by the presence of financial frictions (it is just slightly higher after the 1980s). Agents also believe that imperfect financial markets make the investment process more persistent in the 1950s, 1970s and again in the 1980s-1990s compared to a frictionless economy. In addition to the autoregressive components, agents also revise their beliefs about the expected means of forward-looking variables (given by a constant in the forecasting equations). The presence of a constant brings additional changes in the transmission mechanism compared to the one implied by the RE model, where all the real variables are assumed to have a common trend growth rate and inflation is centered around a fixed inflation objective. Slobodyan and Wouters (2010) interpret the variations of the constant as deviations of agents' expectations from these steady-state values. Figure 3 illustrates that constants vary the most for asset price and investment and reflect rather a cyclical pattern of change in these variables. The fluctuations of the constant for the expected asset price and investment rate are more pronounced in the model with a financial accelerator. Significant shifts in the expected means can add to the macroeconomic volatility and contribute to over-optimistic or -pessimistic developments in agents' expectations. Financial frictions do not have important implications for the perceived persistence of other variables – real consumption, wage and labour – making our results in this respect very similar to Slobodyan and Wouters (2010).

In order to demonstrate more explicitly the joint impact of the financial frictions and adaptive expectations on the transmission mechanism, I compute the persistence implied by the Actual Law of Motion, given by 34. The results are presented in Figure 4. The horizontal solid line shows the persistence implied by the RE model with a financial accelerator, whereas the horizontal dotted line depicts the corresponding value for the model without financial frictions. Thus I compare the implied persistence for RE and "AR(2)+constant" learning models with and without a financial accelerator. Figure 4 demonstrates that the implied inflation persistence in a model with learning and a financial accelerator was higher in the 1950s-1960s, and after the 1980s compared to the analogous model without financial frictions. Introducing a financial accelerator resulted in higher implied asset price persistence during the whole time span. Implied investment growth persistence was also generally higher in a model with financial frictions and differentiated the most from the "noFA" level in the 1970s and after the 1990s. A similar pattern is observed for implied output growth persistence. The dynamics presented in Figure 4 can add to explaining the results of Table 1 and Figures 1 and 2. Specifically, I notice that, on average, the difference in persistence between the RE and learning models with a financial accelerator is lower than the corresponding difference that arises when financial frictions are shut off. In other words, it appears that the RE model with financial frictions does better in capturing the "true" data-generating process and delivers a level of persistence that is closer to the average agents' perceptions about the economy. Indeed, the implied inflation persistence under RE was very close to (time-varying) implied persistence under learning from the middle of the 1980s to the beginning of the 1990s and 2000. The same is true for investment and output. Thus, the gain from modeling the time-varying transmission mechanism declines on some time intervals in the second half of the sample. This explains why the improvement in the data fit under learning relative to the RE model declined compared to the "noFA" specification. Finally, Figure 5 compares the

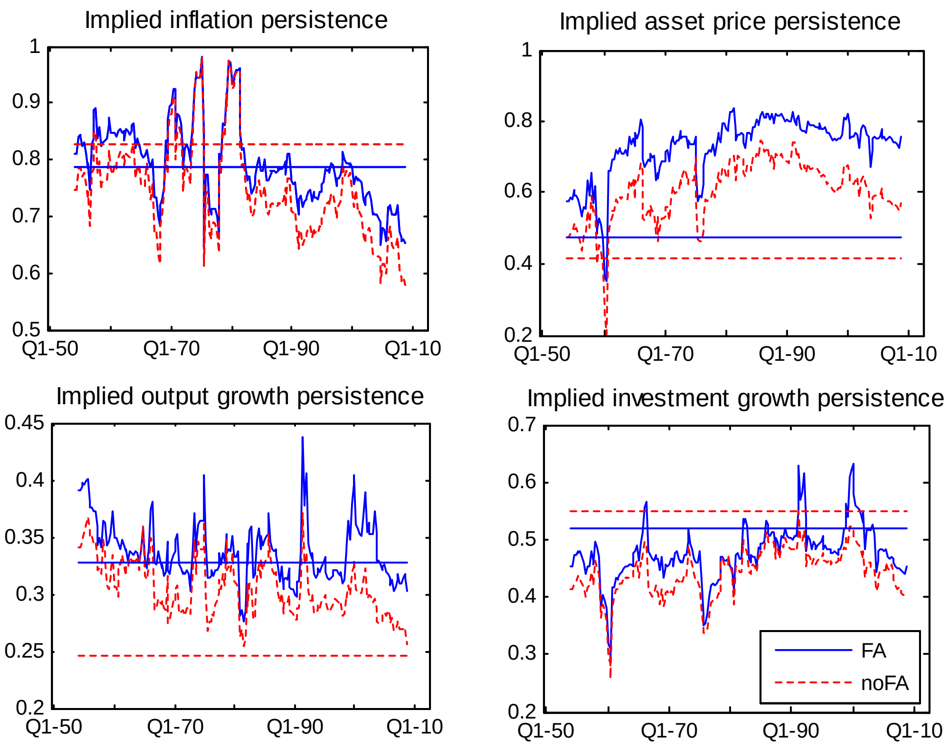

Figure 4: Implied persistence under RE and learning

implied persistence for alternative adaptive learning schemes, which differ in terms of the variables on the RHS of the forecasting equations: "AR(2)+constant", "AR(2)" and "all states" (near MSV). Figure 5 demonstrates that the information set used by agents to form the forecasts has important implications for the implied persistence and thus for the transmission mechanism and the business cycle. In particular, forecasts based on small learning models lead to higher implied persistence. The major difference is observed for the persistence of asset price and investment, the variables which play a crucial role in generating financial accelerator effects. In particular, the forecasting model that incorporates all the state variables on the RHS implies a very low persistence of asset prices and thus would fail to generate significant real effects following financial shocks. The results presented in Figure 5 also indicate that introducing a constant into the forecasting function leads to a smoother transition of agents' beliefs and implied persistence. In a learning model without a constant, agents will associate any developments in observable variables with a change in their persistence, whereas in the "AR(2)+constant" model, some of the volatility may be attributed to a variation of the expected mean. As a result, the "AR(2)" model will generate greater swings in implied persistence and thus more volatile model dynamics. In the specification with a financial accelerator such extra volatility can make the problem with projection facilities (mentioned in the previous sections) more severe. Therefore, the learning model that incorporates a constant is also preferable from the computational point of view.

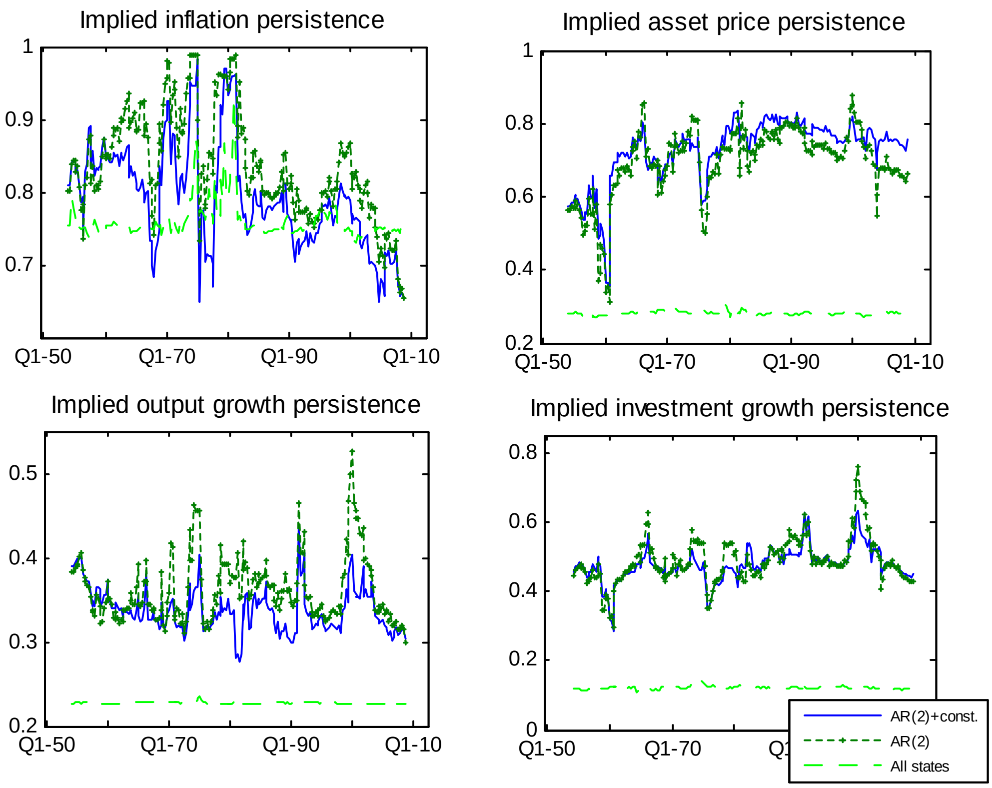

Figure 5: Implied persistence for alternative learning models with a financial accelerator

4.2 Financial accelerator under learning and the transmission mechanism

Implied persistence is an important determinant of the real effects of shocks hitting the economy. In particular, shocks to the inflation rate, which is perceived as a highly persistent process, will lead to stronger and long-lasting responses of inflation. For an inflation targeting central bank, such dynamics would imply a more aggressive monetary policy reaction, which would affect real output to a greater extent. In the financial accelerator framework, agents' perceptions about financial variables such as asset prices may have additional macroeconomic implications. If agents perceive asset prices to be more persistent, financial shocks will result in stronger and more gradual responses of this variable and, hence, a greater impact on households' financial position (net worth) and the external finance premium. Therefore, financial disturbances will entail higher cumulative effects on investment and output. The results presented in the previous subsection indicate that a learning model with "AR(2)+constant" beliefs may have significant implications for the shock transmission due to a higher implied persistence of asset prices, inflation (after the 1980s) as well as real variables relative to the model with RE and also compared to the version without financial frictions.

The previous literature has already provided some insights about the transmission mechanism in models with financial frictions. Christensen and Dib (2008) study the transmission of shocks in the estimated model with RE and a financial accelerator. Unlike Bernanke et al. (1999) their model incorporates a nominal debt contract, allowing for debt deflation effects. Christensen and Dib (2008) find that the financial accelerator mechanism considerably amplifies and propagates the impact of demand-side shocks – monetary policy, money demand and preference shock – on investment and the price of capital. The implications of financial frictions for inflation and output are found to be relatively minor. De Graeve (2008) reports similar effects of the financial accelerator. In particular, the investment response to a preference and monetary policy shocks is stronger relative to the model without financial frictions. In both studies, the financial accelerator mechanism dampens the rise of investment following positive technology and investment supply shocks. This contrasts sharply with the results in Bernanke et al. (1999) and Walentin (2005), in which favorable productivity shocks reduce the premium and therefore boost investment relative to a model without financial frictions. In addition, in the De Graeve (2008) model, the dynamics of investment following investment supply shocks somewhat differs from the results documented in Bernanke et al. (1999) and other existing studies (Walentin, 2005; Christensen and Dib, 2008). He explains the difference in responses by the form of adjustment costs.[8]

In this paper, I compare the implications of financial frictions for the transmission mechanism in the RE and an adaptive learning model based on "AR(2)+constant" forecasting functions. Figures 6, 7 and 8 show the impulse responses under the productivity, risk premium and monetary policy shocks, respectively. In fact, the figures present the time variation of impulse responses and thus reflect the time-varying transmission mechanism under learning. In particular, inflation responded much stronger to shocks around

[8]Bernanke et al. (1999) works with capital adjustment costs, whereas De Graeve (2008) assumes investment adjustment costs. This implies a more gradual and hump-shaped response of investment.

the 1970s, when the perceived inflation was very persistent. The dynamics of inflation is similar to the one documented in Slobodyan and Wouters (2010) because the financial accelerator did not significantly affect inflation persistence in the 1970s. The peaks of the responses of asset prices happen around the 1980s, 1990s and 2000. This corresponds to the dates when agents revised the perceived asset price persistence upwards (see Figure 3 that shows the evolution of beliefs). The very first impulse response (denoted by the thick line) corresponds to the reaction under RE. Figure 6 shows the response under a 1% positive technology shock. The immediate response of output, asset price and investment is lower relative to the model with RE, but becomes more persistent and sizable afterwards. The dynamics of financial variables, i.e. asset prices, net worth and risk premium, differ sharply from the responses under RE. The reduction of the external risk premium is very persistent and therefore explains the more gradual increase in investment and output. The responses of inflation, asset prices and investment the exhibit most volatile dynamics. The reaction of the external finance premium would display more of time variation if the estimated elasticity of the risk premium was higher. Figure 7 shows the responses

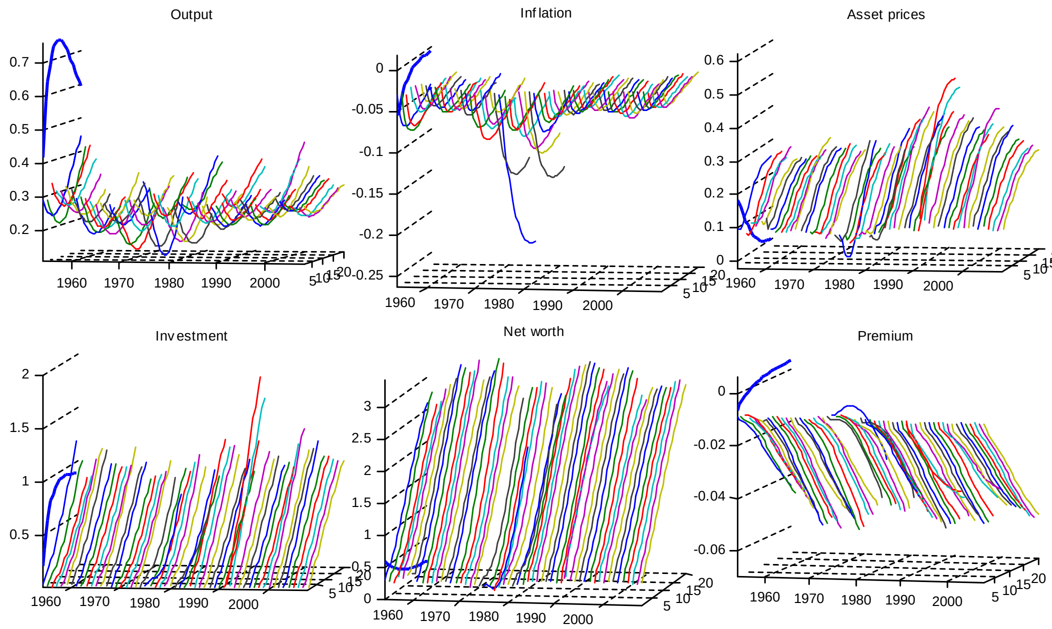

Figure 6: Impulse responses to a productivity shock

to a risk premium shock. The reaction of output, investment and financial variables in the adaptive learning model is stronger relative to the model with rational expectations. Specifically, a sharp fall in asset prices reduces net worth and thus raises the external finance premium, whose immediate reaction is stronger compared to the model with RE. Therefore, the responses of investment and output are also amplified. Figure 7 displays significant time variation in responses to a shock of both financial and real variables, thus demonstrating the implications of the departure from the complete rationality assumption. The peaks in the perceived asset price persistence observed in 1974, the 1980s and the 1990s leads to a dramatic fall in asset prices, which sharply reduces net worth. As a result, even under a relatively low estimated sensitivity of the premium to changes in the entrepreneurial financial health, the gap between the cost of external financing and the risk-free rate shows a significant increase and therefore leads to a stronger impact of the financial accelerator on the real economy. This example clearly illustrates the mutually reinforcing interaction between the financial accelerator and adaptive learning. Finally, I

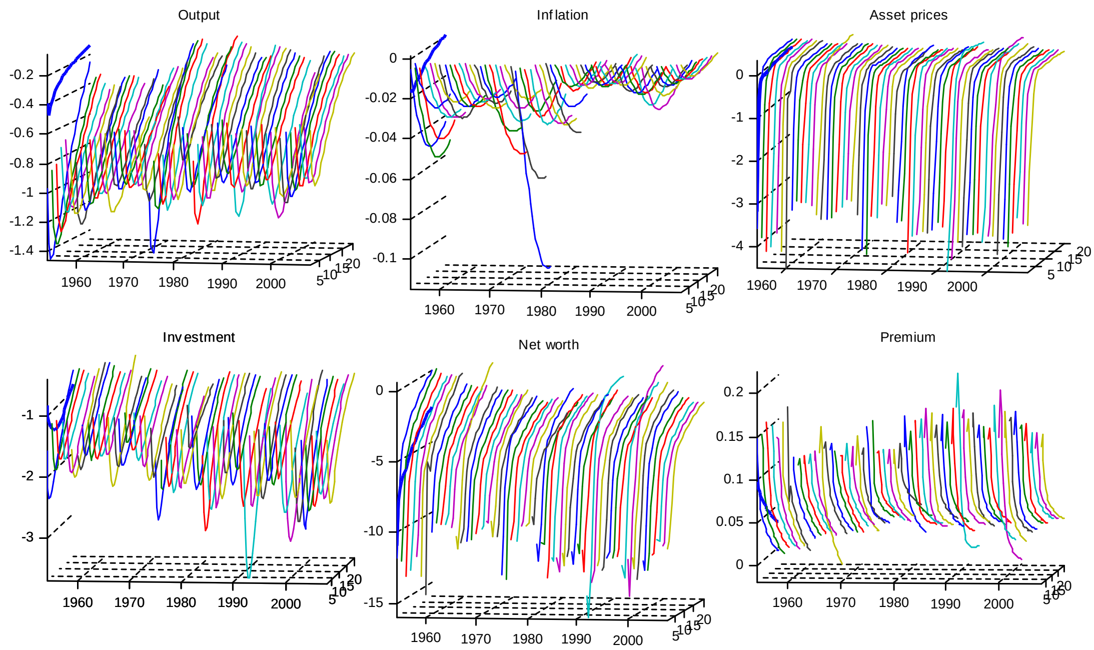

Figure 7: Impulse responses to a risk premium shock

investigate effects of the monetary policy shock, presented in Figure 8. Following monetary tightening, inflation, asset prices, investment and output decline. The immediate reaction of variables under learning is generally lower but much more persistent. At the peak, the responses are considerably amplified relative to the model where financial frictions interact with rational expectations. Again, responses show significant time variation reflecting the evolution of agents' beliefs about the macroeconomy.

4.3 Simulation exercises and sensitivity analysis

In the previous subsection I analyzed the implications of introducing financial frictions into the model with adaptive learning on the basis of the estimated values of the parameters. At the same time, the reported empirical values of the level of financial frictions vary across different studies and may depend on the estimation sample and modeling assumptions. Moreover, the inclusion of observations that cover the most recent financial

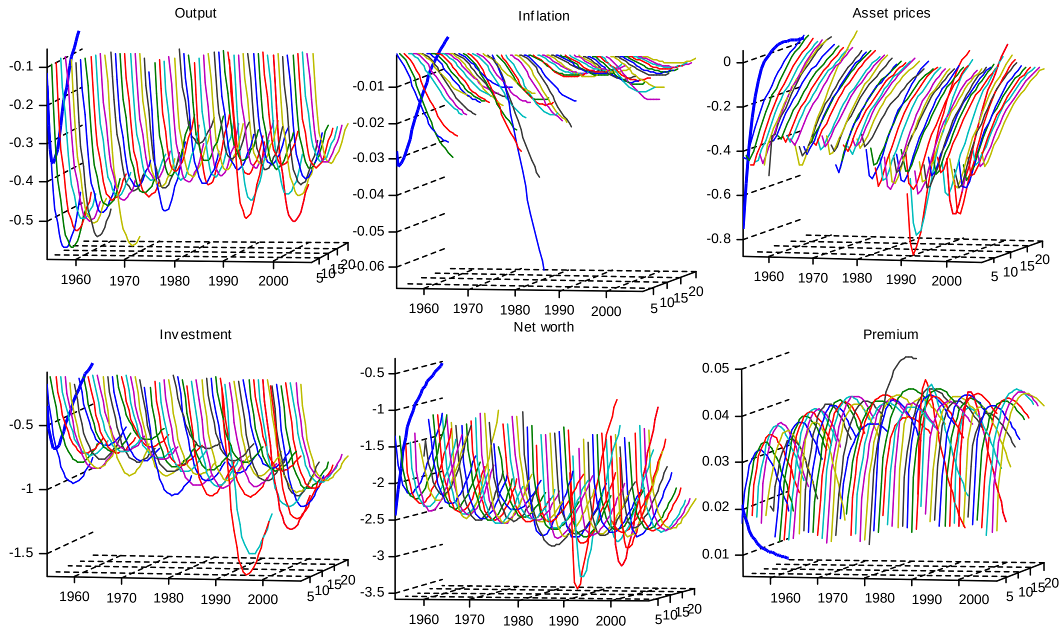

Figure 8: Impulse responses to a monetary policy shock

distress followed by adverse macroeconomic consequences would definitely imply higher estimated values of financial parameters (the observations of the last several years are not included in the sample). Therefore, in this part of the paper I conduct a sensitivity analysis and investigate the impact of the gradual increase of financial frictions on the macroeconomy. In order to perform this task, I fix the parameters for each model at the corresponding posterior mode value and simulate the models for 219 periods, which coincides with the duration of the estimation sample. Additionally, I assume $k/N = 2.5$ (somewhat lower than the estimated value) in order to avoid the problem with projection facilities, which may arise due to the excessive volatility of beliefs under high values of financial-friction parameters. I vary the elasticity of the risk premium: 0.02 (relatively low), 0.04 (average) and 0.06 (relatively high). In each case, I simulate agents' beliefs and calculate the implied (simulated) persistence of inflation, asset prices, growth of output and investment. In order to assess the potential ability of learning to amplify business cycle fluctuations, I compute the difference in the impulse responses to a risk premium shock between the rational expectations and adaptive learning models. I contrast the results for the two alternative learning models that differ in terms of the information sets used by agents to forecast forward-looking variables.

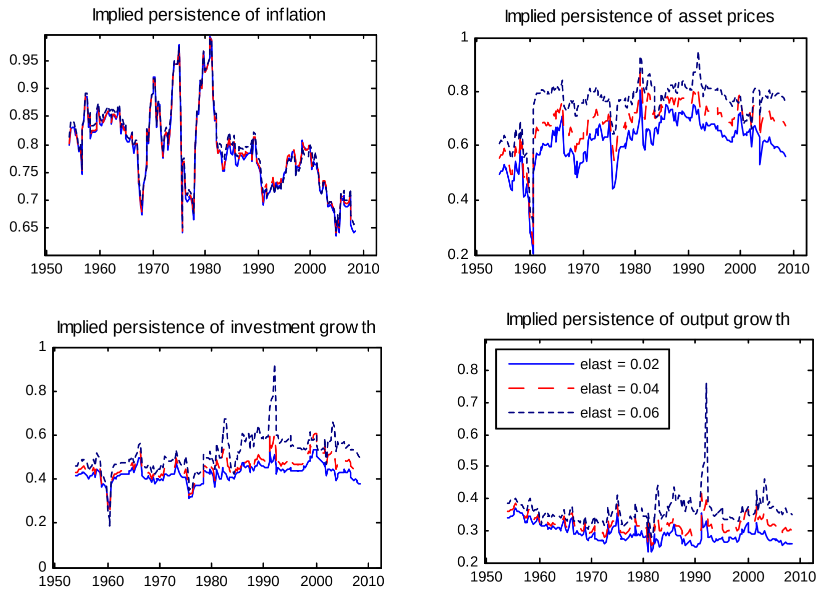

Figure 9: Implied persistence for alternative degrees of financial frictions in the "AR(2)+const." learning model

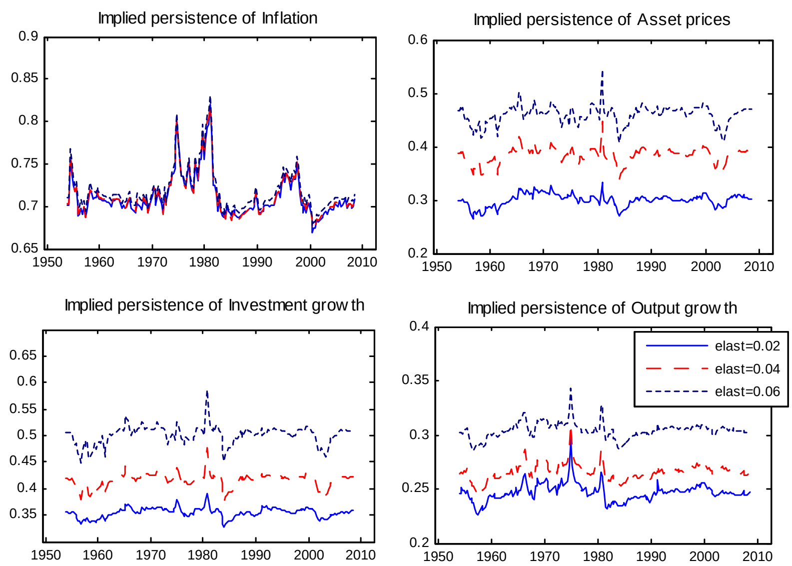

Figure 10: Implied persistence for alternative degrees of financial frictions in the "all states" learning model

Figures 9 and 10 show the implied persistence for the "AR(2)+const." and "all states" learning models for different degrees of financial frictions. The graphs illustrate that the implied inflation persistence is almost independent of the elasticity of the risk premium parameter. At the same time, for the AR(2)+const. learning model, higher financial frictions significantly affect the persistence of asset prices, investment and output growth.

In particular, a greater elasticity of the risk premium increases the level and/or time variation of the implied persistence of these variables, especially on the second half of the sample. The implied persistence simulated for the learning model in which agents employ all the state variables in forecasting functions is generally lower and more stable over time relative to the learning scheme based on small forecasting functions. Specifically, Figure 10 demonstrates that an increase in financial frictions does lead to a somewhat higher level but hardly affects the time variation of the implied persistence in the "all states" model. For impulse responses, such a result would imply lower variability and less pronounced business cycle fluctuations.

Figures 11 and 12 present the differences in the impulse responses to a risk premium shock between the rational expectations and two alternative learning models. The greater the deviation of the line from the zero level, the stronger the reaction of the economy under learning relative to the model with RE. For example, negative values of the differences in the responses of investment (Figure 11) mean that the fall of investment was more pronounced under learning. Figure 11 illustrates that the peak response of all the variables under learning was stronger relative to the responses under RE. The difference in the responses increases as financial frictions become stronger. For the highest value of the parameter elasticity (0.06), the increase of the external finance premium and the corresponding decline of investment is much more sizable under learning compared to the model with RE. Figure 12 demonstrates that the "all states" learning algorithm is rather unsuccessful in amplifying the business-cycle fluctuations even for the highest degree of financial frictions. In particular, under this learning scheme, the risk premium shock leads to a fall of investment that is lower (in absolute value) relative to the negative investment response under RE. Even assuming the highest value of the elasticity parameter does not reverse the results. Therefore, it appears that the type of learning model, and in particular the information set used by learning agents in forecasting, is a more fundamental factor in generating additional macroeconomic volatility than the degree of financial frictions as such.

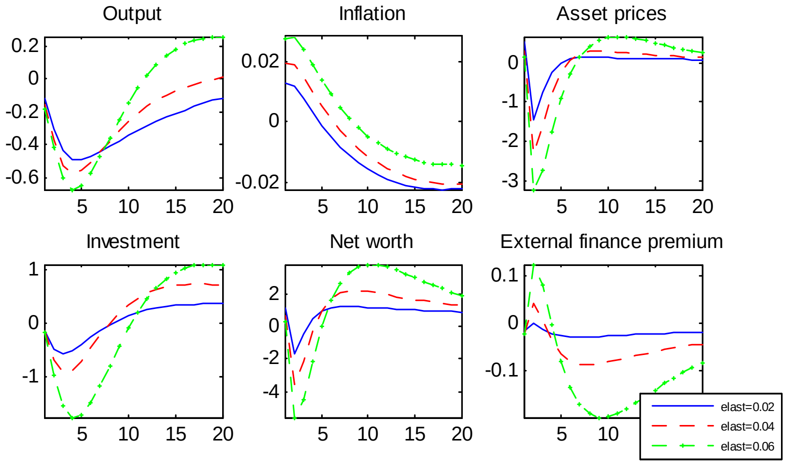

Figure 11: Difference in responses to a risk premium shock between the "AR(2)+const." learning and RE models for alternative degrees of financial frictions

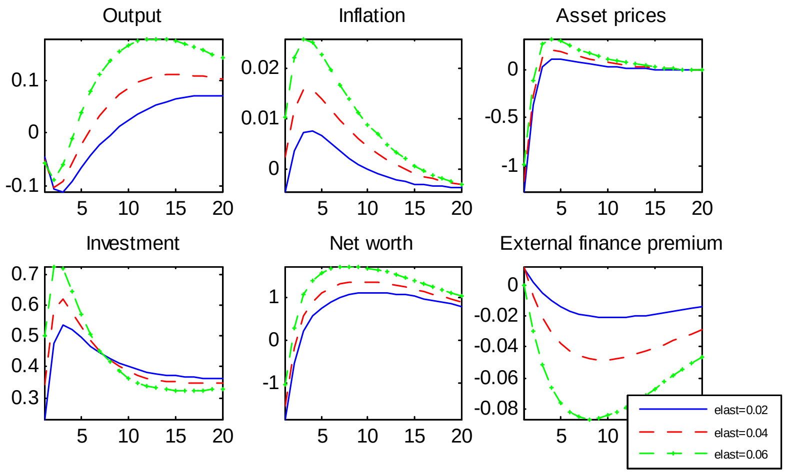

Figure 12: Difference in responses to a risk premium shock between the "all states" learning and RE models for alternative degrees of financial frictions

5 Conclusions and future research

In this paper, I compare the implications of a financial accelerator mechanism for the real economy in models with alternative assumptions about expectation formation. I perform a Bayesian estimation of a medium-scale DSGE model with financial frictions assuming, on the one hand, complete rationality of expectations and, alternatively, several forms of adaptive learning that differ in terms of the information set used by agents to form their predictions. I evaluate and compare the model fit, estimated parameters and the transmission mechanism. The estimation results suggest that both financial frictions and adaptively formed expectations based on very simple forecasting functions add to improved model fit, at least on certain time intervals.

I show that the implications of a financial accelerator for the business cycle may vary depending on the expectation assumptions (RE or forms of learning). The results suggest that a learning scheme based on small forecasting functions is able to amplify the effects of financial frictions relative to a model with RE. I show that the model dynamics under learning is driven to a significant extent by the time variation of agents' beliefs about the evolution of financial variables. Specifically, I demonstrate that perceived asset price persistence in a learning model with simple forecasting equations varies through the cycle and thus differs significantly from the levels implied by the RE and alternative learning schemes. During periods when agents perceive asset prices as being relatively more persistent, shocks that affect this variable lead to more pronounced macroeconomic outcomes. The asset price persistence appears to be particularly important for explaining the investment dynamics. This effect is clearly observed in impulse responses. In particular, increased asset price persistence implies a more pronounced (and persistent) response of investment under the risk premium or monetary policy shocks. Therefore, I argue that certain forms of AL may play a significant role in driving and amplifying macroeconomic fluctuations; it introduces an important time variation and strengthens the real effects of the financial accelerator compared to the assumption of RE. Simulation exercises illustrate that the amplification effect rises more than proportionally as financial frictions become more severe. At the same time, a learning specification in which agents use more information to generate predictions (close to MSV learning) produces very different asset price and investment dynamics. In such a framework, learning cannot significantly alter the real effects of financial frictions implied by the RE model.

The results of the paper allow drawing several conclusions relevant for DSGE modeling and policy analysis. In particular, due to the ability to amplify macroeconomic fluctuations, learning can be a suitable framework to simulate financial crisis scenarios and various policy reactions. Comparison of the data fit for alternative models suggests that AL with a financial accelerator represents the best specification to describe the data-generating process and analyze the shock transmission in the second half of the sample. In addition, the results imply that the link between asset prices and the real economy has become more important and the sensitivity of the economy to financial shocks increased after the middle 1980s. Such an empirical conclusion is supported by the impulse responses of real variables that show higher time variation in the 1990s and 2000-s following monetary and financial shocks. Which economic processes or policy reactions could contribute to the increased propagation of financial shocks is an important question for further research. There exists an opinion that a stable economic environment with low interest rates and inflation could be partly responsible for the adverse dynamics of asset prices and the development of a bubble leading to the crisis. Future research could complement this paper with an analysis of monetary policy in the economy with adaptive learning and financial frictions. One could study how strong an anti-inflationary stance or too-expansionary monetary policy can impact the implied persistence of asset prices as well as the variation of real variables and inflation. In addition, an experiment on policy rules that incorporate the response to asset prices is possible, which would examine whether such rules could deliver better macroeconomic outcomes in terms of monetary and financial stability.

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