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Identification of School Admission Effects Using Propensity Scores Based on a Matching Market Structure*†‡

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May 30, 2020

Abstract

A large literature estimates various school admission and graduation effects by employing variation in student admission scores around schools' admission cutoffs, assuming (quasi-) random school assignment close to the cutoffs. In this paper, I present evidence suggesting that the samples corresponding to typical applications of the regression discontinuity design (RDD) fail to satisfy these assumptions. I distinguish ex-post randomization (as in admission lotteries applicable to those at the margin of admission) from ex-ante randomization, reflecting uncertainty about the market structure of applicants, which can be naturally quantified by resampling from the applicant population. Using data from the Croatian centralized collegeadmission system, I show that these ex-ante admission probabilities differ dramatically between treated and non-treated students within typical RDD bandwidths. Such unbalanced admission probability distributions suggest that bandwidths (and sample sizes) should be drastically reduced to avoid selection bias. I also show that a sizeable fraction of quasi-randomized assignments occur outside of the typical RDD bandwidths, suggesting that these are also inefficient. As an alternative, I propose a new estimator, the Propensity Score Discontinuity Design (PSDD), based on all observations with random assignments, which compares outcomes of applicants matched on ex-ante admission probabilities, conditional on admission scores.

Keywords: RDD, PSDD, school admission effects, lottery
JEL codes: C01, C51

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1. Introduction

The deferred acceptance (DA) algorithm, a major result in market design, has numerous important practical applications. Several countries operate centralized matching markets that implement the DA algorithm to assign students to colleges. In these markets, college applicants submit their school preferences (rankings) along with their (potentially school-specific) admission scores. A growing empirical literature exploits a feature of these college admission systems whereby students with similar admission scores in a neighborhood of a school's admission threshold are or are not offered admission to the schools based on small differences in admission scores. Assuming that for students at the margin of admission, treatment (school assignment) is driven by uncertainty in their admission scores (Lee, 2008), the literature relies on data from such centralized markets and on the regression discontinuity design (RDD) to estimate the causal effects of attending specific schools on various outcomes.[1]

In a typical matching market setting, students submit their preferences (school rankings) knowing their exact admission scores. The matching market mechanism then compares test scores of the whole population of applicants in the order of their rankings and, given school-specific limits on the number of admitted students, it determines the school-specific admission score cutoffs. At the time of the application, there is therefore no individual-level admission score uncertainty; instead, uncertainty of admission (the source of quasi-random assignments to schools) corresponds to the uncertainty of school-specific score cutoffs, which in turn, are entirely determined by the market-level structure of applications, i.e., by the test scores and school rankings of all applicants in the market. From an analyst's perspective, it is natural to quantify the extent of randomness in this structure (and in the implied score cutoffs) by resampling from the applicant population and recording the simulated matching market outcomes.[2] Such resampling then allows one to form an ex-ante probability of admission for each student-school pair — an admission propensity score, calculated, to the best of my knowledge, for the first time in this paper.

I use these propensity scores to assess the key identifying assumption of the RDD approach — that of random assignment in the proximity of school admission cutoffs. The RDD approach assumes that students' applications within a limiting neighborhood of the cutoffs, defined by a particular bandwidth, have similar admission probabilities regardless of the admission outcome, which can be directly tested by comparing the distributions of propensity scores for the treated and the non-treated groups.

Most RDD studies choose a constant arbitrary bandwidth around all school admission cutoff thresholds.[3] To demonstrate that bandwidth choices do not drive

[1] Kirkeboen et al. (2016) and Hastings et al. (2014) estimate labor-market returns of specific fields of study, Lucas and Mbiti (2014) and Abdulkadiroglu et al. (2014) study effects on standardized test scores, and Angrist et al. (2016) evaluate high school attendance effects on college choice. Dustan (2018), Fernandez (2019) and Altmejd et al. (2019a) use this research design to ask about the role of family ties in school choice.

[2] This is feasible in most existing applications of the RDD design to matching markets, in which analysts typically work with the entire applicant population.

[3] To improve the efficiency of the RDD approach, Abdulkadiroglu et al. (2019) use school-specific optimal bandwidths based on Imbens and Kalyanaraman (2012). However, optimality requires the independence of observations assumption, which is not satisfied in the school-choice framework as

the results, these studies typically employ robustness checks repeating the estimation for alternative values of the bandwidth. As an example, Abdulkadiroglu et al. (2014) use bandwidths ranging from roughly a third of the standard deviation up to the full standard deviation, while Kirkeboen et al. (2016) use all data (impose no bandwidths) in their main specification. Propensity scores, constructed by resampling from the applicant population, allow one to also assess whether these bandwidth choices lead the analyst to study outcomes of students who face no (quasi-) randomness in their school assignment. For example, it could be the case that applicants to small schools face more uncertainty in their admission offers than applicants to large programs or schools. Therefore, propensity scores can also be used to inspect whether there are quasi-random assignments outside of the typical bandwidths used in RDD studies.

To illustrate the use of propensity scores in these settings, I calculate (and validate) propensity scores for each student-college pair using data from the Croatian college choice matching market from 2014 to 2018. Next, I evaluate the propensity scores of applicants near school-specific admission cutoffs using bandwidth values typically found in the literature and obtain results that are not consistent with the assumptions of the RDD approach. First, I find that the propensity score distributions within typical bandwidths differ considerably across the treated and the non-treated groups. When considering applications of students who have admission scores at most half a standard deviation away from the school admission cutoffs, the average propensity score for the treated group is 85.8%, compared with 7.6% in the non-treated group. Second, I show that a substantial fraction of applications (roughly 40% in the case of half a standard deviation bandwidth) within the typical bandwidths faces no assignment risk at all (i.e., propensity score equal to 1 or 0). Such extensive differences between propensity score distributions for the applications of the treated and the non-treated students contradict the assumed random assignment to treatment near the admission threshold. Furthermore, that almost half of the applications in RDD comparisons face no assignment uncertainty at all directly violates the Lee (2008) non-trivial assignment probability assumption.

I find that only a drastic reduction in bandwidths — considering observations at most 0.01 standard deviations away from the cutoff, i.e., applying a bandwidth size that is 10 to 50 times smaller than those employed in Abdulkadiroglu et al. (2014) — results in comparable distributions of propensity scores, and less than 1% of students with a deterministic assignment. Focusing on narrow neighborhoods around the admission cutoffs, however, comes at the expense of neglecting observations with non-trivial propensity scores that are located outside of the chosen bandwidths. As an example, suppose that we are studying all students who have a probabilistic assignment to school u (i.e., non-trivial propensity score at school u). A student, who has a non-trivial propensity score at some school ranked higher than school u, which implies a non-trivial propensity score also at school u, has a probabilistic assignment to school u, despite being far above the u-school cutoff. When identifying the effects of admission to school u, a typical RDD estimator will ignore these observations. To illustrate the extent of this, in the Croatian data I employ here, around 1.7% of the

students apply to multiple schools.

total applicant-school dyadic population has a propensity score between 40% and 60%. However, less than 30% of these highly randomized observations are captured within (RDD) samples defined by a 0.01 standard deviation bandwidth.

In sum, to adhere to the RDD assumptions, one needs to use smaller bandwidths, which, however, lead one to ignore much of the quasi-random applications available in matching market data. Hence, the application of the RDD design to matching market data faces fundamental obstacles. As an alternative approach, I propose a new estimator, the propensity score discontinuity design (PSDD), which applies the Rosenbaum and Rubin (1983) propensity score theorem to the matching market setting. By considering the propensity scores, the PSDD extracts the ex-ante uncertainty contained in the market-level structure of applications, and instead of choosing an arbitrary bandwidth, focuses only on the applications whose assignment is (quasi-) random. Crucially, the PSDD takes advantage of the timing of the matching market, recognizing that any potential selection into treatment must be embedded in the students' submitted preferences and in the admission score. Therefore, the selection-on-observables assumption in the standard propensity score theorem, which might seem unrealistic in the school choice setting, is not needed to employ the PSDD, as I show in Section 3. The PSDD estimator studies the outcomes of only those applications that face ex-ante (quasi-) randomness in their school assignment. Identification is therefore, by construction, based on observations with random assignments, both close to and away from the admission cutoff, and not on assuming randomized assignments as a function of distance from admission cutoffs as in the usual RDDs.

The remainder of this paper is structured as follows. Section 2 develops the matching market framework and proves that admission probabilities do not depend on the admission score in a limiting neighborhood around the cutoff. Section 3 develops the PSDD. Section 4 calculates propensity scores using Croatian matching market data and evaluates the typical bandwidths used in the literature. Section 5 concludes.

2. RDD Meets the Matching Market

In this section, I formally define a model of a student-school matching market inspired by Fack et al. (2019), which I then use to apply the quasi-experimental interpretation of RDD (Lee, 2008). The aim is to develop an evaluation tool to assess the appropriateness of the bandwidths used in studies employing the RDD in matching market settings. I adapt the model in Fack et al. (2019) by considering a potentially general form of students' preferences over schools in a market characterized by a finite number of students, and by modelling a student's type as a collection of his admission scores, preferences over schools, and observable and unobservable covariates.[4]

Consider a matching market defined by a set of students I and a set of schools U. Denote the cardinality of the set of students with $|I|$ and suppose that I is constructed by independently drawing $|I|$ students from the distribution H. Next, suppose that a set of schools U is fixed. The objective of the market is to match each student $i \in I$ to exactly one school $u \in U$.

Student $i \in I$ is described by a random vector $i = \{R_{i,v\ v \in U}, >_i, W_i, X_i\}$, where $R_{i,v}$ is the admission score of the student i at school v, $>_i$ describes preferences of student i over (some) schools in U,[5] and W_i and X_i are students i's unobservables and observables, respectively. School $u \in U$ is described by a fixed scalar $u = \{q_u\}$, where q_u is a fixed quota for school u. Given two applications by students i and j, school u gives admission priority to student i if and only if $R_{i,u} > R_{j,u}$.

The timeline of the matching market is as follows:

(i) A set of students I is constructed by $|I|$ independent draws from the distribution H.

(ii) Each student i learns his admission scores $R_{i,v,\ v \in U}$.

(iii) Each student i, based on his preferences $>_i$, admission score $R_{i,v,\ v \in U}$, observable (X_i) and unobservable (W_i) covariates, submits an ordered priority list $S_i := S_{i,l},\ l \in \{1, ..., L_i\}$, where l denotes the priority of the school, and L_i denotes cardinality of the set of schools submitted by student i. For example, if students i's admission score exceeds school's $S_{i,1}$ cutoff, he is offered admission to school $S_{i,1}$. If it is below the school's $S_{i,1}$ cutoff, he is considered for eligibility in school $S_{i,2}$ and so on.

(iv) Given observed students' priority lists $\{S_{i,l}\}, \forall i, l$, and schools' quotas $\{q_u\}, \forall u \in U$, the matching market defines a mapping $T_i : I \mapsto U$, assigning exactly one school $u \in U$ to each student $i \in I$. Denote with c_u the admission cutoff for

[4] In Fack et al. (2019), the student's type does not include observable and unobservable covariates and preferences are assumed to be guided by the von Neumann-Morgenstern utilities. The results do not depend on these extensions of the original model; they are implemented solely due to expositional purposes.

[5] The outside option for a student is not going to any school. If a student's preference over a particular school is not defined, I assume that the outside option is preferred to this school.

school u, which takes a non-zero value only for programs that filled its quota[6]:

$$c_u(I) = \begin{cases} \min_{i:T_i=u} R_{i,u} & \text{if } \sum_i 1_{T_i=u} = q_u \\ 0 & \text{if } \sum_i 1_{T_i=u} < q_u \end{cases}$$

Assume the following properties of the mapping T_i:

- **Non-wastefulness**: $\nexists i \in I$ such that $\exists u \in U$ so that $S_{i,k} = u$ and $S_{i,l} = T_i$ while $k < l$ and $R_{i,u} > c_u$. In words, schools are required to admit students until there are no unfilled vacancies.
- **Transparent assignment**: $\nexists$ a pair of students $i \in I$ and $j \in I$ such that $R_{i,u} > R_{j,u}$ and $T_i = p$, for some $p \in U$, and $T_j = u$ while $S_{i,k} = u$ and $S_{i,l} = p$ while $k < l$. In words, schools are allowed to rank the students only with respect to the (school-specific) admission score.

Denote with I_{-i} the set of students excluding student i. Then, the admission offer of student i at school u is defined as:

$$a_u(S_i, R_i, I_{-i}) = \begin{cases} \mathbb{1}(\text{if } S_{i,l} = u \text{ for some } l \text{ and } R_{i,S_{i,r}} < c_{S_{i,r}}(I)\ \forall r < l \text{ and } R_{i,u} \geq c_u(I)) \\ 0 \text{ otherwise} \end{cases}$$

In words, student i is only offered admission at school u if he listed school u on his priority list, was rejected by all the schools listed above school u, and finally met admission criteria for school u. Further, given that the admission decision for student i, given his submitted priority list and admission scores, depends exclusively on the set of students I, the (ex-ante) probability of student i receiving an admission offer at school u is:

$$P(T_i = u) = \int a_u(S_i, R_i, J_{-i})\, dH(J_{-i}). \tag{1}$$

The matching market implementing the deferred acceptance algorithm corresponds to this framework.

Next, I use the framework described above to adapt the quasi-experimental interpretation of RDD (Lee, 2008) to the case of the matching market. Denote with $g(\cdot)$ the density of the unobservable W_i and introduce the following definition:

Definition 1: Selection on unobservables. *The treatment is subject to selection on unobservables if there is a non-zero correlation between unobservable W_i and the treatment assignment T_i.*

Lee (2008) assumes that there are no discontinuities in the density of unobservables when the admission score equals the cutoff:

Assumption 1: Continuity of unobservables. *The conditional density $g(\cdot|R_{i,u} = c_u)$ is continuous at c_u.*

[6] The cutoffs depend on the market level structure of applicants, as the student with the lowest admission score admitted to a particular school defines that school's cutoff.

In Lee (2008), the cutoff is *perfectly known* before the treatment assignment, and thus the assumption essentially assumes away selection on unobservables. In contrast, in our framework, cutoffs are not known at the time of the application — there is a *cutoff uncertainty*. For example, suppose that the admission score $R_{i,u}$ at school u is drawn from the continuous distribution for each student i. Then the distribution of the cutoff c_u, the q_u-order statistics of school-u applicants, is also continuous. In this case, the Continuity of unobservables assumption is satisfied *almost surely* if the conditional density $g(\cdot|R)$ is discontinuous for, at most, a finite number of admission score values R.

Suppose that we are interested in measuring the effect of attending school u on some outcome Y. RDD strategies focus on those applications, for which $a_u(S_i, R_i, I_{-i}) \equiv 1_{R_{i,u} > c_u}$, where the assignment to school u is, from the ex-post perspective, a function of the school u specific admission score $R_{i,u}$ (from now on I refer to these observations as the *RDD estimation sample*).[7] More formally, the RDD estimation sample is defined as follows:

Definition 2: RDD Estimation Sample. *Suppose that student i is assigned to school $S_{i,k}$, $T_i = S_{i,k}$ for some k. Application $S_{i,l}$ is included in the RDD estimation sample only if $l \leq k$.*

As recognized in Kirkeboen et al. (2016), the outcome Y for student i who enrolled at school u, can only be estimated relative to the outcome at the counterfactual school, which the student would be admitted to if he was rejected admission to school u. To formalize this idea, denote with D_i a dummy variable indicating student i's treatment status (enrolling at school u), i.e. $D_i := 1_{T_i=u}$. Suppose that the outcome Y_k at any school k is a function of the unobservable W_i, i.e. $Y_k = Y(W_i|T_i = k)$. Specifically, for students with applications in the RDD estimation sample, denote with Y_1 and Y_0 the outcomes when attending and not-attending school u, respectively: $Y_1 := Y(W_i|D_i = 1)$ and $Y_0 := Y(W_i|D_i = 0)$. Using the law of iterated expectations, we obtain $E[Y_1 - Y_0|R_{i,u} = c_u] = \sum_{k \in U}(\widehat{Y_{i,u}} - \widehat{Y_{i,k}})P(T_i = k|T_i \neq k)$ where $P(T_i = k|T_i \neq u)$ is the probability of student i being admitted to the school k, given that he was marginally rejected admission to school u.

The following proposition provides the experimental interpretation of the RDD:

Proposition 1. *Suppose that the Continuity of unobservables assumption holds. Then, the following holds for each school-student pair in the RDD estimation sample:*

$$E[Y|R_{i,u} = c_u] - \lim_{x \to c_u^-} E[Y|R_{i,u} = x] = E[Y_1 - Y_0|R_{i,u} = c_u]$$

Proof. See Lee (2008). ■

Under the continuity assumption, the proposition establishes causality by comparing the outcomes of students, with different treatment assignments, in the RDD estimation sample with the school u-specific admission scores within a limiting neighbourhood around the cutoff. A critical aspect of the RDD implementation is choosing a bandwidth

[7] The usual justification of this is that assignment to these schools is not possible.

to define this limiting neighbourhood — reducing the bandwidths excessively, while improving the credibility of Proposition 1, results in diminishing sample sizes. To compromise this empirical tradeoff, a series of papers have proposed procedures to calculate *optimal* bandwidths (i.e., Imbens and Kalyanaraman, 2012). This literature concentrates on cases in which there is only one running variable and does not cover our case of multiple (potentially correlated) cutoffs and admission scores. Therefore, studies applying RDD to the matching markets setting generally use arbitrary bandwidths that are not supported with theory. The following simple adaptation of Proposition 1 can be used to assess the appropriateness of the bandwidth. Intuitively, in the limiting neighbourhood of Proposition 1, as long as the student's admission scores are continuous, the admission score does not change the admission probability significantly. Therefore, to support the bandwidth choice, an analyst can verify that the admission probability distributions of those just above the cutoff (treated students) are similar to admission probability distributions of those just below the cutoff (non-treated students).

Lemma 1. *Suppose that the school u-specific admission score $R_{i,u}$ is drawn from the continuous distribution for each student i. Then, for any application in the RDD estimation sample the following holds:*

$$P(T_i = u | R_{i,u} = c_u) = \lim_{x \to c_u^-} P(T_i = u | R_{i,u} = x).$$

Proof. In the RDD estimation sample, the ex-ante probability of admission to school u for a student with admission score $R_{i,u} = c_u$,

$$P(T_i = u | R_{i,u} = c_u) = \int a_u(S_i, c_u, J_{-i}) \, dH(J_{-i}),$$

which simplifies to

$$P(T_i = u | R_{i,u} = c_u) = \int 1_{c_u > \widehat{c_u}} \, dH(J_{-i}),$$

where the integral goes over the support of possible cutoffs $\widehat{c_u}$ defined by the distribution of $H(J_{-i})$ through:[8]

$$c_u(I) = \min_{j: T_j = u} R_{j,u} \qquad \text{where } \sum_j 1_{T_j = u} = q_u$$

Note that, as the cutoff c_u is simply a q_u-order statistic, the continuity of $R_{j,u}$ for each j, implies the continuity of c_u, so that:

$$\int 1_{c_u > \widehat{c_u}} \, dH(J_{-i}) = \int 1_{c_u > \widehat{c_u}} \, dQ(\widehat{c_u})),$$

for some continuous distribution Q. The lemma now follows from the continuity of Q. ■

[8]Note that the schools where the cutoff is not imposed do not belong to the RDD estimation sample.

3. Propensity Score Discontinuity Design

A key empirical challenge for the identification of school admission effects is accounting for the potential selection on unobservables. For example, unobservable levels of motivation of students might confound the estimate of school-specific labour market returns. In practice, there are two distinct RDD approaches used to account for the selection on unobservables. The more common one is nonparametric, which estimates a local linear regression around the cutoff using pre-defined bandwidth values. This approach assumes that the unobservables W_i are balanced in a local neighbourhood of a fixed admission score:

Assumption 2: Nonparametric identification. *For each school u-specific admission score* $\hat{c}_u$*, there exists* δ *such that* $Y_0, Y_1 \perp W_i$*,* $\forall i \in I$ *for* $R_{i,u} \in \langle \hat{c}_u - \delta, \hat{c}_u + \delta \rangle$*.*[9]

Alternatively, there is a parametric regression approach typically using polynomials to model the running variable (in our case the admission score) over its entire support. This approach assumes that the outcome is orthogonal to the unobservables W_i conditional on polynomials in the admission score:

Assumption 3: Parametric identification. *Outcome is orthogonal to the unobservables conditional on polynomials in admission score:* $Y_0, Y_1 \perp W_i | R_{i,u}, R^2_{i,u}, \dots R^p_{i,u}$*,* $\forall i \in I$*.*

As discussed in the previous section, the RDD uses the *RDD estimation sample*, excluding all the alternatives ranked below a student's (ex-post) admission outcome (as the student did not compete at these schools, the assignment was *impossible* from the ex-post perspective.) This is because the RDD assumes, through Assumption 2 or Assumption 3, that students around the cutoff are "the same" in every aspect except the admission outcome, which is deterministically linked to the school-specific admission score (i.e. a student is offered admission if and only if his admission score is above the school-specific cutoff). For schools ranked below the ex-post admission outcome, this deterministic link between admission score and the assignment is broken — the student is never considered for admission (even if he is above the cutoff for these schools). For this reason, these observations are not included in the RDD estimation sample. However, from the ex-ante perspective, excluding these observations is unnecessary as students who are randomly accepted to a higher-ranked school, are also randomly not assigned to a lower-ranked school.

For example, suppose a student ranked school A as his first choice, followed by school B. Suppose that ex-ante he had a 50% probability of being admitted to school A, and a 50% chance of being admitted at school B, and assume that, ex-post, he was just above school A's cutoff. The RDD concludes that the assignment to school B is ex-post impossible, and therefore excludes this choice from the estimation sample to ensure that the assignment is a deterministic function of only the admission score and the school-specific cutoff. The estimator I am proposing below understands

[9] All results provided in this section hold under the weaker Continuity of unobservables assumption from the previous section, which would require showing that the local PSDD is well defined (analogously to Proposition 1). I decided to focus on the stronger assumption, due to the intuitive appeal, and elegant exposition.

that the assignment to school A was ex-ante just as probable as the assignment to school B, and thus it includes both choices in the estimation sample. In the following paragraph, I provide the motivation the new estimator, by naturally generalizing simple estimators used in settings where admissions are resolved using a lottery at the margin of admission, to incorporate also the uncertainty in cutoffs at the time of the application.

In a typical high-school admission system, admission scores are coarse, and students are divided into a small number of groups with different admission priorities. Admission decisions for the students in the group at the margin of admission are then implemented by breaking the ties (within the group) by an admission lottery. The literature on high-school admission effects typically focuses only on the marginal group where the assignment is explicitly random (e.g., Abdulkadiroglu et al., 2017), which effectively conditions on the market structure of applications that assigned a specific group of applicants to the marginal group. While this ex-post assignment randomization is clearly ideal for the purpose of identification within the marginal group, there are additional sources of ex-ante assignment uncertainty corresponding to uncertainty at the time of the application. In settings where admission scores are highly coarse, the additional ex-ante uncertainty is negligible. To illustrate this, suppose that there are only two admission-score groups: A and B, and only one school prioritizing group A over group B. Suppose also that the school capacity is such that everybody in group A is highly likely to be admitted, while a lottery determines admission from group B. In this case, for a specific student in group B, the dominant component of total ex-ante admission uncertainty is the ex-post lottery draw, since group B is almost certainly the marginal group. In contrast, when admission scores are less coarse, such that there are numerous admission groups, the ex-ante probability depends not only on the ex-post lottery draw applicable to the (smaller) group of marginal applicants, but significantly also to the ex-ante uncertainty of ending up in the marginal group, which depends on the market-level structure of applications. The method I propose below incorporates the ex-ante uncertainty, employing larger sample sizes than the conventional lottery-based estimators since it considers students facing ex-ante probabilistic assignments outside of the marginal admission group. In other words, the method also includes the applications of students outside of the marginal group, as long as their admission is ex-ante sufficiently probabilistic. The remainder of this section formalizes this intuition and generalizes it to the case of a continuous admission score (a typical case in college-admission systems).

Assume the matching market from the previous section and suppose we are interested in the effect of attending a college u on the student i's outcome $Y_i,\ i \in I$. Denote with D_i a dummy variable indicating treatment assignment, $D_i = 1_{T_i=u}$ and let $Y_1 = Y(W_i|D_i = 1)$ and $Y_0 = Y(W_i|D_i = 0)$. Assume the following:

Assumption 4: Ignorability of cutoffs. *The outcome Y is independent of the* *cutoff c_u, conditional on the submitted priority list S_i and admission scores $R_{i,v\ v\in U}$:*

$$Y_1, Y_0 \perp C_u | (S_i, R_{i,v\ v\in U})).$$

In words, the Ignorability of cutoffs assumes that the outcome $Y(W_i)$ does not depend on the realization of the cutoff, conditional on the student's observable characteristics. For example, this will hold if student i's outcome $Y(W_i)$ does not depend on the other students' school assignments, i.e. $Y(W_i) \perp T_j,\ j \in I_{-i}$. It is worth noting that this assumption is also crucial for the identification of the conventional RDD.[10]

Notice that the original Rosenbaum and Rubin (1983) propensity theorem requires the strong ignorability of treatment assumption — it assumes that the selection into treatment is not affected by unobservables.[11] In Proposition 2 below, I adopt the propensity score theorem (Rosenbaum and Rubin, 1983) to the school choice setting, acknowledging that unobservables are reflected in the submitted ordered priority lists and the admission scores. More precisely, given the student's submitted ordered priority list and his admission scores, the admission decision depends only on the realization of the cutoff. Therefore, the student's observable characteristics X_i and the student's unobservable characteristics W_i do not affect his admission outcome D_i, other than through his admission score $R_{i,v\ v\in U}$ and submitted priority list S_i. Consequently, under the assumption that the cutoff realization does not influence the outcome, matching on the propensity scores accounts for the possible selection on unobservables. In other words, unlike in the original propensity score theorem where it is assumed, strong ignorability of treatment follows from the Ignorability of cutoffs.

Proposition 2. *Suppose Ignorability of cutoffs holds. Then treatment assignment is strongly ignorable, in the sense of Rosenbaum and Rubin (1983), given the submitted ordered priority list and the student's admission score:*

$$Y_1, Y_0 \perp D_i | (S_i, R_{i,v\ v\in U}).$$

Therefore, the expected difference in observed outcomes conditional on $P(D_i|(S_i, R_{i,v\ v\in U}))$ is equal to the average treatment effect at $P(D_i|(S_i, R_{i,v\ v\in U}))$:

$$E[Y_1|T = 1, P(D_i|(S_i, R_{i,v\ v\in U}))] - E[Y_0|T = 0, P(D_i|(S_i, R_{i,v\ v\in U}))] =$$

$$E[Y_1 - Y_0|P(D_i|(S_i, R_{i,v\ v\in U}))]$$

.

Proof. The treatment assignment D_i is determined by a mapping Ψ, where $D_i \equiv \Psi(S_i, R_{i,v\ v\in U}, c_{v,v\in U})$; that is, knowing the student's submitted priority list S_i, his

[10] If the Ignorability of cutoffs does not hold, the RDD treatment effect could be driven by the cutoff proximity of the treated students, as their counterfactuals (schools they are assigned if they are just below the treatment) are not necessarily around the cutoff. For example, a student who was marginally declined admission to the cutoff school may be well above the cutoff at his next highest ranked school.

[11] In the school choice setting this might seem unrealistic. For example, suppose that a student is incentivized into a law school, being brought up by a lawyer mother and a lawyer father.

admission scores $R_{i,v\ v\in U}$ and the school-specific cutoffs $c_{v,v\in U}$, one can determine treatment assignment with certainty. Therefore, we obtain:

$$Y_1, Y_0 \perp D_i|(S_i, R_{i,v\ v\in U}) \iff$$
$$Y_1, Y_0 \perp \Psi(S_i, R_{i,v\ v\in U}, c_{v,v\in U})|(S_i, R_{i,v\ v\in U})$$

By assuming Ignorability of cutoffs, i.e. $Y_1, Y_0 \perp c_{v,\ v\in U}|(S_i, R_{i,v\ v\in U})$, the second line above follows directly. Therefore, treatment assignment D_i is strongly ignorable given $(S_i, R_{i,v\ v\in U})$, in the sense of Rosenbaum and Rubin (1983). Using theorem 3 from the same paper, and the Strong ignorability of treatment just obtained, we get:

$$E[Y_1|T = 1, P(D_i|(S_i, R_{i,v\ v\in U}))] - E[Y_0|T = 0, P(D_i|(S_i, R_{i,v\ v\in U}))] =$$

$$E[Y_1 - Y_0|P(D_i|(S_i, R_{i,v\ v\in U}))].$$

■

While Proposition 2 uncovers the treatment effect for students with a particular value of the propensity score, it does not guarantee that there is no heterogeneity in admission scores among these students. More precisely, Theorem 1 in Rosenbaum and Rubin (1983) says that the propensity score is a balancing score, i.e., $(S_i, R_{i,v\ v\in U}) \perp D_1, D_0|P(D_i|(S_i, R_{i,v\ v\in U}))$. Intuitively, given a particular value of the propensity score, the distributions of the submitted priority lists and the admission scores do not differ depending on the treatment assignment. Proposition 2 uses this fact, after proving that submitted priority lists and the admission scores are strongly ignorable, i.e, $Y_1, Y_0 \perp D_i|(S_i, R_{i,v\ v\in U})$, to identify the treatment effect at a particular value of the propensity score (by applying Theorem 3 in Rosenbaum and Rubin, 1983). Therefore, the identification of the average treatment effect at the specified propensity score value is guaranteed due to the balancing property of the propensity score, even though the students who have the same propensity score could potentially have different submitted priority lists and different values of the admission scores. The resulting treatment effect in Proposition 2 is therefore the average over all students with the same propensity score. The following example demonstrates that admission scores can be significantly different for students with the same propensity score.

Example 1. *Suppose that there are two schools A and B, which rank students according to the same admission score. Suppose that student 1, with admission score 100, lists school A as his first priority. Suppose that school A's cutoff is uniformly distributed, with the mean value of 100, so that student 1 has a propensity score for school A of 50%. Suppose that student 2, with admission score 200, lists school A as his second priority, only after school B, which has a uniformly distributed cutoff with the mean value of 200. Even though student 2 has an admission score two times larger than student 1, their propensity scores for school A are the same.*

Example 1 shows that students with the same propensity score can have uncomparable values of the admission score. Therefore, to account for the potential admission score heterogeneity conditional on propensity score (henceforth referred to as heterogeneity), similarly to the conventional RDD, I propose controlling for the admission score, therefore adding the admission score to the conditioning set of Proposition 2:

$$E[Y_1 - Y_0 | P(D_i | (S_i, R_{i,v \ v \in U})), R_{i,v \ v \in U}] \tag{2}$$

Note that, while the RDD utilizes admission scores to deal with both the selection on the unobservables and the heterogeneity, Equation 2 eliminates selection by matching on propensity scores, and uses the admission score only to account for the heterogeneity of students.

Depending on the treatment of the admission score, I define two versions of the PSDD: *Local PSDD* which, similarly to the Nonparametric identification assumption, identifies effects for students with similar admission scores, and the *Global PSDD* which, similarly to the Parametric identification assumption, utilizes the whole sample while controlling for the admission score.

Definition 3: PSDD. *Fix the school u-specific admission score $\hat{c}$, the propensity score value k and an admission score bandwidth δ. There are two versions of the PSDD, depending on the admission score treatment:*

- ***Local PSDD:***

$$PSDD(u, k, \hat{c}, \delta) = E[Y_1 - Y_0 | P(D_i) = k, R_{i,u} \in \langle \hat{c} - \delta, \hat{c} + \delta \rangle],$$

and

- ***Global PSDD:***

$$PSDD(u, k) = E[Y_1 - Y_0 | P(D_i) = k, R_{i,u}].$$

In applications, since there is a limited number of observations at the exact specified propensity score value k, I evaluate the average PSDD over an interval of propensity score values around k. Fix a propensity score bandwidth ϵ and define the *average global PSDD*, $PSDD(u, k, \epsilon)$:

$$PSDD(u, k, \epsilon) = \int_{k-\epsilon}^{k+\epsilon} PSDD(u, \hat{k}) \, d\hat{k}. \tag{3}$$

Average local PSDD, $PSDD(u, k, \hat{c}, \delta, \epsilon)$ is defined analogously:

$$PSDD(u, k, \hat{c}, \delta, \epsilon) = \int_{k-\epsilon}^{k+\epsilon} PSDD(u, \hat{k}, \hat{c}, \delta) \, d\hat{k}. \tag{4}$$

To estimate the average local $PSDD(u, k, \hat{c}, \delta, \epsilon)$, I run the following regression:

$$y_i = \alpha + \rho \cdot D_i, \text{where } PS_i \in \langle k - \delta, k + \delta \rangle \text{ and } R_{i,u} \in \langle \hat{c} - \epsilon, \hat{c} + \epsilon \rangle, \tag{5}$$

where ϵ is the admission score bandwidth for some fixed value of the admission score $\hat{c}$, δ is the propensity score bandwidth for some fixed value of the propensity score k, PS_i is the propensity score of individual i and propensity score, respectively, and ρ is the global average $PSSD(u, k, \delta)$ defined in Equation (4). From a practical perspective, to implement the local PSDD, an analyst can run the usual RDD specification, while restricting the sample to the applications with similar propensity scores. However, unlike in the RDD, where proximity to the cutoff is assumed to eliminate the potential selection on unobservables into schools, the average local PSDD (Equation (5)) eliminates selection on unobservables by restricting the sample to the applications who also have a similar propensity score. Therefore, some applications that are *close to the cutoff* in the RDD sense, might not be included in the PSDD sample. The purpose of restricting the sample to applications with a similar admission score is thus only eliminating heterogeneities in admission scores of students who have similar propensity scores.

To estimate the average global $PSDD(u, k, \delta)$, denote with $Pl_w(\cdot)$ an operator transforming a variable to a polynomial of a fixed degree w, i.e. $Pl_3(PS_i) = \alpha_1 \cdot PS_i + \alpha_2 \cdot PS_i^2 + \alpha_3 \cdot PS_i^3$, and run the following regression:

$$y_i = \alpha + Pl_w(R_{i,u}) + Pl_w(PS_i) + \rho \cdot D_i, \text{where } PS_i \in \langle k - \delta, k + \delta \rangle \tag{6}$$

In some applications, analysts use the Parametric identification assumption 3 employing RDD on the whole data, without using a bandwidth to restrict the RDD sample to those close to the cutoff. In these cases, RDD uses polynomials in the admission score to account for selection into schools. In contrast, the global PSDD (equation (6)) considers only applications with similar propensity scores, i.e. $PS_i \in \langle k - \delta, k + \delta \rangle$, thus explicitly modelling selection on unobservables. Similarly to the above, the purpose of the polynomial in the admission score is thus only to pick up heterogeneity in students' admission scores. In other words, the global PSDD can move away from the cutoff and identify treatment effects by comparing only the applications of students whose assignment, from the ex-ante perspective, is (quasi-) random.

I conclude the section with outlining the procedure for calculating propensity scores. As demonstrated in the previous section, the (ex-ante) probability of student i receiving admission to school u is:

$$P(T_i = u) = \int a_u(S_i, R_{i,u}, J_{-i})\, dH(J_{-i}).$$

Consider the following estimator of the admission probability:

$$\widehat{P(T_i = u)} = \frac{\sum_{r=1}^{N_b} a_u(S_i, R_i, J_{-i,r})}{N_b}, \tag{7}$$

where $J_{-i,r}$ are independent draws from $H^{|I-1|}$, H is the student's distribution (i.e., H determines the student's admission scores and unobservable and observable characteristics), and $|I|$ is the number of students in the application year. The central limit theorem then guarantees the consistency of the estimator (7) as $N_b \to \infty$. Since the set of students I corresponds to the whole population, I assume that the distribution H is completely determined by I, i.e. $H = I$. Then, the independent

$H^{|I-1|}$ draws can be constructed by bootstrapping students with replacement from the set of students I. Below, I provide steps for calculating propensity scores.

Procedure: Calculating assignment probability. Denote with N the cardinality of the set of students I, $N := |I|$ and create a vector of zeros $A_{i,u} = 0$ for each $i \in I, u \in U$. Repeat the following steps N_b times :

(i) Draw N students with replacement from the set I. Denote the generated student sample with $\hat{I}$.

(ii) Given the school's quotas $\{q_u\}, \forall u \in U$ assign exactly one school to each student $i \in \hat{I}$ using the matching algorithm $T_i : \hat{I} \mapsto U$.

(iii) For each student i, matched with some school u, update value $A_{i,u}$ according to $A_{i,u} = A_{i,u} + 1$.

The bootstrapped probability estimate of student i's assignment to a school u is then $P(T_i = u) = A_{i,u}/N_b$.

To implement the proposed bootstrapping procedure, one has to observe a population of schools with their admission quotas (maximum number of admitted students) and a population of students with their submitted priority lists and school-specific admission scores. Since the school-students matching markets under consideration are centralized, this information is usually available to analysts.

4. Empirical Application - DA in Croatia

A large literature estimates various school-graduation effects by employing the RDD around school admission score cutoffs, assuming (quasi-) random school assignment within the implemented RDD bandwidths. In this section, I present evidence suggesting that this key assumption is violated for the bandwidths typically used, using data from the Croatian centralized college admission system from 2014 to 2018. After describing the institutional setup and data, I calculate the propensity scores and evaluate various bandwidths using Lemma 1.

4.1. Institutional Setup and the Data

In Croatia, admissions to all college programs are implemented through a national online platform. Since its introduction in 2010, this platform operates a deferred acceptance (DA) algorithm that ranks students based on their high-school grades and subject-specific elective national-level exams that take place in June, a month after high-school graduation. Students register on the platform in the early spring of their high-school graduation year when universities also list on the platform their program admission quotas along with program-specific weights of subject-specific grades and exams. Students are free to submit their ranked priority lists of up to 10 programs as of registration and update these preference rankings until the system closes for clearing at a predetermined date in mid-July (in 2019, the final deadline was 2 pm on 15$^{\text{th}}$ July).

Students first receive information on their position in various admission queues one week before the final deadline, immediately after receiving their state exam scores and hence, admission scores. The DA algorithm is then regularly updated to show students their current admission position.

I analyze the first preference submission after receiving national exam scores when students are fully aware of their admission scores but do not yet receive the signal about market demand from observing their position in admission queues. This choice is meant to focus on a decision referencing the one-off preference ranking decision in a conventional static DA mechanism with no updating. In addition, by focusing on the first applications students submit after learning their exam performance, I avoid endogeneity issues in admission results that may arise from some students learning about their current admission rankings and being more active in modifying their applications before the deadline. [12] In a recent multi-national study, Altmejd et al. (2019b) argue that the Croatian first preference submissions are structurally similar to the static DA submissions in Sweden and Chile, and find similar siblings' spillover effects on college applications and enrollment in each one of these countries.

Appendix Table 6 shows basic summary statistics for the Croatian DA matching market throughout 2014-2018. The year 2015 is excluded as only the RDD estimation sample is available, which excludes the observations ranked below the admission school — this is not sufficient to calculate propensity scores. Annually, approximately 35,000

[12] I obtained virtually the same results when focusing on the last preference submission.

students enter the matching market, choosing between approximately 620 programs belonging to 49 distinct colleges. An average student applies to approximately six programs, and the average admission rate, calculated as the number of admissions over the number of applications, is just under 0.2.

4.2. Propensity Scores and the RDD

In this section, I generate a function[13] that takes program-specific quotas, and student-specific preferences and admission scores as inputs, performs the DA algorithm, and returns, as output, the matched program T_i for each student i. I validate the function by correctly and completely replicating the actual DA assignments in Croatia. I calculate propensity scores as described in procedure 3, iteratively redrawing students' population, running the DA algorithm, and recording simulated admission outcomes. Student-program-specific propensity scores are then calculated as simple averages of the simulated admission outcomes.[14] The goal of the propensity scores is to extract the ex-ante admission probability for each student-program pair from the complex probability space generated by different programs (and their sizes), students' admission scores, and students' submitted preferences.

The propensity scores predict the actual DA assignments almost perfectly (propensity scores explain 97% of the variation in admission offers), in large part since the majority of the sample (almost 85%) has a deterministic assignment (i.e., propensity score either 0 or 1). In these cases, the propensity score is, by construction, a perfect indicator of the admission.[15]

The RDD approach assumes that students within a limiting neighbourhood of the cutoff have similar admission probabilities regardless of the admission outcome. Using calculated propensity scores and Lemma (1) I evaluate the random assignment assumption using samples of Croatian data defined by the bandwidths typically used in the literature.[16] Most of the RDD studies in the literature choose a constant arbitrary bandwidth, applied to each program-specific cutoff to define a limiting neighborhood. To demonstrate that the bandwidth choice does not drive the results, these studies typically employ robustness checks repeating the estimation for alternative values of the bandwidth. As an example, Abdulkadiroglu et al. (2014) use bandwidths ranging from roughly a third of the standard deviation up to the full standard deviation, while Kirkeböen et al. (2016) use all data (impose no bandwidths) in their main specification.

Figure 1 plots the distribution of propensity scores for the treated and the non-treated group separately[17], and strongly suggests that the bandwidths exceeding half

[13] Codes in Python and R are available upon request

[14] I use 10,000 iterations, ensuring that at the end of the algorithm each additional iteration changes a particular propensity score by at most 0.0001.

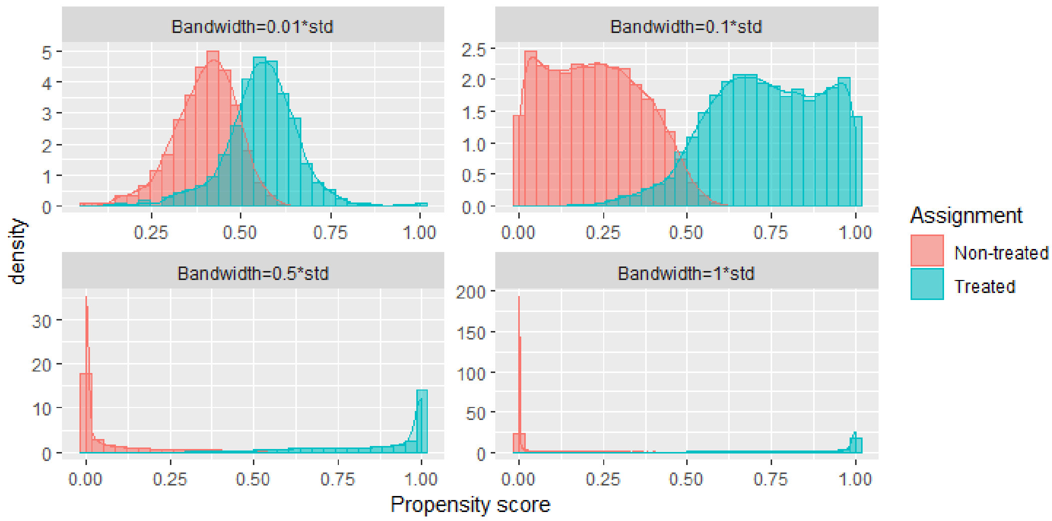
[15] After restricting the sample to applications with propensity scores between 10% and 90%, propensity scores are still a very strong predictor of the admission offer, explaining 81% of the variation

[16] In the appendix I perform the typical Cattaneo et al. (2019) manipulation test around the cutoff, which finds no evidence of discontinuity.

[17] If an application of student i to school s resulted in an admission offer (student i was offered a place in school s), I consider the application treated. Otherwise, I consider the application non-treated.

a standard deviation are excessively large, since using them results in samples in which a sizeable fraction of students is deterministically assigned to a program. More precisely, when considering students with applications that have admission scores at most half a standard deviation away from the school admission cutoffs, 36% of the applications face no assignment risk at all (i.e., their propensity score equals either 1 or 0). When using a full standard deviation bandwidth, 56% of the applications have trivial (0 or 1) propensity scores. Even reducing the bandwidth to 0.1 of the standard deviation results in a sample in which a sizeable fraction of applications face almost no admission risk - 20% of applications have a propensity score higher than 90%.

Figure 1: Distributions of propensity scores for the treated and the non-treated group by RDD bandwidths choice



The figure plots the kernel density of propensity scores in RDD estimation samples defined by different bandwidth values around the cutoffs. The blue (red) histogram plots the distribution for students who were (not) offered admission to the applied school — (non-)treated students. According to Lemma (1), these two distributions should be *similar*. As we increase the bandwidths (to the values typically employed in the literature), the differences between these two distributions become striking.

Further, motivated by Lemma (1), which says that the propensity score, in the limiting neighbourhood around the cutoff, does not depend on the admission score, Figure 1 compares the distributions of the propensity score for the treated and the non-treated groups.[18] It is impossible to compare the applications exactly at the cutoff (all of these applications belong to the treated group). However, one would expect that in a reasonable RDD estimation sample, defined by an appropriate bandwidth, the distributions of the propensity score for the treated and the non-treated groups, as per Lemma (1), are similar. Again, Figure 1 suggests that using a bandwidth of 0.5 standard deviations or more is inappropriate as it results in entirely unbalanced propensity score distributions. For example, when using the bandwidth of 0.5 standard deviations, the average propensity score for the treated group is 85.8%, compared with the 7.6% in the non-treated group. Only a drastic reduction of the bandwidth to a value of around 0.01 standard deviations results in comparable propensity score

[18]By construction (definition of the cutoff), there is a mass of applications exactly at the cutoff (at least 1 application per program). I exclude these applications before plotting the figure.

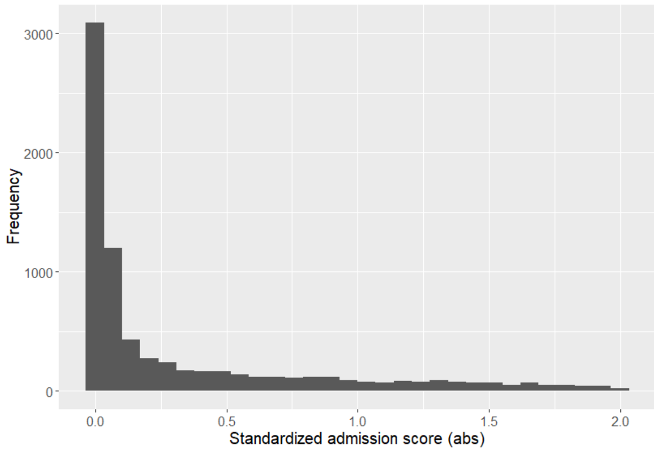
distributions. Additionally, the figure indicates that the implementation of PSDD is feasible for propensity score values close to 50%, since, for these values, both treated and non-treated applications are common (i.e., it is a common support).

The extensive differences between propensity score distributions for the treated and the non-treated students contradict the assumption of random assignment to treatment near the admission threshold. Furthermore, that almost half of applications in "RDD" comparisons face no assignment uncertainty at all directly violates the Lee (2008) non-trivial assignment probability assumption. Since bandwidths used for robustness checks are typically in the range from 0.1 standard deviations to 1 standard deviation, these conclusions hold for them too. Therefore, even if the RDD estimates look stable across robustness checks typically employed, they could be different when using the sample of (quasi-) randomized applications, as the estimation samples differ significantly.

Applying a drastically reduced bandwidth is not a solution either, as any constant bandwidth cannot reflect potentially program-specific cutoff uncertainty. For example, considering only the applications that are at most 0.01 standard deviations away from the cutoff (i.e., 10-50 times smaller bandwidth than in Abdulkadiroglu et al., 2014), which results in 99% of applications within the bandwidths having non-trivial propensity scores, still leaves only 35% of propensity scores between 40% and 60%, as Figure 1 suggests.

Additionally, focusing on narrow neighborhoods around the admission cutoffs comes at the expense of neglecting observations with non-trivial propensity scores that are located outside of the chosen bandwidths. Figure 2 plots the histogram of the distance of the absolute admission score (divided by the standard deviation) from the cutoff for the observations with the propensity scores between 40% and 60% (around 1.7% of the whole sample). Less than 30% of these "highly randomized" observations are captured within the RDD estimation sample defined by a bandwidth of 0.01 standard deviation.

Figure 2: Histogram of admission scores for the applications with random assignment



The figure plots the distribution of the absolute value of the standardized (divided by its standard deviation) admission score for the observations with propensity scores between 40% and 60%. The figure shows that there is a sizeable portion of applications with randomized assignment whose admission score is far from the cutoff. Therefore, the figure suggests that reducing the bandwidths excessively comes at the expense of excluding randomized observations.

In the Croatian case, to adhere to RDD assumptions, one needs to use smaller bandwidths, which, however, leads one to ignore much of the quasi-random assignments available in matching market data. Hence, the application of the RDD design to matching market data faces fundamental obstacles. As described in the previous section, by considering both the admission score and the propensity score, the PSDD, in particular the global PSDD, extracts the ex-ante uncertainty contained in the market-level structure of applications, and instead of choosing an arbitrary bandwidth, focuses on all the applications of students whose assignment is quasi-random, similarly to the lottery-based estimators.

5. Conclusion

This paper provides a new empirical perspective on the nature of assignment uncertainty in centralized matching markets by distinguishing between ex-post randomization reflecting uncertainty after submitting an application, and ex-ante randomization capturing uncertainty at the time of the application. In a typical student-school matching market setting, students submit their applications after learning their admission scores. Therefore, at the time of submitting the application, students are aware of their admission score, and thus the ex-ante uncertainty of admissions is contained in the school-specific score cutoff uncertainty, which in turn is determined by the admission scores and submitted applications of all the market participants. Using this insight, I propose a resampling procedure, which generates the uncertainty of cutoffs by redrawing with replacement from the applicant population and recording the simulated matching market outcomes, to calculate the propensity score for each student-school pair.

I use data from the Croatian DA matching market to compare the distributions of admission propensity scores for treated and non-treated applicants within RDD bandwidths typically used in the literature. I find striking differences that are not in line with the randomized assignment assumption employed in RDD studies. This suggests a drastic reduction of RDD bandwidths and sample sizes. However, the data also implies that the sizeable fraction of quasi-randomized assignments occurs outside of the typical RDD bandwidths. This introduces a trade-off into the RDD implementation. To comply with the RDD assumptions, smaller bandwidths need to be employed. However, small bandwidths ignore a considerable portion of quasi-random variation available in matching market data.

As an alternative approach, I propose a new estimator, the propensity score discontinuity design (PSDD), which applies the Rosenbaum and Rubin (1983) propensity score theorem to the matching market setting. Instead of running the regression using an (arbitrary) bandwidth, identification in PSDD is based on propensity score matching. Therefore, the PSDD focuses exclusively on applications with (quasi-) random treatment assignment, regardless of the proximity to cutoff, extracting the whole ex-ante uncertainty contained in the matching market. Furthermore, while the original propensity score theorem utilizes a strong selection-on-observables assumption, the PSDD elegantly avoids this by acknowledging that any potential selection into treatment must be embedded in the student's submitted preferences and the admission score.

A natural direction for future research is to replicate the results of this paper on a dataset with an outcome variable of interest, such as the Norway college choice dataset where Kirkeboen et al. (2016) estimate labor market returns on different fields of study, and assess the sensitivity of treatment effects with respect to propensity scores. Typical studies run a serious of robustness checks, attempting to replicate the main estimates often using bandwidths that are not sufficiently narrow to exclude all the applications with non-randomized assignments. Therefore, the PSDD estimates might look different from the RDD estimates, even if the RDD estimates look stable across robustness checks typically employed.

Other than being based on ex-ante randomness in assignments, the PSDD offers two advantages over the standard RDD. First, as argued in Kirkeboen et al. (2016), the RDD identifies attendance effects relative to a counterfactual school at the cutoff. To approximate the choice margin, they control for the next most preferred school. In the possible case the next most preferred school is not the correct counterfactual, because this can bias the estimates. As an example, consider a student who was marginally declined in school A, with school B as his next preference. If the student has no chance of being admitted to it, school B bears no information about his counterfactual. Since the propensity scores generate the distribution of all the possible counterfactuals, future research can also concentrate on inspecting and resolving this potential bias. Second, unlike the RDD, which identifies treatment effects only around the cutoffs, the global PSDD specification incorporates all applications with a probabilistic assignment, including those that are potentially well above the cutoff. Future research can use this feature of the PSDD for quasi-experimental identification of away-from-the-cutoff treatment effects.[19] Furthermore, since the PSDD is not tied to the (existence of) cutoffs, it could be used for identification of school-specific treatment effects even in undersubscribed schools that do not have a cutoff.

[19] Angrist and Rokkane (2016) present a method for estimating treatment effects away from the cutoffs in the college choice setting. However, their technique is based on the strong assumption that, conditional on the available covariates, the running variable is ignorable.

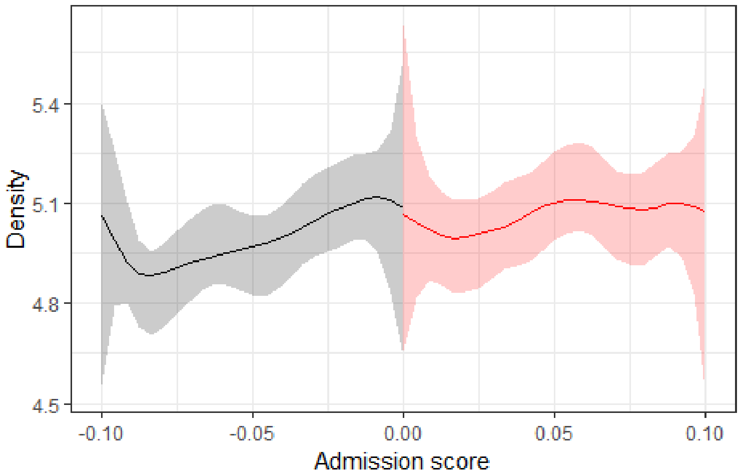
6. Appendix

Table 1: Summary statistics

	Year 2014 (1)	Year 2016 (2)	Year 2017 (3)	Year 2018 (4)
Number of programs	616	620	620	614
Number of colleges	49	49	49	49
Number of applicants	34, 305	34, 518	36, 466	33, 503
Avg. admission score	632.22	648.95	624.47	636.02
	(122.44)	(118.66)	(117.83)	(120.60)
Avg. length of choice list	6.40	6.23	5.58	5.13
	(3.53)	(3.45)	(3.23)	(3.01)
Avg. admission rate	0.15	0.15	0.16	0.18

Notes: The first panel shows the number of programs, colleges and students for each year. The second panel shows the average admission score calculated over all applications in a particular year. The third panel shows the average length of the submitted choice list in a particular year. The final panel shows the average admission rate calculated as the ratio between the number of all applications over the number of all admissions in a particular year. The values in the brackets are standard deviations.

Figure 3: Density of the standardized admission score



The figure implements the manipulation test around the cutoffs employing the local polynomial density estimation method as in Cattaneo et al. (2019). The figure plots a kernel density of standardized admission score centered around school-specific cutoff values using all applications in the data.

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Abstrakt

Mnoho zemí provozuje centralizované systémy přiřazení uchazečů studijních programů do škol. V těchto systémech uchazeči, jejichž bodový výsledek přijímacího řízení je těsně v okolí bodové hranice přijetí do daného programu, jsou do programů přijímáni nebo naopak odmítáni na základě malých kvazi-náhodných rozdílů. Rozsáhlá empirická literatura využívá těchto kvazi-náhodných přiřazení k odhadu efektů přijetí nebo absolvování vysoké školy na různé sledované veličiny. V tomto článku předkládám důkazy naznačující, že vzorky odpovídající obvyklým aplikacím nespojité regrese (dále RDD z anglického regression discontinuity design) v této literatuře nesplňují předpoklady náhodného přiřazení. Rozlišuji ex-post randomizaci (odpovídající loterii uchazečů na hraně přijetí) od ex-ante randomizace, odrážející nejistotu ohledně struktury všech uchazečů v centralizovaném systému, která může být přirozeně kvantifikována opakovaným výběrem z populace uchazečů. S využitím dat z chorvatského centralizovaného systému přijímacích řízení na vysoké školy ukazuji, že ex-ante pravděpodobnosti přijetí se významně liší mezi přijatými a odmítnutými studenty nacházejícími se v obvyklém vzorku používaném pro RDD analýzy. Takový nepoměr v rozdělení pravděpodobnosti přijetí naznačuje, že šířka pásma v okolí hranice přijetí, tj. velikost výběru pro analýzu kvazi-náhodných přiřazení, by měla být oproti současné praxi významně redukována s cílem vyhnout se výběrovému zkreslení. Také ukazuji, že značný podíl kvazi-náhodných přiřazení do přijetí a nepřijetí se nachází mimo typickou šířku RDD pásma, což naznačuje neefektivnost odhadovacích metod. Jako alternativu k RDD metodám navrhuji novou odhadovací metodu Propensity Score Discontinuity Design (PSDD), která využívá všechna pozorování s kvazi-náhodným přiřazením a srovnává výsledky uchazečů porovnatelných co do ex-ante pravděpodobností přijetí do daného programu, tj. pravděpodobností přijetí podmíněných bodovými výsledky příjímacích řízení a strukturou poptávky.

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