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A Note on Jain´s Digital Piracy Model: Horizontal vs Vertical Product Differentiation

Michael Kúnin
Krešimir Žigić

CERGE-EI
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A Note on Jain's Digital Piracy Model: Horizontal vs Vertical Product Differentiation

Michael Kúnin[1] and Krešimir Žigić[2,3]

Abstract

We study how private intellectual property rights protection affects equilibrium prices and profits in a duopoly competition between firms that offer a product variety of distinct qualities (vertical product differentiation) in a setup that is closely related to that put forward by Jain (2008), where firms offer the same qualities in equilibrium (horizontal product differentiation). Consumers may make a choice to buy a legal version, use an illegal copy (if they want to and can), or not use a product at all. Using an illegal version violates intellectual property rights protection and is thus punishable when disclosed. Thus, both private and public (copyright) intellectual property rights protection are available on scene.

Keywords: Vertical and horizontal product differentiation; Software Piracy; Bertrand competition; Private and public intellectual property rights protection

JEL Classification: D43, L11, L21, O25, O34

[1] michael.kunin@cerge-ei.cz, CERGE-EI

[2] CERGE-EI, a joint workplace of the Center for Economic Research and Graduate Education, Charles University, and the Economics Institute of Academy of Sciences of the Czech Republic, Politických vězňů 7, Prague 1, 111 21, Czech Republic. The authors are grateful to Andrea Downing for her superb English editing assistance.

[3] kresimir.zigic@cerge-ei.cz, CERGE-EI

1 Introduction

In an innovative and insightful model, Jain (2008), addresses the issue of digital piracy by end-users and shows that under some plausible conditions firms (providers of digital content) may be better off by not using private intellectual property rights (IPR) protection against piracy. Moreover, he shows that this might be the case even in the absence of any network externalities given that the presence of the latter has often been used as an argument for a lax approach to digital piracy. An important result of his model is the fact that an increase in a firm's IPR protection always leads to a decline in the equilibrium prices, and, consequently, may result in a fall in a firm's profit.

Jain (2008) used the Hotelling model of horizontal product differentiation with two firms located at the end of the Hotelling line and explore a three-stage game. The firms choose the quality of their product in the first stage, strength of private IPR protection in the second stage, and finally, compete in prices in the last stage. Since the focus of our analysis is on IPR protection and its impact on prices and profits, we drop the first stage as it is not important for our discussion. We build a similar model that departs from Jain's (2008) approach in two respects. First, we replace the horizontal product differentiation setup with a vertical one (see Žigić, et al, 2015 for examples of vertical product differentiation in software markets) and second, in addition to a firm's own (or private) IPR protection, we also add public protection, namely, copyright. We would then study how robust Jain's (2008) findings on the impact of private IPR protection on firms' prices and profits are in this new setup.

2 Model Set Up

Developers A and B compete in prices on a particular market and offer product varieties of different quality. Developer A releases a product of quality q_A, while the quality of developer B is q_B and we assume, without loss of generality, that developer A offers higher quality ($q_A > q_B$). Product qualities q_A, q_B are exogenous and cannot be changed by the developers, and the unit variable costs are constant and normalized to zero. One may think about developer A as an already established and known software producer that already operates on other markets. This fact is, in turn, reflected in the preferences of the consumers, who strictly prefer software A over software B if they are offered at the same price. Similarly, developer B can be thought of as a local developer offering lower quality. In other words, we assume that both developers already existed before meeting and competing on the market under consideration. Consequently, both developers are assumed to have already incurred set-up fixed costs and fixed costs associated with software development (R&D costs).[4]These fixed costs are, from our perspective, general and not related to the developer's presence on the particular market under consideration, and we therefore leave them out of the profit function. The probability of being caught using an illegal version is the same for all users, and the level of the penalty is fixed. The penalty and the probability of being caught is known and independent of used product and product prices; thus all users and both developers could calculate the expected penalty for using an illegal version, which we denote as X. Moreover, while we implicitly assume that the regulator choice of optimal IPR is governed by an underlying objective function such as the maximization of social welfare, we do not explicitly study the optimal choice of expected penalty since we focus on the forms of the developers' pricing and IPR protection strategies and their economic implications. Thus, the whole regulator's framework is very simple in our model and translates into one parameter: expected penalty X for illegal users, which also captures the strength of the copyright protection (see Varian's, 2005 survey on the economics of copyrights).

While in principle both *developers* could have access to technology that allows product protection against copying and illegal usage[5], we assume that only a high-quality developer may adopt the protection and this decision is dependent only on the profitability of such a step. A reason for this simplifying assumption could be that hardware protection is not

[4]Alternatively, one can think of these costs as neccesary expenditures to inform the consumers about the existence and quality of their product (like marketing and advertisement). In the language of Duchêne and Waelbroeck (2006) approach, developers rely on information push technologies to diffuse the above pieces of information.

[5]Neither legal nor licence restrictions are assumed for the developer in the case of implementing protection against copying.

available or too costly for a low quality developer, or that the level of public IPR protection is such that it would never be optimal for developer B to adopt protection. In separate supplementary material, however, we provide the complete analysis of the setup where both firms may adopt protection, available on request.

The protection against copying is imperfect, which means that a fraction of the users still have access to the illegal version[6]. Much like in Jain (2008), we say that a developer implements protection at level $c \in [0, 1]$, whereby the level of c represents the fraction of consumers "controlled" by a high quality developer, that is, the share of consumers who are unable to use the software illegally due to the private IPR protection. (In Jain's, 2008 notation $c = 1 - \alpha$)

If c tends to 1 we say that protection becomes perfect and all end users are controlled, while c tending to 0 represents the full public availability of an illegal version[7]. Formally, there is a two-stage game in which a high-quality developer chooses the level of private protection in the first stage, and then two developers compete in prices in the second stage. Thus we use a sub-game perfect equilibrium as a solution concept.

Regarding the developers' cost of incurring protection, Jain (2008) does not give it much attention, since it would not qualitatively change his results (more specifically, adding these costs would only reinforce his findings). While these costs are also not essential for the main argument and the focus of our analysis, we still consider them important for understanding why only the high quality developer incurs these costs. This is because, given the equilibria we focus on, there is no need for a low quality developer to undertake such costs, since the public protection is high enough to protect developer B from piracy. In addition, the private protection of developer A, for whom it is optimal to make private IPR protection also enables developer B to free ride on this protection. Alternately, we can assume that

[6] By eliminating public availability we mean both no access to an illegal version or to an illegal version accompanied by the limited user's skill to install/use the illegal version.

[7] The availability of an illegal version and the ability to break it differs significantly among users and is more dependent on technical skill than on sensitivity to price and quality. The uniform distribution is an analytical simplification that does not harm the nature of the paper.

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developer B does not have the technological capabilty to protect his software from piracy. Thus, we assume that there are convex costs, $h(c)$,of implementing protection c, such that $\Pi_A^* = \pi_A^*(c) - h(c)$ is a concave function reaching its maximum at $c^* \in [0,1]$ (we use the symbol Π for net profit when protection costs are accounted for, while π stands for the price-competition stage profit).

Regarding *consumers*, there are two segments of users, and in each segment consumers differ in their quality sensitivity θ, which has density 1 on $\left[0, \bar{\theta}\right]$. The first segment are the potential copiers ("copier segment") and the second segment are consumers who never opt for an illegal version of software ("non-copier segment"). Regarding the first segment, these are consumers who are willing to copy if they were in a position to do it and, as in Jain (2008), the size of this segment is β (which can be bigger or smaller than a unit). The empirical finding shows that the users in this segment are more price sensitive and have lower willingness to pay (see Cheng et al, 1997) than the consumers in the second segment; so, following Jain (2008), to account for this fact we introduce a discount factor $0 < \delta < 1$ for this segment. Due to the private IPR protection only some of those users have access to both a legal and illegal version, while some users have access only to a legal version. The users with access to both versions prefer the legal version only if the utility from it is higher and their proportion is $1-c$. The utility function of a user θ in the first segment is as follows:

$$U_P(\theta) = \begin{cases} \delta\theta q_i - p_i & \text{... if he buys the legal version of the software} \\ \delta\theta q_i - X & \text{... if he uses the software illegally.} \\ 0 & \text{... if he does not use the software at all.} \end{cases} \tag{1}$$

We also assume that if the price of the legal version of a product exactly equals the expected punishment for using the illegal one, $p_i = X$, then the consumers strictly prefer the legal version—in other words, second-order stochastic dominance applies.

The utility of a non-copier user θ is:

$$U(\theta) = \begin{cases} \theta q_i - p_i & \text{... if he buys the software.} \\ 0 & \text{... if he does not use the software at all.} \end{cases} \tag{2}$$

As we already noted, in principle both developers could implement protection for their product, and so three basic combinations of product protection could occur in the market :

1. None of the developers implement protection. This situation arises when X does not bind in the maximization problems of either A or B, so that in the equilibrium we have $p_B^* \leq p_A^* \leq X$.

2. Developer A implements protection while developer B does not. This situation occurs when pure Bertrand equilibrium is not possible because X would be binding for developer A since $p_B^* \leq X \leq p_A^*$.

3. Both developers implement protections.[8] Finally for low X, both developers would have to introduce protection since pure Bertrand equilibrium would result in $X \leq p_B^* \leq p_A^*$. As already stated, we do not focus on this case in the main text but provide the relevant analysis in separate supplementary material.

We focus on the case where only developer A has the incentive to introduce protection, that is, $p_B^* \leq X \leq p_A^*$. This case seems to be relevant for middle and, perhaps, some high per capita income countries, while the situation associated with zero or very low effective strength of copyright protection is typical in developing countries (see Fig. 1 in Varian 2005). Note that in our set-up, prices are, as is typical, strategic complements (see Tirole, 1989, and Bulow et al., 1985), that is, $\frac{\partial^2 \pi_i}{\partial p_B \partial p_A} > 0$.

[8] Note that the case in which only developer B implements protection never occurs. If B has to implement protection due to the low expected penalty X, then developer A must also implement protection because his product would be the primary target of illegal usage.

3 Demand Function

Before we start with solving the duopoly model backward, we have to work out the demand functions in the potential copier segment that could emerge in the setup under consideration. In the case where $p_B^* \leq X \leq p_A^*$, only developer A has the incentive to implement protection since the product of developer B would only be used legally. As we already mentioned in our model set-up, the illegal version of product A is available only to the fraction $1-c$ of the users' base in a copier segment. Product A is used illegally only by users with $\frac{X}{\delta q_A} \leq \theta$, while users with $\theta \leq \frac{X}{\delta q_A}$ prefer not to use the product at all. The demand for product B consists of users with low sensitivity θ to purchasing product A, who, at the same time, have no access to an illegal version of A, but their θ is high enough to buy product B. These users have $\theta \in (\frac{p_B}{\delta q_B}, \frac{p_A - p_B}{\delta(q_A - q_B)})$, and their fraction is c. Regarding the users with access to an illegal version of product A, there are two main sub-cases that could occur in equilibrium depending on the size of the expected penalty:

1. The first sub-case occurs when there are some users who have illegal access to product A but still want to buy product B, or more formally, the measure of these users is strictly positive with $\theta \in \left(\frac{p_B}{\delta q_B}, \frac{X-p_B}{\delta(q_A-q_B)}\right)$, and so, $\frac{X-p_B}{\delta(q_A-q_B)} > \frac{p_B}{\delta q_B}$. These users would like to purchase product B if X is "large enough" (in the sense that $X > p_B \frac{q_A}{q_B}$). Looking at it from the developers' point of view, developer B competes for the consumers that have illegal access to software (so called "non-controlled" consumers) by aggressively charging a low price so that $p_B^* < \frac{q_B}{q_A} X$. The market coverage is given in Figure 1 .

2. The second sub-case occurs when illegal users always prefer an illegal version of A to the legal version of B, that is, when $\delta\theta q_A - X > \delta\theta q_B - p_B$ for all θ since illegal usage is then more valuable even for the consumer with the lowest valuation. So, X has to be "low" enough, that is, $\frac{X-p_B}{\delta(q_A-q_B)} \leq \frac{p_B}{\delta q_B}$ (or equivalently $X \leq p_B \frac{q_A}{q_B}$) given that $p_B^* \leq X$ still holds. From the perspective of the developers, developer $B's$ price is "too high" to attract the non-controlled consumers and in this situation his profit fully depends on the protection of developer A. The market coverage of this case is presented in Figure

2

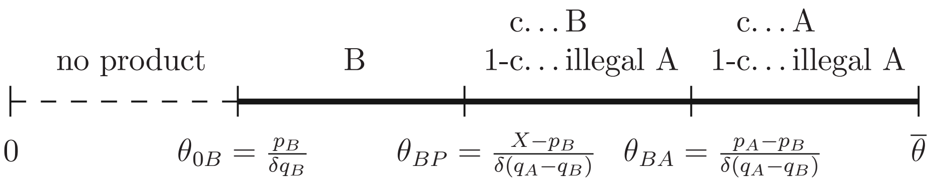


Figure 1: BC, when developer A introduces protection c (Case 1).

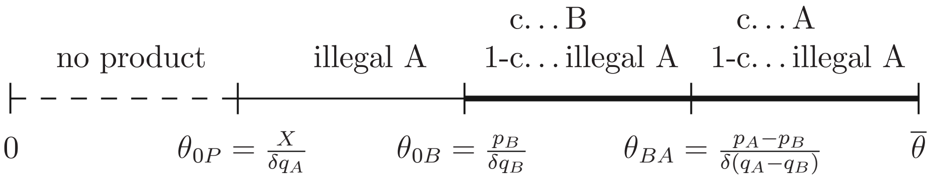


Figure 2: BC, when developer A introduces protection c (Case 2).

Given that the products' demands on the "non-copier segment" is straightforward (that is, $\left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right)$ for A and $\left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right)$ for B), we obtain the total demand for legal versions of both products on both segments by putting all fractions of users together (Subcase 1):

$$D_A = \beta c \left(\bar{\theta} - \frac{p_A - p_B}{(q_A - q_B)\delta}\right) + \left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right) \tag{3}$$

$$D_B = (1 + \frac{\beta c}{\delta}) \left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) + \frac{\beta(1-c)}{\delta} \left(\frac{X - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) \tag{4}$$

If only the users without access to an illegal version of A buy product B, the demand function for developer B is now (Subcase 2):

$$D_B = \left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right)(1 + \frac{\beta c}{\delta}).$$

Interestingly enough, much like in a monopoly (see Žigić at al. 2020), the change in the strength of copyright protection, X, in the sub-case 2, does not affect (at the margin) either the developers' pricing or the IPR protection strategy of developer A. This is because for the particular values of X, developer B does not find it optimal to compete for the illegal ("non-controlled") users of product A but instead focuses (or free rides) on the (lower segment of) users whom developer A prevents from using the software illegally by means of hardware-based protection. So the only target of both firms is the so called "controlled" consumers who legally buy the products and whose fraction is c in both segments of the market.

4 Solving the Model - Equilibrium analysis

4.1 Types of equilibria

As the developers choose their prices to maximize profits, the following can be shown to hold.

Lemma 1 *Each developer can choose its price in a way that obtains a positive profit whatever the other developer's price is.*

Lemma 2 *In any equilibrium, the "non-copier" segment of the market shares of both developers are strictly positive.*

These results mean that the equilibrium market structures are exclusively determined by what happens in the "copier" segment. As shown in Žigić et al. 2020, equilibria with both developers present in the "copier" segment can be classified according to three features: a) the need for private protection to be exercised in equilibrium; b) the character of the optimal solution, that is, whether the profits attained their maxima at the corner or at the interior solution, and; c) the status of product B for non-controlled consumers, that is, whether developer B competes for them or fully depends on developer A's IPR protection.

We call equilibria in which firm B fully depends on the private protection of firm A, "full dependence" equilibria, while those in which this is not the case are called "no full dependence" equilibria. More precisely, given these three features above, there are five possible equilibrium outcomes[9]such that both developers have a positive market share in the "copier" segment that may occur in the set up under consideration:

1. *Unconstrained duopoly*: $p_A^* < X$, which also implies an interior solution for developer A. Then protection is not needed, developer B's profit maximum is also interior, and the outcome coincides with that of the pure Bertrand duopoly.

2. *Constrained duopoly*: $p_A^* = X$, with a corner solution for developer A. Then protection is not needed, developer B's profit maximum is interior, and the outcome coincides with that of the constrained Bertrand duopoly with $p_A^* = X$.

3. *Piracy, no "full dependence"*: $p_A^* > X$, $p_B^* < X\frac{q_B}{q_A}$.

4. *Piracy, interior "full dependence"*: $p_A^* > X$, $X\frac{q_B}{q_A} < p_B^* \leq X$. Then all consumers not controlled by developer A use product P (or nothing).[10]

5. *Piracy, corner "full dependence"*: $p_A^* > X, p_B^* = X$. Then all consumers not controlled by developer A use product P (or nothing), and the equilibrium prices are given by $p_B^* = X$ and p_A^* being the reaction to $p_B^* = X$. Here both X and c enter both developers' profits.

In addition, it might be optimal for developer A to ignore the "copier" segment by setting $c = 0$ and $p_A > X$. In such cases, which typically occur when δ and/or β are small, developer B might either ignore the "copier" segment as well ($p_B > X\frac{q_B}{q_A}$) or enter that segment ($p_B < X\frac{q_B}{q_A}$; if $p_B < X - \delta\bar{\theta}(q_A - q_B)$, in which case developer B would be a monopolist in that segment).

There are three (out of the five) possible equilibrium outcomes with both developers present in the "copier" segment [11]: 1) the piracy no "full dependence" equilibrium; 2) the

[9]Note that in any equilibrium both legal goods have a positive market share. As stated in Žigić et al. 2020, developer B can guarantee a positive market share by setting $p_B = \frac{\min\{p_A, X\}q_B}{2q_A}$; as for developer A, $p_A = \min\{p_B, X\}/2$ does this, which also means that $p_B^* \leq \min\{p_A^*, X\}$ in any equilibrium with $c_A = c$ and $c_B = 0$.

[10]However, unlike in Žigić at al. 2020, the equilibrium prices are not the same as in the pure duopoly in this case.

[11]Note that due to non-continuity and non-unimodality of the profit functions, there are parameter constellations such that more than one equilibrium type can occur.

piracy corner "full dependence" equilibrium, and 3) the piracy, interior "full dependence" equilibrium. We, however, focus on the no "full dependence" equilibria as the only equilibrium a) where the strength of public IPR protection affects (at the margin) both firms'pricing and b) where the high-quality developer sets his private IPR protection strategically. We briefly discuss the other two equilibria (where c appears in equilibrium values) in the appendices.

As for the equilibria in which developer A is absent in the "copier" segment, it holds in all such equilibria that $c = 0$, so while the public protection X may influence the equilibrium prices via developer B's interaction with the pirate product in the "copier" segment, it does not marginally affect the private protection by developer A, which remains zero.

4.2 Deviation to Not Serving the Copier Segment

Before moving to the analysis of the equilibria, we have to check the conditions that none of the developers deviate to serving only "non-copier" segment. Jain (2008) shows that in his model the condition $\delta \geq \frac{1}{2+\beta}$ is sufficient for not deviating to but only serving the the non-copier segment. It turns out that in our model the following holds for developer A.

Proposition 1 *Let (p_A, p_B) be an equilibrium candidate such that both developers enter both consumer segments, maximize their profits taking the other price as given but not considering a deviation to not serving the non-copier segment. Then $\delta \geq \frac{1}{2+c\beta}$ is sufficient for developer A not to make such a deviation.*

Proof. *First assume the equilibrium candidate is such that developer A maximizes its profit, i.e. it is not a constrained duopoly.*
Given p_B, developer A's reaction functions and profits are

$$p_A = \frac{1}{2}\left(p_B + \frac{(1+c\beta)\delta}{c\beta+\delta}\left(q_A - q_B\right)\bar{\theta}\right), \Pi_A = \frac{\left(\left(c\beta+\delta\right)p_B + \left(1+c\beta\right)\delta\left(q_A-q_B\right)\bar{\theta}\right)^2}{4\delta\left(c\beta+\delta\right)\left(q_A-q_B\right)}$$

when both consumer segments are served and

$$p_A = \frac{1}{2}\left(p_B + (q_A - q_B)\bar{\theta}\right), \Pi_A^0 = \frac{\left(p_B + (q_A - q_B)\bar{\theta}\right)^2}{4(q_A - q_B)}$$

when only the non-copier segment is served. Then

$$\Pi_A - \Pi_A^0 = \frac{c\beta(q_A - q_B)\bar{\theta}^2}{4(c\beta + \delta)}((2 + c\beta)\delta - 1) + \frac{1}{2}c\beta\bar{\theta}p_B + \frac{c\beta}{4\delta(q_A - q_B)}p_B^2.$$

Here the terms containing p_B are non-negative whereas the first term is non-negative iff $\delta \geq \frac{1}{2+c\beta}$, which completes the proof.

The constrained duopoly case is analysed in the Appendix. ■

As for developer B, note that in the cases of duopoly and interior "full dependence" the profit function when serving both segments, which has an interior maximum given p_A, is exactly $(1 + c\beta/\delta)$ times the profit function when only serving the non-copier segment; therefore such a deviation never occurs in these cases. The "no full dependence" case for developer B is analysed in the Appendix. However, in the corner "full dependence" case, i.e., when $p_B = X$, it is obvious that if X is sufficiently low then developer B would prefer to set a price $p_B > X$ and only serve the non-copier segment.

4.3 The piracy no "full dependence" equilibrium

The piracy no "full dependence" equilibrium, that is at the center of our attention, occurs within the Subcase 1 presented above. Thus, we start with determining the range of the expected penalty values X such that this sub-case is the Nash equilibrium in prices. Namely, sub-case 1 is not an equilibrium if (i) at least one developer's profit, given the other developer's price choice, does not have a local maximum in the relevant price range. Also, it is not an equilibrium if (ii) there is a local maximum in the relevant range, but at least one developer is better off deviating to a price outside the range (e.g., developer A can be better off deviating to $p_A = X$). Finally, it is not an equilibrium if (iii) developer A is better off not entering the "copier" segment at all (see Proposition 1). Note that there is no deviation by developer B to only serving the "non-copier" segment if c is not "too low" as shown in the Appendix. More specifically, $c \geq 1/9$ is sufficient and the no "full dependence" equilibrium typically does not occur at such low values of c—see the numerical example below.

Intuitively, for developer A to charge a high price $p_A > X$, the value of X should be small enough so that developer A prefers introducing protection than simply lowering the price to X. For developer B to charge a low price $p_B < X\frac{q_B}{q_A}$, X should be large enough so that developer B prefers charging a low price to both charging an intermediate price $X\frac{q_B}{q_A} \leq p_B \leq X$ or charging a high price $p_B > X$ and introducing protection. In addition, the "copier" segment should be attractive enough in the sense of β and δ being high enough.

If this equilibrium occurs, then the equilibrium prices are

$$\begin{aligned} p_A^* &= \frac{\beta\left(c\beta+\delta\right)X\left(1-c\right)q_B+2\bar{\theta}\left(1+c\beta\right)\delta\left(\beta+\delta\right)q_A\left(q_A-q_B\right)}{\left(c\beta+\delta\right)\left(4\left(\beta+\delta\right)q_A-\left(c\beta+\delta\right)q_B\right)}, \\ p_B^* &= q_B\frac{2\beta X\left(1-c\right)+\bar{\theta}\delta\left(1+c\beta\right)\left(q_A-q_B\right)}{4\left(\beta+\delta\right)q_A-\left(c\beta+\delta\right)q_B}. \end{aligned}$$

It is straightforward to show that if $\delta = 1$ and $\beta \to \infty$, then these prices converge to those in Žigić et al. (2020), and that (i) and (ii) hold for a non-empty range of X when c, β, and δ are sufficiently high.

4.4 Other equilibrium structures

As for other equilibrium structures, there are two distinct cases. First, there are duopoly structures with $p_A \geq X$, where there is no interaction between public and private protection. Second, there are "full dependence" structures, where the outcome is basically the same as in Jain (2008). Specifically, of the two effects we consider below, the consumer base effect and the price sensitivity effect, the latter always dominates under such equilibrium structures, so that an increase in private protection by developer A results in a decrease in developer A's price and, in the interior "full dependence" case, in developer B's price (recall that in the corner "full dependence" case, $p_B = X$). The formulae can be found in the Appendix.

5 Vertical versus horizontal product differentiation

To make our comparison with Jain's, (2008) model as insightfull as possible, we, as claimed above, focus on the most interesting "no-full dependence" equilibrium, in which the private IPR protection enters both prices and profits of both developers and in which developer A chooses his private IPR protection strategically. Roughly speaking, this equilibrium occurs when the copier segment is large enough, price sensitivity is not very small and the copyright protection is such that it pays off for developer B to compete for the users who have access to the illegal version (i.e.X is in the "midrange" of permissable values; see the numerical example below). Following Jain (2008), we put the comparative statics analysis with respect to c in the form of a Proposition:

Proposition 2 *Equilibrium prices $p_A^*(c)$, $p_B^*(c)$ and the profit $\pi_B^*(c)$ show in general non-monotonic behavior in private IPR protection c. They increase in c for a high enough discount factor δ and decrease otherwise.*

The Proof can be obtained on request in the form of a Mathematica file.

The reason behind the above result is that there are two opposing effects at work at the copier segment of the market. First, strengthening of the private IPR protection by the developer A enables him to broaden the base of his end users and thus to increase the price of his product. An increase in A's protection, in turn, has also direct positive impact on firm $B's$ market share since $A\prime s$ protection applies also on consumers with lower valuation who opt for the product B. Moreover, the competitive segment, $1-c$, on which developer B competes for (potential) illegal users of product A, shrinks as c increases, enabling developer B to also increase his price. We name the above effect as the *consumer base effect.* In other words, in the absence of price sensitivity (i.e. $\delta = 1$) this would be the only effect, and so developer A acts strategically and softens the price competition by overinvesting in c and (in jargon) displays pacifistic "fat cat" behavior (see Fudenberg and Tirole, 1984). However, since $\delta < 1$, there is also a second effect, which works in the opposite direction. Imposing private protection on the fraction of (potentially) copying consumers would tend to lower prices in equilibrium since any increase in the fraction of price sensitive consumers would, ceteris paribus, require a lower price in the absence of price discrimination. Clearly, if this *price sensitivity effect* is very strong then it dominates and equilibrium prices would be adversely affected by imposing IPR protection. In Jain, (2008), however, the impact of private IPR protection on equilibrium price is always negative for any value of discounting factor lower than a unit (and zero for $\delta = 1$) and so the second effect (price sensitivity) dominates across all permissible values of δ and β.

In light of the above intuition for *Proposition* 1 it is also insightful to qualify our findings in relation to the size of the copier segment (i.e, a change in β). If the copier segment gets very large (β tends to infinity), then only it matters, and so the significance of IPR protection (consumers' base effect) is of critical importance and has an undoubtedly positive effect on equilibrium prices (and the profit of developer B) as long as $\delta \geq \frac{1}{2+c\beta}$. If, on the other hand, the price sensitivity effect is extremly strong (δ tends to zero) it might more than offset the first, positive, consumers' base effect or, when $\delta < \frac{1}{2+c\beta}$, it may lead developer A to abandon the copier segment setting $c = 0$ and focus only on the more profitable non-copier segment (in which there is no piracy). In other words, there are critical values of δ at which there is a switch of the sign of dp^*/dc from negative to positive as δ moves from zero to one, that is, $\frac{\partial^2 p_A}{\partial \mathbf{c} \partial \boldsymbol{\delta}} > 0$. (It is straightforward to show that dp_A/dc is positive for $\delta = 1$ and negative for $\delta = 0$, irrespective of the size of β).

Lemma 3 *The higher the size of the copier market segment, the more important the consumers' base effect is for developer A, given the size of the price sensitivity* $\delta \geq \frac{1}{2+c\beta}$*. Thus* $\frac{\partial^2 p_A}{\partial \mathbf{c} \partial \boldsymbol{\beta}} > 0$ *when* β *is large enough.*

The Proof can be obtained under request in the form of a Mathematica file.

Regarding developer B, the effect of the copier segment size on $\frac{\partial p_B}{\partial c}$ is generally ambiguous due to the competition from the illegal product which may or may not be offset by the consumers' base effect.

Last but not least, unlike in Jain's (2008) symmetric model where the equilibrim price is the same for both developers (so the change in equilibrium prices due to a rise in IPR protection for both firms is the same and has always the same, negative, sign); in our setup, however, it is quite possible that an increase of private IPR protection would have an opposite impact on the equilibrium prices. In particular, it is quite conceivable that $\frac{dp_B}{dc} > 0$ while $\frac{dp_A}{dc} < 0$ when the price sensitivity is strong enough to more than offset the consumer base effect for firm A but still not strong enough for the same effect for firm B.[12] The reason for this, as we discussed above, is that developer B has higher benefits from this protection at the margin than developer A . Moreover, a marginal increase in c is not associated with any marginal costs for developer B.

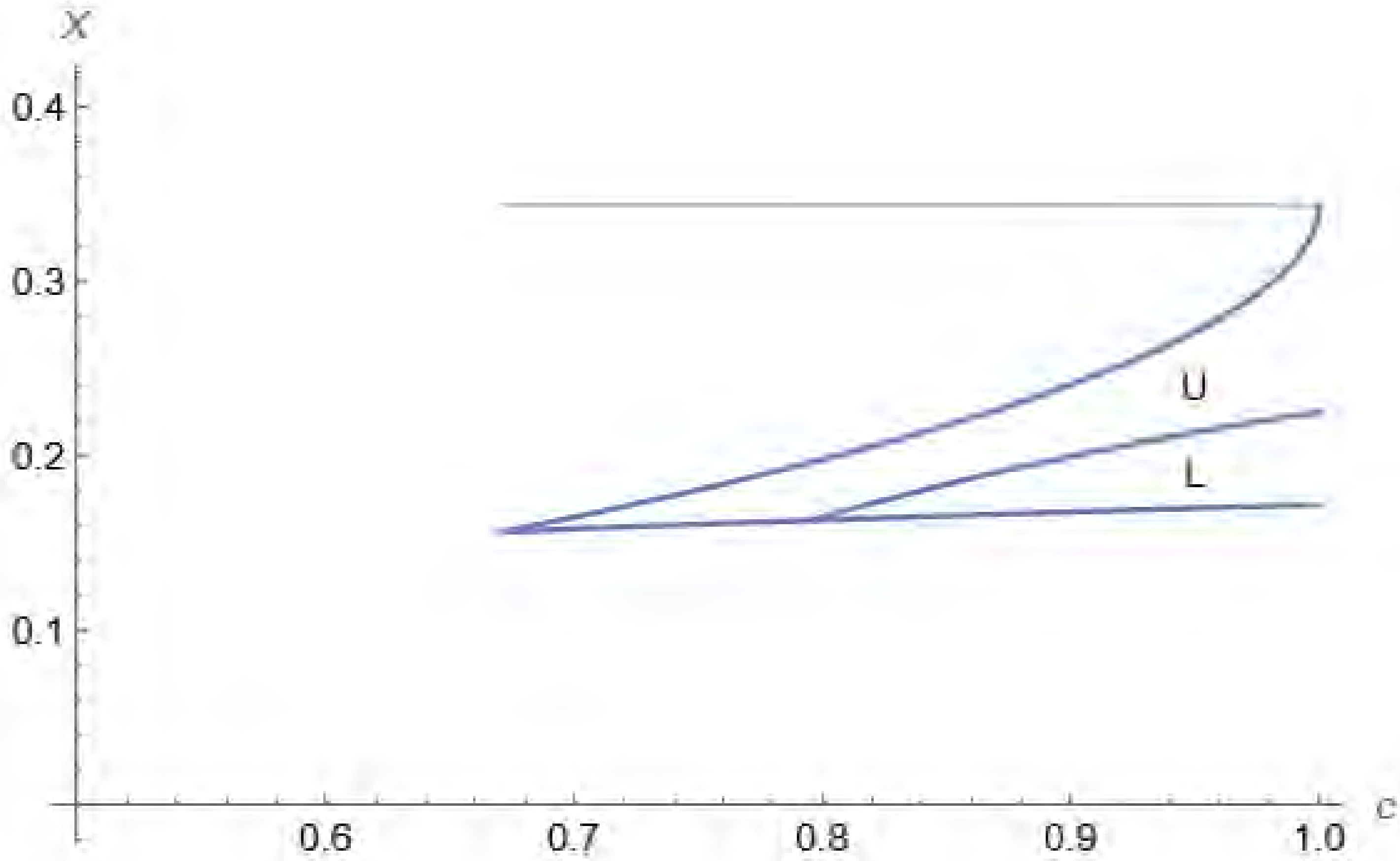
In our model, on the other hand, the first effect (increase in market share) is predominant unless the discount factor is so low as to counterweigh it. In other words, if consumers in the potential copying segment are very price sensitive, then the second effect would take over and $\frac{dp_A}{dc} < 0$ **.** When this is the case, then the strategic effect of firm A would imply an underinvestment in private protection and trigger the "lean and hungry look" strategy. Consequently, for "rather small" δ, it would be optimal for developer A not to invest in private IPR protection at all. To conclude, there is in general a non-monotonic relationship between private IPR protection and equilibrium prices in our extended model.

5.1 Numerical Example

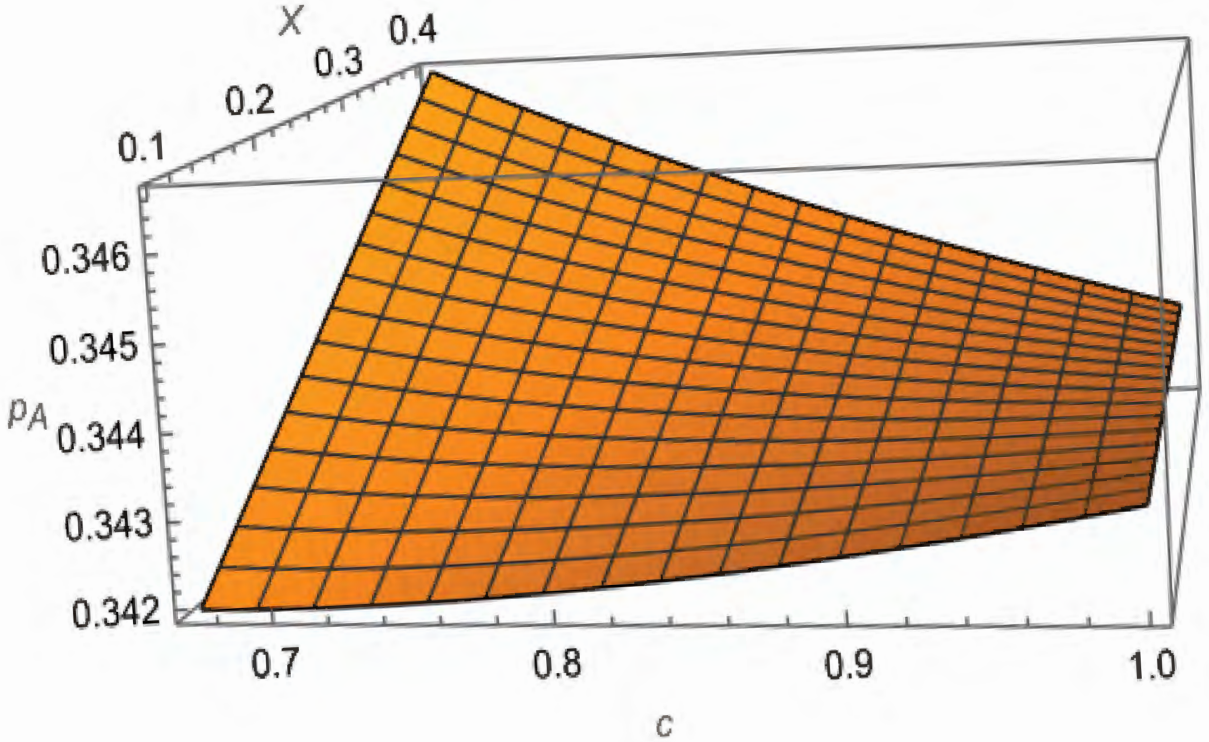
In order to illustrate the key findings in Proposition 2 and other important comparative statistics results, we use the following parameter values: $\bar{\theta} = 1$, $q_A = 1$, $q_B = 0.2$, $c = 0.9$, $\delta = 0.8$, $\beta = 10$. Under these values, developer A's unconstrained duopoly price (which is also the value of X above which an unconstrained duopoly occurs), is $\bar{X} = 0.343080$ (all numerical values are approximate). We show that the piracy "no full dependence"

[12] For $\frac{dp_B}{dc} < 0$ to hold, the discount factor has to be substantially lower than the critical δ for $\frac{dp_A}{dc} < 0$ since developer B benefits even more from A's protection. For example, it can be shown that $\frac{dp_A}{dc} < 0$ for the entire "no full depencence" equilibrium range for $\delta < \frac{5-\beta}{6}$, but the corresponding condition for $\frac{dp_B}{dc} < 0$ is $\delta < \frac{1-2\beta}{3}$.

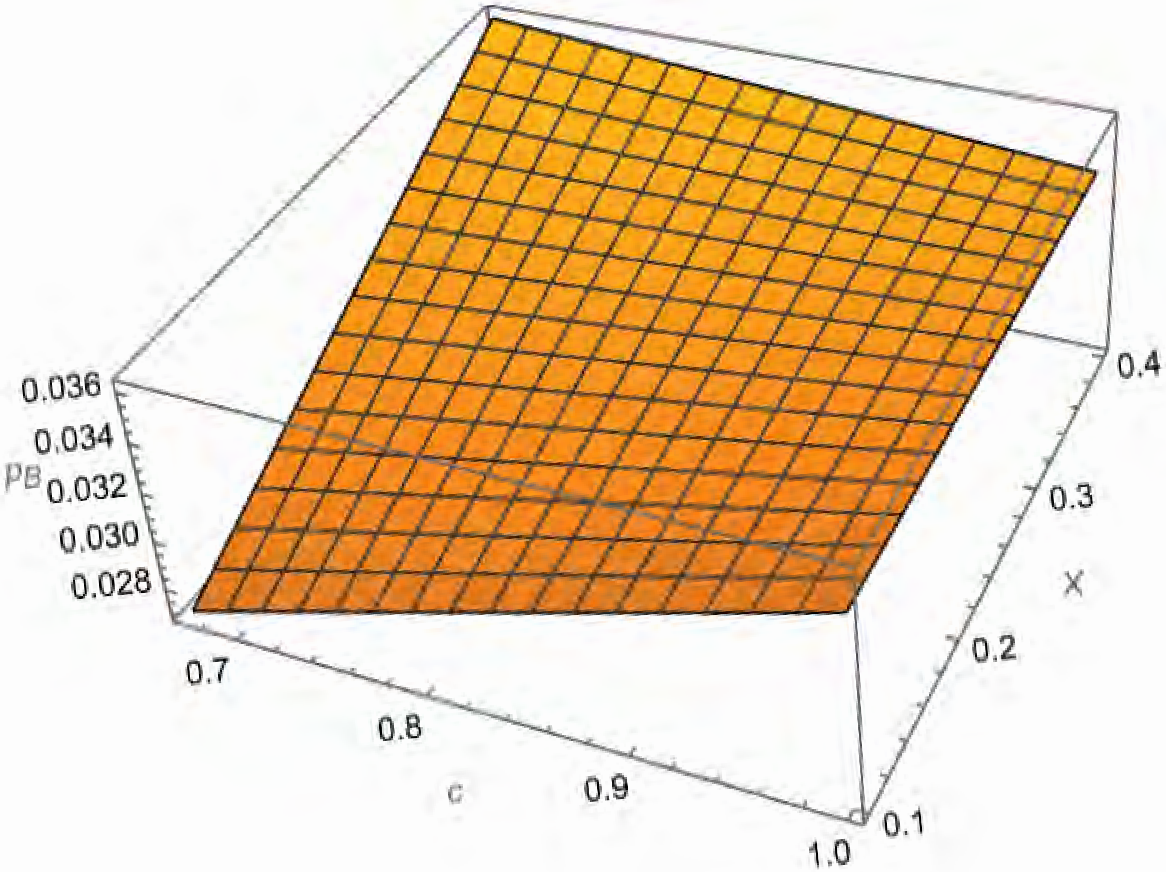
equilibrium for these concrete values occurs in the range $0.167267 < X < 0.240178$ within the $0 \leq X \leq \bar{X}$ interval. At these parameter values p_B increases in c (so does π_B^*) in the entire "no full dependence" range, whereas p_A displays a non-monotonic behavior (see the Graphs below). It increases in c when X is low enough but falls in c for larger values given the "no full dependency" interval (which changes with the value of c) .



In this graph, the top thin horizontal line stands for the pure duopoly price of developer A in our numerical example, so for X above this value the outcome is one of pure duopoly. The second uppermost and the lowermost curves are the maximum and the minimum levels of X, as functions of c, such that neither developer wants to deviate to another market structure (recall that $\delta > \frac{1}{2+c\beta}$ in this case so there is no deviation to not serving the copier segment either). The areas labelled "U" and "L" correspond to the parameter ranges where $dp_A/dc < 0$ and $dp_A/dc > 0$ respectively. The following two graphs show this general behavior of the two "no full dependence" equilibrium prices. It can be observed that in the "no full dependence" area, p_A is non-monotonic in c whereas p_B increases in c.



"No full dependence," p_A



"No full dependence," p_B

As for the other interesting comparative statics results, we, much like in Jain (2008), find that $dp_i^*/d\beta < 0$ while $dp_i^*/d\delta > 0$ where $i = A, B$ under the "no full dependence" equilibrium for all applicable X and c in the above example. Here an increase in β means that the consumer base effect strengthens, i.e., there are relatively more consumers in the copier segment, so both developers decrease their prices. Conversely, an increase in δ means that the price sensitivity effect weakens, so both developers increase their prices.

6 Conclusion

The main purpose of this Note is to study the impact of the firm's private IPR protection on the equilibrium pricing in a setup where there are two segments of consumers; the non-copier segment, which never opts for piracy, and the copier segment, which considers digital piracy as a potential option. An end-user of the copier segment would use piracy i) if she is capable of circumventing the installation key (or other hardware protection) and ii) if this would be beneficial for the end user. Jain (2008) used the above setup in the symmetric horizontal differentiation duopoly model and shows that an increase in private IPR protection is always associated with a decrease in the equilibrium price, due to the existence of the (more) price sensitive copier segment. Thus, the key assumption for his result is the very price sensitivity in the copier segment, and, consequently, when this price sensitivity is "very large", it pays off not even to introduce any protection and serve only the non-copier segment. Žigić, et al (2020), on the other hand, show that in the related duopoly model of end user piracy where there is only a copier segment the impact of private IPR protection on the equilibrium prices is always positive due the fact that firms increase the market base by increasing private protection and can therefore charge higher prices (see Žigić, et al.,2020). Unlike Jain, (2008), they use a model of the vertical product protection.

In this Note we extend the model of Žigić, et al.,(2020) by mimicking Jain's,(2008) setup in adding the non-copier segment and assuming that the consumers in the copier segment are more price sensitive than those in the non-copier segment and, much like Jain,(2008) we parametrize the size of the copier segment. Our main finding is summarized in Proposition 2 which states that the impact of private protection on the equilibrium pricing crucially depends on the intensity of the price sensitivity and the size of the copier segment. The bigger the copier segment is, and the less price sensitive consumers are in this segment, the more the market base effect would dominate and so stronger IPR protection would result in higher prices. Alternatively, for strong price sensitivity and a "not so large" copier segment, Jain's, (2008) negative effect of protection on prices would prevail. Thus, the model we put forward in this Note nests in a sense both Jain's, (2008) and Žigić, et al.'s, (2020) findings on the impact of private IPR on firms' pricing.

Since our above results crucially hinge on the equilibria in which both firms are active in both segments, as an insightful aside to our analysis we provide rigorous conditions for the firms not to deviate to only serving the non-copier segment and summarize this findings in the form of Proposition 1 and related Appendices.

The important reason that our results are somewhat different than those of Jain, (2008), is that, besides private IPR protection, we also include public IPR protection (copyright) in our analysis, which enhances the magnitude of the first effect - increasing the market base. Recall that, unlike in Jain, (2008), in our model private protection of level c by firm A also applies to the subsegment of potential copiers with low valuation who would then opt to buy product B. In Jain (2008), however, private IPR protection of one firm does not directly protect the other firm from the end users' piracy. Thus, the effect of an increase in c is much larger in our asymmetric model of vertical product differentiation than in Jain's (2008) model of symmetric horizontal product differentiation, where the firms fully cover the market in equilibrium and share it equally.[13] More specifically, an increase in c in our setup not only directly increases both firms' share but also shrinks the competitive subsegment, $1 - c$, of developer B, where the size of public protection X enables firm B to compete for the (potential) low-end illegal users who are capable of acquiring the high quality software but may prefer the legal, unprotected version of the low quality software if the price is low enough[14] (that is, $p_B < X\frac{q_B}{q_A}$).

Finally, having both private and public IPR protection in the model, it would be possible to study another very important issue, e.g. the interaction of the two forms of protections within and across the different equilibria discussed above. More specifically, it would be important for policy makers to know when these two forms of protections are complements and when they act as substitutes to each other. Žigić,et al., (2020) focus on this important subject. Looking at their results from the perspective of this Note, we can say that, by a continuity argument, their findings would also hold (at least) in this enlarged model for the situation where there is a large copier segment and not "too many" price sensitive consumers.

[13] Note that in our asymmetric equilibrium $c_A = c^*$ and $c_B = 0$ wheras in Jain's (2008) symmetric equilibrium $c_A = c_B = c^*$.

[14] If, however, we, like Jain (2008), exclude public IPR protection, and have, like him only private IPR protection together with the segment of never copying consumers with higher willingness to pay than the potential copiers, then Jain's result carries over qualitatively in our vertical differentiation setup.

Abstrakt

Zkoumáme, jakým způsobem soukromá ochrana intelektuálních vlastnických práv ovlivňuje rovnovážné ceny a zisky při duopolní konkurenci mezi firmami, které nabízí produkty lišící se kvalitou (vertikální diferenciace produktu) v prostředí podobném článku Jain (2008), kde firmy nabízí při tržní rovnováze produkty stejné kvality (horizontální diferenciace produktu). Spotřebitelé volí mezi legální verzí, nelegální kopií (pokud chtějí a zároveň mohou) nebo úplným nepoužíváním produktu. Použití ilegální kopie porušuje intelektuální vlastnická práva a je proto potrestatelné v případě odhalení. Soukromá i veřejná (copyright) ochrana intelektuálních vlastnických práv je tedy dostupná.
Klíčová slova: vertikální a horizontální diferenciace produktu, softwarové pirátství, Bertrandova konkurence, soukromá a veřejná ochrana intelektuálních vlastnických práv

APPENDIX

A Basic Model

A.1 General notes for all appendices

Most of the calculations in this paper were performed using *Mathematica* and other similar software. The *Mathematica* file is available upon request.

In almost all model situations here, profit functions are concave (quadratic, or, in singular cases, linear) in the respective choice variables, so that an interior solution is always a (local) maximum. In the remaining situations, profit functions are explicitly assumed to be concave in the main text. Thus, second-order conditions always hold in equilibrium, so they are omitted everywhere below.

A.2 Indifferent users

From the user utility function it follows that indifferent users are characterized by the following quality sensitivities. The notation θ_{YZ}, where Y and Z can be one of $\{0, A, P, B\}$ implies that the users with $\theta < \theta_{YZ}$ strictly prefer Y to Z, and the users with $\theta > \theta_{YZ}$ strictly prefer Z to Y. Throughout this appendix, "product P" refers to the illegal version of product A.

As in the basic model, for thresholds not involving the illegal products,

$$\theta_{0A} = \frac{1}{\delta}\frac{p_A}{q_A},\ \theta_{0B} = \frac{1}{\delta}\frac{p_B}{q_B},\ \theta_{BA} = \frac{1}{\delta}\frac{p_A - p_B}{q_A - q_B}.$$

For thresholds involving product P, note that the decision between P and A is made on the basis of prices alone. The remaining thresholds are

$$\theta_{0P} = \frac{1}{\delta}\frac{X}{q_A},\ \ \theta_{BP} = \frac{1}{\delta}\frac{X - p_B}{q_A - q_B}.$$

In all these cases, $\delta = 1$ for the non-copier segment and $0 < \delta \leq 1$ for the copier segment.

Also recall that the illegal product is available only to the fraction of the copier segment not controlled by developer A.

A.2.1 The price-quality ratio rule

The following general result can be easily shown to hold.

Lemma 4 *If there is a good of quality q_A available at price p_A and a good of quality $q_B < q_A$ available at price p_B, then a necessary condition exists for consumers to buy good B, namely the price per unit of quality is strictly lower for the lower quality good, i.e., $\frac{p_B}{q_B} < \frac{p_A}{q_A}$.*

Proof. The claim directly follows from $\theta_{BA} - \theta_{0B} > 0$. ■

This result was implicitly used in previous chapters, and the equilibrium prices complied with it. However, in this chapter, profit functions are not unimodal, and an analysis of deviations requires the Lemma above explicitly.

Corollary 1 *No consumer with access to P prefers B to P if $p_B \geq X\frac{q_B}{q_A}$.*

B Equilibrium prices and profits

The detailed calculations can be found in the *Mathematica* file available upon request.

B.1 Unconstrained duopoly equilibria

These equilibria occur iff the expected punishment X is above developer A's equilibrium price, so the effective control rate in the copier segment is $c = 1$ at no cost. The prices and profits are those of a standard Bertrand duopoly adjusted by the presence of the two consumer segments. Note that if $\delta = 1$ or $\beta = 0$, then the prices coincide with the standard Bertrand duopoly prices, and the profits are those of the standard Bertrand duopoly multiplied by $(1+\beta)$.

$$\begin{aligned}
p_A^* &= 2\bar{\theta} q_A \frac{(q_A - q_B)}{(4q_A - q_B)} \frac{(1+\beta)\,\delta}{(\beta+\delta)}, \\
p_B^* &= \bar{\theta} q_B \frac{(q_A - q_B)}{(4q_A - q_B)} \frac{(1+\beta)\,\delta}{(\beta+\delta)}, \\
\Pi_A^* &= 4\bar{\theta}^2 q_A^2 \frac{(q_A - q_B)}{(4q_A - q_B)^2} \frac{(1+\beta)^2\,\delta}{(\beta+\delta)}, \\
\Pi_B^* &= \bar{\theta}^2 q_A q_B \frac{(q_A - q_B)}{(4q_A - q_B)^2} \frac{(1+\beta)^2\,\delta}{(\beta+\delta)}.
\end{aligned}$$

B.2 Constrained duopoly equilibria

In these equilibria, $p_A^* = X$ so $p_B^* = X\frac{q_B}{2q_A}$. The equilibrium profits are given by

$$\begin{aligned}
\Pi_A^* &= X\,(1+\beta)\,\bar{\theta} - \frac{(2q_A - q_B)\,X^2\,(\beta+\delta)}{2q_A\,(q_A - q_B)\,\delta}, \\
\Pi_B^* &= \frac{q_B X^2\,(\beta+\delta)}{4q_A\,(q_A - q_B)\,\delta}.
\end{aligned}$$

B.3 "No full dependence" equilibria

If this equilibrium occurs, then the equilibrium prices and profits are

$$\begin{aligned}
p_A^* &= \frac{\beta\,(c\beta+\delta)\,X\,(1-c)\,q_B + 2\bar{\theta}\,(1+c\beta)\,\delta\,(\beta+\delta)\,q_A\,(q_A - q_B)}{(c\beta+\delta)\,(4\,(\beta+\delta)\,q_A - (c\beta+\delta)\,q_B)}, \\
p_B^* &= q_B \frac{2\beta X\,(1-c) + \bar{\theta}\delta\,(1+c\beta)\,(q_A - q_B)}{4\,(\beta+\delta)\,q_A - (c\beta+\delta)\,q_B}, \\
\Pi_A^* &= \frac{\left(\beta\,(c\beta+\delta)\,X\,(1-c)\,q_B + 2\bar{\theta}\,(1+c\beta)\,\delta\,(\beta+\delta)\,q_A\,(q_A - q_B)\right)^2}{(q_A - q_B)\,\delta\,(c\beta+\delta)\,(4\,(\beta+\delta)\,q_A - (c\beta+\delta)\,q_B)^2}, \\
\Pi_B^* &= q_A q_B\,(\beta+\delta)\,\frac{\left(2\beta X\,(1-c) + \bar{\theta}\delta\,(1+c\beta)\,(q_A - q_B)\right)^2}{(q_A - q_B)\,\delta\,(4\,(\beta+\delta)\,q_A - (c\beta+\delta)\,q_B)^2}.
\end{aligned}$$

B.4 Interior "full dependence" equilibria

Unlike in Žigić et al (2020), the prices no longer coincide with the unconstrained duopoly prices (unless $c = 1$), and can easily be shown to decrease in c.

$$\begin{aligned}
p_A^* &= 2\bar{\theta} q_A \frac{(q_A - q_B)}{(4q_A - q_B)} \frac{(1 + c\beta)\,\delta}{(c\beta + \delta)}, \\
p_B^* &= \bar{\theta} q_B \frac{(q_A - q_B)}{(4q_A - q_B)} \frac{(1 + c\beta)\,\delta}{(c\beta + \delta)}, \\
\Pi_A^* &= 4\bar{\theta}^2 q_A^2 \frac{(q_A - q_B)}{(4q_A - q_B)^2} \frac{(1 + c\beta)^2\,\delta}{(c\beta + \delta)}, \\
\Pi_B^* &= \bar{\theta}^2 q_A q_B \frac{(q_A - q_B)}{(4q_A - q_B)^2} \frac{(1 + c\beta)^2\,\delta}{(c\beta + \delta)}.
\end{aligned}$$

B.5 Corner "full dependence" equilibria

In these equilibria, $p_B^* = X$ so $p_A^* = \frac{1}{2}\left(X + (q_A - q_B)\,\delta\bar{\theta}\frac{1+c\beta}{c\beta+\delta}\right)$, which can easily be shown to decrease in c. The equilibrium profits are given by

$$\begin{aligned}
\Pi_A^* &= \frac{\left((c\beta + \delta)\,X + (q_A - q_B)\,(1 + c\beta)\,\delta\bar{\theta}\right)^2}{4\,(q_A - q_B)\,\delta\,(c\beta + \delta)}, \\
\Pi_B^* &= \frac{X}{2}\left((1 + c\beta)\,\bar{\theta} - \frac{(2q_A - q_B)\,(c\beta + \delta)\,X}{(q_A - q_B)\,q_B\delta}\right).
\end{aligned}$$

C Proof of Proposition 1 for the constrained duopoly case

As stated in the proof in the main text, this case differs from the rest in that the maximum of developer A's profit is not interior, so more extensive analysis is required.

Recall that developer A's profit function is given by

$$\Pi_A(p_A) = \begin{cases} \Pi_A(p_A; 1), & p_A \leq X, \\ \Pi_A(p_A; c), & p_A > X, \end{cases}$$

where the cases that never occur in equilibrium are omitted and

$$\Pi_A(p_A;c) = p_A\left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right) + c\beta p_A\left(\bar{\theta} - \frac{p_A - p_B}{(q_A - q_B)}\frac{1}{\delta}\right)$$

when both consumer segments are served and by

$$\Pi_A^0(p_A) = p_A\left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right)$$

when only the non-copier segment is served.

Given p_B, denote $p_A(c)$ the price maximizing $\Pi_A(p_A;c)$ and p_A^0 the price maximizing $\Pi_A^0(p_A)$. In our equilibrium classification, the global maximum of $\Pi_A(p_A)$ is attained at $p_A = p_A(1)$ in the case of un unconstrained duopoly, at $p_A = p_A(c)$ in all cases where $p_A > X$, and at $p_A = X$ in the case of a constrained duopoly. This last non-interior maximum implies that (i) $p_A(1) > X$, i.e., that the unconditional maximum of $\Pi_A(p_A;1)$ is attained outside the range $p_A \leq X$ and (ii) no deviation to $p_A > X$ is profitable, i.e.,

$$\Pi_A(X;1) \geq \max_{p_A > X} \Pi_A(p_A;c).$$

now recall that

$$p_A(c) = \frac{1}{2}\left(p_B + \frac{(1+c\beta)\delta}{c\beta + \delta}(q_A - q_B)\bar{\theta}\right)$$

is decreasing in c when p_B is given, i.e., $p_A(c) > p_A(1)$ when $c < 1$. Hence, $p_A(c) > p_A(1) > X$, so that the maximum $\max_{p_A > X}\Pi_A(p_A;c)$ is interior, i.e., attained at $p_A(c)$.

To complete the proof, observe that the proof in the main text is equivalent to showing that $\Pi_A(p_A(c);c) \geq \Pi_A^0(p_A^0)$ when $\delta \geq \frac{1}{2+c\beta}$, and here we showed that in the case of a constrained duopoly $\Pi_A(X;1) \geq \Pi_A(p_A(c);c)$.

D Deviation to not serving the non-copier segment by developer B in the "no full dependence" case

Recall that developer B's profit when both consumer segments are served (which implies $p_B \leq X$) is given by

$$\Pi_B(p_B) = \begin{cases} \Pi_B^N(p_B), & p_B \leq X\frac{q_B}{q_A}, \\ \Pi_B^F(p_B), & X\frac{q_B}{q_A} < p_B \leq X, \end{cases}$$

where the cases that cannot occur in equilibrium are omitted and

$$\begin{aligned} \Pi_B^N(p_B) &= (1+\frac{\beta c}{\delta})\left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right)p_B + \frac{\beta(1-c)}{\delta}\left(\frac{X - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right)p_B, \\ \Pi_B^F(p_B) &= (1+\frac{\beta c}{\delta})\left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right)p_B. \end{aligned}$$

(Here "N" and "F" stand for "no full dependence" and "full dependence" respectively.) If developer B only serves the non-copier segment, then

$$\Pi_B(p_B) = \Pi_B^0(p_B) = \left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right)p_B.$$

Just as in Žigić,et al., (2020), the maximum when both segments are served is never attained at $p_B = X\frac{q_B}{q_A}$, so a "no full dependence" equiliibrium candidate implies an interior local maximum of $\Pi_B^N(p_B)$,

$$p_B = p_B^N = \frac{(p_A(c\beta + \delta) + X(1-c)\beta)\,q_B}{2(\beta+\delta)\,q_A},$$

which satisfies $p_B \leq X\frac{q_B}{q_A}$ iff

$$p_A \leq p_A^N = X\left(1 + \frac{\beta+\delta}{c\beta+\delta}\right)$$

and results in the profit of

$$\Pi_B^N\left(p_B^N\right) = \frac{\left(p_A\left(c\beta+\delta\right)+X\left(1-c\right)\beta\right)^2 q_B}{4\delta\left(\beta+\delta\right)q_A\left(q_A-q_B\right)}.$$

Also, this interior maximum is global for $p_B \leq X$, i.e. there is no profitable deviation to the "full dependence" range. The maximum in the "full dependence" range can be either interior or corner at $p_B = X$. In the former case, there is no deviation to not serving the copier segment since the argument in the main text, $\Pi_B^F\left(p_B\right) = \left(1+\frac{\beta c}{\delta}\right)\Pi_B^0\left(p_B\right)$, applies. In the latter case, the condition $p_A \leq p_A^N$ cannot be improved without further assumptions on the model parameters and a direct comparison is needed.

The deviation price and profit are given by

$$p_B^0 = \frac{p_A q_B}{2q_A},\ \Pi_B^0\left(p_B^0\right) = \frac{p_A^2 q_B}{4q_A\left(q_A-q_B\right)}.$$

Then $\Pi_B^N\left(p_B^N\right) - \Pi_B^0\left(p_B^0\right)$ is a positive multiple of

$$\left(1-c\right)^2\beta X^2 + 2\left(1-c\right)\left(c\beta+\delta\right)Xp_A + \left(c^2\beta-\delta+2c\delta\right)p_A^2$$

This expression is quadratic in p_A as well as non-negative and non-decreasing in p_A at $p_A = 0$, so if it is non-negative at $p_A = p_A^N$ then it is non-negative at all applicable values of p_A. Substituting $p_A = p_A^N$ results in a positive multiple of

$$4c^2 + \left(3c^2+6c-1\right)\left(\delta/\beta\right) + 4c\left(\delta/\beta\right)^2.$$

The last expression is always positive when $3c^2+6c-1 \geq 0$, i.e., when $c \geq \frac{2\sqrt{3}-3}{3} \approx 0.154701$. Otherwise, its minimum occurs at $\left(\delta/\beta\right) = \frac{1-6c-3c^2}{8c}$, and the minimum value equals

$$\frac{\left(1-c\right)^3\left(9c-1\right)}{16c}.$$

Thus, a sufficient condition for developer B to never deviate from a "no full dependence" equilibrium candidate to not serving the copier segment is $c \geq \frac{1}{9}$. Note that while it is shown in Žigić et al (2020), that such equilibria can only occur at much higher values of c, this is not the case here due to the presence of the non-copier segment. It is also interesting that the condition only depends on c, just like several "no full dependence" equilibria–related conditions in Žigić et al (2020).

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