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Yuliya Rychalovska
Sergey Slobodyan
Raf Wouters

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Yuliya Rychalovska* Sergey Slobodyan‡ Raf Wouters§

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Abstract

The use of survey information on inflation expectations as an observable in a DSGE model can substantially refine identification of the shocks that drive inflation. Optimal integration of the survey information improves the model forecast for inflation and for other macroeconomic variables. Models with expectations based on an Adaptive Learning setup can exploit survey information more efficiently than their Rational Expectations counterparts. The resulting time-variation in the perceived inflation target, in inflation persistence and in the sensitivity of inflation to various shocks provide a rich and consistent description of the joint dynamics of realized and expected inflation. Our framework produces a reasonable interpretation of the post-Covid inflation dynamics. Our learning model successfully identifies the more persistent nature of the recent inflation surge.

JEL Classification: C5, D84, E3

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*University of Namur, email: yuliya.rychalovska@unamur.be

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‡CERGE-EI, email: sergey.slobodyan@cerge-ei.cz

§National Bank of Belgium, email: rafael.wouters@nbb.be

1 Introduction

Given the central role of inflation expectations in economics and policy analysis, it is important to have a modeling framework that incorporates realistic dynamics of agents' inflation predictions that align with empirical data. Evidence on expectations enables development of a credible theory of expectation formation and serves as a useful data source. Despite the recent post-COVID episode, which was characterized by extraordinary inflation developments, expectations from the Survey of Professional Forecasters (SPF) are still regarded as the most informative inflation predictions, incorporating valuable and timely information. Central banks closely monitor inflation expectations observed in surveys or distilled from financial yields, and stress that it is absolutely necessary for inflation expectations to remain anchored around the long run inflation objective. The stability of inflation expectations is an important indicator of the central bank's credibility and its capacity to achieve its inflation target.

The objective of this paper is to model the joint dynamics of inflation survey expectations and realized macroeconomic data within a structural general equilibrium model. We propose a framework that allows us to effectively exploit the valuable content of surveys and to improve upon survey forecasts, particularly during periods marked by systematic prediction errors. We illustrate that the observation of inflation expectations from the Survey of Professional Forecasters improves the estimation outcomes of the standard New-Keynesian DSGE model and provides useful insights for explaining and predicting inflation dynamics, and for understanding the expectation formation mechanism.

To make the most effective use of the information embedded in inflation survey forecasts, we explore the idea that this data supports understanding of how agents perceive the impact of fundamental shocks on future economic conditions and price movements. In particular, the survey data on inflation expectations assist separate identification of the innovations in the persistent component of the inflation markup process. These innovations constitute only a small fraction of the high frequency volatility in inflation and are hard to distinguish from the noisy transitory component without the additional timely information present in survey expectations. By resolving this filtering problem, the model forecast for inflation tends to benefit from the useful content of survey expectations and the overall fit and forecasting performance of the model improve substantially. Further, models with expectations based on the Adaptive Learning (AL) setup can exploit the joint dynamics of survey forecasts and realized inflation more efficiently than their Rational Expectations counterparts. With appropriate specification of the forecasting (or belief) rules that incorporate the signals from survey evidence, agents update their beliefs about the roles of observed and latent signals for future inflation as function of systematic forecast errors. The resulting time-variation in the perceived inflation target, in inflation persistence, and in the sensitivity of inflation to various shocks provide a rich and consistent description of the dynamics in realized and expected inflation. In addition, we demonstrate that our framework with separately identified persistent mark-up shock produces a reasonable interpretation of post-Covid inflation dynamics. The AL model successfully identifies the persistent nature of the recent inflation surge.

1.1 Literature review

Inflation expectations have played a fundamental role in models of inflationary dynamics since the seminal work of Friedman (1968) and Lucas (1972). In the current generation of macro models, inflation expectations drive actual price and wage setting through the forward looking New Keynesian Phillips curve, which is the central equation in monetary DSGE models. Analyses of the role of inflation expectations during periods of persistently high inflation in the 1970s and 1980s has drawn significant attention. Clarida et al. (2000) illustrated that the lack of commitment and credibility could have led to insufficient anchoring of inflation expectations in the 1970s. Expectations that shift independently of economic fundamentals can contribute to undesirable macroeconomic effects and suboptimal policy outcomes. Chari, Christiano, and Eichenbaum (1998) and Albanesi, Chari, and Christiano (2003) explain persistent episodes of high or low inflation by "expectation traps" that arise due to the absence of commitment in monetary policy.

Recent literature has assigned a crucial role to inflation expectations in explaining the behavior of inflation during the Great Recession and the subsequent recovery period. When the nominal rate is constrained by a zero lower bound, inflation expectations have the capacity to influence aggregate demand via expected real returns and intertemporal substitution. Coibion and Gorodnichenko (2015) suggest that stability in expectations, in particular on the household side, led to the relatively stable inflation realisations during the Great Recession, as household inflation expectations were more responsive to increases in oil prices from 2009 to 2012. This allowed to avoid the onset of deflationary dynamics. Del Negro, Giannoni and Schorfheide (2015) argue that inflation expectations remained anchored during the Great Recession because monetary policy managed to maintain expectations of rising future marginal costs. Understanding the ways inflation expectations are formed and how they develop over time is crucial for evaluating the risk of convergence to a low/high inflation steady state. Survey expectations may contain signals relevant for interpreting how agents formulate their beliefs and which solution path they select.

Survey expectations on inflation are very informative. A comprehensive review of various inflation forecasting models and survey data by Ang, Bekaert and Wei (2007) and Faust and Wright (2013) have documented the superior forecasting performance of survey expectations for inflation. The high quality of survey forecasts likely results from a large amount of information that is processed in an efficient manner with sufficient flexibility to adjust over time. This evidence motivates the inclusion of data on inflation expectations into the standard datasets on which we estimate macromodels. In this way, the dynamics of inflation expectations are analysed together with realized inflation data to pin down the transmission mechanism of the various shocks more precisely. The superior information results in consistent estimates of the state of the economy and of expectations, inflation expectation process in particular. Thus, a growing body of literature has incorporated survey data into estimated models. Survey expectations have been used successfully as a proxy for inflation expectations in single-equation estimates of the NKPC: see, e.g. Roberts (1995), Adam & Padula (2011). In these analyses, survey data are treated as exogenous, eliminating the need to explain how these expectations are formed. Coibion et al.(2018) argue that surveys of inflation expectations improve NKPC estimates and contribute to improved inflation forecasting. Fuhrer (2017) examines inflation survey data from the SPF in the context of a semi-structural model and shows that incorporating surveys improves parameter identification and reduces excessive reliance on lagged dependent variables and correlated structural shocks.

In the literature integrating surveys into structural macromodels, the main discussions center on: understanding the information content revealed by surveys; devising strategies to integrate survey data and to reconcile it with model-based inflation forecasts to fully exploit its valuable information; and modeling appropriate expectation formation mechanisms.[1]

Eusepi and Del Negro (2011) introduce exogenous inflation target shocks in their RE model to improve the match between model and survey forecasts. Though it improves the model fit, the approach does allow them to explore the content of survey forecasts effectively. Similarly, De Graeve et al(2009) illustrate how an inflation target shock is necessary for matching inflation expectations in the yield structure. Del Negro and Schorfheide (2013) use long run inflation expectations as the observable and an inflation target shock as the modelling device. Eusepi et al (2015), on the other hand, use short and medium term survey expectations and learning about the long run inflation target to model inflation expectations.

The availability of survey data enables assessment of the validity of rational (model-consistent) expectations against alternatives that allow for more flexibility and provide more insight on how agents formulate their beliefs and how they adjust them when confronted with new data and changes in their environments. In particular, broad literature has used survey data to illustrate deviations in agents' expectations from the complete rationality assumption. [2]

Numerous studies have emphasized the success of the AL approach, viewed as one of the alternatives to the prevailing paradigm of Rational Expectations (RE), in replicating the patterns observed in survey data on expectations.[3] A

[1] There is broader literature that investigates what type of price setting models are consistent with the inflation expectations in survey data: i.e., models with sticky information or heterogeneous beliefs, such as Mankiw and Reis (2003) and Branch (2007). See also Coibion and Gorodnichenko (2015) for more evidence on information rigidities in survey data and their theoretical interpretation.

[2] Notable contributions include studies by Roberts (1997), Mankiw et al. (2004), and literature following Coibion and Gorodnichenko (2015).

[3] Earlier studies that consider a learning approach as an alternative expectation hypothesis include the work of Sargent (1999), who explored a rational versus adaptive expectations framework to interpret postwar US inflation. He argued that a model based on adaptive expectations better captures the key features of the FED policy-making and was more suc-

paper that is closer to ours is Ormeno and Molnar (2015). These authors use survey data on inflation expectations as an observable for the model expectations via an additional measurement equation. Their results illustrate that the survey contains information that is not present in macro data, and that this can improve the model forecast. They also show how an AL approach based on small forecasting models is more flexible in exploiting the information than a fully rational expectations model. The paper, however, does not explain the nature of information revealed by the survey and what adjustments to the model specification can optimize the integration of survey data.

Other recent papers have employed both learning and surveys to infer the most suitable learning mechanism and to endogenize long-term trends in expectations. Hommes et al (2023) illustrate that simple AR(1) forecasting rules provide the best fit to short-term survey data on inflation. Gati (2023) uses survey data on inflation expectations to discipline the degree of unanchoring of inflation expectations and to estimate the sensitivity of expectations to new information in a model with learning. Carvalho et al. (2023) formulate a model with learning dynamics and endogenous inflation drift, which evolves as a function of agents' beliefs. Their study demonstrates that the model, when estimated using solely inflation and short-term forecasts from professional surveys, accurately predicts observed measures of long-term inflation expectations.

Through these examples, it is evident that Rational Expectation models often rely on exogenous shocks in the inflation target to incorporate survey expectations, whereas Learning models explain the long-run drift in expectations through the updating of agents' belief processes, in particular their perceptions about the inflation target.

The extensive body of literature on AL contributes to the broader field of studies that utilize survey data to test various theories of expectation formation and to understand their macroeconomic implications. In particular, a growing literature explores models with various forms of information rigidities which can generate the observed properties of expectations and also have the potential to explain the dispersion in survey expectations across agents. Angeletos et al. (2020) provide a unified framework that explains a number of these deviations in observable expectations of many macroeconomic variables reported in Coibion and Gorodnicienko (2015), Bordalo et al. (2020), and Kohlhas and Walther (2021), among many others. Coibion et. al (2018) argue for "careful (re)consideration of the expectations formation process" and review the expectations framework alternative to FIRE. Among the competing alternatives, they discuss the potential of learning models to successfully capture important deviations from the complete rationality assumption. Cavallo et al (2017) explore

cessful in explaining the rise and fall of American inflation. Orhanides and Williams (2005a) exploit the AL mechanism to generate endogenous fluctuations in expectations formation and explain the "excess sensitivity" of long-term inflation expectations and nominal interest rates to aggregate shocks that is observed in the data. Eusepi and Preston (2011) and Orphanides and Williams (2005b) exploit survey data on forecasts to discipline the belief dynamics and estimate the parameters of the learning process. They illustrate the ability of constant-gain learning models to closely match SPF expectations.

the evidence from a series of survey experiments to test the relevant sources of informational frictions. They find evidence in support of the rational inattention theory as well as of cognitive limitations. Recent papers that utilize surveys to study the implications of deviations from FIRE, such as dispersed information, inattention, and myopic behavior, include the works of Melosi (2017), Hajdini (2020), and Chou et al. (2023).

1.2 This paper

In this paper, we contribute to the growing literature described above and exploit survey data to discipline the dynamics of expectations and to study the implications of alternative expectation formation mechanisms. We start with a standard rational expectations macromodel and concentrate on the optimal exploitation of the information in survey forecasts to improve the overall fit and forecast of the model.

Three features of our approach help us to overcome the limitations of survey data and existing studies. First, we use short run SPF survey forecasts for inflation as an observable variable. These one-quarter-ahead forecasts exhibit fewer of the inefficiencies that are typically observed in survey data for longer forecast horizons. They also contain timely information that is complementary to the standard macro dataset and that is crucial for achieving our objective. Second, we consider model specifications that are flexible enough to exploit the information in survey forecasts. Our introduction of a more general markup shock process is one ingredient that is well identified by the observations of survey forecasts. Finally, we consider a model version in which agents use an AL scheme as an alternative to the Rational Expectations approach. This assumption relaxes the full information and model consistent expectation restrictions. Instead, agents formulate their expectations based on historical realizations of macro variables that they try to interpret in real time, given their prior information on the model as background structure. Our learning model provides valuable insights about how agents form and update their beliefs.

To illustrate the benefits of our approach, we exploit our setup to analyze the dynamics of inflation and inflation expectations during the post-Covid period. Our model is able to shed more light on the question of why high inflation has been such a surprise and provides an interpretation of the behavior of professional forecasters around the pronounced inflation surge of 2021-2022. We show that inflation expectations remained broadly anchored because SPF agents perceived inflation as being driven by stable data-generating process, in line with the rationality assumption. We also illustrate that an AL mechanism more successfully detects the time-varying nature of fundamental processes that drive inflation. As a result, an AL model can outperform professional forecasters in terms of predictive accuracy and can offer a deeper understanding of economic dynamics.

The outline of the paper is as follows. First, we document the main features of the survey and real-time data, and illustrate the discrepancy between inflation expectations in the surveys and the expectations implied by standard

DSGE models. We also demonstrate that including survey expectations in the model by adding only a measurement equation is not sufficient. In section three, we explain how the introduction of two markup shocks, one i.i.d. and another persistent, is extremely helpful for efficient integration of the survey data into the model. We show this first in a standard Rational Expectations model and discuss the remaining issues in this context. Then we briefly present our Adaptive Learning approach and show how updating of beliefs accounts well for the time-varying properties of the joint dynamics in realised and expected inflation. Finally, we evaluate our models across the Covid pandemics period.

2 Survey expectations versus model expectations

In this section, we first document some properties of SPF-inflation forecasts and their relation to real-time releases of realized inflation data. Then, we show that survey expectations deviate substantially from the expectations that are implicitly present in two DSGE models, the Smets and Wouters (2007) model estimated with rational or model-consistent expectations (RE) and the Slobodyan and Wouters (2012) model in which the estimation assumes that agents are using AL approach.[4] The latter model uses an AL setup in which agents update their perceived forecasting models over time as new data becomes available with a Kalman filter learning scheme; the paper shows that beliefs based on simple AR(2) forecasting models capture time-varying persistence in the inflation process well. We document the differences between the expectations implied in these models by plotting these forecasts against the SPF forecasts and by computing the statistical properties of the forecasts errors. SPF typically outperforms the model forecasts for inflation. Therefore, we re-estimate the models using survey expectations as an observable for the model expectations, allowing for measurement error in the observation equations. This results in substantial and systematic measurement error and the model forecasts are only marginally improved. Thus the original model specification clearly lacks the flexibility required to effectively exploit the information that is available in the survey forecasts.

2.1 Comparing model expectations and SPF forecasts.

It is important to note that we re-estimated our models using real-time data. Including SPF-forecasts in the model requires us to specify the model in real time, so that model expectations are based on the same information set that was available to survey participants when they formulated their expectations. As illustrated in Table 1, over the sample since 1971q1, the revision between the first and the second (final) releases for the quarter-on-quarter GDP deflator inflation has an RMSE of 0.11 (0.23). These revisions are of the same order of magnitude as the one-quarter-ahead SPF forecast error with a standard error of

[4] We refer to the original articles for the detailed model specification and estimation results. We provide more information on the learning setup in section 2 and in appendix A.

0.25. The magnitude of the data revisions is thus significant, and real-time data issues could not be ignored when we include survey forecasts into the model.[5]

Table 1: Statistical properties of the inflation revisions and SPF forecast errors.

	Full sample			Prediction sample		
Revisions	MEAN	MAD	RMSE	MEAN	MAD	RMSE
$\pi_t^{r1} - \pi_t^{r2}$	-.02	.07	.11	-.01	0.04	0.06
$\pi_t^{r1} - \pi_t^{rf}$	-.02	.17	.23	-.03	0.13	0.18
$\pi_t^{r2} - \pi_t^{rf}$	.01	.17	.23	-.02	0.12	0.15
SPF statistics						
$\pi_{t+1\|t}^{SPF} - \pi_{t+1}^{r1}$	.03	.20	.25	.03	.16	.20
$\pi_{t+1\|t}^{SPF} - \pi_{t+1}^{r2}$	.01	.20	.26	.02	.15	.18
$\pi_{t+1\|t}^{SPF} - \pi_{t+1}^{rf}$	.01	.18	.23	-.00	.15	.19
SPF for longer horizons						
$\pi_{t+2\|t}^{SPF} - \pi_{t+2}^{r1}$	.03	.24	.32	.04	.18	.21
$\pi_{t+3\|t}^{SPF} - \pi_{t+3}^{r1}$	.04	.27	.37	.06	.18	.22
$\pi_{t+4\|t}^{SPF} - \pi_{t+4}^{r1}$	.05	.29	.42	.07	.19	.24

Note: π^{r1}, π^{r2}, and π^{rf} are the first, second, and final available quarterly releases for GDP deflator inflation. $\pi_{t+1|t}^{SPF}$ is the SPF nowcast, and $\pi_{t+1+i|t}^{SPF}$ the 1, 2, and 3-quarters ahead forecast for $i = 1:3$, respectively. The *Full* sample 1972q1-2019q4 starts with quarter for which the GDP deflator inflation and GDP forecasts of sufficient quality are available in the Survey of Professional Forecasters (1971q4), plus four pre-sample quarters used in the estimated models. The *Prediction* sample 1996q1-2019q4 is the typical time interval that we use in the out-of-sample model forecast tests presented in the paper.

The models are re-estimated with real time data for GDP-deflator inflation and the GDP growth rate.[6] For the other five observables (growth rates of consumption, investment, and real wages, total hours worked, and the Fed-funds rate) we still use the final data, because the survey forecasts for these variables start later.[7],[8] Agents in the model are assumed to observe the first and the second releases of these series: the second release is taken as the 'true' measure for inflation and GDP growth. The first release is assumed to contain a simple *i.i.d.* measurement error $\xi^{\pi r}$. Therefore, for inflation the measurement equations are:

$$\begin{aligned} \pi_t^{r1} &= \overline{\pi} + \widetilde{\pi}_t + \xi_t^{\pi r}, \\ \pi_t^{r2} &= \overline{\pi} + \widetilde{\pi}_{t-1}, \end{aligned}$$

[5] See also Croushore (2010) for an evaluation of survey forecasts of inflation using real-time data.

[6] Our exercise is based on inflation expectations as measured by the GDP-deflator series. This choice coincides with the data used in the original Smets and Wouters (2007) and Slobodyan and Wouters (2012a, b) models. We intend to test the robustness of our results with CPI and PCE expectations in the future.

[7] Real-time data and SPF data are downloaded from the Philadeplphia Fed web-site https://www.philadelphiafed.org/research-and-data/real-time-center/

[8] We use the full suit of SPF expectations and the corresponding RT data in our companion project, Rychalovska et al. (2023).

and similarly for GDP growth:

$$\begin{aligned} dy_t^{r1} &= \overline{\gamma} + \widetilde{y}_t - \widetilde{y}_{t-1} + \xi_t^{yr}, \\ dy_t^{r2} &= \overline{\gamma} + \widetilde{y}_t - \widetilde{y}_{t-1}. \end{aligned}$$

When the agents in the model form their expectations for quarter $t+1$, the information set includes the first release of the data for quarter t and the second release for quarter $t-1$. This timing assumption is an approximation of the information structure available to the SPF participants: survey forecasts for $t+1$ are collected after the first release of data for quarter t is published. Of course data processing and publication takes time, and surveys are collected when quarter $t+1$ is already ongoing; more precisely, during the first half of the second month of quarter $t+1$. That is why the SPF forecast for quarter $t+1$ is also called *nowcast*. Nowcasts can reflect information that became available only after the end of the quarter t. This timely nature of the information set available to SPF-participants might contribute to the excellent forecasting performance of the surveys, which is evident from Table 1. The SPF nowcasts' RMSFE of 0.25 for the *Full* sample (1972q1-2019q4) and 0.20 for our out-of-sample *Prediction* sample (1996q1-2019q4) are important benchmarks for our later model forecasts. For comparison, the RMSFE of a no-change forecast ($E_t^* \pi_{t+1} = \pi_t$) for the first release is equal to 0.36 for the *Full* and 0.29 for the *Prediction* sample.

Table 2 collects the outcomes of the standard rationality tests applied to these survey forecasts. Following Mankiw et al. (2003) and Coibion and Gorodnichenko (2015), we test whether the survey forecast errors are persistent and predictable by the forecast, by other information available at the time of forecasting (actual inflation and interest rates), and by the forecast revisions. We run the tests using inflation expectations over the following four quarters as is usually done in the literature, but we also consider one-quarter-ahead expectations. We consider both the complete sample over which the survey data are available, and our shorter out-of-sample prediction sample.

For annual inflation forecast errors, we reproduce the well-documented deviations from the Full Information Rational Expectations hypothesis. Note, however, that these results are not necessary robust over shorter samples, as deviations from rationality for the annual SPF errors have become less severe since 1996. One explanation could be that survey participants had already observed sufficient information about the ongoing quarter, so that traditonal arguments that explain the forecast limitations do not apply: easily observable public information might dominate dispersed private signals and model uncertainty in the production of the nowcast. The exceptional prediction quality of the nowcast provides an additional argument for concentrating on this concept when we integrate SPF survey information into our DSGE models.[9] This approach is also consistent with the standard forecasting practice in many policy institutions,

[9] See also Del Negro and Schorfheide (2013) for an illustration of how short term model forecasts can be improved by conditioning on nowcasts for inflation, output, and interest rates. Carvalho et al (2023) also use one- and two-quarter ahead survey forecasts to produce long run model forecasts that are consistent with their survey counterpart.

where a model forecast is typically augmented with judgemental - read: survey based - interventions, mainly for a very short time horizon.[10]

Table 2: Test statistics for SPF forecast errors

	Annual inflation forecast				One quarter ahead forecast			
	Full sample		Prediction sample		Full sample		Prediction sample	
persistence: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta \left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$								
α	-.110	(.131)	-.143	(.102)	-.027	(.020)	-.031	(.022)
β	**.386**	(.124)	**.318**	(.139)	.071	(.107)	-.213	(.174)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta \pi^{r1}_{t+h\|t}$								
α	-.380	(.192)	.174	(.430)	-.069	(.036)	-.016	(.113)
β	.062	(.076)	-.200	(.261)	.046	(.052)	-.021	(.244)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta \pi^{r1}_{t+h\|t} + \gamma \pi^{r1}_{t-1} + \delta r_{t-1}$								
α	-.324	(.222)	.134	(.411)	-.071	(.037)	-.066	(.123)
β	.249	(.190)	-.297	(.302)	.178	(.109)	.345	(.280)
γ	.022	(.132)	.204	(.226)	-.078	(.084)	-.274	(.134)
δ	**-.585**	(.146)	-.212	(.187)	-.034	(.027)	.002	(.039)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta \left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$								
α	-.153	(.122)	-.194	(.087)	-.029	(.018)	-.026	(.021)
β	**.600**	(.173)	**.442**	(.208)	.369	(.173)	-.026	(.424)

Note: For annual inflation forecasts $\pi^{r1}_{t+h|t}$ equals $\sum_{h=1,4}(E_t \pi^{r1}_{t+h})$. Newey-West corrected standard errors are in brackets. Bold slope coefficients are statistically significant at a 95% level.

The estimated parameters for the real-time versions of the two DSGE models are documented in Table B1 in Appendix B. We refer to these models as the RE-SW07-9obs and AL-SW12-9obs: relative to the original versions with seven observables, these versions include two additional real-time data series (the second releases of inflation and output growth) as observables. The estimated parameter values are reasonably comparable to the original Smets and Wouters (2007) and Slobodyan and Wouters (2012b) models, despite the use of real-time data and the longer sample. The estimated standard errors for the measurement errors $\xi^{\pi r}$ and ξ^{yr} are almost identical under RE and under AL, at 0.12 for GDP inflation and 0.19 for GDP growth.

Figure 1 illustrates the inflation expectations that are implicitly present in these real-time models. The forecasts for $t+1$ are of particular interest, as they appear directly in the first-order conditions that describe the decision rules of the agents. The upper panel of the figure presents projected inflation trajectories at each point in time for the next 4 quarters. These are true out-of-sample forecasts, as the underlying models are estimated on the datasets that are available at the moment that the forecast is made. Each forecast starts

[10] In the robustness exercise in Appendix C, we use an alternative timing assumption. All results are robust when we use the next quarter survey forecast as an observable, but the prediction errors for inflation become larger, and the model forecasting gains from survey data decrease, because the information content of the survey data declines.

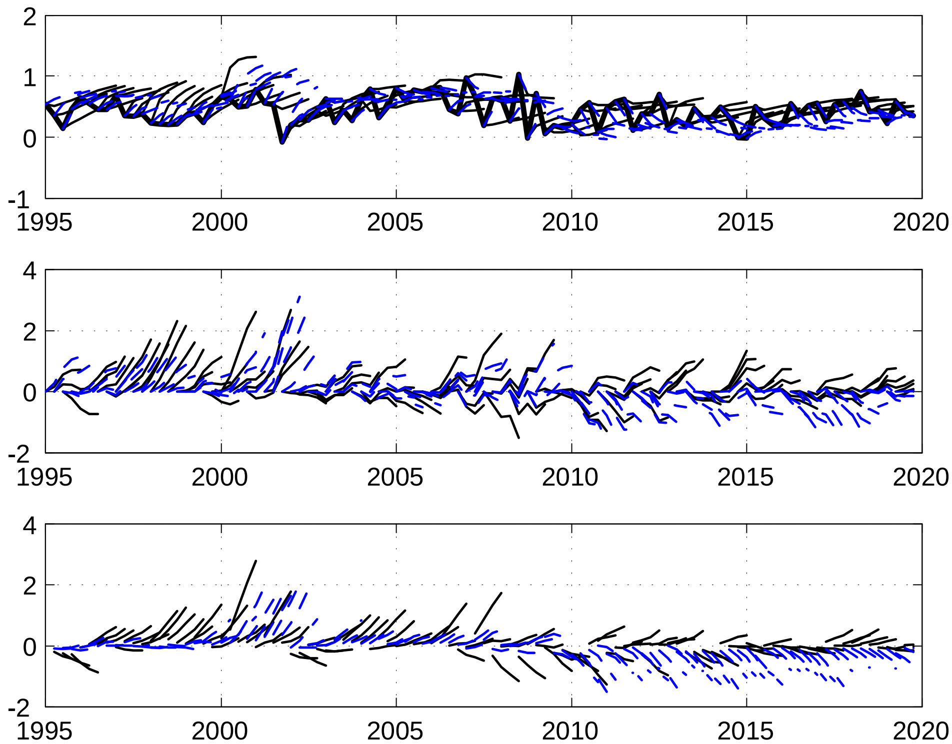
from the last available observation for the first release π_t^{r1} (thick solid line). In RE-SW07-9obs (thin black solid line), these model forecasts are consistent with the agents' expectations. In AL-SW12-9obs (thin blue dashed line), the model forecasts produced by the Actual Law of Motion process (ALM-forecasts) are plotted; in general, these are not equivalent to the agents' own forecasts (PLM forecasts), but the deviations remain modest.[11]

In the middle panel, actual inflation realisations π^{r1} are subtracted from the model forecasts, and the *cumulative* difference over the next 4 quarters is plotted. Around 2000, thre are a few instances in which the 1Q-ahead model forecasts are under-predicting first releases, while longer horizon forecasts are over-predicting them, as the cumulative difference shifts from negative to positive. In general, however, the forecast errors tend to be of the same sign at different horizons, which is clearly seen before 2000.

The lower panel of the figure displays the cumulative deviations between the models and the SPF forecasts. Here, there are often large deviations between the two forecasts. Both models, RE-SW07-9obs in particular, tend to predict higher inflation than the SPF nowcast for most of the period between 1996 and 2002, and again from 2004 to 2007. Since the start of the Great Recession, both models, but most notably AL-SW12-9obs, produce forecasts lower than the survey. At the very end of the sample, the RE model slightly over-predicts the SPF. The deviations between model and survey forecasts are very persistent for most of the projection trajectories, as well as across projections as time proceeds.

[11] With our solution procedures, ALM forecasts are easily rolled forward for longer horizons. Depending on the specification of the belief models, how to produce longer horizon PLM forecasts it is less evident. A systematic study of longer horizon ALM and PLM forecasts is presented in the follow-up study Rychalovska et al. (2024).

Figure 1: Model and SPF forecasts for 1 to 4 quarters ahead versus realized inflation (first release)



Note: Upper panel: model forecasts up to 4 quarters ahead, relative to the first release π_t^{r1} (thick solid line). Middle panel: cumulative model forecast errors relative to π_t^{r1}. Bottom panel: cumulative deviations of the model forecasts relative to the SPF nowcasts. Thin black solid line: RE-SW2007-9obs, thin blue dashed line: AL-SW12-9obs.

The statistics reported in Table 3 further document the inflation forecasting performance of the RE-SW2007-9obs and AL-SW2012-9obs models re-estimated with real-time data. The forecast errors of the RE model are large compared to the SPF nowcasts on all three criteria we consider (mean, MAD, and RMSFE). The mean error increases systematically with the forecast horizon in the out-of-sample forecasts that cover the more recent sub-period since 1996, probably because the inflation target estimated in the RE model is biased upward by data in the 70s and early 80s. It is important to note that the RE forecast deviates substantially from the SPF nowcast. The root of mean squared difference between the two forecasts for quarter $t+1$ is 70% to 80% of the SPF nowcast error's RMSFE: the difference between the two forecasts is almost as large as the SPF forecast error itself. Testing the equivalence of the two forecasts using the

Diebold-Mariano test clearly rejects the hypothesis that the two forecasts are equivalent for horizons from one to four, as the SPF significantly outperforms the model forecast.

Table 3 compares the same statistics for the AL-SW2012-9obs. In this model, the inflation forecast errors are large and worse than those of the SPF in terms of MAD and RMSE. The root of mean squared difference between the two forecasts is as much as 80% of the SPF forecast error. The Diebold-Mariano test indicates the superiority of the SPF forecasts over all forecast horizons considered. The longer horizon forecasts deteriorate more under AL than under RE: for long term forecasts, the structure imposed on the RE forecasts seems to pay off, while the flexibility of the AL beliefs can become costly. These results apply to the ALM-forecasts in the AL-model. However, in this model, the PLM forecasts are also relevant. It is these PLM expectations that enter into the agents' decision rules when they make the actual price decision. The PLM based on the small forecasting model does a good out-of-sample forecasting job in this model, at least compared to the ALM-forecast.

Table 3: Forecast Statistics for the RE-SW2007-9obs and AL-SW2012-9obs models with SPF observable

	Full sample						Prediction sample					
$t+1$ forecast	Mean		MAD		RMSFE		Mean		MAD		RMSFE	
	RE	AL	RE	AL	RE	AL	RE	AL	RE	AL	RE	AL
$\pi^{r1}_{t+1} - \pi^{AL\_PLM}_{t+1\|t}$		.00		.25		.33		.02		.18		.24
$\pi^{r1}_{t+1} - \pi^{M}_{t+1\|t}$	-.02	-.01	.26	.26	.33	.34	-.05	-.01	.21	.20	.26	.26
$\pi^{r2}_{t+1} - \pi^{M}_{t+1\|t}$	.00	.00	.26	.26	.34	.35	-.04	.00	.21	.20	.25	.25
$\pi^{rf}_{t+1} - \pi^{M}_{t+1\|t}$	-.00	-.00	.22	.23	.28	.31	-.02	.02	.19	.20	.24	.24
longer horizons												
$\pi^{r1}_{t+2} - \pi^{M}_{t+2\|t}$	-.03	-.02	.28	.30	.37	.40	-.09	-.02	.23	.23	.28	.28
$\pi^{r1}_{t+3} - \pi^{M}_{t+3\|t}$	-.03	-.01	.30	.33	.40	.44	-.13	-.03	.23	.24	.30	.30
$\pi^{r1}_{t+4} - \pi^{M}_{t+4\|t}$	-.03	-.01	.32	.36	.43	.49	-.16	-.04	.24	.26	.30	.33
Model vs. SPF	rel. RMSE, %				DM-test		rel. RMSE, %				DM-test	
	RE		AL		RE	AL	RE		AL		RE	AL
$horizon = 1$	71.02		75.98		4.70	4.48	80.03		79.36		4.53	3.69
$horizon = 2$	56.64		68.58		3.09	3.22	83.41		94.85		3.05	2.25
$horizon = 3$	47.27		63.79		2.07	2.91	83.95		105.21		3.15	2.07
$horizon = 4$	44.51		65.43		1.24	2.52	79.94		107.59		1.95	1.75

Note: Statistics for the Full sample 1972q1-2019q4 are based on in-sample predictions (with 4 presample observations excluded), while the results for 1996q1-2019q4 are based on out-of-sample predictions with recursively estimated models. Relative RMSE at horizon h is defined as $\frac{RMSE(\pi^{M}_{t+h|t} - \pi^{SPF}_{t+h|t})}{RMSFE(\pi^{SPF}_{t+h|t} - \pi^{r1}_{t+h})} \times 100$, where $M \in \{RE, AL\}$. For the learning model, $\pi^{M}_{t+h|t}$ denote ALM model forecasts. DM-test is the Diebold-Mariano test for equal accuracy between the model and the SPF forecast, with positive numbers indicating better performance (lower RMSFE) for the SPF.

In discussing the forecasting performance of the two models, one should note that the root mean squared forecast errors measured against the final data are smaller than for the first and second releases. These results confirm the good inflation forecast performance reported in the original published versions of these models. The DSGE forecasts outperformed various VAR models in terms of inflation forecasts, with the most visible gains at the longer horizons. Note also that the inflation forecast errors of the models estimated on real-time data are still smaller than the naive no-change RMSFEs of 0.36 and 0.29 for the one-quarter ahead forecast over the complete and the shorter sample, respectively.

In Tables B5 and B6 in Appendix B we also report test statistics for rationality of the model forecasts for inflation. Compared to the corresponding SPF-statistics reported in Table 2, the model forecast errors display similar degrees of predictability over the complete sample. For the shorter - more recent - Prediction period, the RE model fails on all tests. This finding might be due to misspecification of the RE model, which is unable to adjust to the time-varying dynamics in the inflation expectations. However, the results also suggest that one should interpret forecast rationality tests applied over short intervals with care.

2.2 Including an SPF nowcast as an observable with a measurement error

Here we present results for the two models re-estimated with the SPF nowcast for $t+1$ as an additional observable.[12] We integrate the SPF survey data as observable for the expected variable as follows:

$$\pi_t^{f0} = \overline{\pi} + E_t \widetilde{\pi}_{t+1} + \xi_t^{\pi f}, \tag{1}$$

where $\xi_t^{\pi f}$ is an i.i.d. measurement error (ME) between the observed SPF nowcast $\pi_t^{f0} = \pi_{t+1|t}^{SPF}$ and the model forecast $E_t \widetilde{\pi}_{t+1}$, which is expressed in terms of deviation from the inflation target $\overline{\pi}$.

The estimated parameters for these model versions, denoted RE-ME-10obs and AL-ME-10obs for estimation under RE and AL, respectively, are available in Table B2. The *i.i.d.* measurement error $\xi_t^{\pi f}$ has a standard error of 0.18 in both RE and AL versions. Including the survey data in the model in this elementary way does change some of the estimated parameters and standard

[12] Roberts (1995) was one of the first attempts to use survey forecasts as instruments for expectations in estimation of the NKPC. Adam and Padula (2011), Smith (2009), and Nunes (2010) confirmed that survey forecasts can be used succesfully as proxies for expected inflation, though they also point out the limitations of their information content. In this literature, survey expectations are treated as exogenously given observables. Fuhrer (2015) discussed how survey expectations can be endogenized and observed an important role for instrinsic persistence in expectations. See also Coibion et al. (2017) for a discussion of the literature on inflation expectations and surveys. In our approach, actual and survey data are treated as observables in the measurement equations, and their dynamics are fully endogenized by the state transition equations.

errors of the shocks. The most striking changes are the higher degree of nominal stickiness. In particular, the Calvo-probability for prices is significantly higher with observations on the SPF expectations under both RE and AL.

Table 4 illustrates that models with observable expectations perform better on all statistics related to the inflation forecast. By minimizing the measurement error on the SPF nowcasts, the model forecasts are refined to align more closely with the survey forecasts; the model forecast performance measured against the ex-post inflation realisations also improves. Obviously, the model inflation forecasts benefit from the excellent prediction potential of the survey data.

Table 4: Forecast Statistics for the RE-ME-10obs and AL-ME-10obs models with SPF observable

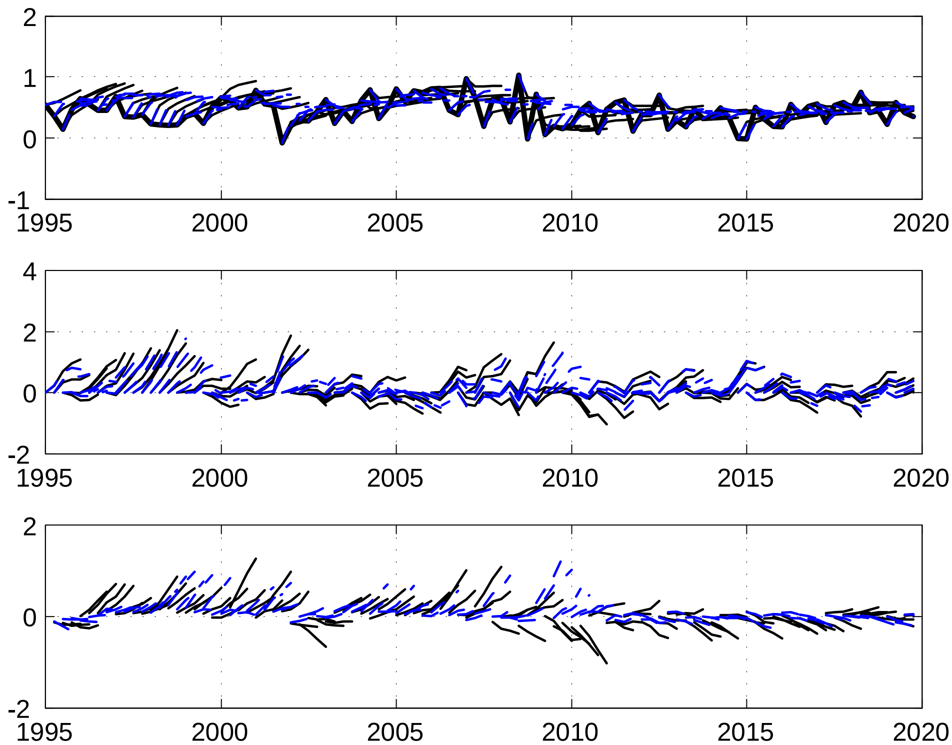
	Full sample						Prediction sample					
$t+1$ forecast	Mean		MAD		RMSFE		Mean		MAD		RMSFE	
	RE	AL	RE	AL	RE	AL	RE	AL	RE	AL	RE	AL
$\pi^{r1}_{t+1} - \pi^{AL\_PLM}_{t+1\|t}$		-.03		.23		.30		-.05		.18		.23
$\pi^{r1}_{t+1} - \pi^{M}_{t+1\|t}$	-.02	-.04	.24	.23	.31	.30	-.04	-.07	.19	.19	.23	.23
$\pi^{r2}_{t+1} - \pi^{M}_{t+1\|t}$	.00	-.02	.25	.24	.33	.31	-.03	-.06	.18	.18	.22	.22
$\pi^{rf}_{t+1} - \pi^{M}_{t+1\|t}$	-.00	-.02	.21	.20	.27	.26	-.01	-.04	.18	.17	.21	.22
longer horizons												
$\pi^{r1}_{t+2} - \pi^{M}_{t+2\|t}$	-.02	-.05	.27	.25	.36	.33	-.07	-.10	.20	.20	.25	.24
$\pi^{r1}_{t+3} - \pi^{M}_{t+3\|t}$	-.02	-.05	.29	.27	.39	.36	-.09	-.11	.20	.20	.26	.25
$\pi^{r1}_{t+4} - \pi^{M}_{t+4\|t}$	-.01	-.04	.30	.30	.42	.41	-.11	-.11	.21	.21	.26	.26
Model vs. SPF	rel. RMSFE, %				DM-test		rel. RMSFE, %				DM-test	
	RE		AL		RE	AL	RE		AL		RE	AL
$horizon = 1$	56.56		53.48		4.31	3.58	51.50		49.82		3.63	2.73
$horizon = 2$	44.92		43.61		2.56	0.59	53.30		52.43		2.23	2.00
$horizon = 3$	38.17		39.02		1.31	-0.28	55.22		43.06		2.41	1.78
$horizon = 4$	38.53		42.17		0.65	0.38	54.17		46.31		1.28	1.27

Note: See Table 3

Compared to the RE-ME-10obs, the AL-ME-10obs model does a relatively good job in the out-of-sample prediction exercise over the period since 1996. The out-of-sample inflation forecasts of AL-ME-10obs are more in line with the survey forecast according to the relative RMSFE criteria, but they are still outperformed by the survey, according to the DM test. In terms of longer horizon inflation forecasts, the AL (AL-ME-10obs) model is now superior to the RE (RE-ME-10obs) model, while the opposite was true for the 9obs-models.

Figure 2 presents out-of-sample forecasts for the RE and AL models with 10 observables. Compared to Figure 1, the predictions of both models are now much closer to the SPF nowcasts, while there is no discernible difference regarding the inflation forecast errors (upper and middle panels).

Figure 2: Model forecasts with SPF as an observable with measurement error



Note: See Fig.1. Thin solid line: RE-ME-10obs, thin dashed line: AL-ME-10obs.

The relative success of the AL-ME-10obs model in capturing the overall dynamics of the inflation process is confirmed by the marginal likelihood comparison summarized in Table 5. [13],[14] Overall, the AL models with 9 and 10 observables have better marginal likelihood than the RE models. By relaxing the RE-restrictions and assuming that expectations are based on small belief models that are updated over time depending on new realisations, the AL-models have some extra flexibility that is useful for forecasting. This flexibility is particularly helpful in reconciling the observed survey forecast and realized

[13] The estimation sample in Table 5 is 1971q1-2015q3, but we evaluated the models on the full pre-Covid pandemics period of 1971q1-2019q4. The first four periods are used as a presample and do not contribute to the likelihood calculation during estimation; therefore, all Tables reporting *Full* sample statistics use the period 1972q1-2019q4 to facilite comparison with the estimated models.

[14] We also estimate the 2MU-10 obs models on the Full sample and report the resulting parameters in Table B.9. The comparison with Table B.4 shows that restriction of the estimated sample to 1971q1-2015q3 does not generate significant changes in the estimated parameters or in the model rankings.

inflation in the 10obs model. This becomes most obvious when we calculate the marginal likelihood for the original 9 variables implied by the 10obs model. For the AL-models, the marginal likelihood for this common block deteriorates only slightly relative to the 9obs model (-951 versus -943), while for the RE-models, including the survey nowcast in the model worsens the marginal likelihood of the common 9 observables significantly more (-999 versus -965). When it must comply with the nowcasts, the AL model is more flexible in delivering predictions consistent with the survey, while retaining its overall good forecasting performance, but the RE-model loses in overall performance.

That AL-models with simple AR(2) PLM beliefs do a reasonably good job of mimicking survey expectations was also suggested in Slobodyan and Wouters (2012b). This observation is consistent with experimental evidence on expectations forecasting.[15] Small forecasting models provide a good respresentation of how agents formulate their expectations. However, it would be surprising if small models were able to reproduce the potentially rich information set underlying SPF forecasts. To illustrate the role of the belief specification under AL, we also consider a slightly more complicated belief model in which the AR(2) specification is augmented with the marginal cost variable in the inflation PLM, leaving the remaining PLMs unchanged. This Phillips Curve-based specification can capture the basic relation between inflation and its underlying macroeconomic determinants. While this augmented belief model (AR2+MC) produces some gains for the standard 9obs model (-934 versus -943), it becomes even more informative for the model with observed survey forecasts. The marginal likelihood of the AL model with marginal cost (MC) in the beliefs improves by 26 units relative to the model with a basic AR(2) PLM specification. The estimated standard error for $\xi_t^{\pi f}$, the measurement error on SPF-expectations, drops from 0.18 to 0.15. The Phillips curve relation seems to have some relevance in forecasting the relatively smooth inflation expectation variable, though it was not very informative for predicting the highly volatile realized inflation process.

Table 5: Marginal likelihood of alternative model specifications

[15] See Hommes and Zhu (2014) for a review of arguments in favor of small forecasting models.

	Full sample		Prediction sample	
	9obs	10obs	9obs	10obs
RE-SW07-9obs	-965.22		-361.25	
AL-SW12-9obs (AR2)	-943.42		-340.96	
AL-SW1-9obs2 (AR2+MC)	-934.41		-317.78	
RE-ME-10obs	-999.56	-911.05	-374.78	-302.57
AL-ME-10obs (AR2)	-951.29	-883.46	-337.62	-282.83
AL-ME-10obs (AR2+MC)	-941.46	-857.51	-324.84	-260.00
RE-2MU-10obs	-944.76	-839.96	-344.84	-267.14
AL-2MU-10obs (AR2)	-959.35	-894.51	-338.60	-287.04
AL-2MU-10obs (AR2+MC)	-951.46	-866.45	-333.57	-272.42
AL-2MU-10obs (AR2+MC+UC)	-915.48	-787.18	-312.83	-228.69

Note: We follow Warne et al. (2016) in calculating the marginal likelihood for a subset of the variables.

We can conclude from this section that only adding an SPF nowcast as an observable for the inflation expectations in our DSGE models does not produce significant improvement. Reducing the discrepancy between the SPF nowcast and the model forecasts leads to some interesting changes in the estimated parameters, and the inflation forecasts improve. However, the measurement errors are large and persistent, and are correlated with other structural innovations in the model. There is no evidence that the additional observable leads to better identification of shocks or parameters that could improve the overall model performance. This implies that refining the model specification is necessary to ensure proper transmission of valuable information from survey forecasts.

3 Reconciling model and survey expectations

To exploit rich content from survey expectations more effectively, we need more flexibility in the specification of inflation dynamics. In the Smets and Wouters (2007) model, the price and wage markup shocks are modelled as ARMA processes. For the price markup, this process is written as

$$\begin{aligned} \mu_t^p &= \rho_\mu^p \cdot \mu_{t-1}^p - \theta_\mu^p \cdot \varepsilon_{t-1}^p + \varepsilon_t^p \Leftrightarrow \\ \mu_t^p &= \mu_t^{p.ar} + \varepsilon_t^p, \\ \mu_t^{p.ar} &= \rho_\mu^p \cdot \mu_{t-1}^{p.ar} + \left(\rho_\mu^p - \theta_\mu^p\right) \varepsilon_{t-1}^p. \end{aligned}$$

This specification implies that *the same* innovation ε_t^p is driving the volatile high frequency MA-component on the one hand and the persistent low frequency AR-component on the other hand. This ARMA specification works well to capture the complex exogenous shock process in the actual price and wage dynamics.[16] As long as the dataset is limited to the standard seven macrovariables, there is no need to distinguish between separate innovations driving the high and the low

[16] See also Ang, Bekart and Wei (2007) and Stock and Watson (2007) for more evidence supporting the ARMA specification for forecasting inflation dynamics.

frequency components. These innovations are simply not identified individually by the standard seven observables. In the AL version of Slobodyan and Wouters (2012b), the price and wage markup shocks reduce to *i.i.d.* processes, such that

$$\mu_t^p = \varepsilon_t^p.$$

The time-varying AR(2) beliefs generate the dynamics required for matching the observed price and wage persistence. Importantly, in this setup, one exogenous innovation is also sufficient to describe the exogenous shock process.

Observing the survey expectations, which are most likely based on a broader and more timely information set, allows us to precisely distinguish the pure *i.i.d.* and the persistent components in the markup processes and the corresponding innovations.[17] Therefore, we specify the price and wage markup processes as a combination of a persistent AR ($\mu_t^{p.ar}$) and a separate *i.i.d.* shocks ($\mu_t^{p.iid}$), each with their own innovation:

$$\begin{aligned} \mu_t^p &= \mu_t^{p.ar} + \mu_t^{p.iid}, \\ \mu_t^{p.ar} &= \rho_\mu^p \cdot \mu_{t-1}^{p.ar} + \varepsilon_{t-1}^{p.ar}, \\ \mu_t^{p.iid} &= \varepsilon_t^{p.iid}. \end{aligned} \tag{2}$$

We further assume that the innovation to the persistent shock process ($\varepsilon_{t-1}^{p.ar}$) is already observed publicly in the quarter prior to its actual impact on price setting. This assumption is not crucial for the results presented below, but the model fit improves when we use the "news" specification instead of a contemporaneous innovation. Examples of this type of event are oil shocks or other commodity shocks that are observed in world prices before they actually enter into retail prices, or announced changes in regulated prices and taxes that are communicated in advance of actual implementation. We use the same dual process as defined in equation (2) for the wage markup shock, to maintain symmetry.

We use the same measurement equation for the SPF survey nowcast as in equation (1). We discuss the implication of this new specification first for the RE-setup of Smets and Wouters (2007) and in the next section for the AL-setup of Slobodyan and Wouters (2012b).[18]

3.1 Integrating survey data in the augmented RE-SW2007 model

Table B3 in the Appendix B summarizes the estimated parameters for this RE model with two markup shocks estimated on ten observables including the SPF

[17] This specification of the markup process is consistent with the noisy-information model used in Coibion and Gorodnichenko (2015), and therefore also with the observed predictability of ex-post forecast errors by ex-ante forecast revisions in the SPF for inflation.

[18] Note that these two separate markup innovations are not identified as long as the survey expectations are not included in the data file. The marginal likelihood of the model with this additional shock is identical to the model with the ARMA structure under both RE and AL. Both the filtered and the smoothed innovations are highly correlated and identified only weakly .

nowcast (RE-2MU-10obs model). The estimated standard deviation of the measurement error for the SPF nowcast reduces to 0.04, against 0.18 in the model specification with measurement error only (RE-ME-10obs). The nominal price stickiness parameter is estimated to be high. This tendency towards more stickiness was already present in the RE-model with only measurement error. Under RE, high price stickiness seems important for matching the survey expectations. In the wage setting process, the wage markup shock is close to a random walk while nominal stickiness remains relatively low.

Table 6 documents the inflation forecast performance of this model, which improves in all dimensions. The model forecast statistics are now very similar to those of the SPF. The RMSFE of forecast errors for the one quarter ahead inflation rate aproaches the benchmark SPF performance. The DM-test confirms that the two forecasts are not significantly different at a shorter sample. At longer horizons, the model forecasts remain very similar to the SPF ones, although these are not observed in the model. Figure 3 provides a corresponding plot of the cumulative forecast deviations for the out-of-sample forecasts. For the RE model, there are very few differences between the SPF and the model forecasts since 1996. These features of the model forecast are confirmed by the rationality test (reported in Table B7 in the Appendix). While observing and utilizing timely information, the model also inherits the inefficiencies present in the SPF, and thus the test statistics for the RE-model forecasts are in line with the test outcomes for the SPF forecasts.

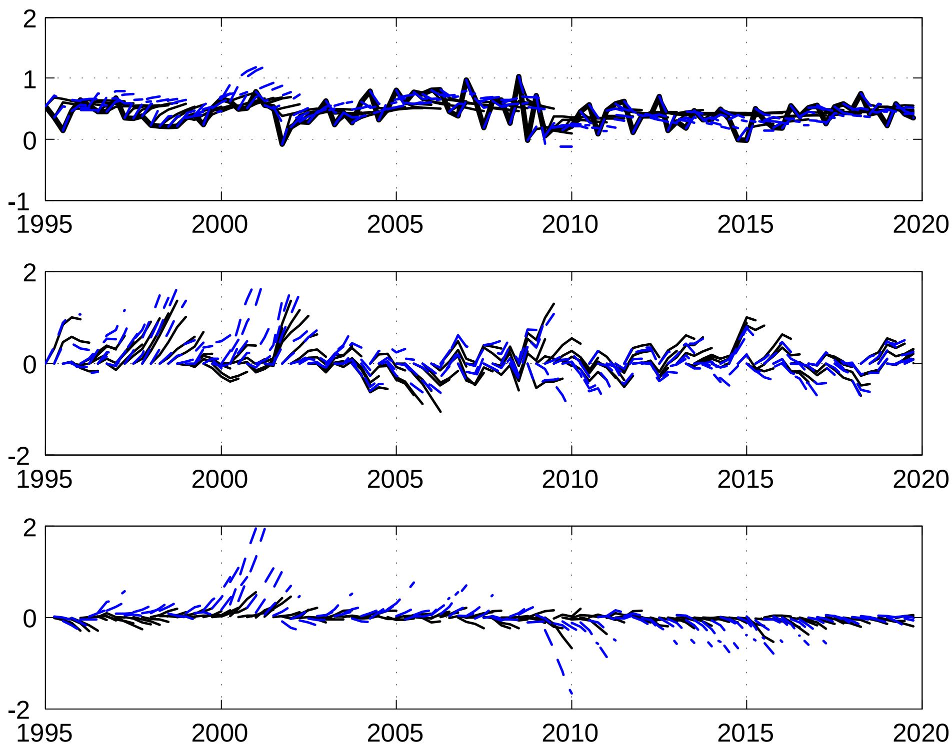
Table 6: Forecast Statistics for the RE-2MU-10obs and AL-2MU-10obs models with SPF observable

	Full sample						Prediction sample					
$t+1$ forecast	Mean		MAD		RMSFE		Mean		MAD		RMSFE	
	RE	AL	RE	AL	RE	AL	RE	AL	RE	AL	RE	AL
$\pi^{r1}_{t+1} - \pi^{AL\_PLM}_{t+1\|t}$		-.03		.20		.25		-.03		.16		.20
$\pi^{r1}_{t+1} - \pi^{M}_{t+1\|t}$	-.03	-.05	.20	.20	.26	.26	-.03	-.05	.16	.17	.20	.20
$\pi^{r2}_{t+1} - \pi^{M}_{t+1\|t}$	-.01	-.02	.20	.20	.27	.26	-.02	-.04	.15	.16	.19	.19
$\pi^{rf}_{t+1} - \pi^{M}_{t+1\|t}$	-.01	-.03	.18	.18	.23	.22	.00	-.02	.15	.15	.19	.19
longer horizons												
$\pi^{r1}_{t+2} - \pi^{M}_{t+2\|t}$	-.03	-.04	.25	.24	.33	.32	-.04	-.05	.18	.19	.22	.24
$\pi^{r1}_{t+3} - \pi^{M}_{t+3\|t}$	-.03	-.03	.27	.25	.36	.35	-.05	-.06	.19	.20	.23	.25
$\pi^{r1}_{t+4} - \pi^{M}_{t+4\|t}$	-.03	-.02	.29	.28	.39	.41	-.05	-.07	.19	.21	.23	.26

Model vs. SPF	Full sample: rel. RMSFE, %		DM-test		Prediction sample: rel. RMSFE, %		DM-test	
	RE	AL	RE	AL	RE	AL	RE	AL
$horizon = 1$	5.20	34.33	2.62	0.46	5.11	45.25	1.44	2.73
$horizon = 2$	31.83	53.67	0.37	0.04	28.13	59.10	0.86	1.96
$horizon = 3$	29.77	56.93	-0.73	-0.56	33.59	68.30	1.19	2.12
$horizon = 4$	33.92	61.04	-0.63	0.10	33.04	70.55	-0.37	1.11

Note: See Table 3

Figure 3: Model forecast with SPF observed and two markup shocks



Note: See Fig.1. Thin solid line: RE-2MU-10obs, thin dashed line: AL-2MU-10obs.

The new structure with two markup shocks gives the model precisely the flexibility necessary to jointly fit the realized inflation process and the survey nowcast. The highly volatile *i.i.d.* markup component with a standard deviation of 0.24 explains the volatile high-frequency component of actual inflation. As illustrated by the impulse response functions in Figure 4, inflation is only affected by this innovation on impact, and returns to its pre-shock level in the next period, with a very small negative correction afterwards. This implies that the shock is almost irrelevant for the $t + 1$ inflation forecast, and the spillover effects to the real variables are minimal. On the other hand, the persistent autoregressive price and wage markup shock components with standard errors of 0.03 and 0.01 and persistence of 0.78 and 0.99, respectively, are crucial for capturing innovations in the survey nowcasts. These "news" shocks already have an impact on actual prices at time t, consistent with the forward-looking nature of the price setting problem. The magnitude of this short run impact effect is substantially smaller than for the *i.i.d.* component: one half for the price and one third for the persistent components of the wage shock. Over time, the wage shock begins to dominate, as it is a quasi-permanent shock.

The conditional covariance decomposition presented in Table 7 indicates that the four markup shock components each have their own role in the inflation process. The *i.i.d.* price markup $\varepsilon^{p.iid}$ explains the one-quarter-ahead forecast error in realised inflation (67%) but is irrelevant for expectations. The persistent price markup shock $\varepsilon^{p.ar}$ is crucial for the short term forecast error in the survey expectations (65%), and, consistently, is also important for realized inflation over the medium term horizon of one or two years ahead. The role of the persistent wage markup shock $\varepsilon^{w.ar}$ builds up only gradually, but it is dominant in the long run, and explains 78% of the inflation expectations and 58% of the realized inflation variance at the 10-year horizon. Note that the persistent component of the wage shock has only minor effects on short term wage developments. Therefore, it is not surprising that the precise timing of the innovations to this component (in both the "news" and the contemporaneous specification) is difficult to identify. In fact, innovations of the persistent wage and price components are highly correlated (0.79) between themselves, but not with the corresponding *i.i.d.* markup innovations. This observation is important, as it raises questions about the correct interpretation of the persistent wage markup shock.

Table 7: Conditional variance decomposition for the RE-2MU-10obs model

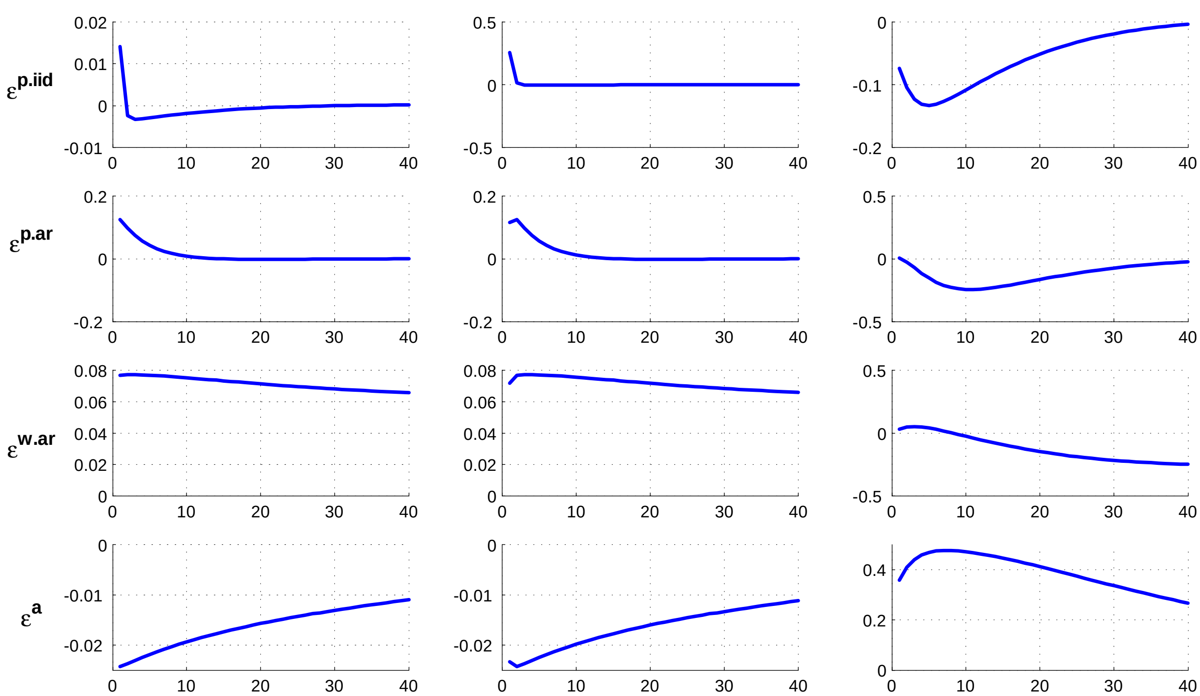
	ε^a	ε^b	ε^g	ε^{qs}	ε^m	$\varepsilon^{p.iid}$	$\varepsilon^{w.iid}$	$\varepsilon^{p.ar}$	$\varepsilon^{w.ar}$	$\xi^{\pi r}$	$\xi^{\pi f}$
1 quarter horizon											
π_f0	2.44	0.14	0.69	0.00	0.52	0.82	0.29	64.66	24.55	0	5.89
π_r1	0.56	0.03	0.15	0.00	0.12	66.78	0.08	13.98	5.33	12.98	0
w	0.05	0.86	0.10	0.05	0.84	9.63	86.60	1.58	0.29	0	0
y	18.71	41.39	20.33	5.04	13.24	0.80	0.33	0.01	0.15	0	0
1 year horizon											
π_f0	3.46	0.18	1.06	0.00	0.73	0.36	0.31	53.97	37.68	0	2.25
π_r1	1.50	0.08	0.44	0.00	0.32	43.66	0.16	29.89	15.50	8.46	0
w	0.22	3.74	0.83	0.52	5.47	6.07	72.73	6.16	4.25	0	0
y	17.07	39.17	6.53	10.96	24.28	1.19	0.14	0.45	0.21	0	0
10 year horizon											
π_f0	4.26	0.08	1.54	0.04	0.42	0.11	0.11	14.56	78.33	0	0.54
π_r1	3.28	0.07	1.17	0.03	0.34	18.39	0.11	14.65	58.40	3.56	0
w	2.57	5.06	7.78	3.65	16.69	2.50	17.49	10.47	33.78	0	0
y	34.13	15.30	3.20	11.01	24.65	1.14	0.04	4.84	5.68	0	0

Note:

To illustrate this interpretation further, we consider a model with an additional quasi-permanent inflation target shock in the monetary policy reaction function. The model specification is similar to the one used in Smets and Wouters (2003). Del Negro and Eusepi (2011) argued in favour of such a target shock to explain inflation survey expectations in the context of a DSGE model.[19] This exogenous inflation target process has become a popular modelling device to explain the low frequency inflation trend in RE-DSGE models. In Table B4 we report the estimation outcomes for this specification. The marginal likelihood of the model improves slightly, from -840 for RE-2MU-10obs to -834. The target shock substitutes for the persistent wage markup shock, whose persistence declines drastically from almost unity to a value of 0.40, curtailing its impact. The long term inflation trend, which is common to expectations and realisations, is now completely explained by the exogenous target shock, while under the wage markup shock interpretation, it implies a severe trade-off problem for monetary policy, as is typical for cost-push shock situations. Without further information from additional labour market variables (as in Gali, Smets and Wouters 2014) and/or about the monetary policy objective, the RE has a hard time differentiating among these alternative interpretations of the long run inflation component.

Figure 4: IRF functions for *iid* and AR price markup, AR wage markup, and productivity shocks

inflation expectations	inflation	output



Note:

We turn next to discussing this model's performance on other dimensions.

[19] De Graeve et al (2009) made a similar argument for the target shock to achieve consistent integration of long term interest rates in these models.

Allowing for the two separate markup innovations boosts the log marginal likelihood of the 10obs model by 61 units relative to the RE-ME-10obs model with measurement error only. Of course, the improved fit of inflation expectations constitutes an important contribution to this gain. Still, evaluating the marginal likelihood of the original 9 observables, we observe that RE-2MU-10obs model also outperforms both the original RE-SW07-9obs and RE-ME-10obs. Thus, the overall performance of the model is improved under this specification, and the information from survey expectations helps us to predict the other variables in the economy as well. This result is further documented in Table 8, which presents the forecasting results for the individual variables. Over the complete sample, the in-sample RMSFE indicates that the main gains are concentrated in the inflation block. Similarly, in the recent period, the out-of-sample forecast gains are concentrated in the price, wage, and interest rate block, but the forecasts of other real variables (investment, output, and hours) deteriorate slightly.

Table 8: Forecast performance of the RE-2MU-10obs model

	π^{r1}	π^{r2}	π^{f0}	dy^{r1}	dy^{r2}	dc	$dinve$	$hours$	dw	r
1972q1-2019q4										
Mean	-0.03	0.02	0.00	-0.17	0.01	0.07	0.20	-0.04	-0.02	-0.04
MAD	0.20	0.08	0.10	0.46	0.16	0.46	1.18	0.41	0.56	0.15
RMSFE	0.26	0.11	0.15	0.61	0.20	0.62	1.65	0.53	0.75	0.22
1996q1-2019q4										
Mean	-0.03	0.01	-0.01	-0.31	0.01	-0.08	-0.20	-0.28	0.03	-0.09
MAD	0.16	0.05	0.06	0.45	0.15	0.39	0.97	0.42	0.69	0.11
RMSFE	0.20	0.07	0.09	0.54	0.19	0.52	1.38	0.55	0.90	0.14
log lik score	0.92	2.04	1.60	-0.06	1.15	0.11	-0.86	0.09	-0.44	1.24
Comparison to RE-ME-10obs model										
rel. RMSFE	0.86	0.99	0.61	1.01	1.00	0.97	0.96	1.07	0.96	0.93
Δlog lik score	0.12	0.00	0.39	0.01	0.00	0.02	0.04	-0.04	0.05	0.06
Comparison to RE-SW07-9obs model										
rel. RMSFE	0.74	1.10		1.04	1.00	0.97	1.02	1.04	0.99	0.96
Δlog lik score	0.23	0.01		-0.01	0.00	0.05	0.04	0.02	0.12	0.05

Note: Results reported for the Full sample 1972q1-2019q4 are calculated as in-sample results, while the results for the Prediction sample 1996q1-2019q4 are out-of-sample forecasts with recursive estimation for every period. Forecast error is defined as realization minus the forecast. In the comparison panels, values for the relative RMSFE smaller than one mean that the RE-2MU-10obs model has smaller forecast errors, while positive values for the relative log score mean higher log likelihood score for the RE-2MU-10obs model. Log likelihood scores are per period.

Overall, the nominal block performs very well in this RE setup with two markup shocks. The model exploits survey information efficiently and improves the forecast of nominal variables. However, the RE model forecasts display predictability issue similar to SPF predictions. Moreover, the RE model imposes a constant variance-covariance structure on the data, while we know from reduced form exercises that the nature of the inflation process changes over time. Therefore, the question is whether a structure with two shocks with different persistence but constant variance is optimal. The adaptive learning setup can improve on precisely these dimensions.

3.2 Integrating survey data in the AL-SW2012 model

In this section we reiterate the main steps of our Kalman filter based AL-algorithm and discuss the assumptions that we make on the forecasting rules that represent the beliefs of the agents in the AL-model. The simple AR2 specification that we retained in SW2012 is not able to optimally exploit the rich information structure from the survey. We reformulate the forecasting/belief rules so that there is a role for expectation signals in the agents beliefs. The estimation results are presented in the last subsection.

3.2.1 Adaptive Learning and belief specifications

As in Evans & Honkapohja (2001), we assume that the economic agents do not have perfect knowledge of the reduced form parameters of the model when they form expectations about the future. Therefore, they forecast future values of the forward variables in the model (y^f) with linear functions of endogenous model variables.[20] One-period ahead forecasts generated by these models are substituted for the expectations in the model.[21] The general logic of this adaptive learning approach works as follows.

The model is represented as

$$\overline{\alpha} + A_0 y_{t-1} + A_1 y_t + A_2 E_t y_{t+1} + B\epsilon_t = 0, \tag{3}$$

where y_t is a vector of endogenous and exogenous model variables. The RE solution of this system is presented as a VAR(1) process,

$$y_t = \mu + T y_{t-1} + R\epsilon_t.$$

Under adaptive learning, agents assume that the forward-looking variables are linear combinations of some variables in the vector y_{t-1}. This assumption is known as the Perceived Law of Motion, or PLM:

$$y_t^f = \beta_{t-1}^0 + \underline{\boldsymbol{\beta}}_{t-1}^T y_{t-1}. \tag{4}$$

By rolling forward the PLM, we obtain the agents' expectations of forward-looking variables as

$$E_t y_{t+1}^f = \beta_{t|t-1}^0 + \underline{\boldsymbol{\beta}}_{t|t-1}^T y_t.$$

We then plug these expectations into the model representation (3), and solve the resulting purely backward-looking model to produce the Actual Law of Motion, or ALM:

$$y_t = \mu_t + T_t y_{t-1} + R_t \epsilon_t. \tag{5}$$

[20] Our adaptive learning models are realized in a specialized DYNARE toolbox. Therefore, we follow the Dynare notation in our formulae.

[21] This adaptive learning approach is referred to as Euler Equation learning, as opposed to the infinite horizon or anticipated utility approach (see Evans et al 2013 and Eusepi and Preston 2015).

The model transmission mechanism (μ, T, and R) thus becomes a function of time-varying coefficients in the agents' forecasting equations (β_t^0 and $\underline{\boldsymbol{\beta}}_t$), called *beliefs*. The beliefs can be updated using any convenient adaptive algorithm. In the literature, Recursive Least Squares (RLS) and the Kalman filter have proven to be the most often used. In the previous paper (Slobodyan and Wouters 2012b), we utilized Bayesian Kalman filter learning as a flexible learning mechanism for a set of forecasting variables.[22] The beliefs models were specified as small forecasting models in which the set of variables the agents use to form their forecasts is much smaller than the Minimum State Variable (MSV) set needed to achieve a rational expectations forecast. A simple AR(2) specification turned out to be sufficient to capture the time-varying persistence in expectations that is useful to explain the dynamics in the observed macrodata. The forecasting equation for inflation was:

$$\pi_t^f = \begin{pmatrix} 1 & \pi_{t-1} & \pi_{t-2} \end{pmatrix} \cdot \underline{\boldsymbol{\beta}}_{\pi,t-1} + u_{\pi,t}, \tag{6}$$

As noted, including the marginal cost as an additional regressor in the prediction model is very useful when we observe the survey expectations in the 10obs model. This suggests that, in order for the AL-models to exploit the information from the survey data more efficiently, we must include additional variables in the belief models. A simple AR(2) belief model cannot capture the rich information structure of the survey data that we observed in the analysis of the RE-model. Therefore, we consider a specification for beliefs that includes all independent determinants that affect the inflation dynamics in the structural model equations. This means that we have to include not only the lags of inflation and the marginal cost, but also unobserved innovations of the markup process into the belief specification. It is precisely by including these innovations into the beliefs that identification of the separate markup disturbances becomes possible under AL.[23]

$$\pi_t^f = \begin{bmatrix} 1 & \pi_{t-1} & \pi_{t-2} & mc_{t-1} & \varepsilon_{t-1}^{p.ar} & \varepsilon_{t-2}^{p.iid} \end{bmatrix} \underline{\boldsymbol{\beta}}_{\pi,t-1} + u_{\pi,t}, \tag{7}$$

To maintain symmetry, we use a similar model for beliefs about the wage process (with the marginal rate of substitution replacing the marginal cost):

$$w_t^f = \begin{bmatrix} 1 & w_{t-1} & w_{t-2} & mrs_{t-1} & \varepsilon_{t-1}^{w.ar} & \varepsilon_{t-2}^{w.iid} \end{bmatrix} \beta_{w,t-1} + u_{w,t}. \tag{8}$$

3.2.2 Estimation results with survey data and two-component markup shocks under AL

The estimated parameters of this AL-2MU-10obs model are standard (see Table B3 in Appendix B): the stickiness in both prices and wages is high but not too extreme. The innovations in the persistent markup shock have small standard deviations, 0.04 for prices and 0.03 for wages, and reasonable persistence of,

[22] See appendix A for more detail on the setup of our learning approach.

[23] Note that we must include the *iid* innovation with two lags to secure independence among the RHS-regressors and to avoid singularity in the covariance matrix.

respectively, 0.78 and 0.65. The measurement error in the inflation expectations is further reduced to 0.01. This indicates that valuable content from surveys is effectively conveyed and model forecasts benefit from timely information. Of course, in this setting the PLM coefficients are also crucial for understanding the inflation dynamics, both in terms of persistence and of volatility. In the AL context, transmission via the endogenous belief coefficients is more important for inflation dynamics than via the exogenous persistence in the shocks, which is crucial under RE.

The one-quarter ahead inflation forecast of this AL model resembles the equivalent SPF inflation nowcast (see Figure 4 and Table 6). The accuracy of the two forecasts is not significantly different according to the DM-test. For longer horizons, the quality of the inflation forecast is less impressive, both relative to the SPF, although the difference is not significant, and relative to the RE model (RE-2MU-10obs). As suggested before, this can result from the flexibility of the AL coefficients and the lack of parameter restrictions on the belief model. We can partially solve this problem by incorporating longer horizon forecasts into the list of observables, as we illustrate further below. It is also noteworthy that the rationality test for the forecasts produced by this AL model are relatively successful: Table B8 in the Appendix B displays fewer significant results for persistence or predictability in the forecast errors than with forecasts from the other models considered, and from the SPF forecasts, at both the one-quarter and the annual horizon.

The marginal likelihood of this model is superior to all previous models. The improvement relative to the best AL-model with measurement error only (AL-ME-10obs AR2+MC) is of the order of 70 units (from -857 to -787); with respect to the best RE-model (RE-2MU-10obs), the improvement is 50 units. The model also does an excellent job for the 9 original variables: here the improvement is of the order of 26 to 29 units. Note that the augmented belief equation for inflation is crucial for this excellent marginal likelihood result: the marginal likelihoods for the models with AR(2) and AR(2)+MC specifications for the PLM beliefs are much smaller. Thus, the information about the nature of the markup shocks must be incorporated into the belief equations. In this way, we provide the agents in the model with the same timely information that the survey participants possess. When observing the survey forecast for time $t+1$ in the course of time t, the agents in the model can correctly identify the nature of the markup shocks. This information about the persistent components of the shocks determines their contemporaneous actions and their expectations for the next period. As in the RE-model, the survey data are extremely informative for distinguishing the more persistent markup shocks from the *i.i.d.* component in the inflation dynamics. When there is a large revision in the one-period ahead survey forecast, this typically leads to an innovation in the persistent markup shock. The correlation between the first difference of the SPF nowcast, π^{f0}, and the persistent price markup's innovation in the complete sample is 0.82. Moreover, time-varying beliefs allow for flexible filtering of SPF nowcast innovations into the price mark-up components, which can be important when the data-generating process changes over time. Section 4, which illustrates the application of our approach to the analysis of inflation dynamics in the post-Covid period, emphasizes the importance of time variation and shows the ability of AL to address the limitations of the RE setup.

These impressive gains in marginal likelihood are explained by two features of the forecast distribution: the mean forecast precision and the time-varying volatility. The standard out-of-sample prediction statistics of this model: mean, MAD, and RMSFE, presented in Table 9, are excellent. This applies to both the inflation variables and the real variables. The AL-model outperforms or is equal to the RE-model in RMSFE on all variables except consumption and output growth. Compared to the simpler AL-ME-10obs model, there is an overall gain except for investment and second release data, and gains for the inflation releases but not for other variables relatively to AL-SW12-9obs. Given the time-varying covariance structure, the likelihood score of the forecasts becomes informative as it weights the forecast errors by their conditional variances. The impact of this correction on the forecast score is the largest for the inflation expectation variable. While in terms of RMSFE the results are marginally worse than for the RE-model, in terms of log likelihood score, the AL forecasts by far dominate the RE outcomes.

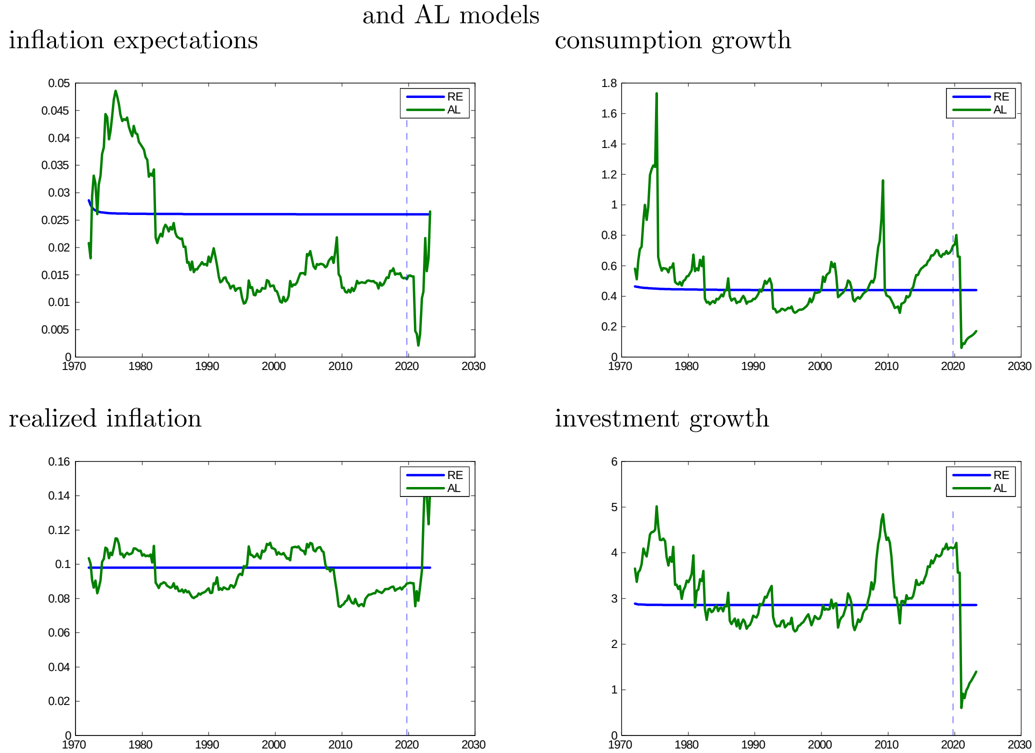
To further illustrate the impact of the time-varying covariance matrix on the likelihood/posterior evaluation, we perform the following experiment. We use the time-varying covariance matrices for the one-period ahead forecast errors from the AL-model to evaluate the likelihood of the RE-prediction errors. The log posterior value of the RE-model, evaluated at the parameters corresponding to the RE-mode, improves with this correction for time variation in the prediction uncertainty from -718 to -673. This value can be compared with the log posterior of the AL-model at the mode of -664. Re-evaluating the AL-models with the fixed covariance structure of the RE model results in a deterioration of the log posterior to -716, which is still slightly better than the log posterior mode of the RE-model. Thus, a large fraction of the improvement in the log posterior value can be attributed to the time-varying covariance matrix that is generated by updating the beliefs over time. One could expect that most of this gain is realized in the beginning of the sample, which is characterized by large variation in the covariance matrix, as illustrated below. However, we confirm the same result when we repeat the exercise over the Prediction sample: the log posterior of the RE-model for that sub-sample improves from -268 to -237 when evalated with the time-varying AL-covariance structure. On the other hand, the AL model deteriorates from -241 to -265 when we use the constant covariance structure of the RE-model. Time-variation in the covariance structure is very important, but the AL model still outperforms the RE-model even when it is evaluated with the same constant covariance structure.

Table 9: Forecast performance of the augmented AL-2MU-10obs model

	π^{r1}	π^{r2}	π^{f0}	dy^{r1}	dy^{r2}	dc	$dinve$	$hours$	dw	r
1972q1-2019q4										
Mean	-0.05	0.02	0.00	-0.19	0.01	0.05	0.20	-0.06	0.02	-0.02
MAD	0.20	0.07	0.10	0.48	0.15	0.50	1.16	0.40	0.54	0.13
RMSFE	0.26	0.11	0.15	0.64	0.20	0.67	1.60	0.52	0.72	0.21
1996q1-2019q4										
Mean	-0.05	0.01	-0.01	-0.18	0.01	-0.01	0.07	-0.14	-0.02	-0.06
MAD	0.17	0.05	0.08	0.38	0.15	0.40	1.01	0.36	0.68	0.08
RMSFE	0.20	0.07	0.10	0.47	0.19	0.55	1.33	0.46	0.87	0.12
log lik score	0.91	2.05	1.75	-0.04	1.16	0.08	-0.84	0.20	-0.38	1.30
Comparison to AL-ME-10obs model										
rel. RMSFE	0.88	1.00	0.86	0.94	1.00	0.94	1.00	0.89	0.97	0.84
Δlog lik score	0.13	0.02	0.34	0.02	0.00	0.05	-0.01	0.07	0.04	0.08
Comparison to AL-SW12-9obs model										
rel. RMSFE	0.79	0.99		1.03	1.01	1.00	1.07	1.04	0.98	1.08
Δlog lik score	0.14	0.04		-0.02	-0.01	-0.04	-0.07	-0.01	0.5	0.03

Notes: See Table 8.

Figure 5: Conditional variance for forecasts of selected observables in the RE and AL models



Note: Conditional variance of one-step-ahead forecast errors. Blue line: RE-2MU-10obs model, green line: AL-2MU-10obs model. Vertical dash line: end of the Full sample.

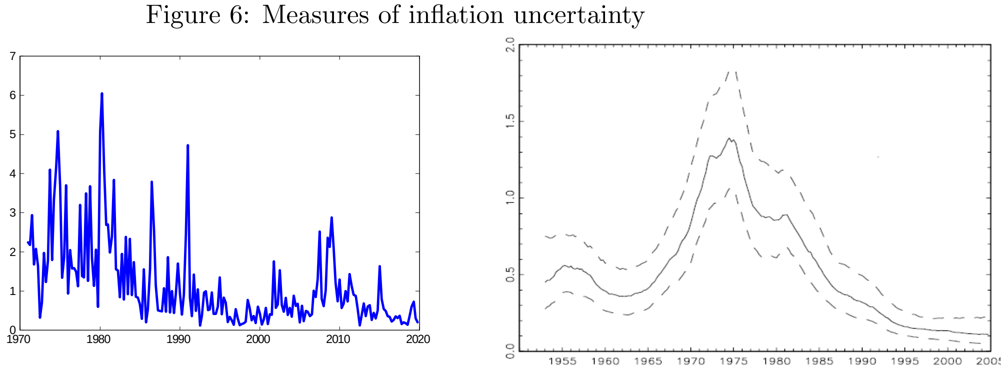
3.2.3 Analysis of the time variation in the AL model

Figure 5 illustrates the time profile of the conditional variance for selected variables. We observe crucial variations in the one-period forecast error variance of the inflation expectations. This variance was as much as three times higher in the 1970s than it has been since 1995. It is interesting that the variance increased again slightly from 2005 onwards with a peak in 2010, but has since declined. The conditional variance is a purely backward-looking object that reacts to past volatility of the data; with inflation expectations holding low and stable after the Great Recession, the uncertainty in inflation expectations has been reduced. The profile in this uncertainty is also consistent with other indicators of forecast uncertainty: for instance, the squared inter-quartile dispersion among individual forecasts in the SPF survey follows a similar historical development (Figure 6, left panel). It also resembles the stochastic volatility process of the variance for the persistent unobserved component in the Stock and Watson UC-SV model for inflation (Stock and Watson 2007-JMCB) as illustrated in the right panel of Figure 6.

The conditional variance in actual inflation realisations follows a more complex profile: it inherits the uncertainty peak in the seventies from the inflation expectations component, but it has an additional peak during the period 1995-2007. The conditional variance in consumption growth is representative of all other real variables: its profile is strongly affected by a cyclical updating process with a positive outlier in the mid-seventies.

Note also that the conditional variance of inflation expectations is typically less than the conditional variance of inflation by a factor of three or more, for both AL and RE models. As a result, the forecast errors for prediction of π^{f0} are significantly lower than those for π^{r1}, see Tables 8 and 9. This also explains why, in the periods of systematic differences between observed inflation and inflation expectations, such as the inflation peak of the 1970s and the post-Covid pandemic period, both models are rather successful in forecasting π^{f0} but significantly under-predict π^{r1}: see section 4.

Figure 6: Measures of inflation uncertainty



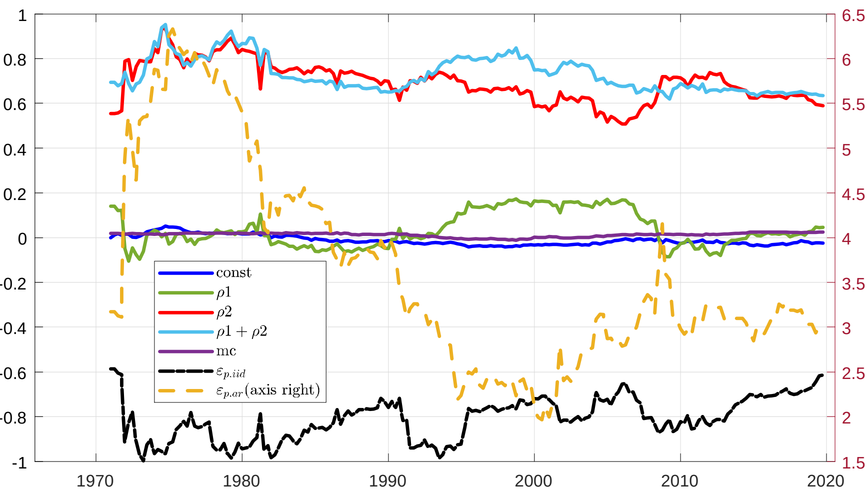
Note: Left panel: Squared inter-quartile range of SPF nowcasts of GDP deflator inflation, variable PGPD D2(T). Right panel: Stock and Watson (2007) variance of permanent inflation component.

This time-variation in the conditional variances is explained by updating in the belief models. In Figure 8, we plot the time variation in the belief coefficients of the forecasting models for inflation, consumption, and investment. Starting with inflation beliefs, the persistence parameter and the constant follow the same profile as documented in SW2012. The updating in the constant follows the surprise in the inflation realization systematically: unexpected higher inflation leads to positive updating in the constant and vice-versa. The updating in the constant is very important for the long run inflation trend, and the scale of this coefficient in Figure 7 is therefore misleading. The updating in the persistence parameter (the sum of ρ_1 and ρ_2, the coefficients of the two lagged inflation terms in the beliefs) reacts in a slightly more complicated way, because it depends on the forecast error and on the level of inflation: in periods when inflation is higher than the long-run mean implied by the belief coefficients, a positive inflation surprise will generate an upward adjustment in the perceived persistence of inflation. However, when inflation is low, a positive surprise in realised inflation leads to lower perceived persistence. Note also the opposite adjustment in the first (ρ_1) and second (ρ_2) autocorrelation coefficients: ρ_1 is particularly important for the impact effect of all shocks, including the highly volatile *i.i.d.* markup shock. The beliefs on the markup shocks also adjust in a similar direction: the coefficient of the persistent markup mimics the updating in the constant, while the changes in the *i.i.d.* markup innovation are somewhere in between the updating in the constant and the persistence.

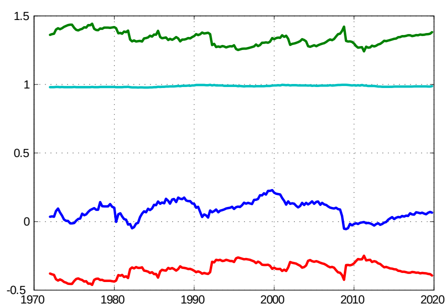
Clearly, large and repeated inflation surprises in the same direction substantially affect the belief coefficients. Understanding this time-variation in the long run perceived inflation target and the perceived persistence and shock sensitivity of inflation is highly relevant for correct interpretation of inflation expectations in the monetary policy analysis.

Figure 7: Time-varying belief coefficients for inflation, consumption and investment, AL-MU-10obs model

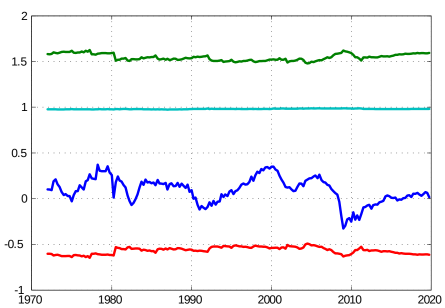
PLM beliefs about inflation



PLM beliefs about consumption



PLM beliefs about investment



The adjustments in simple beliefs for consumption and investment are also very interesting: these variables are perceived as almost unit root processes with a highly time-varying drift in the constant and a time-varying autocorrelation term in the growth process.[24] These beliefs generate a strong cyclical and skewed accelerator process in investment and consumption. In booms, both the constant drift coefficient and the first order autocorrelation coefficient in the growth rate tend to adjust positively, implying a higher long-term mean, and thus reflect higher confidence and optimism in the expectations. Once a negative shock interrupts the growth cycle, the beliefs about the constant in the growth rate decrease rapidly, but the second order persistence parameter ρ_2 adjusts only with a delay. This means that the negative shocks are perceived as relatively persistent, and their impact is extrapolated into the future. Negative

[24]If the beliefs about level are given by $y_t = \mu + \rho_1 y_{t-1} + \rho_2 y_{t-2}$, with $\rho_1 + \rho_2 \approx 1$, then the beliefs about growth rate are $\Delta y_t \approx -\rho_2 \Delta y_{t-1}$.

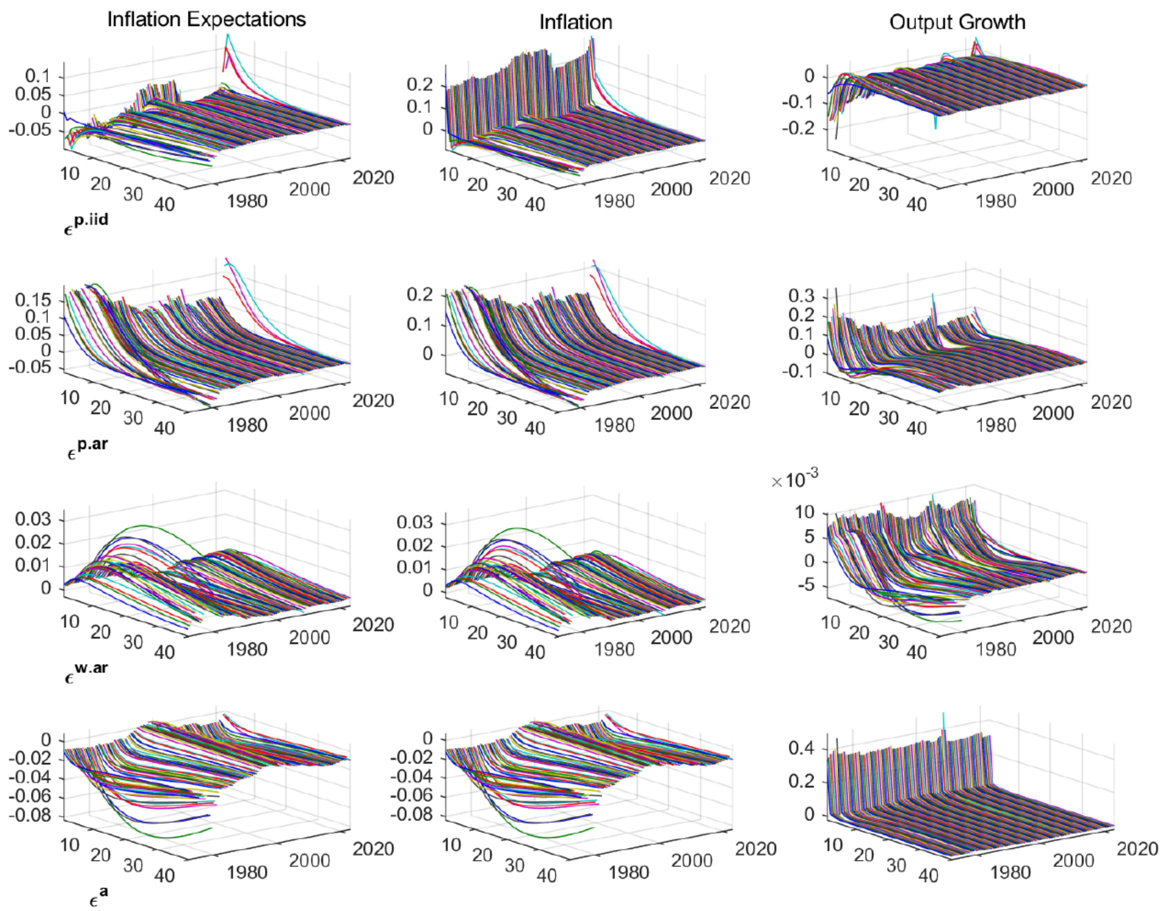
shocks in the beginning of a recession are therefore amplified and contribute to asymmetry in the growth rate over the cycle.[25]

The first order autoregressive parameter in the consumption beliefs, ρ_1, is of additional importance, because it interacts with the habit coefficient in the consumption Euler equation. When $\rho_1 - 1$ approaches the habit parameter, habits and growth rate extrapolation become reinforcing mechanisms that make consumption extremely sensitive to interest rate fluctuations. This explains the peak in the conditional variance of consumption in the mid seventies, visible in Figure 5. Note also how the Great Recession has a huge impact on the drift growth factor in consumption and investment beliefs, and how long it took for these beliefs to re-adjust in the recovery.

The time-varying impulse response functions in Figure 8 confirm this amplifying or attenuating effect of the belief coefficients on the transmission mechanism of the various shocks over the cycle.

[25] These implications for consumption and investment expectations should be verified by the survey expectations as well. We plan to do this in a follow-up paper.

Figure 8: Time-varying IRF functions in the AL-model:
iid and AR price markup, AR wage markup, and productivity shocks



Note: Left column: inflation expectations, middle column: inflation, right column: output growth. Top row: price markup *iid* shock; second row: price markup persistent shock; third row: wage markup persistent shock; bottom row: productivity shock.

The time variation in the IRFs of the various shocks on inflation expectations and inflation realisation follows a similar time profile. The updating in the perceived inflation persistence and in the impact coefficients of the persistent markup innovations - which update in the same direction - are crucial for these dynamics. High impact effects and high persistence in the inflation belief models explain the high sensitivity and persistence in the seventies and the gradual moderation of the response later on. Inflation expectations and the actual inflation response are generally consistent with each other, which is not automatically guaranteed in an AL-context. The time-profile in the IRF on the *i.i.d.* markup shock's component deviates from the other shocks. The belief coefficient on this component, plus (most importantly) the first order autoregressive coefficient ρ_1 which shows a notable increase between 1995 and 2010, is responsible for this specific time variation. This component, which is important for high frequency inflation fluctuations, explains the second peak between 1995 and 2005 in the variance of the one-period ahead inflation uncertainty; see Figure 5.

Table 10: Conditional variance decomposition for the AL-2MU-10obs model

	ε^a	ε^b	ε^g	ε^{qs}	ε^m	ε^{piid}	ε^{wiid}	ε^{par}	ε^{war}	$\xi^{\pi r}$	$\xi^{\pi f}$
1 quarter horizon											
π^{f0}	0.40	0.02	0.00	0.00	0.00	4.44	1.04	93.27	0.03	0	0.80
π^{r1}	0.12	0.00	0.00	0.00	0.00	68.89	0.31	17.27	0.00	13.40	0
w	0.00	0.14	0.00	0.00	0.03	14.94	83.49	0.09	1.31	0	0
y	19.66	46.70	17.55	2.79	9.68	0.68	0.39	2.55	0.01	0	0
1 year horizon											
π^{f0}	2.58	0.45	0.01	0.00	0.11	6.75	6.16	83.37	0.25	0	0.31
π^{r1}	1.07	0.18	0.01	0.00	0.05	47.60	2.60	40.21	0.11	8.78	0
w	0.01	1.87	0.00	0.00	0.47	11.53	81.73	0.72	3.67	0	0
y	8.98	64.68	5.02	1.22	16.99	1.24	0.31	1.54	0.01	0	0
10 year horizon											
π^{f0}	12.07	14.86	0.36	0.34	6.17	5.45	15.76	43.35	1.54	0	0.10
π^{r1}	8.54	9.90	0.26	0.23	4.12	28.12	11.48	31.51	1.13	4.70	0
w	0.35	37.61	0.06	0.03	15.57	6.34	32.25	3.77	3.01	0	0
y	11.44	56.38	3.48	0.23	22.98	0.90	1.08	3.44	0.10	0	0

Note: The average decomposition over the Full sample is presented.

Table 10 illustrates that the impact effects of the wage markup shocks on inflation and on inflation expectations are strongly reduced relative to the RE-model. Most importantly, the effects of these shocks are now transitory, though they were responsible for the long term inflation trend in the RE-model. In fact, in this AL model, the long term inflation trend is no longer explained by exogenous shocks. It is learning about the constant in the belief equation that explains the trend. This means that all shocks can contribute to long term inflation expectations, depending on how the updating in the inflation beliefs is affected. [26] This decomposition illustrates that, for short and medium-term dynamics, the AL and the RE models provide similar interpretations in terms of shock contributions, except that the wage shocks are less important. At the long forecast horizon, all shocks now contribute to the inflation variance. There is also a non-negligible role for demand shocks such as the risk premium and monetary policy shocks. However, the learning responses must be added on

[26] The table reports the average of the variance decomposions computed at every time period of the complete sample, keeping the transmission mechanism constant. Therefore, nonlinear effects such as the shocks affecting the constant, which will will affect future expectations in the next period, are ignored. We consider these nonlinear interactions in detail in Rychalovska et al. (2024).

top of this static analysis, making the contributions of different shocks highly nonlinear and non-additive.[27]

In sum, the AL model provides a more informative analysis of inflation dynamics than the RE model. The flexibility of the AL model allows for better identification of the persistent process in inflation dynamics through time-varying beliefs about the constant, persistence, and impact coefficients of the shock components. In section 4, we show how this feature of the AL model helps us to interpret the recent inflation dynamics. Moreover, instead of explaining the long term inflation trend by exogenous shocks, it is the expectations and the updating of beliefs that are now crucial for inflation anchoring. This basic AL-result is robust across various specifications. For instance, adding an inflation target shock in this AL-model does not change the results as it did for the RE-model. The results of this exercise are shown in Table B4: the inflation target shock remains minimal, and the marginal likelihood of this model does not improve relative to the model without target shock.

4 The post-Covid inflation surge

In recent years we have seen a rather uncommon pattern in the joint evolution of inflation and inflation expectations. After remaining low and stable for several decades, inflation increased sharply in the post-Covid period, inviting some comparisons with episodes in 1970s and 1980s (Blanchard 2022, Reis 2021). Recent studies have attributed this high and persistent level of inflation to the combination of different disturbances, such as supply chain pressures caused by the rapid recovery from the pandemic, surge in energy prices, and the war in Ukraine, coupled with very tight labor markets and rising demand. Some literature reports evidence of structural changes in the economy in this recent period. In particular, Harding et al (2023) illustrate the presence of non-linearities in the Phillips curve during and after the Covid pandemic, and resulting stronger transmission of cost-push and demand shocks to inflation. Ball et al (2022) point to a stronger pass-through into core inflation from past shocks to headline inflation. The complex nature of the inflation process significantly impacted the ability of policymakers and professional forecasters to generate reliable projections of inflation. As a result, in recent years we have witnessed unusually poor inflation forecasts, characterized by significant and persistent prediction errors.

In this section, we investigate how our models interpret inflation experience during the pandemic and in the post-Covid recovery period. We assess the ability of the RE and AL models to foresee the uncommon inflation developments relative to professional forecasters, who consistently underestimated inflation. By exploring our setup with separate transitory and persistent mark-up shock components, we can gain a more detailed understanding of the forces driving inflation. Furthermore, the AL framework, which incorporates a time-varying transmission mechanism, allows us to capture changes in the properties of the

[27] In SW2012, we illustrated how the learning responses react to various shocks depending on the state of the economy.

data-generating process of inflation in recent years. Additionally, it provides the means to assess the implications of these changes for shock propagation. Our analysis contributes to very recent literature that examines the evolution of inflation and expectations around the Covid period. The paper most relevant to our work is Bae et al (2024). The paper uses micro-level data from the Michigan Survey of Consumers to analyze the dynamics of inflation expectations since 2021. They show that persistently high and heterogeneous expectations of consumers with less education and lower income are mainly responsible for widening of the distribution in the recent period. They also illustrate that a simple estimated AL model (based on autoregressive forecasting functions) is able to capture the evolution of inflation expectations over time for different demographic groups well. Their model interprets the surge of inflation in 2021 as primarily the result of a persistent price markup shock.

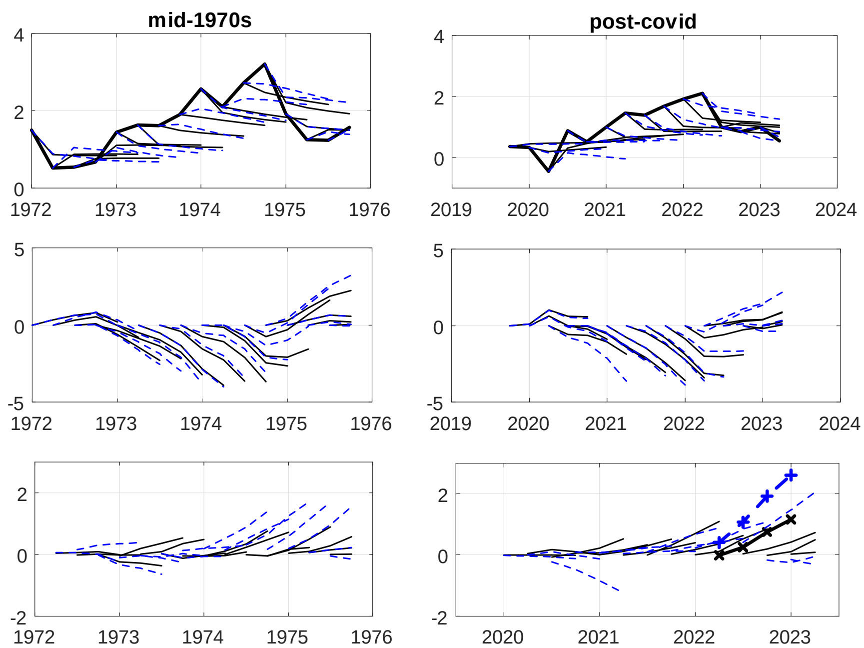
We re-evaluate our RE-2MU-10obs and AL-2MU-10obs models on the sample extended throughout 2023q2, maintaining the parameters fixed at the level obtained on the baseline estimation sample.[28] To account for the exceptional magnitude of the crisis, we modify the structure of the fundamental shock processes by introducing heteroskedasticity adjustment around the Covid recession period. In particular, we introduce the scaling factor $exp(\gamma)$ that increases the variance of the structural shocks during 2020q2 and 2020q3. The scaling factor multiplies the elements of the variance-covariance matrix of fundamental shocks in the primary Kalman filter. In addition, for the AL model, we modify the secondary Kalman Filter, which is responsible for the belief adjustment. Specifically, we introduce the theoretically consistent heteroscedasticity intervention with the same scaling factor to allow for increased volatility of PLM forecast errors and perceived uncertainty. The value of the scaling factor equal to $exp(5)$ is chosen to maximize the model fit.

The hair plot in Figure 9 compares the models' forecasting performance (up to 4 quarters ahead) relative to realized inflation and relative to SPF across two subperiods characterized by excessively high inflation: the mid-1970s and post-Covid. The figure illustrates a striking similarity between these two episodes, both in the magnitude and in the persistence of the inflation surge. The pattern of the 1 to 4Q model forecasts shown on the upper panel of Figure 9 illustrates that the AL model generates systematically higher predictions in both subperiods compared to the model with RE. The bottom panel of Figure 9, which displays the cumulative deviations between forecasts generated by the models and SPF forecasts, shows that the RE model essentially replicates inflation expectations generated by professional forecasters, particularly in the short term. This is true even in periods when SPF systematically underpredicts observed inflation, as during the inflation peak of 1972-1975 and the post-pandemic period. In contrast, the AL model can produce expectations that differ more significantly from the SPF, exhibiting a smaller degree of under-prediction in

[28] For this exercise, we omit the 'news' specification for the inflation and wage persistent shock processes. Utilizing contemporaneous innovations seems more appropriate to capture the specific period of the COVID-19 pandemic with abrupt changes in inflation. However, as discussed in Section 2, this assumption is not crucial for the results.

periods of substantial positive inflation surprises.[29] As an illustration, the bold lines with crosses on the lower right panel of Figure 9 demonstrate that, up to 4 quarters ahead, the AL model predicts higher inflation than both the SPF and the RE model at the peak of the inflation surge in 2022q2.

Figure 9. Model forecast in periods of elevated inflation



Note: See Fig.1.

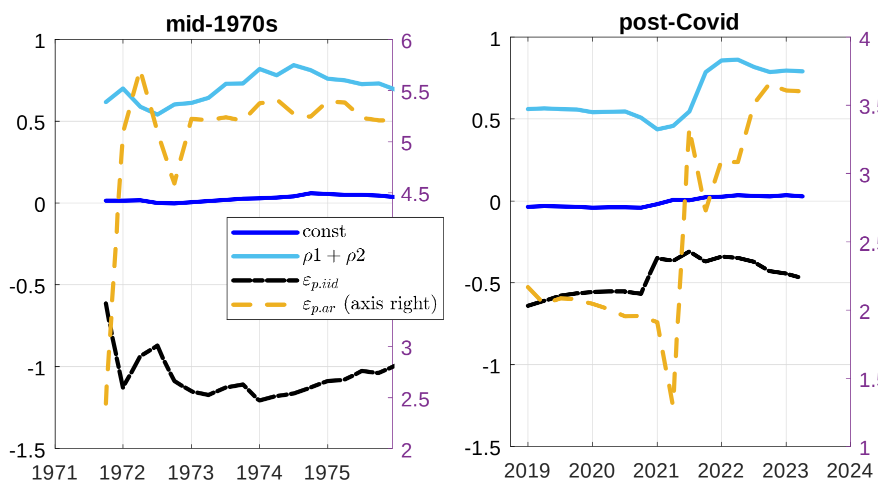
Which features of the AL model determine its superior ability to anticipate periods of high inflation? We argue that the property that allows the AL model to generate amplified inflation projections is its modified transmission mechanism based on belief updating. The more flexible expectation formation mechanism enables the model to pick up the time-varying nature of fundamental processes driving inflation and to more effectively capture and anticipate shifts in inflation levels and volatility. As a result, the AL model can better identify the persistent nature of inflation shocks in the 1970s and the post-covid period.

[29] The behavior of the RE model, which closely tracks the SPF expectations, is explained in section 2.2.2: as is obvious from the Figure 5, conditional variance of π^{r1} is more than three times larger than that of π^{f0}, which is translated into a significantly larger sensitivity of the likelihood to observation errors in the SPF expectations variable. The estimated parameters are selected in such a way as to ensure that the time series of π^{f0} is fitted significantly better than π^{r1}.

The AL model, in contrast, generates much higher conditional variances of inflation expectations for these periods, which allows it to be more flexible when the two observables disagree materially. Measures of forecast bias and forecast RMSFE in Tables 8 and 9 confirm this view.

Figure 10 shows the time variation of inflation PLM beliefs for constant, persistence and price markup shock components over two subperiods. Figure 10 indicates that, following the sharp increase in inflation in the mid-1970s and in 2021, agents significantly revised their perceptions about inflation persistence and the contribution of the persistent markup shock upwards.

Figure 10.PLM beliefs for inflation

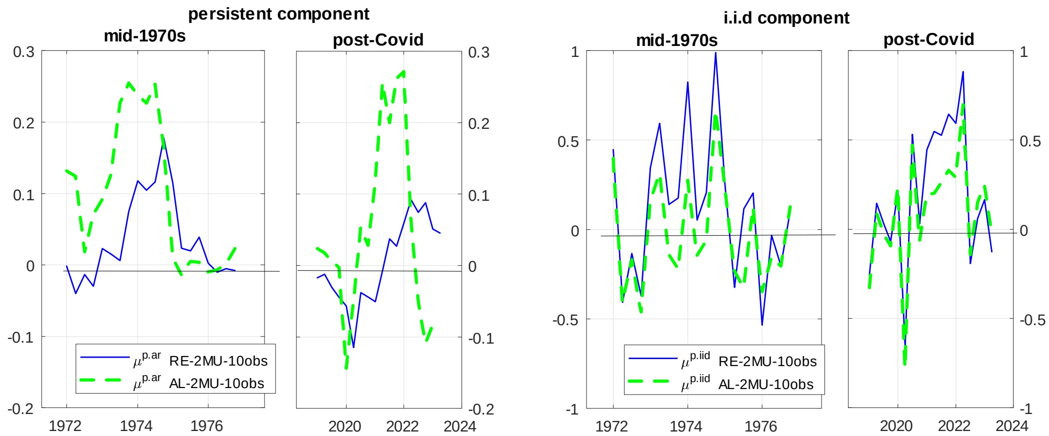


Due to modified model dynamics driven by the time variation of beliefs, Kalman filter attributes a larger share of inflation surprise to the effect of the persistent shock process, so that the autoregressive shock component is exploited more intensively in explaining inflation. In contrast, in the RE model featuring a stable transmission mechanism, the Kaman filter interprets the repeated surprises as mainly an i.i.d process.

To illustrate the difference in identification of underlying inflation shocks, Figure 11 plots the components of the price mark-up shock identified by models with rational and imperfectly rational beliefs. While the sum of the persistent and transitory components is essentially the same, there are significant differences in the identified persistent component: the AL model relies on the persistent component to a larger degree to explain the behavior of observed inflation, while the RE model utilizes a greater increase in the i.i.d. component.

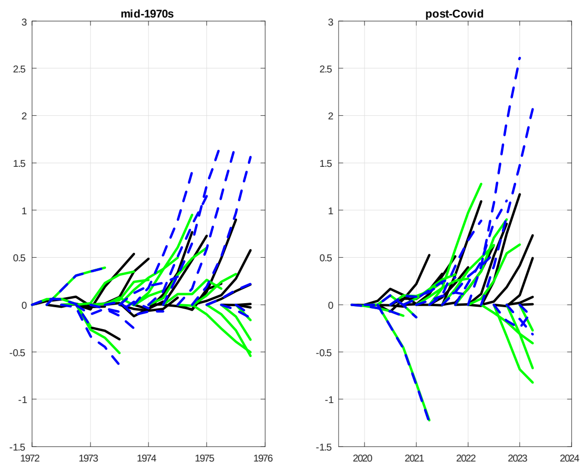
In other words, the sustained rise of inflation in 1970s and 2021-2022 surprised the SPF forecasters, who were slow to revise their predictions upwards, and the RE model has interpreted this discrepancy as a sequence of large and systematically positive i.i.d. mark-up shocks, which increased inflation but not SPF expectations. Indeed, Figure 11 indicates that the behavior of the i.i.d. component becomes remarkably systematic and temporary correlated with the persistent component, particularly around the recent inflation surge. The RE model explores the systematic pattern of the i.i.d. component to a larger extent in explaining the rise in inflation in 2021-2022.

Figure 11. Components of identified price mark-up shocks



The ability of the AL model to identify the persistent component of the price-mark-up shock as a more important driver of inflation and to generate higher inflation forecasts in the mid-1970s and post-Covid is explained by its time-varying transmission mechanism. The important role of belief updating under AL is confirmed by Figure 12, which compares the performance of learning models with and without updating relative to RE and SPF. To calculate the forecasts under stable transmission mechanims, we stop the belief updating process for 1972-1976 and 2019q3-2023q2. The green line shows the cumulative difference between the predictions from the AL model without updating and from SPF forecasts. As before, the black line is the cumulative difference between the RE model and SPF forecasts at horizons 1 to 4, dashed blue lines are the corresponding difference between the AL model with updating and SPF. Figure 12 emphasizes the important effect of updating: at the peak of the inflation increase in 1974 and in 2022, the AL model with time variation strongly over-predicts the SPF, while the AL model with frozen beliefs tends to produce forecasts close to the RE model predictions. Therefore, updating is crucial to generate rising inflation persistence and higher inflation forecasts in the mid-1970s and in the post-Covid period.

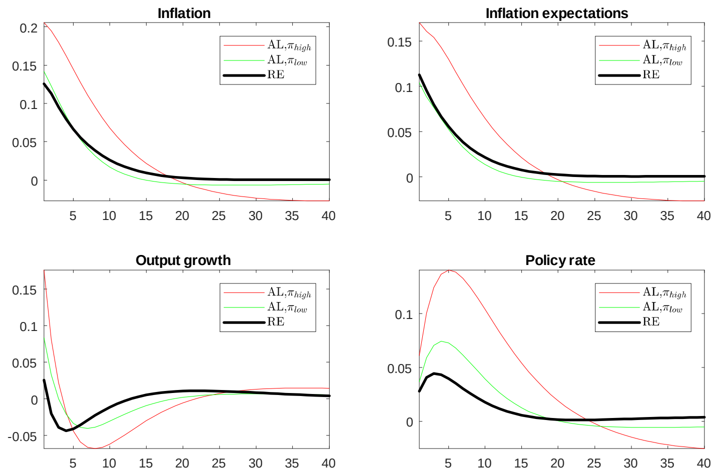
Fig.12 SPF and model forecasts: the role of updating



Note: Cumulative deviations of the model forecasts relative to SPF nowcasts. Green line: AL-2MU-10obs without updating. Black solid line: RE-2MU-10obs. Blue dashed line: AL-2MU-10obs with updating.

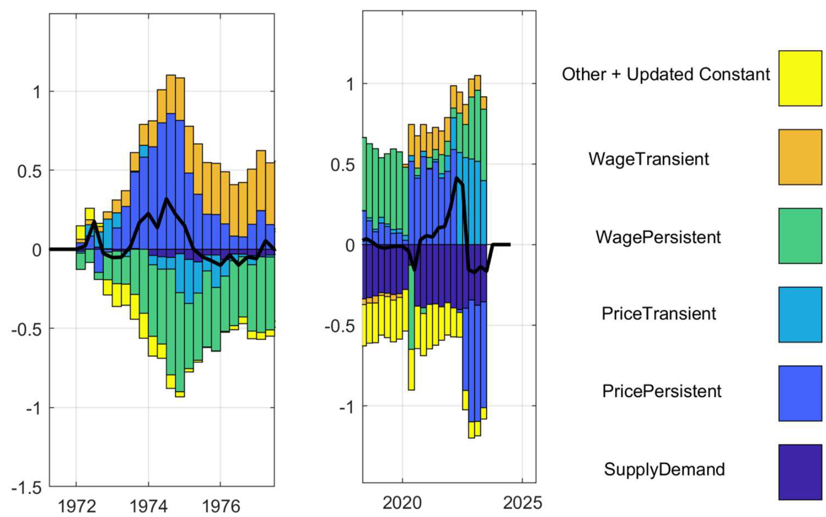
The importance of time variation is further emphasized in figure 8, which demonstrates that transmission mechanism under AL can have important consequences for shock propagation. In particular, the AL model based on belief updating process and variation in the perceived inflation persistence in particular, can produce nonlinear responses to shocks in periods of low(stable) and high inflation. Figure 13 shows the averaged AL impulse responses to a persistent markup shock in the states of high and low inflation, and thereby summarizes the key insight of figure 8. We define a period of high inflation, as annual inflation above 6%. This implies that our sample includes 2 periods of high inflation: the mid-1970s and the post-Covid period 2021-2022. Figure 13 illustrates that inflation and inflation expectations become much more sensitive to persistent markup shocks in the period of high inflation. Therefore, the AL model has the potential to forecast significant drifts in inflation and poses a higher risk of unanchoring inflation expectations in periods of high inflation than does the RE model. As a result, monetary policy may need to respond more aggressively to deal with supply shocks in periods of elevated inflation, leading to more pronounced negative real consequences and thus more severe policy trade-offs in the longer run. This conclusion is similar in spirit to the result shown in Harding et al (2023), who propose a structural macro model with a non-linear Phillips curve to study inflation dynamics in the post-covid period. They illustrate that a non-linear model can generate stronger transmission of the cost-push shock in 2022, when inflation was high, than can a linear model.

Figure 13. Average impulse responses to $\varepsilon^{p.ar}$: high and low inflation



Finally, we analyse the real-time historical decomposition of the models' forecast errors, which provides a comprehensive overview of the key drivers of unexpected inflation developments. Figure 14 presents the difference in the decomposition of the AL and RE models' 1Q forecast errors. The black solid line reflects the difference in the models' forecast errors, with positive values indicating higher predictions generated by the AL model. To produce the decomposition, we run the model utill time "t", generate the forecast for the periods "$t + k$"$(k = 1 : 4)$, and then compute the forecast errors using realized values. Conditional on the information set at time "t", we calculate the contribution of the various groups of shocks to the forecast errors. We consider the following main groups of structural innovations in our analysis: iid and persistent price markup shocks; iid and persistent wage markup shocks; other supply and demand shocks. Figure 14 emphasizes the dominant role of the persistent price markup shock in the AL model compared to the RE model in explaining the episodes of heightened inflation in the mid-1970s and 2021-2022. Moreover, during the post-Covid period, we observe that the AL model effectively utilizes a combination of persistent and i.i.d price mark-up shocks to capture the sustained surge in inflation followed by a reversal in the inflation trend. Specifically, in the AL model, the time variation of beliefs, in particular higher perceived inflation persistence, enhances the propagation all of the shocks affecting inflation. As a result, the innovations to the transitory markup component can have longer-lasting effects and thereby contribute to more pronounced inflation dynamics. This argument is further supported by Figure 8, which illustrates the dynamic responses to shocks over the sample, including the post-Covid era. Under the RE framework, the impact of iid markup shock dissipates within one or two periods, while the shock may have longer-lasting effects under AL when persistence is high. Our result is consistent with other recent studies, which emphasize the dominant role of supply shocks in explaining recent inflation experiences, [30] and, more specifically, the importance of the pass-through of headline inflation shocks to core inflation, resulting in more persistent inflation dynamics (Ball et al, 2022).

Figure 14. Difference in the historical decomposition of inflation forecast errors



Note: AL-2MU-10obs - RE-2MU-10obs

Our framework allows us to address two more important questions: Why did SPF forecasters systematically miss inflation during the first peak in mid-1970s and again post-Covid? Was the underprediction due to certain changes in the nature of the underlying shocks that forced the forecasters to be inaccurate?

Based on our models, we suggest the following interpretation. The data generating process in these two periods of elevated inflation became much more complex. In particular, in 2021-2022, inflation was affected by an unusual sequence of positive i.i.d. innovations (which can be associated with short-lived energy shocks) in addition to autocorrelated disturbances (which can reflect the impact of rising energy prices on all components of inflation, prolonged supply chain disruption, global factors, tight labor markets and other more persistent supply side drivers). Due to repeated transitory innovations, the i.i.d. shock began to behave in a persistent manner, which made it very hard to distinguish between the separate components of the mark-up process. In both episodes of systematic forecast errors, SPF participants did not entertain the possibility that 'independent' innovations were now almost collinear, and they continued to expect the long sequence of positive i.i.d. shocks to end, and therefore continued to 'see through' this sequence. This interpretation is consistent with the stance that prevailed in the beginning of the inflation surge among the policymakers

[30] In the context of the non-linear Phillips curve, some studies also discuss the role of demand factors, which could explain about 20-25% of an inflation surge.

and academics. In particular, in his speech on the economic outlook in August 2021, the Fed Chair Powell emphasized the complexity of inflation dynamics, but also declared “[t]he absence so far of broad-based inflation pressures”. Ball et al (2021) presented estimates that “underlying inflation could rise to about 2.5-3% by 2023” and claimed that “there is little risk of a 1960s-type inflationary spiral”. Therefore, inflationary pressure was mostly attributed to the effects of the transitory energy price shocks, while the importance of persistent factors that affected inflation at the same time was underestimated and not well understood. According to our model, it is the mixture of different shocks and the temporary change in the data generating process of fundamental shocks driving inflation that confused agents. Given that the RE model’s predictions closely mirror the SPF forecasts, it seems that professional forecasters behaved similarly to rational agents who perceived the economy as a stable data-generating process. Instead, AL agents did recognize the shift in the nature of the inflation shocks and adjusted forecasts to the changed economic conditions more rapidly than SPF participants.

We conduct the following exercise as an additional check of the "confused forecasters" hypothesis. For the last 10 periods of the extended sample, 2021q1-2023q2, we create an alternative time series for the inflation forecasts, in which the difference between π^{r1} and π^{f0} is reduced by a factor of two. In this counterfactual world, the forecasters still under-predict the quick rise of inflation, but to a significantly smaller degree. We evaluate the RE and AL models, without re-estimating them, using this new data, and investigate the identified innovations to the two components of the price mark-up shock. Consistently with the "confusion" explanation, less under-prediction of inflation leads to a significantly smaller i.i.d. component and to a larger persistent component. In particular, under RE, persistent innovation increases by as much as 100%. In addition, the correlation between the two innovations in the 2020q1-2023q2 period decreases significantly. Thus, it is indeed the sudden temporary deviation of the correlation structure of the innovations from the assumed independence that is responsible for the SPF forecasters being unable to track inflation correctly during the post-Covid episode.

To sum up, our RE model interprets the mid-1970s and the post-pandemic inflation experience as low-probability event of a few periods in which both components of the price mark-up shock were systematically positive. Within this interpretation, agents reacted only to the relatively small persistent component and saw through the large i.i.d. component. However, as they were unable to predict that the i.i.d. component would remain positive for a number of periods, they made systematic errors, under-predicting inflation. AL agents were more successful in identifying the persistent nature of the inflation process via the higher perceived inflation persistence and increased role of the autocorrelated component of the mark-up shock in their PLM. The better fit of the AL model indicates that such identification is better supported by the data. The measure of inflation expectations generated by our AL model deviates from the SPF forecasts during periods of elevated inflation. Specifically, from 2021 to 2022, it exhibits greater sensitivity to higher realized inflation, resembling the characteristics of household survey expectations in this regard. The time-varying transmission mechanism reveals stronger propagation of supply shocks in periods of elevated inflation.

5 Conclusion

A proper integration of survey expectations - as measured by the SPF - into a DSGE model makes it possible to identify transitory and persistent shocks in inflation separately. Improving the efficiency of the model filter improves forecasts for both inflation and for other macrovariables. Under AL, updating of belief models augmented with timely information signals from survey data, generate time-varying estimates of the perceived inflation target, persistence, and sensitivity to shocks. In this way, the model captures the joint dynamics in the first and second moments of realized and expected inflation.

Our exercise has some interesting methodological implications for the efficient application of AL in empirical macromodels. In Slobodyan and Wouters (2012b), we argued that small belief models were sufficient to capture the important time-variation in inflation expectations and persistence. Simple expectations models are also favoured by experimental studies (Hommes and Zhu 2014). The results in this paper suggest that expectations contain detailed information on the precise nature of the latest inflation developments. Belief models must be sufficiently flexible to capture this information, so it may be necessary to augment belief specifications with a minimum set of latent factors that summarize and transmit all relevant signals.

Our framework provides an interpretation of the behavior of professional forecasters during the recent inflation surge. We show that inflation expectations remained broadly anchored because SPF agents perceived inflation as being driven by stable data-generating process, in line with the rationality assumption. We demonstrate that a more flexible expectation formation mechanism implied by AL enables a model to pick up the time-varying nature of fundamental processes driving inflation and to more effectively anticipate the latest inflation rise. The model can therefore partially correct SPF forecasters' under-prediction of inflation in the post-Covid era.

More research is necessary to test alternative specifications of the belief models in AL. There seems to be a trade-off between simple specifications on the one hand and sufficiently informative specifications on the other. Another issue is whether agents update their beliefs gradually over time or whether a regime switching setup that allows for sharp adjustments in beliefs is more appropriate. Whether expectations are determined only by a backward-looking updating process or there is also a role for forward-looking expectation shocks is another remaining question. Finally, we would like to test whether time-variation delivered by the adaptive learning process is consistent with the time-variation as detected in reduced form models with time-variation in coefficients and volatilities.

This paper focuses on inflation expectations. In followup work, we test whether survey data on other variables, such as consumption, investment, and wages are equally important for model performance.

References

Adam, K. and M. Padula (2011), "Inflation dynamics and subjective expectations in the United States", *Economic Inquiry*, 49(1), 13–25.

Albanesi, S., Chari, V. V., and L. J. Christiano (2003), " Expectation Traps and Monetary Policy, " The Review of Economic Studies, 70(4), 715–741.

Ang, A. , Bekaert, G., and M. Wei (2007), "Do macro variables, asset markets, or surveys forecast inflation better?," *Journal of Monetary Economics*, 54(4), 1163-1212.

Angeletos, G.M, H. Zhen, and K.A. Sastry (2020), "Imperfect Macroeconomic Expectations: Evidence and Theory," NBER Chapters, in: NBER Macroeconomics Annual 35: 1-86.

Bae, E., Hodge, A., and A. Weber (2024), "U.S. Inflation Expectations During the Pandemic, " IMF working paper 24/25.

Ball, L., Leigh, D., and P. Mishra (2022), "Understanding U.S. Inflation During the COVID Era," IMF Working Papers 2022/208, International Monetary Fund.

Ball, L., Gopinath, G., Leigh, D., Mishra, P., and A. Spilimbergo (2021), "US Inflation: Set for Take-Off?", VoxEU. 07 May 2021.

Bordalo, P., N. Gennaioli, Y. Ma, and A. Shleifer (2020), "Overreaction in Macroeconomic Expectations," American Economic Review 110 (9): 2748-82.

Branch, W.A., (2007), "Sticky information and model uncertainty in survey data on inflation expectations". *Journal of Economic Dynamics and Control*, 31, 245–276.

Carvalho, C, Eusepi, S., Moench, M., and B.Preston (2023), "Anchored Inflation Expectations," *American Economic Journal: Macroeconomics*, 15 (1), 1-47.

Cavallo, A., Cruces, G., and R. Perez-Truglia (2017), "Inflation Expectations, Learning, and Supermarket Prices: Evidence from Survey Experiments, " American Economic Journal: Macroeconomics 9 (3): 1–35.

Cerrato, A., and G. Gitti (2022), "Inflation Since COVID: Demand or Supply ", mimeo.

Chari, V. V. , Christiano, L.J.,and Eichenbaum, M., (1998), "Expectation Traps and Discretion," Journal of Economic Theory, Elsevier, vol. 81(2), pages 462-492, August.

Chou, J., Easaw, J., and P. Minford (2023), "Does inattentiveness matter for DSGE modeling? An empirical investigation," Economic Modelling, Volume 118, 2023.

Clarida, R., Gali, J., and Gertler, M. (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory, " The Quarterly Journal of Economics, 115(1), 147–180.

Cochrane J. (2016), "Do Higher Interest Rates Raise or Lower Inflation?", mimeo.

Coibon, O. and Y. Gorodnichenko (2015), "Is The Phillips Curve Alive and Well After All? Inflation Expectations and the Missing Disinflation" American Economic Journal – Macroeconomics 7(2015), 197-232.

Coibion, O. and Y. Gorodnichenko (2015), "Information rigidity and the expectations formation process: A simple framework and new facts", American Economic Review 2015, 105(8): 2644–2678.

Coibion, O., Y. Gorodnichenko, and R. Kamdar (2018), "The Formation of Expectations, Inflation, and the Phillips Curve, " Journal of Economic Literature 56 (4): 1447–91.

De Graeve, F. & Emiris, M. & Wouters, R. (2009), "A structural decomposition of the US yield curve," Journal of Monetary Economics, Elsevier, vol. 56(4), pages 545-559, May.

Del Negro, M. and Eusepi, S. (2011), "Fitting observed inflation expectations," Journal of Economic Dynamics and Control, Elsevier, vol. 35(12), pages 2105-2131.

Del Negro, M., M. Giannoni and F.Schorfheide (2015), "Inflation in the Great Recession and New Keynesian Models," American Economic Journal: Macroeconomics, American Economic Association, vol. 7(1), pages 168-96, January.

Del Negro, M. and Schorfheide, F. (2013), "DSGE Model-Based Forecasting," Handbook of Economic Forecasting, Elsevier.

Eusepi, S., M. Giannoni, and B. Preston (2015), "The Limits of Monetary Policy with Long-term Drift in Expectations, " mimeo, The University of Melbourne.

Eusepi, S. and B. Preston (2011), "Expectations, Learning, and Business Cycle Fluctuations, " American Economic Review 101 (6): 2844–72.

Eusepi, S. and B.Preston (2016), "The Science of Monetary Policy: An Imperfect Knowledge Perspective", NYFed Staff Report No. 782, June 2016.

Evans, G., and S. Honkapohja (2013): “Learning as a Rational Foundation for Macroeconomics and Finance”, Ch. 2 in Rethinking Expectations: The Way Forward for Macroeconomics, Roman Frydman and Edmund S. Phelps (eds.), Princeton University Press.

Evans, G. and B. McGrough (2016), "Interest Rate Pegs in New Keynesian Models", April 2016.

Evans, G, K. Mitra and S. Honkapohja (2016), "Expectations, Stagnation and Fiscal Policy", May 2016.

Faust J. and J.H. Wright (2013), "Forecasting inflation, " Handbook of economic forecasting, vol. 2, Elsevier (2013), pp. 2-56 chapter 1.

Friedman, M. (1968), “The Role of Monetary Policy,” American Economic Review 58, pp. 1-17.

Fuhrer, J. (2017), "Expectations as a Source of Macroeconomic Persistence: Evidence from Survey Expectations in a Dynamic Macro Model, " Journal of Monetary Economics 86 (April): 22–35.

García-Schmidt, M. and M. Woodford (2015), "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis," NBER Working Papers 21614, National Bureau of Economic Research, Inc.

Gati, L. (2023), "Monetary Policy & Anchored Expectations. An Endogenous Gain Learning Model," Journal of Monetary Economics, Elsevier, vol. 140(S), pages 37-47.

Hajdini, I., (2020), "Misspecified Forecasts and Myopia in an Estimated New Keynesian Model, " mimeo, Drexel University.

Harding, M., J. Linde, and M.Trabandt (2023), "Understanding post-covid inflation dynamics ," Journal of Monetary Economics, Volume 140, Supplement.

Hommes, C. and Zhu, M. (2014), "Behavioral learning equilibria," Journal of Economic Theory, Elsevier, vol. 150(C), pages 778-814.

Hommes, C., K. Mavromatis, T. Özden, and M. Zhu (2023), "Behavioral learning equilibria in New Keynesian models," Quantitative Economics, 14: 1401-1445.

Kohlhas, A.N., and A. Walther (2021), "Asymmetric Attention," American Economic Review, 111 (9): 2879-2925.

Lucas, R. E. (1972), "Expectations and the Neutrality of Money," Journal of Economic Theory 4.

Mankiw, N. G., and R. Reis (2003), "Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve." Quarterly Journal of Economics 117 (4), 1295-1328.

Mankiw, G., R. Reis and J. Wolfers (2004), "Disagreement about Inflation Expectations", NBER Macroeconomics Annual 2004, Volume 18.

Melosi, L., (2017), "Signalling Effects of Monetary Policy," Review of Economic Studies, 84(2), 853-884.

Milani, F. (2011), "Expectation Shocks and Learning as Drivers of the Business Cycle", The Economic Journal, 121 (May), 379–401.

Milani, F. and A.Rajbhandari (2012), "Observed Expectations, News, and the Business Cycle", mimeo.

Monti, F. (2010), "Combining Judgment and Models", Journal of Money, Credit, and Banking, Volume 42, Issue 8.

Nunes, R. (2010), "Inflation Dynamics: The Role of Expectations", Journal of Money, Credit and Banking, Vol. 42, No. 6.

Orphanides, A., and J.C. Williams (2005a), "Inflation scares and forecast-based monetary policy, " Review of Economic Dynamics 8 (2005) 498–527.

Orphanides, A., and J.C. Williams (2005b), "Imperfect Knowledge, Inflation Expectations, and Monetary Policy, " In The Inflation-Targeting Debate, edited by Ben S. Bernanke and Michael Woodford, 201–48. Chicago and London: University of Chicago Press.

Ormeno, A. and K. Molnar (2015), "Using Survey Data of Inflation Expectations in the Estimation of Learning and Rational Expectations Models", Journal of Money, Credit and Banking, Vol. 47, No. 4.

Powell, J.H. (2023), "Monetary Policy Challenges in a Global Economy," International Monetary Fund, speech delivered on November 09, 2023.

Roberts, J.M. (1995), "New Keynesian Economics and the Phillips Curve", Journal of Money, Credit and Banking, Vol. 27, No. 4, Part 1 (Nov., 1995), pp. 975-984

Roberts, J.M. (1997), "Is Inflation Sticky", Journal of Monetary Economics 39 (1997) 173 196.

Rychalovska, Y., Slobodyan, S., and R. Wouters (2023), "Professional Survey Forecasts and Expectations in DSGE Models ", CERGE-EI Working Papers wp766, 2023.

Sargent, T. J. (1999), " The Conquest of American Inflation. " Princeton University Press.

Slobodyan, S. and R. Wouters (2012a), "Learning in an estimated medium-scale DSGE model," *Journal of Economic Dynamics and Control*, 36(1), 26-46.

Slobodyan, S. and R. Wouters (2012b), "Learning in a Medium-Scale DSGE Model with Expectations Based on Small Forecasting Models," *American Economic Journal: Macroeconomics*, 4(2), 65-101.

Smets, F., Warne, A. and R. Wouters (2014), "Professional forecasters and real-time forecasting with a DSGE model," International Journal of Forecasting, Elsevier, vol. 30(4), pages 981-995.

Smets, F. and R. Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," American Economic Review, American Economic Association, vol. 97(3), pages 586-606, June.

Smith, G.W. (2009), "Pooling forecasts in linear rational expectations models", Journal of Economic Dynamics & Control, 33(2009), p.1858–1866.

Stock, J. and M. Watson (2007), "Why Has U.S. Inflation Become Harder to Forecast?," Journal of Money, Credit and Banking, Blackwell Publishing, vol. 39(s1), pages 3-33, 02.

Warne, A., Coenen, G., and Christoffel, K. (2016), "Marginalized Predictive Likelihood Comparisons of Linear Gaussian State-Space Models with Applications to DSGE, DSGE-VAR, and VAR Models", Journal of Applied Econometrics, doi: 10.1002/jae.2514.

Woodford, M. (2013), "Macroeconomic Analysis without the Rational Expectations Hypothesis," Annual Review of Economics.

Appendix A: Technical details of the Kalman Filter learning procedure of Slobodyan and Wouters (2012b)

The precise learning procedure is defined as follows. Agents estimate the forecasting model at each point in time given the information set available at that time. We assume that they use an efficient Kalman filter updating mechanism[31]. They believe that the coefficients β (a vector obtained by stacking all β_j) follow a vector autoregressive process around $\overline{\beta}$ (which will be specified later): $vec\left(\beta_t - \overline{\beta}\right) = F \cdot vec\left(\beta_{t-1} - \overline{\beta}\right) + v_t$, where F is a diagonal matrix with $\rho \leq 1$ on the main diagonal[32]. Errors v_t are assumed to be *i.i.d.* with variance-covariance matrix V.

We can write the forecasting model in the following SURE format[33]:

$$\begin{bmatrix} y_{1t}^f \\ y_{2t}^f \\ \vdots \\ y_{mt}^f \end{bmatrix} = \begin{bmatrix} X_{1,t-1} & 0 & \dots & 0 \\ 0 & X_{2,t-1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_{m,t-1} \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \vdots \\ \beta_{m,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix}, \quad (9)$$

The errors $u_{j,t}$ depend on a linear combination of the true model innovations ϵ_t and therefore they are likely to be correlated, making the variance-covariance matrix non-diagonal: $\Sigma = E\left[u_t \cdot u_t^T\right]$. With the above notation, the Kalman filter updating and transition equations for the belief coefficients and the corresponding covariance matrix are given by

$$\begin{aligned} \beta_{t|t} &= \beta_{t|t-1} + P_{t|t-1} X_{t-1} \left[\Sigma + X_{t-1}^T P_{t|t-1} X_{t-1}\right]^{-1} \times \left(y_t^f - X_{t-1}^T \beta_{t|t-1}\right), && (10a) \\ & \quad \textit{with } (\beta_{t+1|t} - \overline{\beta}) = F \cdot (\beta_{t|t} - \overline{\beta}). \\ P_{t|t} &= P_{t|t-1} - P_{t|t-1} X_{t-1} \left[\Sigma + X_{t-1}^T P_{t|t-1} X_{t-1}\right]^{-1} \times X_{t-1}^T P_{t|t-1}, && (10b) \\ & \quad \textit{with } P_{t+1|t} = F \cdot P_{t|t} \cdot F^T + V. \end{aligned}$$

These best estimates for the beliefs ($\beta_{t|t-1}$) are then substituted for β_t in (9) to generate expectations of forward-looking variables, $E_t y_{t+1}^f$. Plugging these expectations into (3), we obtain a purely backward-looking representation of the

[31] Sargent and Williams (2005) showed that, even if the Kalman filter and constant gain learning are asymptotically equivalent on average, their transitory behaviour may differ significantly. In particular, a Kalman filter tends to result in much faster adjustment of agents' beliefs. With faster adjustment of beliefs, we are able to better understand whether the initial beliefs or time–varying coefficients matter more for the improved model fit.

[32] ρ is restricted to be the same for the seven forward-looking variables. Allowing for a variable specific autocorrelation provides some extra flexibility but also larger parameter uncertainty.

[33] The SURE format and the corresponding GLS estimator are necessary to obtain an efficient estimator of the complete forecasting model because the variables appearing on the RHS in each equation are not identical.

model (5).[34] The resultant time-dependent matrices μ_t, T_t, and R_t replace the constant equivalents in the RE solution. These matrices now depend on both the structural parameters of the decision problem (Θ) and on the best estimates of the forecasting model ($\beta_{t|t-1}$), and contain all necessary information to describe the dynamics and propagation of the shocks in the model under learning. In terms of adaptive learning literature, equation (5) represents the Actual Law of Motion (ALM) of the model.

To initialize this Kalman filter for the belief coefficients, we need to specify $\beta_{1|0} = \overline{\beta}$, $P_{1|0}$, Σ, and V. In our baseline approach, all these expressions are derived from the correlations between the model variables implied by the RE Equilibrium evaluated for the corresponding structural parameter vector Θ. In other words, the initial beliefs are assumed to be model consistent.[35]

Using the fact that $\widehat{\beta}_{OLS} = \left(X^T X\right)^{-1} X^T y$ is unbiased, we use the theoretical moment matrices $E\left[X^T X\right]$ and $E\left[X^T y\right]$ from the RE solution and set $\beta_{1|0} = (E\left[X^T X\right])^{-1} \cdot E\left[X^T y\right]$. Given $\beta_{1|0}$, we calculate Σ as

$$\Sigma = E\left[\left(y_t^f - X_{t-1}^T \beta_{1|0}\right)\left(y_t^f - X_{t-1}^T \beta_{1|0}\right)^T\right],$$

again using the RE theoretical moments. Finally, $P_{1|0}$, the initial guess about the mean square forecast error of the belief coefficients, and V, the variance–covariance matrix of shocks v_t to these coefficients, are both taken to be proportional to $\left(X^T \Sigma^{-1} X\right)^{-1}$:[36] $P_{1|0} = \sigma_0 \cdot \left(X^T \Sigma^{-1} X\right)^{-1}$, and $V = \sigma_v \cdot \left(X^T \Sigma^{-1} X\right)^{-1}$. This initialization leaves just three parameters, σ_0, σ_v, and ρ, to fully describe the learning dynamics, but in practice, we can keep σ_0, σ_v fixed and optimize over ρ only.

[34] Note that we expand the state vector y in this representation with additional lags that occur in the forecasting models.

[35] An alternative approach would be to derive the initial beliefs and the underlying moment matrixes from the restricted expectations equilibrium. Given our under-parameterized beliefs, this equilibrium deviates from the REE and requires the solution of the underlying ODE. Computationally this procedure was not feasible in the estimation context.

[36] $\left(X^T \Sigma^{-1} X\right)^{-1}$ is equal to $Var\left[\widehat{\beta}_{GLS}\right]$ where $\widehat{\beta}_{GLS} = \left(X^T \Sigma^{-1} X\right)^{-1} X^T \Sigma^{-1} y$, which gives an efficient estimator for the SURE model. Given knowledge of theoretical moments and of Σ, the matrix $\left(X^T \Sigma^{-1} X\right)^{-1}$ could be readily calculated.

Appendix B: Additional Tables

Table B1: Prior and posterior distributions 9obs models.

Parameter		Prior distribution			9obs-RE Metropolis Chain			SW2007 Posterior	9obs-AL Metropolis Chain			SW2012 Posterior
		type	mean	std.dev.	mean	5%	95%	mode	mean	5%	95%	mode
Calvo prob. wages	ξ_w	B	0.50	0.10	0.78	0.68	0.88	0.73	0.80	0.75	0.84	0.84
Calvo prob. prices	ξ_p	B	0.50	0.10	0.70	0.62	0.78	0.65	0.74	0.68	0.80	0.65
Indexation wages	ι_w	B	0.50	0.15	0.56	0.33	0.77	0.59	0.25	0.12	0.38	0.21
Indexation prices	ι_p	B	0.50	0.15	0.16	0.06	0.25	0.22	0.46	0.27	0.65	0.19
Gross price markup	ϕ_p	N	1.25	0.12	1.58	1.46	1.71	1.61	1.53	1.40	1.66	1.56
Capital production share	α	N	0.30	0.05	0.19	0.16	0.22	0.19	0.18	0.15	0.22	0.17
Capital utilization cost	ψ	B	0.50	0.15	0.71	0.56	0.87	0.54	0.56	0.34	0.76	0.56
Investment adj. cost	φ	N	4.00	1.50	4.25	2.58	5.82	5.48	3.24	2.11	4.33	3.23
Habit formation	$\varkappa$	B	0.70	0.10	0.65	0.51	0.79	0.71	0.64	0.53	0.76	0.68
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.51	1.08	1.97	1.59	1.76	1.24	2.16	1.58
Labor supply elast.	σ_l	N	2.00	0.75	1.77	0.97	2.57	1.92	2.22	1.58	2.89	1.77
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	1.27	-1.01	3.57	-0.10	2.69	1.04	4.54	0.83
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.18	0.08	0.28	0.16	0.18	0.07	0.28	0.17
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.32	0.29	0.35	0.43	0.40	0.35	0.45	0.41
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.95	0.99	0.98	1.00	0.99
Risk premium shock	ρ_b	B	0.50	0.20	0.47	0.23	0.70	0.18	0.58	0.38	0.74	0.55
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.84	0.77	0.92	0.71	0.48	0.38	0.58	0.51
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.99	1.00	0.97	0.99	0.99	1.00	0.97
Price markup shock	ρ_p	B	0.50	0.20	0.96	0.93	0.99	0.90				
Wage markup shock	ρ_w	B	0.50	0.20	0.94	0.90	0.99	0.97				
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.59	0.43	0.74	0.52	0.64	0.49	0.78	0.54
Stationary tech. shock	σ_a	G	0.20	0.15	0.44	0.39	0.48	0.45	0.44	0.40	0.50	0.46
Risk premium shock	σ_b	G	0.20	0.15	0.18	0.11	0.25	0.24	0.20	0.15	0.24	0.15
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.37	0.31	0.43	0.45	0.43	0.37	0.48	0.45
Gov't cons. shock	σ_g	G	0.20	0.15	0.53	0.48	0.58	0.52	0.51	0.46	0.56	0.50
Price markup shock	σ_p	G	0.20	0.15	0.18	0.14	0.21	0.14	0.20	0.19	0.23	0.15
MA(1) price markup shock	ϑ_p	B	0.50	0.20	0.82	0.73	0.90	0.74				
Wage markup shock	σ_w	G	0.20	0.15	0.40	0.35	0.44	0.24	0.36	0.32	0.41	0.23
MA(1) wage markup shock	ϑ_w	B	0.50	0.20	0.89	0.83	0.97	0.88				
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.76	0.59	0.93	0.81	0.59	0.44	0.73	0.64
Inflation response	r_π	N	1.50	0.25	1.59	1.35	1.84	2.03	1.55	1.19	1.87	1.75
Output gap response	r_y	N	0.12	0.05	0.05	0.03	0.08	0.08	0.10	0.05	0.15	0.15
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.17	0.13	0.20	0.22	0.12	0.09	0.15	0.14
Mon. pol. shock std	σ_r	G	0.20	0.15	0.24	0.21	0.26	0.24	0.22	0.20	0.24	0.22
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.09	0.02	0.16	0.12	0.12	0.03	0.19	0.10
Interest rate smoothing	ρ_R	B	0.75	0.10	0.81	0.78	0.85	0.81	0.93	0.89	0.96	0.89
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12		0.11	0.10	0.12	
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.21	0.19	0.22		0.21	0.19	0.23	
Learning persistence	φ	U	0.00	1.00					0.99	0.98	1.00	0.97
Log marginal likelihood					MCMC	-965.22			MCMC	-943.42		

Note: models are evaluated over the period 1971Q1 - 2015Q3 using the first four observations as a pre-sample.

Table B2: Prior and posterior distributions 10obs-ME models.

Parameter		Prior distribution			10obs-RE Metropolis Chain			10obs-AL Metropolis Chain		
		type	mean	std.dev.	mean	5%	95%	mean	5%	95%
Calvo prob. wages	ξ_w	B	0.50	0.10	0.80	0.74	0.87	0.84	0.78	0.89
Calvo prob. prices	ξ_p	B	0.50	0.10	0.87	0.80	0.93	0.76	0.69	0.83
Indexation wages	ι_w	B	0.50	0.15	0.50	0.29	0.71	0.26	0.10	0.41
Indexation prices	ι_p	B	0.50	0.15	0.13	0.05	0.21	0.47	0.32	0.63
Gross price markup	ϕ_p	N	1.25	0.12	1.50	1.37	1.63	1.49	1.36	1.62
Capital production share	α	N	0.30	0.05	0.17	0.14	0.20	0.17	0.14	0.21
Capital utilization cost	ψ	B	0.50	0.15	0.65	0.47	0.82	0.52	0.27	0.77
Investment adj. cost	φ	N	4.00	1.50	3.86	2.12	5.82	2.65	1.66	3.84
Habit formation	$\varkappa$	B	0.70	0.10	0.58	0.44	0.71	0.60	0.51	0.69
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.05	0.73	1.41	1.77	1.45	2.11
Labor supply elast.	σ_l	N	2.00	0.75	1.60	0.79	2.42	1.96	1.24	2.69
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	0.18	-2.22	2.73	2.87	1.20	4.62
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.18	0.08	0.28	0.18	0.07	0.28
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.35	0.31	0.38	0.40	0.36	0.43
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.99	0.98	1.00
Risk premium shock	ρ_b	B	0.50	0.20	0.91	0.86	0.99	0.65	0.54	0.76
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.69	0.54	0.84	0.38	0.23	0.54
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.98	1.00	0.99	0.99	1.00
Price markup shock	ρ_p	B	0.50	0.20	0.93	0.87	0.98			
Wage markup shock	ρ_w	B	0.50	0.20	0.99	0.98	1.00			
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.61	0.46	0.76	0.62	0.47	0.76
Stationary tech. shock	σ_a	G	0.20	0.15	0.35	0.31	0.38	0.45	0.41	0.49
Risk premium shock	σ_b	G	0.20	0.15	0.08	0.04	0.10	0.17	0.13	0.22
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.38	0.31	0.45	0.42	0.36	0.47
Gov't cons. shock	σ_g	G	0.20	0.15	0.52	0.47	0.57	0.51	0.47	0.55
Price markup shock	σ_p	G	0.20	0.15	0.21	0.19	0.23	0.24	0.21	0.26
MA(1) price markup shock	ϑ_p	B	0.50	0.20						
Wage markup shock	σ_w	G	0.20	0.15	0.43	0.39	0.47	0.37	0.34	0.41
MA(1) wage markup shock	ϑ_w	B	0.50	0.20						
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.66	0.52	0.81	0.68	0.55	0.79
Inflation response	r_π	N	1.50	0.25	1.73	1.45	2.04	1.69	1.40	1.97
Output gap response	r_y	N	0.12	0.05	0.05	0.00	0.09	0.09	0.06	0.13
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.19	0.15	0.22	0.13	0.09	0.16
Mon. pol. shock std	σ_r	G	0.20	0.15	0.24	0.22	0.27	0.22	0.20	0.24
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.05	0.01	0.09	0.11	0.03	0.19
Interest rate smoothing	ρ_R	B	0.75	0.10	0.83	0.79	0.87	0.89	0.86	0.93
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12	0.11	0.10	0.12
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.21	0.19	0.23	0.21	0.19	0.22
m.e. π_f1	$\sigma_{\pi f1}$	G	0.20	0.15	0.15	0.14	0.17	0.18	0.16	0.19
Learning persistence	φ	U	0.00	1.00				0.98	0.97	1.00
Log marginal likelihood					MCMC	-910.87		MCMC	-885.94	

Note: see Table B1.

Table B3: Prior and posterior distributions 2MU-10obs models.

Parameter		Prior distribution			10obs-RE Metropolis Chain			10obs-AL Metropolis Chain		
		type	mean	std.dev.	mean	5%	95%	mean	5%	95%
Calvo prob. wages	ξ_w	B	0.50	0.10	0.79	0.73	0.86	0.89	0.85	0.92
Calvo prob. prices	ξ_p	B	0.50	0.10	0.91	0.87	0.94	0.83	0.78	0.88
Indexation wages	ι_w	B	0.50	0.15	0.38	0.18	0.57	0.25	0.10	0.39
Indexation prices	ι_p	B	0.50	0.15	0.07	0.03	0.11	0.06	0.03	0.10
Gross price markup	ϕ_p	N	1.25	0.12	1.45	1.33	1.56	1.50	1.38	1.61
Capital production share	α	N	0.30	0.05	0.20	0.17	0.23	0.18	0.15	0.21
Capital utilization cost	ψ	B	0.50	0.15	0.69	0.453	0.85	0.72	0.57	0.87
Investment adj. cost	φ	N	4.00	1.50	1.85	1.00	2.67	1.72	1.24	2.17
Habit formation	$\varkappa$	B	0.70	0.10	0.46	0.35	0.57	0.52	0.45	0.59
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.35	1.03	1.66	1.43	1.26	1.59
Labor supply elast.	σ_l	N	2.00	0.75	1.58	0.75	2.39	1.83	1.06	2.60
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	0.54	-1.15	2.25	3.37	2.37	4.37
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.17	0.07	0.27	0.18	0.09	0.27
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.36	0.32	0.39	0.39	0.35	0.42
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.99	0.98	1.00
Risk premium shock	ρ_b	B	0.50	0.20	0.83	0.74	0.94	0.80	0.73	0.88
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.86	0.77	0.95	0.39	0.29	0.49
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.98	0.99	0.99	0.99	1.00
Price markup shock	ρ_{par}	B	0.50	0.20	0.77	0.59	0.93	0.79	0.68	0.91
Wage markup shock	ρ_{war}	B	0.50	0.20	1.00	0.99	1.00	0.43	0.09	0.77
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.65	0.50	0.80	0.65	0.50	0.79
Stationary tech. shock	σ_a	G	0.20	0.15	0.45	0.40	0.49	0.44	0.39	0.48
Risk premium shock	σ_b	G	0.20	0.15	0.11	0.08	0.15	0.12	0.11	0.14
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.44	0.40	0.54	0.32	0.27	0.24
Gov't cons. shock	σ_g	G	0.20	0.15	0.52	0.47	0.57	0.51	0.46	0.55
Price markup shock-iid	σ_{piid}	G	0.20	0.15	0.24	0.22	0.26	0.26	0.23	0.28
Price markup shock-ar	σ_{par}	G	0.20	0.15	0.03	0.01	0.05	0.03	0.02	0.04
Wage markup shock-iid	σ_{wiid}	G	0.20	0.15	0.43	0.39	0.47	0.39	0.35	0.43
Wage markup shock-ar	σ_{wae}	G	0.20	0.15	0.01	0.00	0.01	0.04	0.01	0.08
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.62	0.46	0.77	0.60	0.50	0.69
Inflation response	r_π	N	1.50	0.25	1.46	1.17	1.73	1.65	1.37	1.93
Output gap response	r_y	N	0.12	0.05	0.13	0.08	0.17	0.05	0.02	0.08
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.23	0.19	0.27	0.15	0.12	0.19
Mon. pol. shock std	σ_r	G	0.20	0.15	0.24	0.22	0.27	0.22	0.20	0.24
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.06	0.01	0.10	0.11	0.03	0.19
Interest rate smoothing	ρ_R	B	0.75	0.10	0.87	0.84	0.91	0.90	0.87	0.94
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12	0.11	0.10	0.12
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.21	0.19	0.22	0.21	0.19	0.22
m.e. π_f1	$\sigma_{\pi f1}$	G	0.20	0.15	0.04	0.01	0.06	0.02	0.01	0.04
Learning persistence	φ	U	0.00	1.00				0.97	0.96	0.98
Log marginal likelihood					MCMC	-840.90		MCMC	-790.51	

Note: see Table B1.

Table B4: Prior and posterior distributions 2MU-10obs + inflation objective shock.

Parameter		Prior distribution			RE-10obs Metropolis Chain			AL-10obs Metropolis Chain		
		type	mean	std.dev.	mean	5%	95%	mean	5%	95%
Calvo prob. wages	ξ_w	B	0.50	0.10	0.89	0.86	0.92	0.89	0.86	0.92
Calvo prob. prices	ξ_p	B	0.50	0.10	0.90	0.87	0.93	0.83	0.79	0.87
Indexation wages	ι_w	B	0.50	0.15	0.39	0.18	0.58	0.24	0.08	0.39
Indexation prices	ι_p	B	0.50	0.15	0.11	0.06	0.16	0.06	0.03	0.09
Gross price markup	ϕ_p	N	1.25	0.12	1.43	1.32	1.54	1.50	1.39	1.62
Capital production share	α	N	0.30	0.05	0.19	0.16	0.21	0.19	0.16	0.23
Capital utilization cost	ψ	B	0.50	0.15	0.64	0.47	0.83	0.70	0.56	0.85
Investment adj. cost	φ	N	4.00	1.50	2.04	1.00	2.89	1.47	1.04	1.82
Habit formation	$\varkappa$	B	0.70	0.10	0.52	0.41	0.63	0.50	0.44	0.56
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.29	0.95	1.60	1.50	1.30	1.68
Labor supply elast.	σ_l	N	2.00	0.75	2.35	1.62	3.08	1.90	1.21	2.61
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	-0.06	-2.04	1.87	3.20	-0.17	4.87
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.19	0.08	0.30	0.24	0.08	0.38
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.34	0.30	0.37	0.39	0.35	0.42
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.99	0.98	1.00
Risk premium shock	ρ_b	B	0.50	0.20	0.83	0.75	0.93	0.79	0.73	0.86
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.91	0.86	0.97	0.34	0.23	0.45
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.99	0.99	0.99	0.99	1.00
Price markup shock	ρ_{par}	B	0.50	0.20	0.25	0.03	0.46	0.77	0.66	0.91
Wage markup shock	ρ_{war}	B	0.50	0.20	0.40	0.09	0.69	0.41	0.11	0.73
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.63	0.49	0.77	0.67	0.53	0.80
Stationary tech. shock	σ_a	G	0.20	0.15	0.45	0.41	0.50	0.44	0.39	0.48
Risk premium shock	σ_b	G	0.20	0.15	0.10	0.08	0.13	0.12	0.10	0.14
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.48	0.33	0.62	0.34	0.28	0.39
Gov't cons. shock	σ_g	G	0.20	0.15	0.51	0.47	0.56	0.51	0.46	0.55
Price markup shock-iid	σ_{piid}	G	0.20	0.15	0.24	0.22	0.26	0.26	0.23	0.29
Price markup shock-ar	σ_{par}	G	0.20	0.15	0.06	0.04	0.08	0.04	0.02	0.05
Wage markup shock-iid	σ_{wiid}	G	0.20	0.15	0.42	0.38	0.45	0.39	0.34	0.43
Wage markup shock-ar	σ_{wae}	G	0.20	0.15	0.04	0.00	0.07	0.06	0.00	0.12
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.61	0.45	0.77	0.59	0.47	0.71
Inflation response	r_π	N	1.50	0.25	1.62	1.26	1.96	1.78	1.40	2.17
Output gap response	r_y	N	0.12	0.05	0.07	0.02	0.13	0.04	0.01	0.06
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.22	0.18	0.26	0.16	0.13	0.19
Mon. pol. shock std	σ_r	G	0.20	0.15	0.22	0.19	0.25	0.22	0.20	0.24
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.07	0.01	0.10	0.10	0.02	0.18
Interest rate smoothing	ρ_R	B	0.75	0.10	0.91	0.88	0.95	0.90	0.88	0.93
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12	0.11	0.10	0.12
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.21	0.19	0.22	0.21	0.19	0.22
m.e. π_f1	$\sigma_{\pi f1}$	G	0.20	0.15	0.03	0.00	0.06	0.02	0.00	0.04
Learning persistence	φ	U	0.00	1.00				0.97	0.96	0.98
Inflation target shock	$\sigma_{\overline{\pi}}$	G	0.20	0.15	0.12	0.10	0.13	0.01	0.00	0.02
Log marginal likelihood					MCMC	−833.52		MCMC	−803.93	

Note: see Table B1.

Table B5: Test statistics for RE-9obs model forecast errors

	average annual inflation forecast				one quarter ahead forecast			
	Full sample		Prediction sample		Full sample		Prediction sample	
persistence: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$								
β	**.366**	(.142)	.194	(.129)	-.038	(.093)	-.256	(.147)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\pi^{r1}_{t+h\|t}$								
β	.082	(.072)	**-.648**	(.195)	-.018	(.066)	**-.660**	(.109)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\pi^{r1}_{t+h\|t} + \gamma\pi^{r1}_{t-1} + \delta r_{t-1}$								
β	-.685	(.523)	-.414	(0.236)	-.422	(.215)	**-.586**	(.230)
γ	**.797**	(.333)	.125	(0.204)	.255	(.158)	-.127	(.213)
δ	**-.545**	(.236)	-.456	(0.240)	.082	(.043)	.026	(.054)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$								
β	**.415**	(.197)	-.083	(.142)	-.186	(.148)	**-.473**	(.190)

Note: For annual inflation forecasts $\pi^{r1}_{t+h|t} = \sum_{h=1,4}\pi^{RE}_{t+h|t}$. Newey-West corrected standard errors are in brackets. Bold slope coefficients are statistically significant at 95% level.

Table B6: Test statistics for AL-9obs model forecast errors

	average annual inflation forecast				one quarter ahead forecast			
	Full sample		Prediction sample		Full sample		Prediction sample	
persistence: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$								
β	**.464**	(.148)	**.504**	(.115)	.010	(.088)	-.152	(.172)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\pi^{r1}_{t+h\|t}$								
β	-.146	(.077)	**-.772**	(.147)	-.075	(.063)	**-.675**	(.123)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\pi^{r1}_{t+h\|t} + \gamma\pi^{r1}_{t-1} + \delta r_{t-1}$								
β	**-1.078**	(.332)	**-.410**	(.183)	**-.701**	(.232)	-.549	(.361)
γ	**1.052**	(.217)	.085	(.131)	**.461**	(.166)	-.147	(.260)
δ	-.395	(.269)	**-.740**	(.242)	.096	(.054)	.005	(.074)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$								
β	.386	(.220)	-.080	(.290)	-.172	(.139)	-.489	(.257)

Note: For annual inflation forecasts $\pi^{r1}_{t+h|t} = \sum_{h=1,4}\pi^{AL\_ALM}_{t+h|t}$. Newey-West corrected standard errors are in brackets. Bold slope coefficients are statistically significant at a 95% level.

Table B7: Test statistics for RE-2MU-10obs model forecast errors

	average annual inflation forecast				one quarter ahead forecast			
	Full sample		Prediction sample		Full sample		Prediction sample	
persistence: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$								
β	**.385**	(.137)	.215	(.154)	.092	(.105)	-.218	(.173)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\pi^{r1}_{t+h\|t}$								
β	-.007	(0.080)	-0.475	(0.329)	.046	(.052)	-.020	(.244)
predictability:$\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\pi^{r1}_{t+h\|t} + \gamma\pi^{r1}_{t-1} + \delta r_{t-1}$								
β	.157	(.243)	-.875	(.688)	.177	(.117)	.427	(.293)
γ	.118	(.193)	.381	(.326)	-.072	(.092)	**-.304**	(.136)
δ	**-.782**	(.206)	-.170	(.256)	-.038	(.027)	-.007	(.040)
predictability:$\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$								
β	**.721**	(.157)	.436	(.261)	.483	(.191)	-.014	(.453)

Note: For annual inflation forecasts $\pi^{r1}_{t+h|t} = \sum_{h=1,4} \pi^{RE}_{t+h|t}$. Newey-West corrected standard errors are in brackets. Bold slope coefficients are statistically significant at a 95% level.

Table B8: Test statistics for AL-2MU-10obs model forecast errors

	average annual inflation forecast				one quarter ahead forecast			
	Full sample		Prediction sample		Full sample		Prediction sample	
persistence: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$								
β	.174	(.138)	.182	(.112)	.086	(.089)	-.130	(.133)
predictability: $\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\pi^{r1}_{t+h\|t}$								
β	-.072	(0.043)	**-0.546**	(.175)	-.008	(.045)	-.170	(.146)
predictability:$\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\pi^{r1}_{t+h\|t} + \gamma\pi^{r1}_{t-1} + \delta r_{t-1}$								
β	-.387	(.234)	.169	(.271)	.013	(.113)	.126	(.209)
γ	.464	(.224)	**-.368**	(.114)	-.006	(.096)	-.177	(.128)
δ	**-.508**	(.168)	-.637	(.239)	-.012	(.032)	-.060	(.040)
predictability:$\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t} = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$								
β	.127	(.113)	.115	(.140)	.273	(.166)	.105	(.292)

Note: For annual inflation forecasts $\pi^{r1}_{t+h|t} = \sum_{h=1,4} \pi^{AL\_ALM}_{t+h|t}$. Newey-West corrected standard errors are in brackets. Bold slope coefficients are statistically significant at a 95% level.

Table B9: Prior and posterior distribution for 2MU-10obs models over the Full sample 1971q1-2019q4.

Parameter		Prior distribution			10obs-RE Metropolis Chain			10obs-AL Metropolis Chain		
		type	mean	std.dev.	mean	5%	95%	mean	5%	95%
Calvo prob. wages	ξ_w	B	0.50	0.10	0.71	0.64	0.78	0.89	0.85	0.93
Calvo prob. prices	ξ_p	B	0.50	0.10	0.91	0.87	0.95	0.84	0.79	0.89
Indexation wages	ι_w	B	0.50	0.15	0.36	0.17	0.56	0.27	0.11	0.42
Indexation prices	ι_p	B	0.50	0.15	0.07	0.03	0.11	0.06	0.03	0.09
Gross price markup	ϕ_p	N	1.25	0.12	1.45	1.33	1.56	1.50	1.38	1.61
Capital production share	α	N	0.30	0.05	0.20	0.17	0.23	0.18	0.15	0.21
Capital utilization cost	ψ	B	0.50	0.15	0.69	0.54	0.85	0.69	0.55	0.85
Investment adj. cost	φ	N	4.00	1.50	2.65	2.00	3.47	2.01	1.27	2.68
Habit formation	$\varkappa$	B	0.70	0.10	0.51	0.41	0.61	0.56	0.47	0.64
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.31	1.01	1.60	1.28	1.14	1.41
Labor supply elast.	σ_l	N	2.00	0.75	1.10	0.25	1.82	1.80	1.03	2.58
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	0.43	-1.24	2.05	0.73	-2.26	3.29
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.18	0.08	0.28	0.18	0.07	0.28
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.34	0.31	0.37	0.37	0.34	0.40
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.99	0.98	0.99
Risk premium shock	ρ_b	B	0.50	0.20	0.82	0.73	0.92	0.81	0.74	0.89
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.82	0.73	0.91	0.40	0.29	0.50
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.98	0.99	0.99	0.99	0.99
Price markup shock	ρ_{par}	B	0.50	0.20	0.84	0.75	0.94	0.83	0.73	0.94
Wage markup shock	ρ_{war}	B	0.50	0.20	0.99	0.99	1.00	0.49	0.15	0.84
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.66	0.52	0.79	0.68	0.55	0.82
Stationary tech. shock	σ_a	G	0.20	0.15	0.44	0.39	0.48	0.43	0.39	0.46
Risk premium shock	σ_b	G	0.20	0.15	0.11	0.08	0.14	0.11	0.10	0.13
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.37	0.31	0.42	0.31	0.26	0.35
Gov't cons. shock	σ_g	G	0.20	0.15	0.50	0.46	0.55	0.48	0.44	0.52
Price markup shock-iid	σ_{piid}	G	0.20	0.15	0.24	0.22	0.26	0.25	0.23	0.27
Price markup shock-ar	σ_{par}	G	0.20	0.15	0.02	0.01	0.03	0.03	0.02	0.04
Wage markup shock-iid	σ_{wiid}	G	0.20	0.15	0.43	0.38	0.47	0.38	0.34	0.42
Wage markup shock-ar	σ_{wae}	G	0.20	0.15	0.01	0.01	0.01	0.04	0.01	0.08
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.63	0.47	0.78	0.62	0.50	0.74
Inflation response	r_π	N	1.50	0.25	1.51	1.20	1.82	1.63	1.34	1.91
Output gap response	r_y	N	0.12	0.05	0.13	0.08	0.17	0.04	0.01	0.06
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.22	0.18	0.25	0.16	0.12	0.19
Mon. pol. shock std	σ_r	G	0.20	0.15	0.23	0.21	0.25	0.21	0.20	0.23
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.06	0.01	0.10	0.12	0.03	0.20
Interest rate smoothing	ρ_R	B	0.75	0.10	0.88	0.85	0.92	0.90	0.87	0.93
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12	0.11	0.10	0.12
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.20	0.18	0.22	0.20	0.18	0.22
m.e. π_f1	$\sigma_{\pi f1}$	G	0.20	0.15	0.04	0.03	0.06	0.02	0.01	0.03
Learning persistence	φ	U	0.00	1.00				0.97	0.96	0.98
Log marginal likelihood					MCMC	-843.45		MCMC	-782.16	

Note: models are evaluated over the period 1971Q1 - 2019Q4 using the first four observations as a pre-sample.

Appendix C: Robustness exercises and extensions

Different timing of SPF forecasts

In our main analysis, we used the survey forecast for the current quarter ($\pi^f_{t+1|t}$) as the observable for our model expectations ($E_t\pi_{t+1}$) as presented in Eq. (1). This survey forecast or nowcast is collected around the end of the first month of the quarter $t+1$ and thus contains information that is not available to agents when they make their decision for quarter t. This provides the survey with a time advantage relative to the model forecast, which is based only on the first releases for quarter t and second releases of $t-1$ data for the macro-aggregates. For this reason, we also experimented with an alternative setup, in which we use the survey forecast for the next quarter ($\pi^f_{t+1|t-1}$) as observable for the model expectations ($E_t\pi_{t+1}$). This forecast is collected at time t and is used as an observable for the model forecast.

As documented in Table C1, the time disadvantage of this survey measure has a clear impact on the forecast performance. Over the complete sample, the forecast errors with respect to the different releases and for different horizons are much larger than for the nowcast timing documented in Table 1. However, remarkably, for the sample after 1996, the forecast performance remains very similar to the nowcast timing. This result reflects the change in dynamic properties of the inflation process over our sample period.

Table C1: Statistical properties of the SPF forecasts errors with alternative timing

	Full sample			Prediction sample		
	Mean	MAD	RMSFE	Mean	MAD	RMSFE
SPF statistics						
$\pi^{r1}_{t+1}-\pi^{SPF}_{t+1\|t-1}$	-0.03	0.24	0.32	-0.04	0.18	0.21
$\pi^{r2}_{t+1}-\pi^{SPF}_{t+1\|t-1}$	-0.01	0.25	0.34	-0.03	0.17	0.21
$\pi^{rf}_{t+1}-\pi^{SPF}_{t+1\|t-1}$	-0.02	0.23	0.29	-0.01	0.17	0.22
SPF for longer horizons						
$\pi^{r1}_{t+2}-\pi^{SPF}_{t+2\|t-1}$	-0.04	0.27	0.37	-0.06	0.18	0.22
$\pi^{r1}_{t+3}-\pi^{SPF}_{t+3\|t-1}$	-0.05	0.29	0.41	-0.07	0.19	0.24
$\pi^{r1}_{t+4}-\pi^{SPF}_{t+4\|t-1}$	-0.05	0.32	0.44	-0.08	0.20	0.25

From the marginal likelihood results in Table C2, we can first observe that the model specification with two markup shocks, an *i.i.d.* process and a persistent autoregressive component, again appears to be a very flexible structure capable of simultaneously matching both realized and expected inflation data. There is a significant improvement in the log marginal likelihood when we move from the specification with measurement error for the survey expectations to the setup with two structural markup components, in particular in the RE setup. But we do not observe the same large gain in forecasting power under AL, when the agents are allowed to use the detailed shock information in their belief models, as we did when the nowcast expectations were included in the dataset. Our interpretation is that the lagged survey forecasts do not contain the same timely information as the nowcast data, and identification of the shocks based on this outdated information is not helpful for improving the forecasting performance. This is consistent with the observation that the marginal likelihood of the common 9 variables is worse in the models estimated on 10 observables than in the models estimated on these 9 observables alone. Restricting the model expectations to be consistent with the lagged survey forecast has a cost for overall performance, even in the model with flexible model specification and AL. This finding suggests that survey forecasts are useful to improve the model forecast mainly because they incorporate new information. The exercise also confirms the importance of inflation expectations for overall model performance: correctly identifying these expectations allows us to improve the dynamic interactions between nominal and real variables in the economy.

Table C2: Marginal likelihood of models with alternative timing

	Full sample		Prediction sample	
	9obs	10obs	9obs	10obs
RE-SW07-9obs	-965.22		-361.25	
AL-SW12-9obs (AR2)	-943.42		-340.96	
AL-SW12-9obs (AR2+mc)	-934.41		-317.78	
RE-ME-10obs	-1016.24	-896.61	-367.65	-288.06
AL-ME-10obs (AR2)	-978.38	-896.37	-343.66	-295.82
AL-ME-10obs (AR2+mc)	-954.71	-836.91	-329.92	-256.46
RE-2MU-10obs	-979.76	-852.09	-353.77	-269.17
AL-2MU-10obs (AR2+mc+UC)	-962.65	-823.94	-336.92	-250.89

Note: see Table 5.

Longer-term SPF forecasts

The robustness of the results was also successfully tested on datasets with real-time data for all observed series and on datasets with survey expectations for several horizons.

The idea of this exercise is to improve the AL model forecasts, which perform relatively worse than the RE on forecasting inflation over multiple quarters. This might suggest that there is too much flexibility in the belief specification and updating. One way to overcome this problem is to add survey expectations about future quarters to the list of observables. Up to this point, we have only experimented with one additional observable for inflation two quarters ahead. The beliefs can either contain two separate belief equations for each of these forecasts (unrestricted model) or the two-quarter forecast can be written as a restricted belief equation, where the restriction imposes consistency with the one period ahead forecast coefficients.[37] In both cases, the results are promising in that they improve on the inflation forecast without distorting the other implications of the model. In future work, we also plan to add long term inflation expectations to further discipline updating in the belief coefficients. By doing so, long run inflation expectations will no longer be purely backward looking via the updating process, but some role may remain for forward-looking expectation shocks related, for example, to changes in the monetary or fiscal policy context. This type of model extension can improve the precision of long term

[37] The RHS-variables in the two period ahead forecast could be substituted consistent with the PLM equations for π_t, w_t, r_{Kt} (thus generating values of mc_t) and the exogenous shocks processes (for productivity and price markup shocks).

inflation forecasts, but it should be noted that the long term inflation forecasts of our best model are not significantly different from the survey forecasts in their present form, according to the Diebold-Mariano test.

Abstrakt

Použití informací z průzkumů o inflačních očekáváních jako pozorovatelné veličiny v DSGE modelu může podstatně zpřesnit identifikaci šoků, které ovlivňují inflaci. Optimální začlenění informací z průzkumů zlepšuje prognózu inflace a dalších makroekonomických proměnných v modelu. Modely s očekáváními založené na nastavení adaptivního učení mohou využívat informace z průzkumů efektivněji než jejich protějšky založené na racionálních očekáváních. Výsledná časová variabilita vnímaného inflačního cíle, perzistence inflace a citlivosti inflace na různé šoky poskytuje bohatý a konzistentní popis společné dynamiky realizované a očekávané inflace. Náš rámec poskytuje rozumnou interpretaci postkovidové dynamiky inflace. Náš model učení úspěšně identifikuje trvalejší povahu nedávného prudkého nárůstu inflace.

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Phone: + 420 224 005 153
Email: office@cerge-ei.cz
Web: https://www.cerge-ei.cz/

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